

BSM Kaon Mixing at the Physical Point

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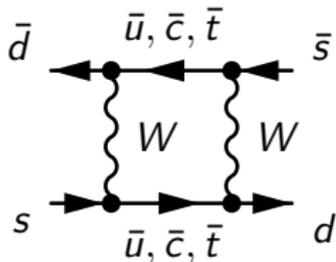
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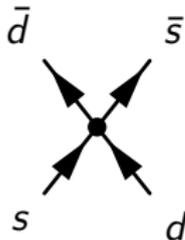
Kaon Mixing



- One-loop FCNC interaction
- Mediated by two W boson
- Related to indirect CP violation parameter ϵ_K
- Gives constraints on CKM matrix

- OPE to separate long and short distance contribution
- Effective $\Delta S = 2$ four-quark operator

$$O^{\Delta S=2} = [\bar{s}_a \gamma_\mu (1 - \gamma_5) d_a] [\bar{s}_b \gamma_\mu (1 - \gamma_5) d_b]$$



Beyond the Standard Model

Beyond the standard model we can form a complete (BSM) basis of 5 model independent Four-Quark operators.

$$\mathcal{H}^{\Delta S=2} = \sum_{i=1}^5 C_i(\mu) O_i(\mu) + \sum_{i=1}^3 C_i(\mu) \tilde{O}_i(\mu)$$

$$O_1 = [\bar{s}_a \gamma_\mu (1 - \gamma_5) d_a] [\bar{s}_b \gamma_\mu (1 - \gamma_5) d_b] \quad (1)$$

$$O_2 = [\bar{s}_a (1 - \gamma_5) d_a] [\bar{s}_b (1 - \gamma_5) d_b] \quad (2)$$

$$O_3 = [\bar{s}_a (1 - \gamma_5) d_b] [\bar{s}_b (1 - \gamma_5) d_a] \quad (3)$$

$$O_4 = [\bar{s}_a (1 - \gamma_5) d_a] [\bar{s}_b (1 + \gamma_5) d_b] \quad (4)$$

$$O_5 = [\bar{s}_a (1 - \gamma_5) d_b] [\bar{s}_b (1 + \gamma_5) d_a]. \quad (5)$$

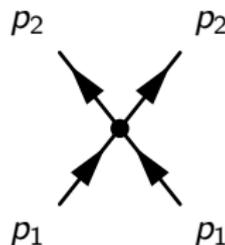
Non-Perturbative Renormalisation

- Use an intermediate MOM/SMOM renormalisation scheme.¹

$$O_i(\mu)^{\overline{MS}} = C_{ij}^{\overline{MS} \leftarrow \text{MOM}}(\mu) \left(\lim_{a^2 \rightarrow 0} \frac{Z_{jk}^{RI}(\mu)}{Z_q^2} O_k(a) \right)$$

$$Z^{RI}(\mu) \hat{P}[\Lambda(p^2)]|_{p^2=\mu^2} = \Lambda(p^2)^{\text{tree}}$$

- Non-exceptional kinematics

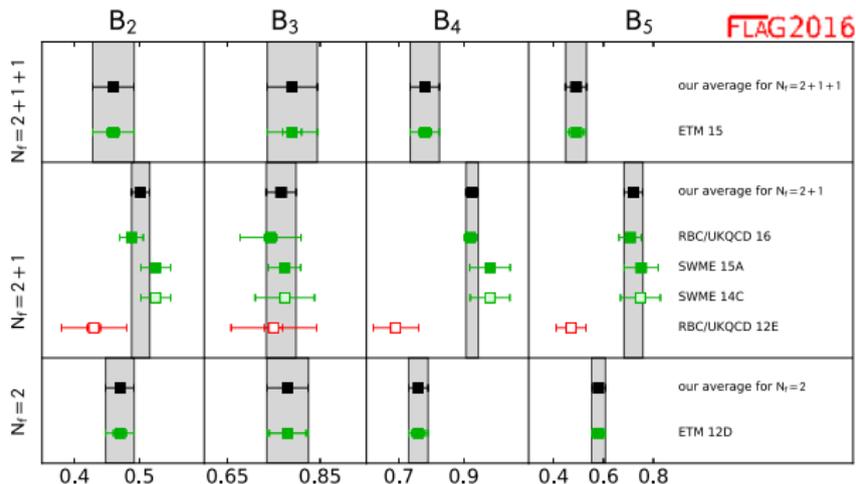


$$p_1 \neq p_2$$

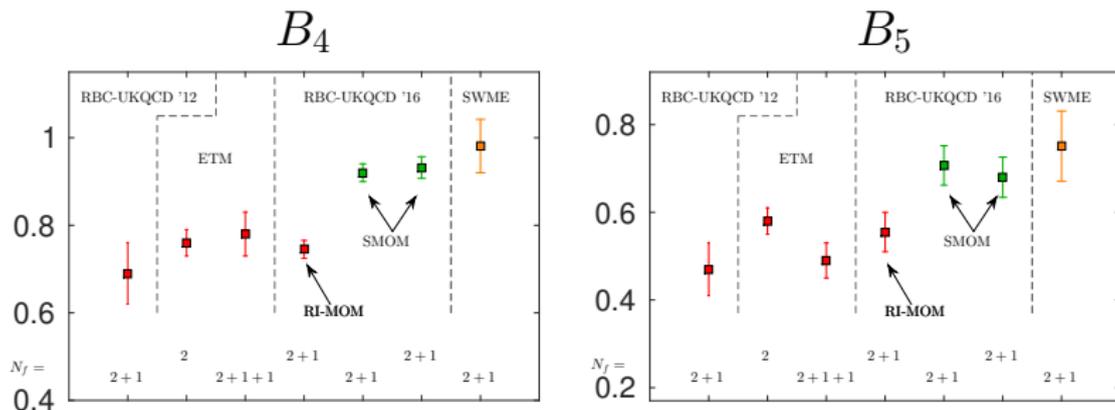
$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

¹Martinelli et al, 1994, arXiv:9411010

Tensions in previous results



In Garron et al ² it was proposed that source of tension come from choice of kinematics in renormalisation.



Figures provided by Nicolas Garron

Original study included only 2 lattice spacings and unphysical pion masses. We want to extend on this to reduce uncertainties.

²Garron et al, RBC-UKQCD, 16 arXiv:1609.03334

Simulations

- Iwasaki Gauge Action
- 2+1f DWF QCD
- 3 lattice spacings
- 2 ensembles with physical pions
- New: $a^{-1} = 2.8\text{GeV}$ with 230 MeV pion mass
- extra lattice spacing + physical pion added³

β	name	L/a	T/a	$a^{-1}[\text{GeV}]$	$m_\pi[\text{MeV}]$	am_l^{sea}	am_s^{val}	am_s^{phys}	smearred sources?
2.13	CO	48	96	1.730(4)	139	0.00078	0.0358	0.03580(16)	yes
2.13	C1	24	64	1.785(5)	340	0.005	0.03224	0.03224(18)	no
2.13	C2	24	64	1.785(5)	430	0.01	0.03224	0.03224(18)	no
2.25	M0	64	128	2.359(4)	139	0.000678	0.02661	0.02539(17)	no
2.25	M1	32	64	2.383(9)	300	0.004	0.02477	0.02477(18)	no
2.25	M2	32	64	2.383(9)	360	0.006	0.02477	0.02477(18)	no
	F1	48	96	2.774(10)	230	0.002144	0.02144	0.02132	no

³compared to arXiv:1609.03334

Quantities Measured

The SM bag parameter:

$$B_1(\mu) = \frac{\langle \bar{P} | \mathcal{O}_i(\mu) | P \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

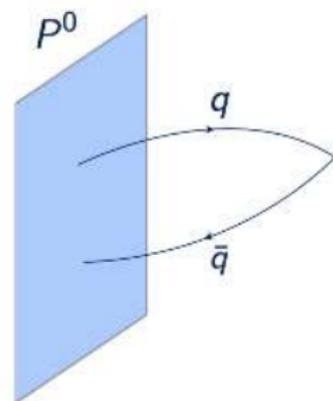
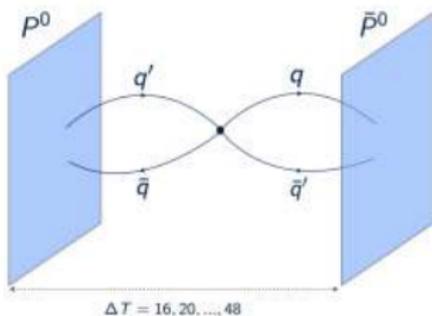
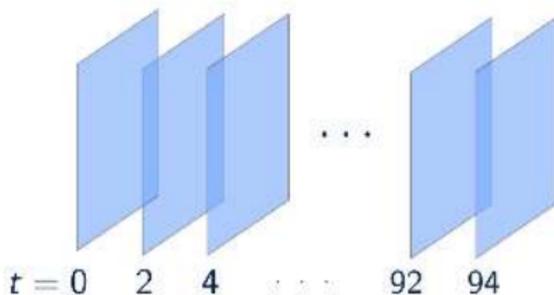
Define a ratio parameter:

$$R_i\left(\frac{m_P^2}{f_P^2}, a^2, \mu\right) = \left[\frac{f_K^2}{m_K^2} \right]_{\text{exp}} \left[\frac{m_P^2}{f_P^2} \frac{\langle \bar{P} | \mathcal{O}_i(\mu) | P \rangle}{\langle \bar{P} | \mathcal{O}_1(\mu) | P \rangle} \right]_{\text{lat}}$$

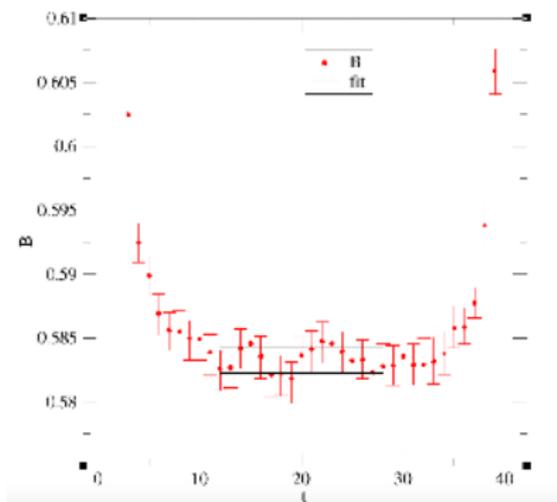
such that when $a^2 \rightarrow 0$ and $m_P^2/f_P^2 \rightarrow m_K^2/f_K^2$ it reduces to:

$$R_i(\mu) = \frac{\langle \bar{K} | \mathcal{O}_i(\mu) | K \rangle}{\langle \bar{K} | \mathcal{O}_1(\mu) | K \rangle}$$

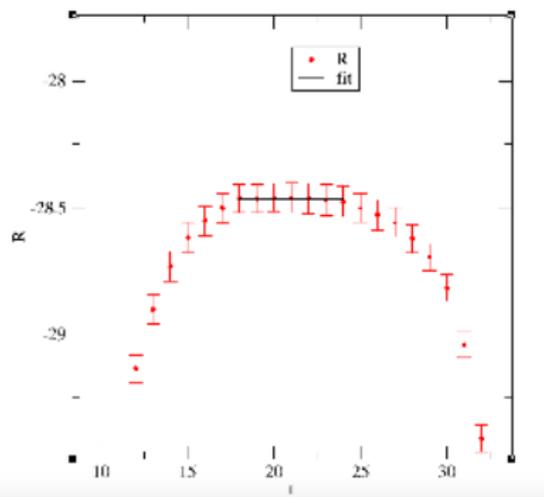
Source-Sink Time Separations



Three Point Correlator Fits



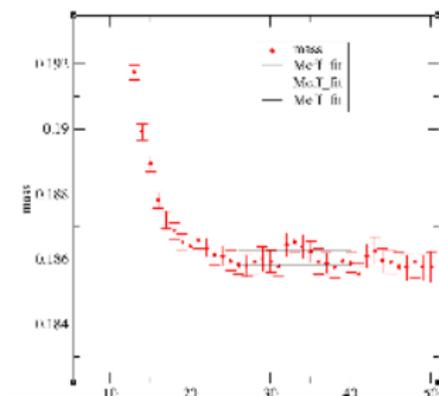
$$(a) \frac{C_{2pt}^{3pt}(t, t_{sink})}{C_{2pt}^{AP}(t) C_{2pt}^{AP}(t_{sink} - t)} \rightarrow \frac{\langle \bar{P} | O_i | P \rangle}{\langle P | A \rangle \langle A | P \rangle}$$



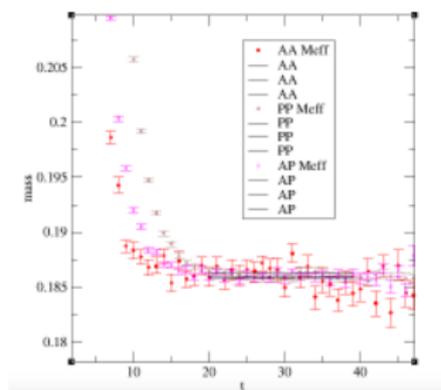
$$(b) \frac{C_i^{3pt}(t, t_{sink})}{C_1^{3pt}(t, t_{sink})} \rightarrow \frac{\langle \bar{P} | O_i | P \rangle}{\langle \bar{P} | O_1 | P \rangle}$$

Two Point Correlator Fits

$$C_{O_1 O_2}(t, t_i) \rightarrow \frac{a^3}{2m_P} \langle 0 | O_1 | P \rangle \langle 0 | O_2 | P \rangle (e^{E_P t} \pm e_P^E (T - t))$$



(a) Single channel (PP) fit



(b) Multi Channel (PP, AA, PA) fit

decay constants - $\langle 0 | Z_A A_4^{Local} | P \rangle = m_P f_P$.

Extrapolation to Physical Point and Continuum

Extrapolate to the physical point in a global fit with two methods:

- Chiral PT fit ansatz

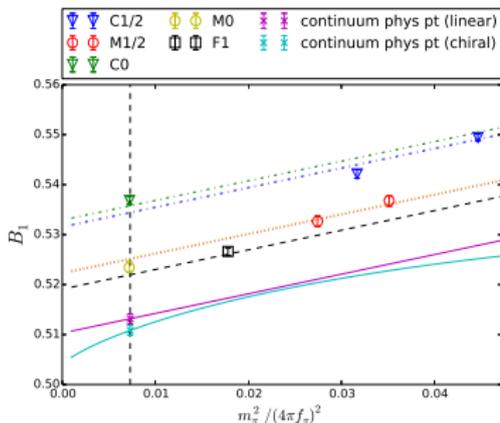
$$Y(a^2, m_{||}^2/f_{||}^2) = Y(0, m_{\pi}^2/f_{\pi}^2) \left[1 + \alpha_i a^2 + \frac{m_{||}^2}{f_{||}^2} \left(\beta_i + \frac{C_i}{16\pi^2} \log \left(\frac{m_{||}^2}{\Lambda^2} \right) \right) \right]$$

- Linear fit ansatz

$$Y(a^2, m_{||}^2/f_{||}^2) = Y(0, m_{\pi}^2/f_{\pi}^2) \left[1 + \alpha_i a^2 + \beta_i \frac{m_{||}^2}{f_{||}^2} \right]$$

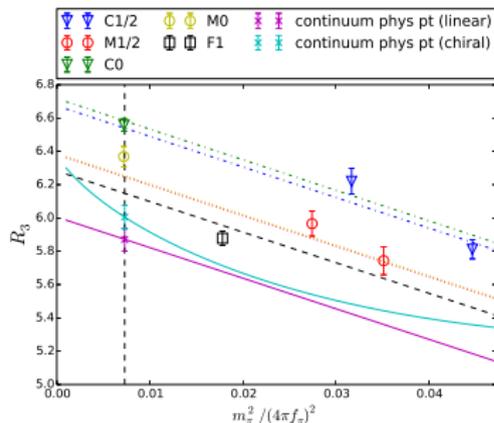
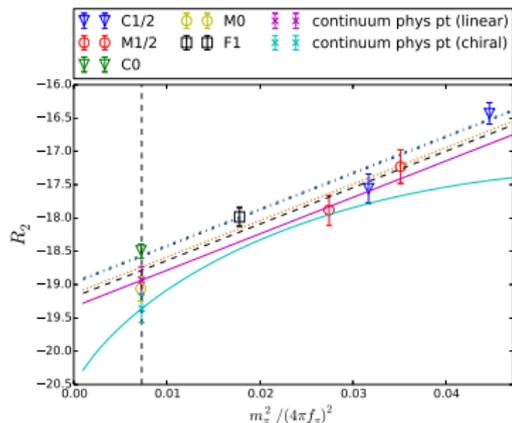
Bag Parameter Plots

Preliminary results for B_K in \overline{MS} at 3 GeV RI-SMOM $^{\gamma,\gamma}$ intermediate scheme.



Ratio Parameter Plots I

Preliminary results for R_1 and R_2 in \overline{MS} at 3 GeV RI-SMOM $^{\gamma,\gamma}$ intermediate scheme.



Ratio Parameter Plots II

Preliminary results for R_3 and R_4 in \overline{MS} at 3 GeV RI-SMOM γ,γ intermediate scheme.

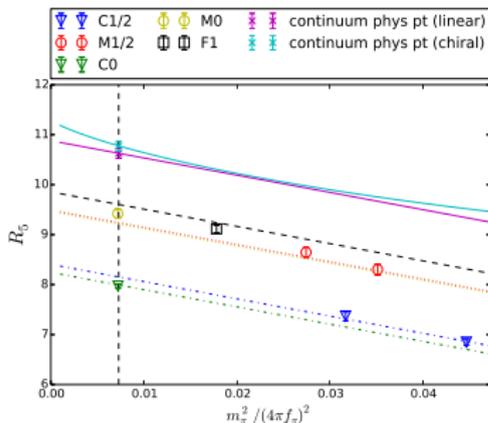
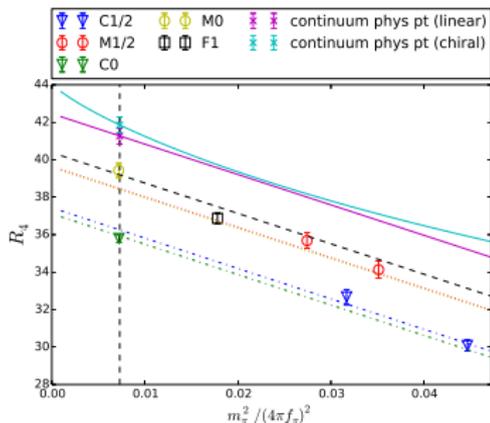


Table of Preliminary Results

Table: Preliminary continuum physical point results for the standard model bag parameter, and ratios of the BSM to SM operators are shown here.

	phys pt (lin)	χ^2 /dof	phys pt (chir)	χ^2 /dof	RBC-UKQCD 16 ⁴
B_1	0.5132(10)	1.51(95)	0.5108(10)	2.6(12)	0.523(9)(7)
R_2	-18.93(21)	1.4(11)	-19.36(21)	1.9(12)	-19.11(43)(31)
R_3	5.872(69)	1.6(12)	6.005(70)	2.0(13)	5.76(14)(16)
R_4	41.27(42)	2.2(13)	41.85(42)	2.0(13)	40.12(82)(188)
R_5	10.63(10)	1.5(11)	10.78(10)	1.4(11)	11.13(21)(83)

⁴arXiv:1609.03334

Summary

- Expanded our calculation to include 3 lattice spacings
- Achieved consistency with our previous results
- Reduced statistical error on the results.
- Going Forward:
 - Use of gaussian smeared sources
 - Perform NPR for M0 and C0
 - Systematic error calculations

Acknowledgements

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