Axial U(1) symmetry at high temperature in 2-flavor lattice QCD

Presenter:
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from JLQCD Collaboration:
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Outline of this talk

1. Motivation
2. Axial U(1) symmetry and Dirac spectrum
3. Results

    3-1: $U(1)_A$ susceptibility at finite T

    (3-2: Topological susceptibility) $\Rightarrow$ Next talk by Y. Aoki
4. Summary

Related talks from JLQCD:
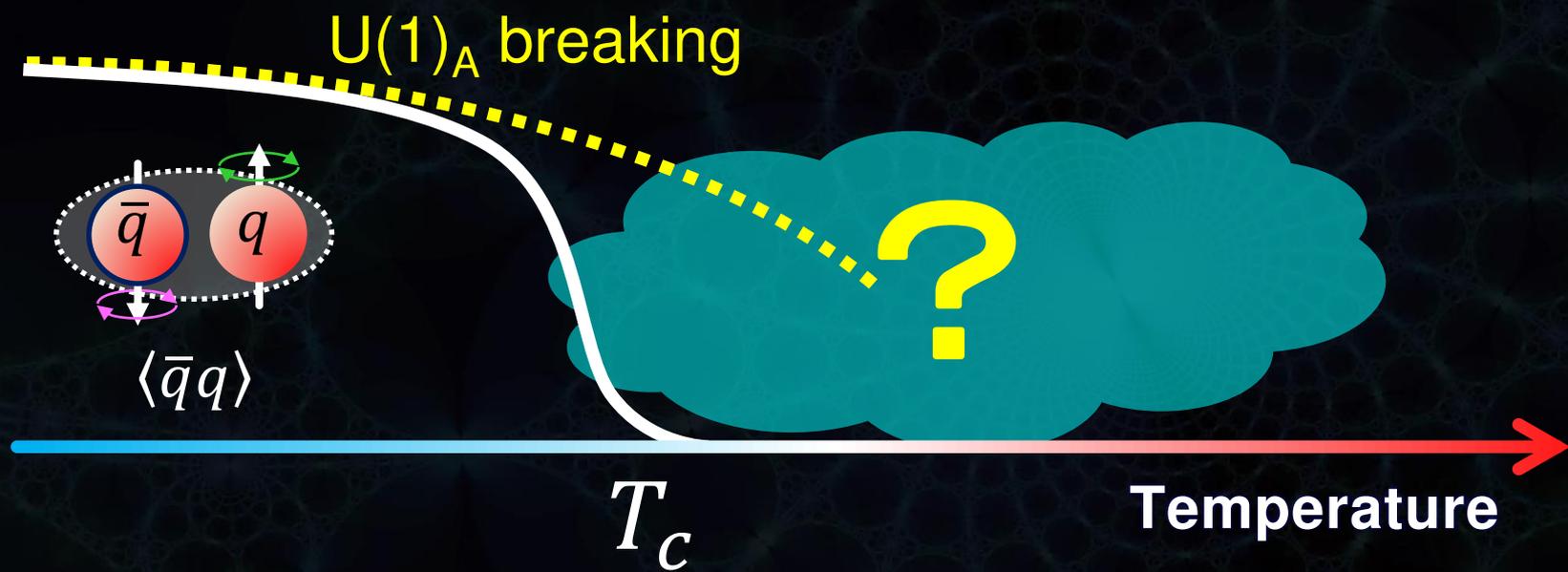
H. Fukaya (Thu. plenary) – Summary of finite T

H. Fukaya (Thu.) – Topological susceptibility at zero T

C. Rohrhofer (Wed.) – Mesonic collelator at finite T

Motivation: Is U(1)$_A$ symmetry restored above T$_c$?

- For $T > T_c$, chiral symmetry breaking by $\langle \bar{q}q \rangle$ is restored
  ⇒How about U(1)$_A$ symmetry?

$$\Delta_{\pi-\delta} = \int_0^\infty d^4x \left[\pi^a(x)\pi^a(x) - \delta^a(x)\delta^a(x) \right]$$

U(1)$_A$ breaking

$\bar{q}$ q

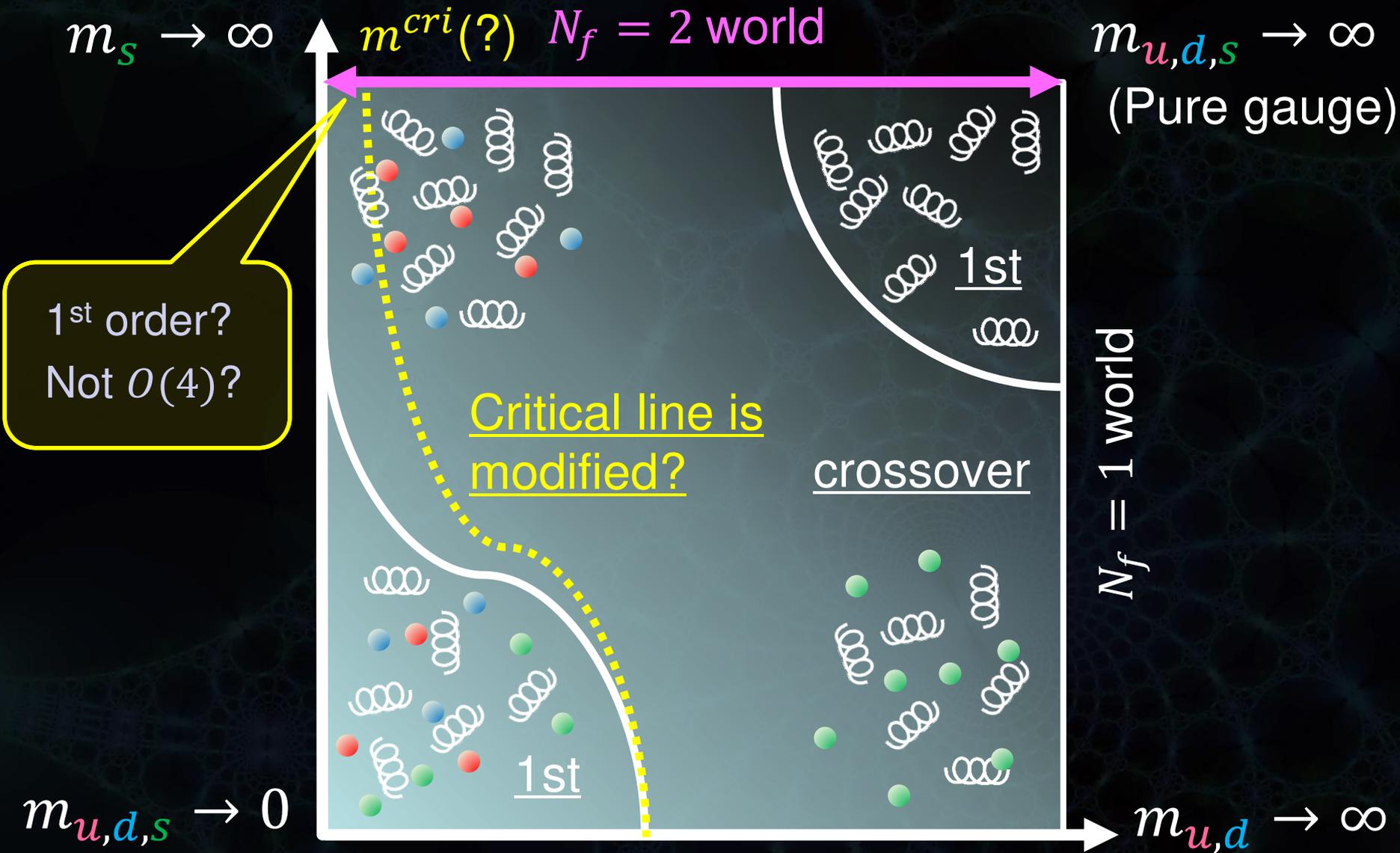$\langle \bar{q}q \rangle$

?

T_c

Temperature

Motivation: Is U(1)$_A$ symmetry restored above T$_c$?

- For $T \gg T_c$, U(1)$_A$ symmetry will be restored because of dilute instanton gas picture (Gross-Pisarski-Yaffe, 1981)
- HotQCD (DW, 2012) broken
- JLQCD (topology fixed overlap, 2013) restored
- TWQCD (optimal DW, 2013) restored
- LLNL/RBC (DW, 2014) broken
- Dick et al. (overlap on HISQ, 2015) broken
- JLQCD (reweighted overlap from DW, 2016) restored

⇒ It is still an unsettled problem

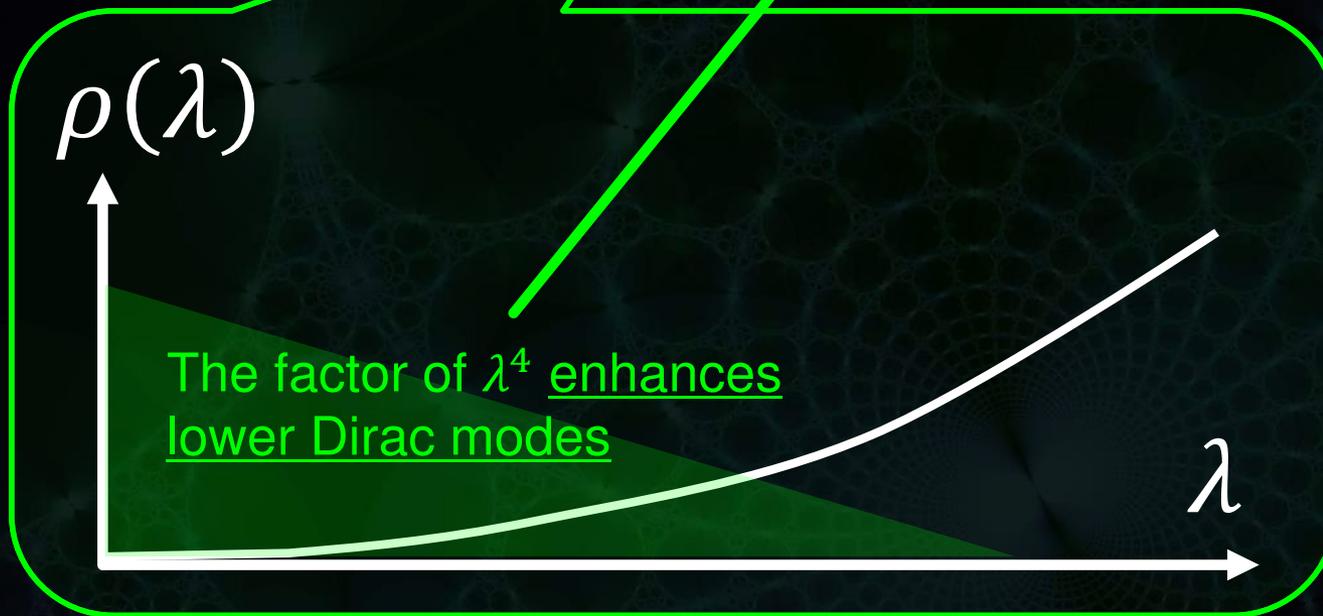**If U(1)$_A$ is restored…**

the Colombia plot is modified?

$m_s \to \infty$ $m^{cri}(?)$ $N_f = 2$ world $m_{u,d,s} \to \infty$

(Pure gauge)

1st order?
Not $O(4)$?

$N_f = 1$ world

Critical line is modified?

crossover

1st

$m_{u,d,s} \to 0$ 1st $m_{u,d} \to \infty$

U(1)$_A$ susceptibility from Dirac spectra

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda \, \rho(\lambda) \, \frac{2m^2}{(\lambda^2 + m^2)^2}$$

$\rho(\lambda)$

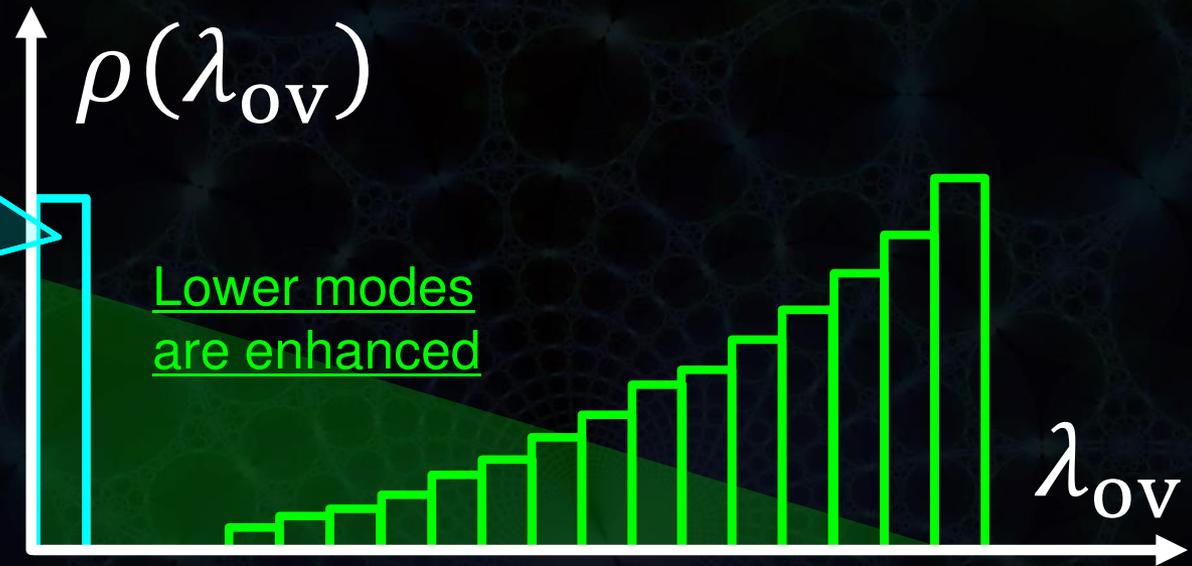The factor of λ^4 enhances lower Dirac modes

λ

Cf.) Banks-Casher relation: $\langle \bar{q}q \rangle = \lim_{m\to 0} \int_0^\infty d\lambda \, \rho(\lambda) \, \frac{2m}{\lambda^2 + m^2}$

Observables: Dirac spectra on the lattice

$$\Delta_{\pi-\delta} = \int_0^\infty d\lambda \, \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \quad \Longrightarrow \quad \Delta_{\pi-\delta}^{\mathrm{ov}} \equiv \frac{1}{V(1-m^2)^2} \sum_i \frac{2m^2(1-\lambda_{\mathrm{ov}}^{(i)4})^2}{\lambda_{\mathrm{ov}}^{(i)4}}$$

Practically, (physical) zero-modes will be dominant

⇒ <u>instanton contributions</u> in finite box

$$\rho(\lambda_{\mathrm{ov}})$$

<u>Lower modes are enhanced</u>

$$\lambda_{\mathrm{ov}}$$

Practical observable:
Subtracted zero-modes

$$\overline{\Delta}_{\pi-\delta}^{\mathrm{ov}} \equiv \frac{1}{V(1-m^2)^2} \sum_{i>0-mode} \frac{2m^2(1-\lambda_{\mathrm{ov}}^{(i)4})^2}{\lambda_{\mathrm{ov}}^{(i)4}}$$
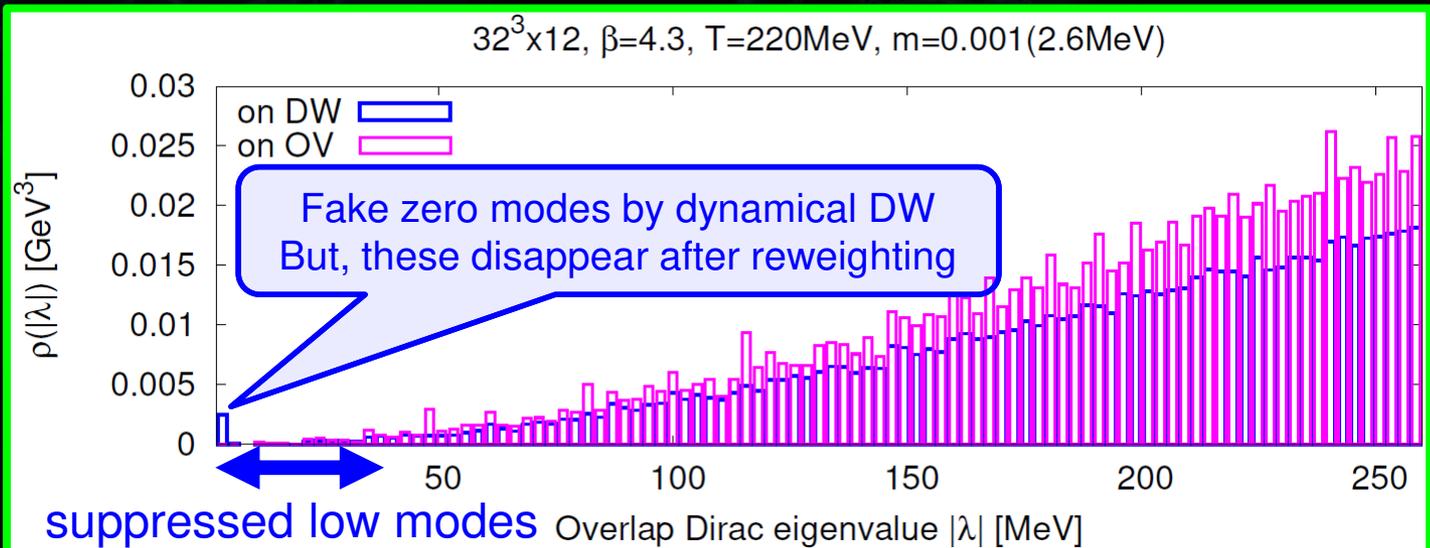
(Instanton contribution becomes zero at $V \to \infty$: $\overline{\Delta}_{0-mode}^{\mathrm{ov}} = \frac{2N_0}{Vm^2} (\propto 1/\sqrt{V})$)

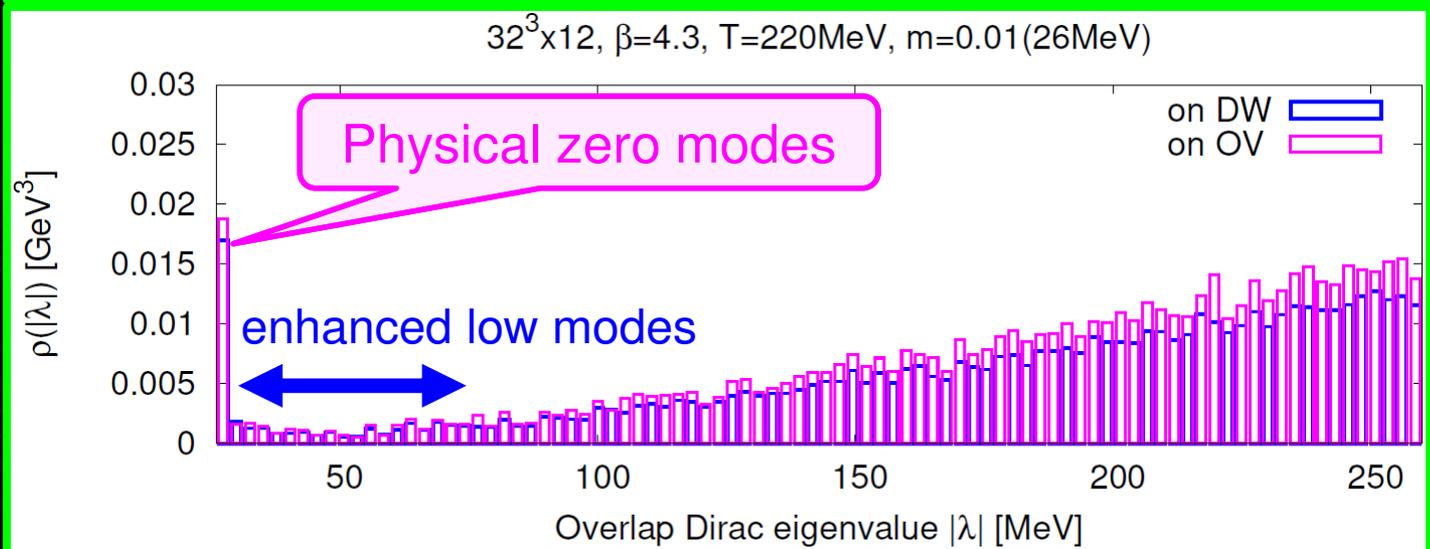# Project of U(1)$_A$ at finite T by JLQCD collaboration

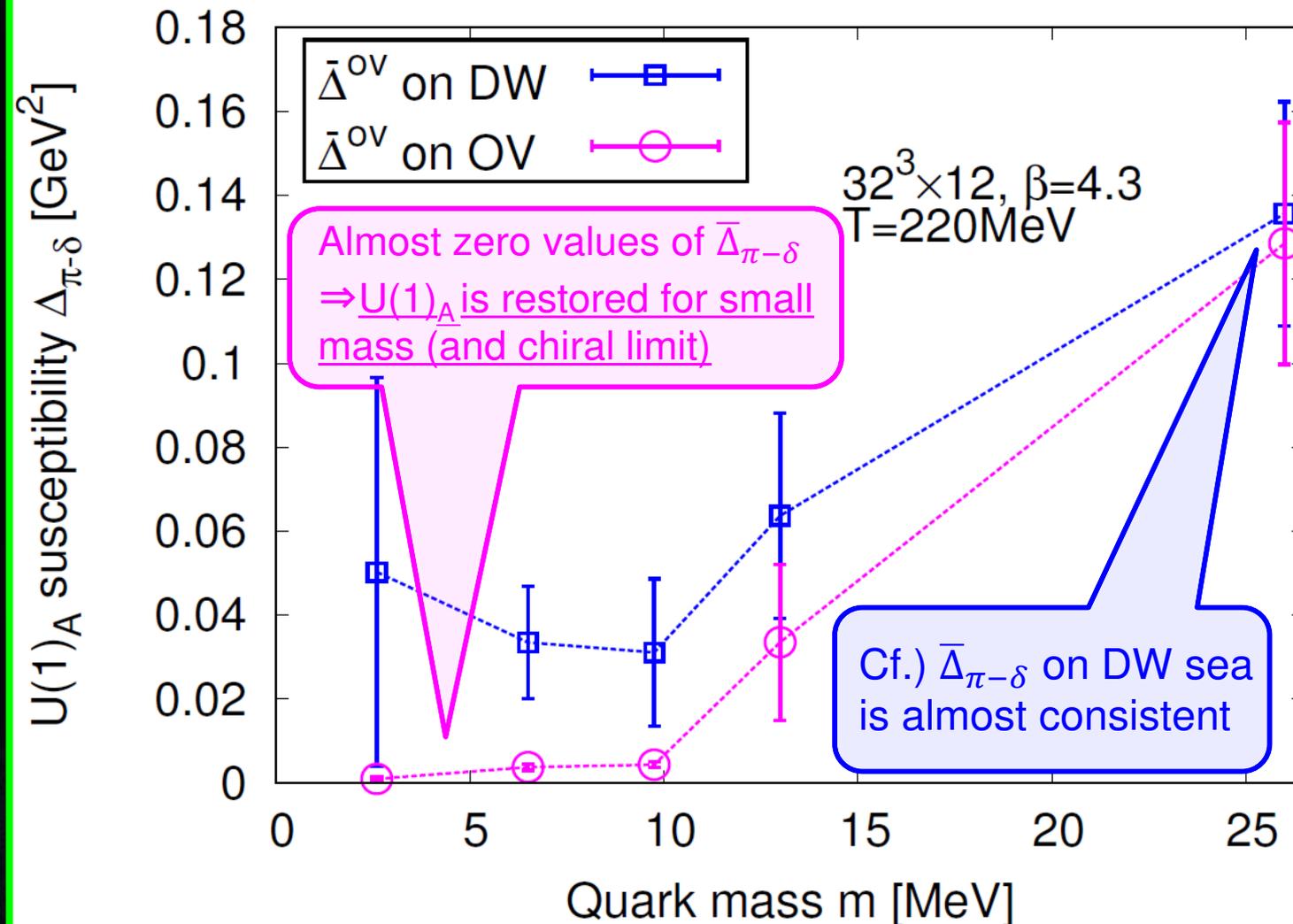|	Valence/Sea quarks	Setup
G. Cossu et al. PRD87 (2013)	OV on OV (Topology fixed sector)	
A. Tomiya et al. [arXiv:1612.01908]	DW on DW OV on DW OV on (reweighted) OV	1/a=1.7GeV
This work	OV on DW OV on (reweighted) OV	1/a=2.6GeV (More fine)

Results (overlap Dirac spectra)

m=2.6MeV

m=26MeV

U(1)$_A$ symmetry at T = 220MeV

Inset top: $32^3\times12$, $\beta=4.3$, T=220MeV, m=0.001(2.6MeV)

on DW
on OV

$\rho(|\lambda|)$ [GeV^3]

Overlap Dirac eigenvalue $|\lambda|$ [MeV]

$32^3\times12$, $\beta=4.3$
T=220MeV

Inset right: 32^3x12, $\beta=4.3$, T=220MeV, m=0

$\rho(|\lambda|)$ [GeV^3]

Overlap Dirac eigenvalue
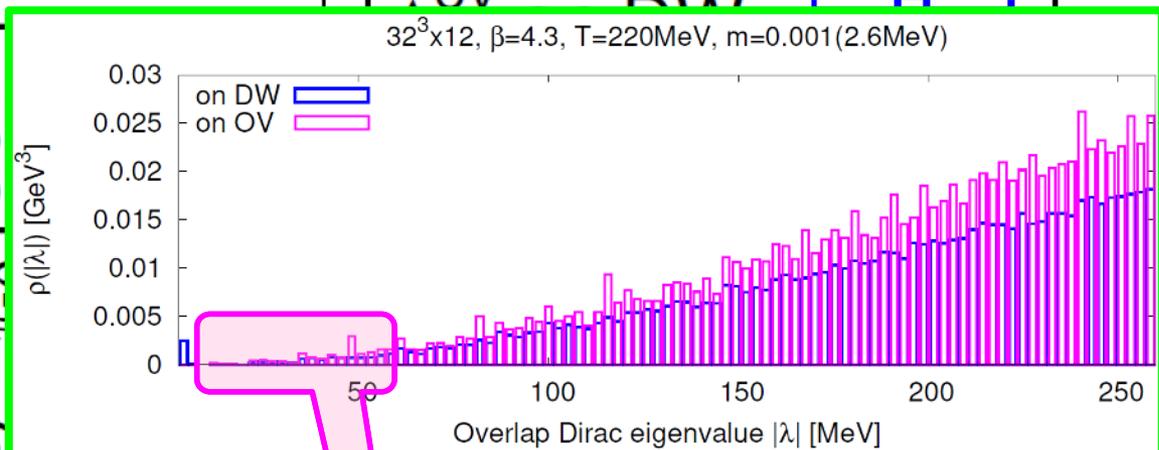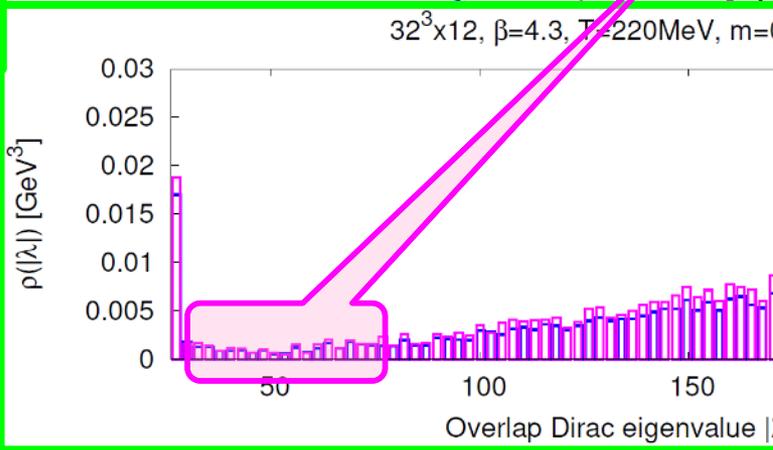
$\Rightarrow$ Finite $\overline{\Delta}_{\pi-\delta}$ comes from <u>enhancement of lower Dirac modes</u>
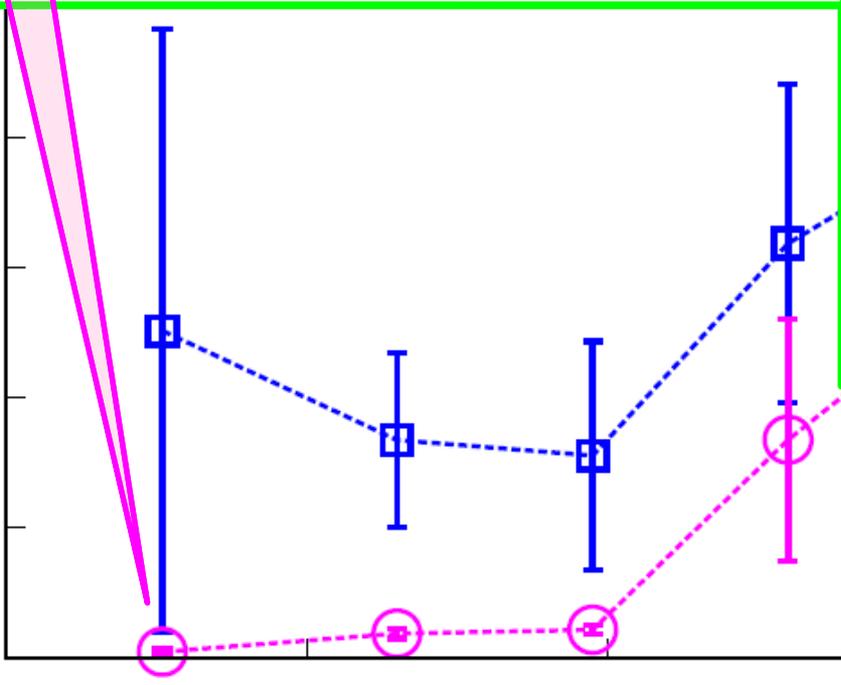
$\Rightarrow$ Zero $\overline{\Delta}_{\pi-\delta}$ comes from <u>suppression of lower Dirac modes</u>
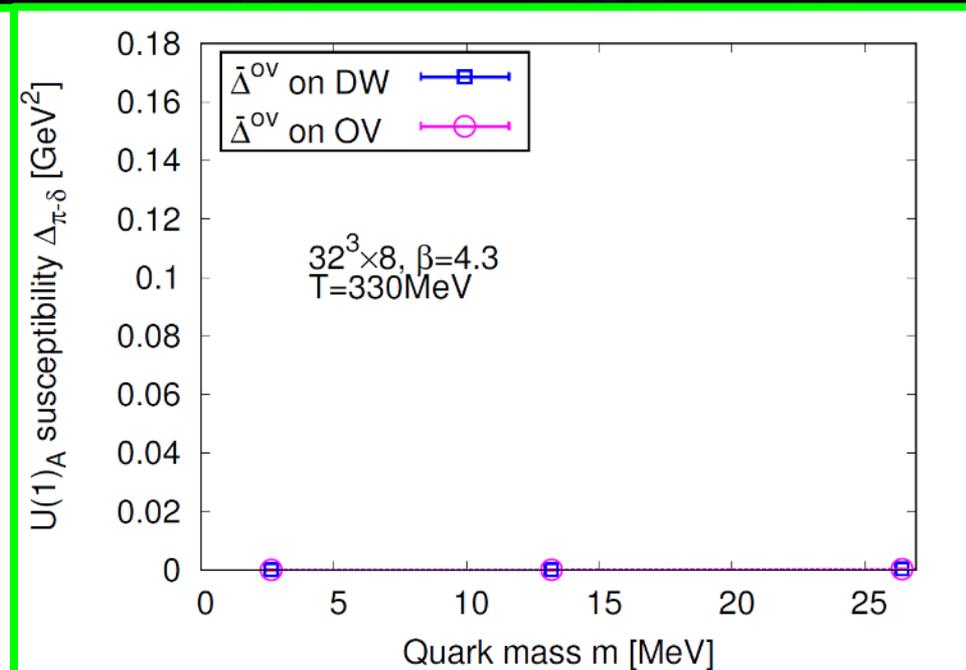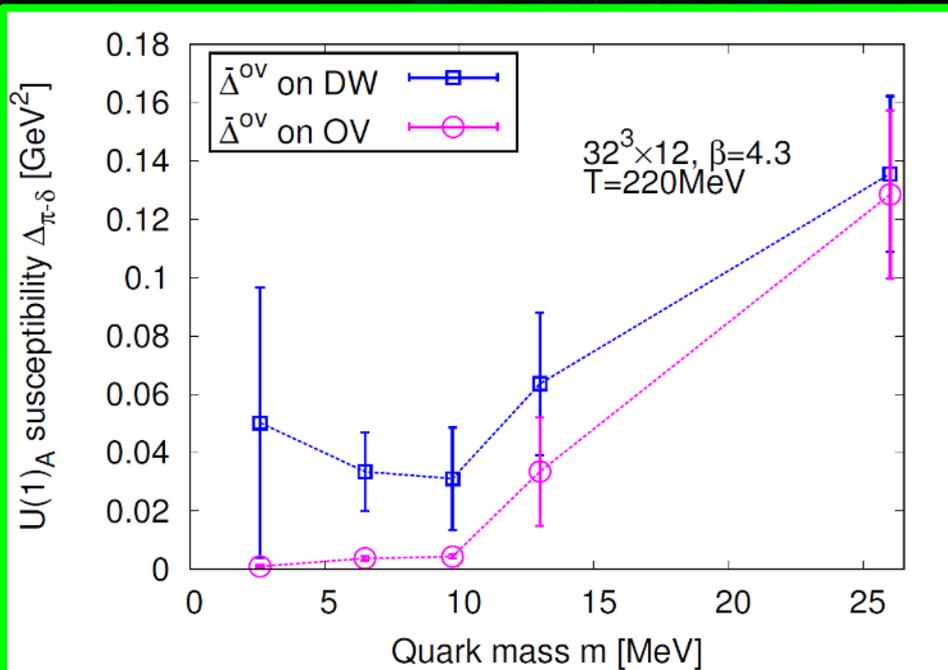
Quark mass m [MeV]

Temperature dependence (220 or 330MeV)
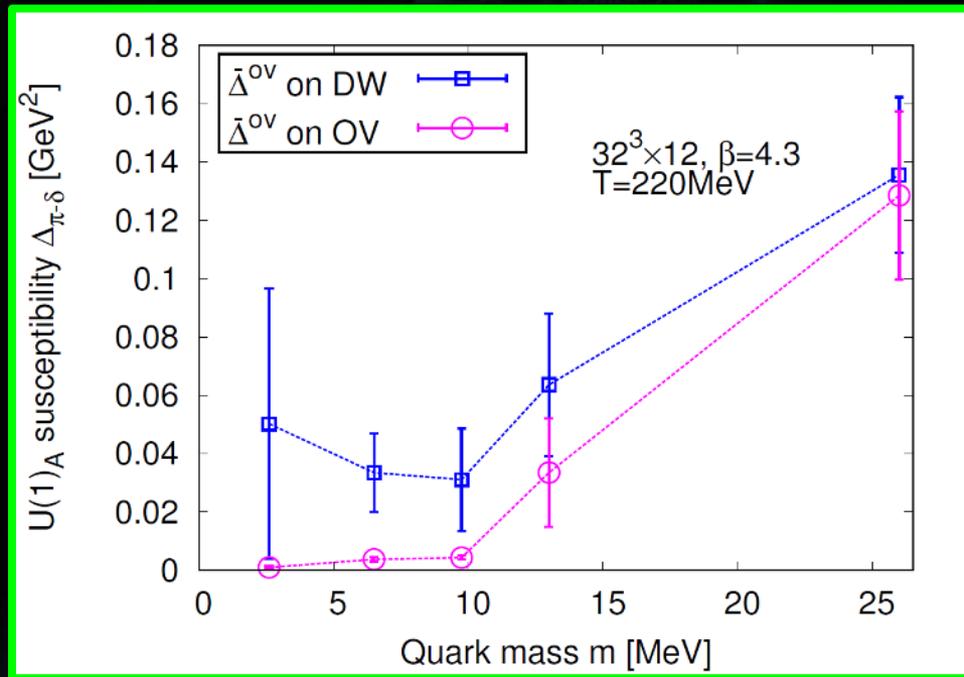
T=220MeV   T=330MeV

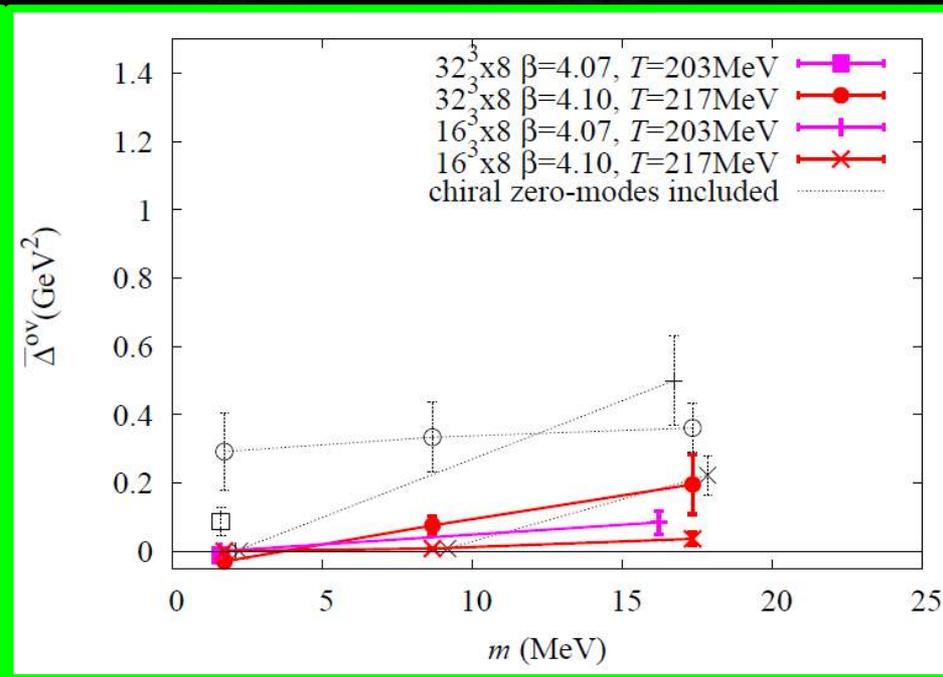⇒ With increasing T, U(1)$_A$ symmetric region becomes wider

Comparison with <u>Coarse lattice</u>

JLQCD, preliminary (2017)
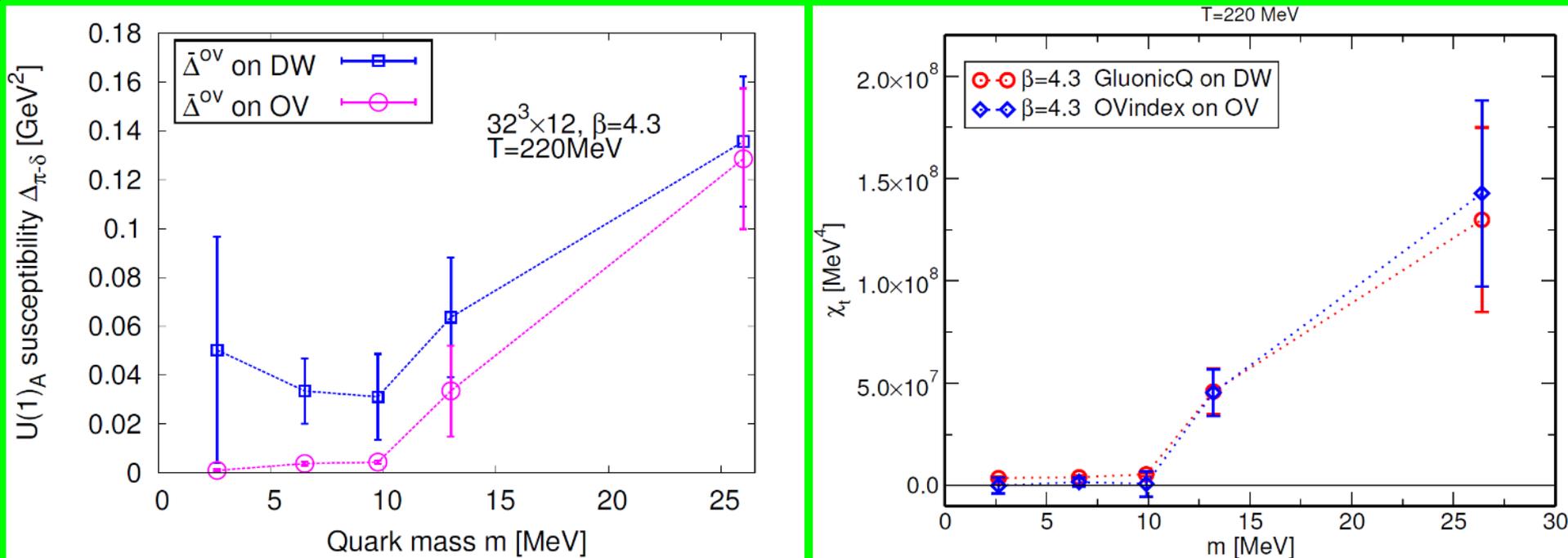
A. Tomiya et al. (JLQCD) [arXiv:1612.01908]

1/a=2.6GeV

1/a=1.7GeV

⇒Results from two lattice are consistent

Comparison with <u>Topological susceptibility</u>

- Q_t is related to Dirac operator by the index theorem:
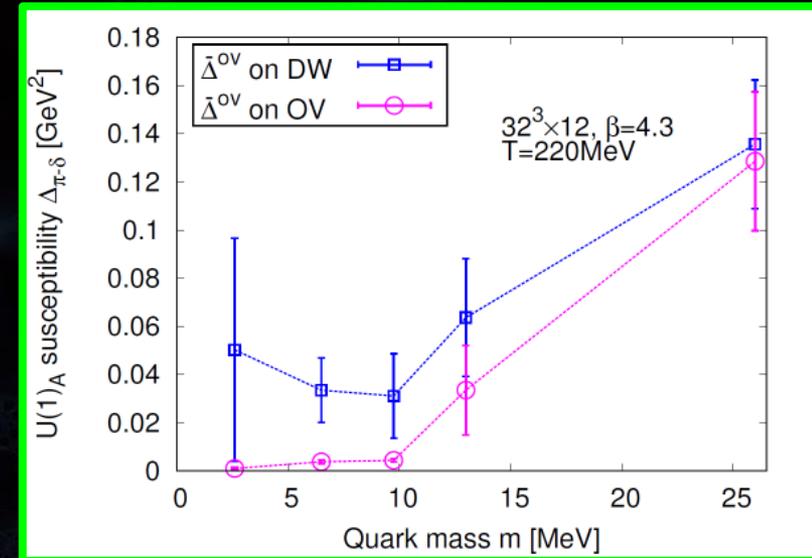
$$Q_t = n_+ - n_- = index(D)$$

The behavior seems to be similar? ⇒Next talk by Y. Aoki

Summary and Outlook

- U(1)$_A$ susceptibility at high T for $N_f = 2$ was simulated from OV Dirac spectra on the chiral sea quarks

$\Rightarrow$ <u>It seems to be restored in the chiral limit</u> for T=220-330MeV

- Relation to topological susceptibility $\Rightarrow$ Next talk
- Functional form of $\Delta(m)$ (powers of m …?)
- Going to near T_c (critical exponent for chiral transition?)
- Going to $N_f = 2 + 1$ sector

Backup