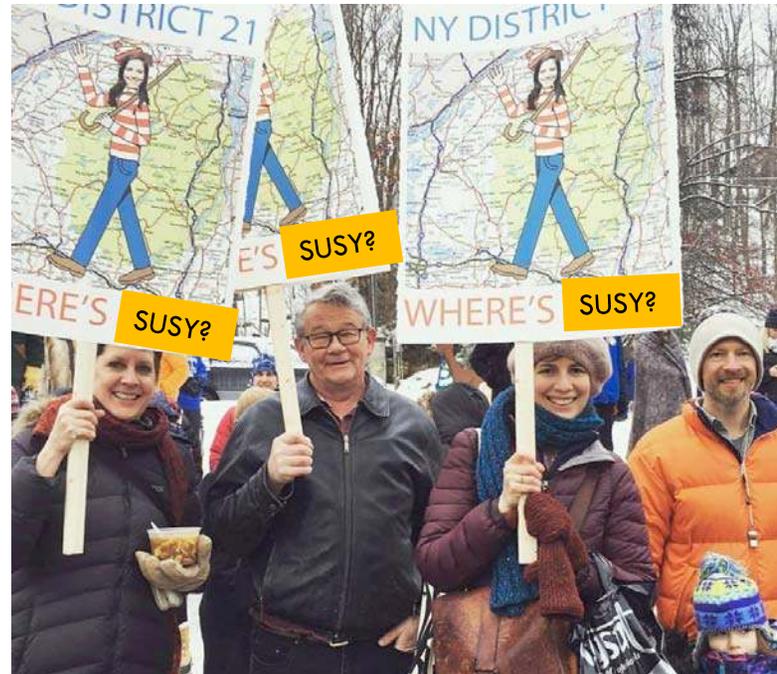# TRUNCATION OF LATTICE N=4 SUPER YANG-MILLS

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DESPERATELY SEEKING SUSY

WILSON FERMION N=4

Can impose SO(4), gauge invariance

$$S = \int d^4x \, \text{Tr}\left\{\frac{1}{2g_r^2}F_{\mu\nu}F_{\mu\nu} + \frac{i}{g_r^2}\overline{\lambda}_i\overline{\sigma}^\mu D_\mu\lambda_i + \frac{1}{g_r^2}D_\mu\phi_m D_\mu\phi_m + m_\phi^2\phi_m\phi_m\right.$$

$$+ m_\lambda(\lambda_i\lambda_i + \overline{\lambda}_i\overline{\lambda}_i) + \kappa_1\phi_m\phi_m\phi_n\phi_n + \kappa_2\phi_m\phi_n\phi_m\phi_n + y_1(\lambda_i[\phi_{ij},\lambda_j] + \overline{\lambda}_i[\phi_{ij},\overline{\lambda}_j])$$

$$\left. + y_2\epsilon_{ijkl}(\lambda_i[\phi_{jk},\lambda_l] + \overline{\lambda}_i[\phi_{jk},\overline{\lambda}_l])\right\}$$

$$+ \int d^4x \left\{\kappa_3(\text{Tr}\phi_m\phi_m)^2 + \kappa_4\text{Tr}\phi_m\phi_n\text{Tr}\phi_m\phi_n\right\}$$

8-dimensional parameter space to fine-tune in

Notice rescaling of fields exploited for first three terms---we do that in our twisted theory too.

The twisted, Q invariant lattice action takes the form

$$S = \frac{1}{2g^2}(\mathcal{Q}\lambda + S_{\text{closed}})$$

$$\lambda = \sum_x a^4 \text{Tr}(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a - \frac{1}{2}\eta d)$$

$$S_{\text{closed}} = -\frac{1}{4}\sum_x a^4 \epsilon_{abcde}\text{Tr}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}(x)$$

$$\mathcal{Q}S = 0$$

Observes the notion that anything correct should be simple.
Four terms --- will result in four coefficients to fine tune.

WHAT'S THE DIFFERENCE?

Of course we have to say how the derivatives are implemented:

$$\mathcal{F}_{ab}(x) = \mathcal{D}_a^{(+)}\mathcal{U}_b(x) = \mathcal{U}_a(x)\mathcal{U}_b(x+e_a) - \mathcal{U}_b(x)\mathcal{U}_a(x+e_b)$$

$$\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(x) = \mathcal{U}_a(x)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x-e_a)\mathcal{U}_a(x-e_a)$$

$$\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}(x) = \epsilon_{abcde}\chi_{de}(x+e_a+e_b)[\chi_{ab}(x)\overline{\mathcal{U}}_c(x-e_c)$$
$$-\overline{\mathcal{U}}_c(x-e_c+e_a+e_b)\chi_{ab}(x-e_c)]$$

$$\mathcal{U}_a = \tfrac{1}{a} + \mathcal{A}_a \qquad\qquad e_a \leftrightarrow A_4^*$$

$$gl(N,\mathbb{C}) \qquad \mathcal{A}_a = A_a + iB_a$$

SCALAR SUPERSYMMETRY

The supersymmetry transformation is:

$$\mathcal{Q}\mathcal{U}_a = \psi_a, \quad \mathcal{Q}\psi_a = 0, \quad \mathcal{Q}\overline{\mathcal{U}}_a = 0$$

$$\mathcal{Q}\chi_{ab}(x) = -\overline{\mathcal{F}}_{ab}(x) \equiv \overline{\mathcal{U}}_b(x+e_a)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x)$$

$$\mathcal{Q}\eta = d, \quad \mathcal{Q}d = 0$$

BIANCHI IDENTITY FOR Q CLOSED TERM

$$\epsilon_{mncde}\overline{\mathcal{D}}_c^{(-)}\overline{\mathcal{F}}_{mn}(x+e_c)=0$$

$$\epsilon_{mncde}\overline{\mathcal{D}}_c^{(-)}\overline{\mathcal{F}}_{mn}(x+e_c)=\epsilon_{mncde}[\overline{\mathcal{F}}_{mn}(x+e_c)\overline{\mathcal{U}}_c(x)-\overline{\mathcal{U}}_c(x+e_m+e_n)\overline{\mathcal{F}}_{mn}(x)]$$

$$\overline{\mathcal{F}}_{mn}(x)=\overline{\mathcal{U}}_n(x+e_m)\overline{\mathcal{U}}_m(x)-\overline{\mathcal{U}}_m(x+e_n)\overline{\mathcal{U}}_n(x)$$
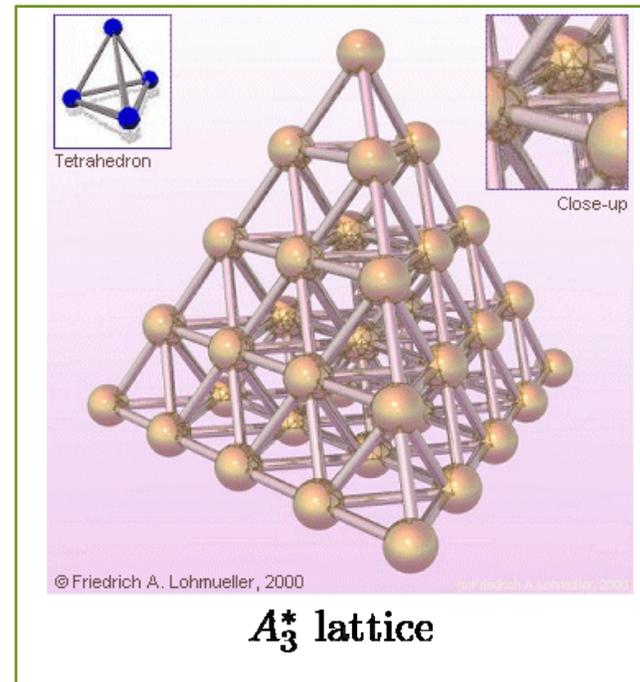
then algebra

PRIMITIVE VECTORS

$$e_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)$$

$$e_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)$$

$$e_3 = \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)$$

$$e_4 = \left(0, 0, -\frac{3}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)$$

$$e_5 = \left(0, 0, 0, -\frac{4}{\sqrt{20}}\right)$$

A_4^* lattice

S_5 point group symmetry

Tetrahedron

Close-up

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A_3^* lattice

TRUNCATION

$$\det \mathcal{U}_m(x) = \det(G(x)\mathcal{U}_m(x)G^\dagger(x + e_m))$$

provided $\det G(x) = 1$ for all x.

Hence we can consistently truncate to SL(N,C). All we lose is the U(1) gauge invariance, if we do it right. The surviving gauge group is SU(N). This is very helpful because the U(1) modes (scalar and gauge) are a royal pain.

The U(1) is 1 out of N^2 modes so we expect its effects to vanish in the large N limit.

Thus we expect the exact Q to be restored in this limit.

Furthermore, the coupling of the U(1) sector to the SU(N) sector is only through irrelevant vertices $ag^2 = a\lambda/N$.

So at fixed 't Hooft coupling, we expect $1/N^2$ suppression of the Q violation.

The dynamic truncation version of our code ("detlink") includes the term

$$\Delta S = \kappa \kappa_{\text{link}} \sum_{x,a} |\det \mathcal{U}_a(x) - 1|^2 \qquad \kappa = N/\lambda$$

We also have a version ("hybrid") that simply enforces the unimodular condition on the link fields.

We find that they give equivalent results at large κ_{link} .

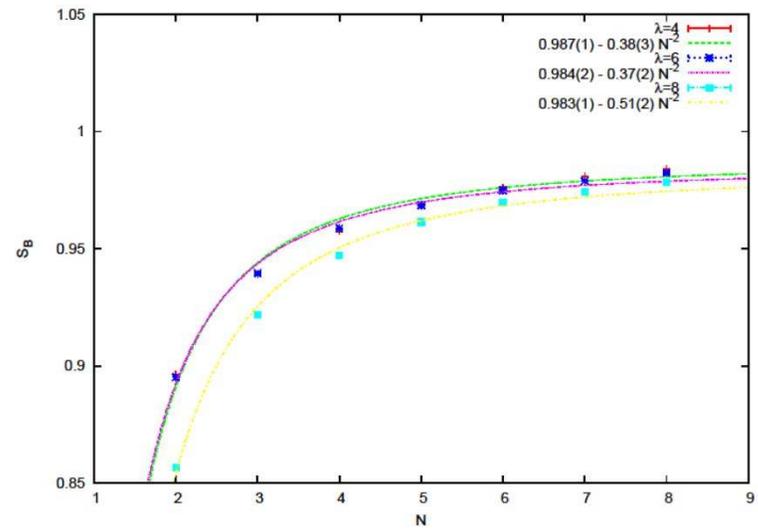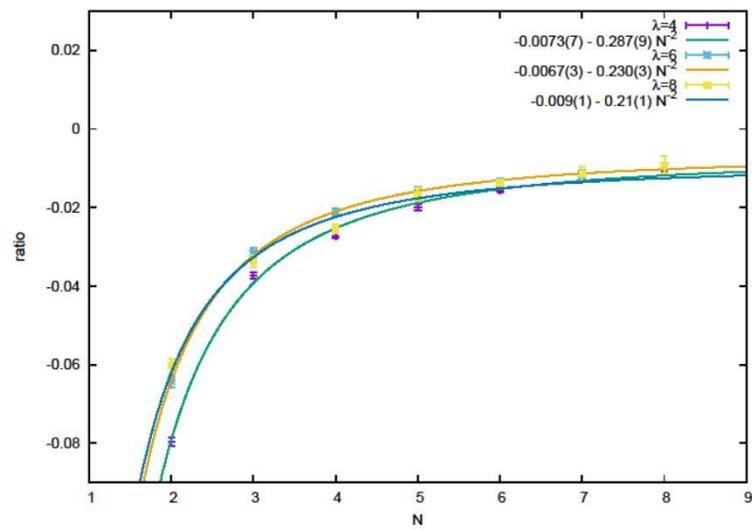It is good to have the dynamical option when providing analytical arguments about the large N behavior.

We measure two Ward identities that should be exact when Q is restored.

$$\frac{\langle S_B \rangle}{\frac{9}{2}N^2 V} = 1 \qquad\qquad \text{ratio} = \langle \text{ward} \rangle / \langle \text{bilinear} \rangle$$

$$\langle \text{ward} \rangle = \sum_x \langle \text{Tr}(\sum_b (\mathcal{U}_b \overline{\mathcal{U}}_b - \overline{\mathcal{U}}_b \mathcal{U}_b) \sum_a \mathcal{U}_a \overline{\mathcal{U}}_a) \rangle - \langle \text{bilinear} \rangle$$

$$\langle \text{bilinear} \rangle = \sum_x \langle \text{Tr}\eta \sum_a \psi_a \overline{\mathcal{U}}_a \rangle$$

Q RESTORATION

CONCLUSIONS

Lattice formulation with very little fine-tuning, due to symmetries

Understanding of renormalization

BPS solitons under study

Highly optimized code with good scaling

SL(N,C) truncation

Adequate computing resources

Lower-dimensional finite T/gauge-gravity duality

Operator dimensions