The new method to extract the spectra of irreducible representations of lattice

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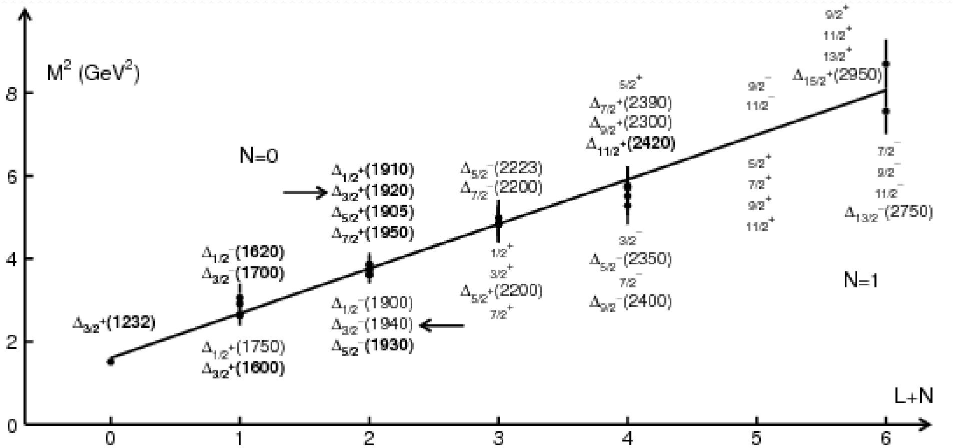
Outline

- Motivation
- New method
- Pion-Pion with I=2
- Summary and Outlook

Motivation : How important of High partial wave state

- Regge Trajectory



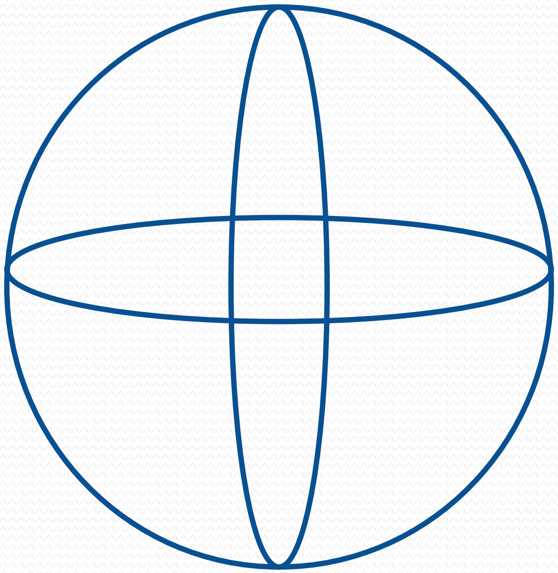
- High momentum cross section

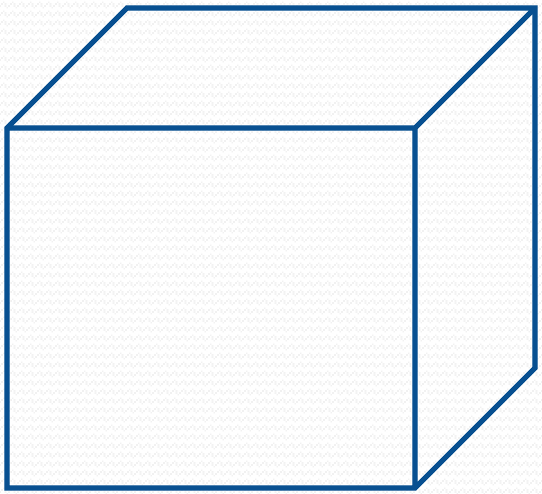
$$\sigma \sim p^{2l+1}$$

- Nucleon-Nuclei scattering

P wave phase shifts are the key of "A_y-puzzle" in nucleon-deuteron scattering.

Motivation : How to get High partial wave state in the finite volume ??





Symmetry SO(3) O_h

$L^p = 0^+, 1^-, 2^+, 3^-,$ ……

$A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$

Motivation : Infinite ⟷ Finite

SO(3)	O_h
0^+	A_1^+
1^-	T_1^-
2^+	$E^+ \oplus T_2^+$
3^-	$A_2^- \oplus T_1^- \oplus T_2^-$
4^+	$A_1^+ \oplus E^+ \oplus T_1^+ \oplus T_2^+$
5^-	$E^- \oplus 2T_1^- \oplus T_2^-$
6^+	$A_1^+ \oplus A_2^+ \oplus E^+ \oplus T_1^+ \oplus 2T_2^+$
7^-	$A_2^- \oplus E^- \oplus 2T_1^- \oplus 2T_2^-$
8^+	$A_1^+ \oplus 2E^+ \oplus 2T_1^+ \oplus 2T_2^+$
9^-	$A_1^- \oplus A_2^- \oplus E^- \oplus 3T_1^- \oplus 2T_2^-$

O_h	SO(3)
A_1^+	$0^+, 4^+, 8^+,$ ....
A_2^+	$6^+,$ ....
E^+	$2^+, 4^+, 6^+, 8^+,$ ....
T_1^+	$4^+, 6^+, 8^+,$ ....
T_2^+	$2^+, 4^+, 6^+, 8^+,$ ....
A_1^-	$9^-,$ ....
A_2^-	$3^-, 7^-,$ 9-, ...
E^-	$5^-, 7^-,$ 9-, ...
T_1^-	$1^-, 3^-,$ 5-, 7^-, 9-, ...
T_2^-	3^-, 5-, 7^-, 9-, ...

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0^+	A_1^+
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O_h	SO(3)
A_1^+	0^+ , 4^+ , 8^+ , ...
A_2^+	6^+ , ...
E^+	2^+ , 4^+ , 6^+ , 8^+ , ...
T_1^+	4^+ , 6^+ , 8^+ , ...
T_2^+	2^+ , 4^+ , 6^+ , 8^+ , ...
A_1^-	9^- , ...
A_2^-	3^- , 7^- , 9- , ...
E^-	5^- , 7^- , 9- , ...
T_1^-	1^- , 3^- , 5- , 7^- , 9- , ...
T_2^-	3^- , 5- , 7^- , 9- , ...

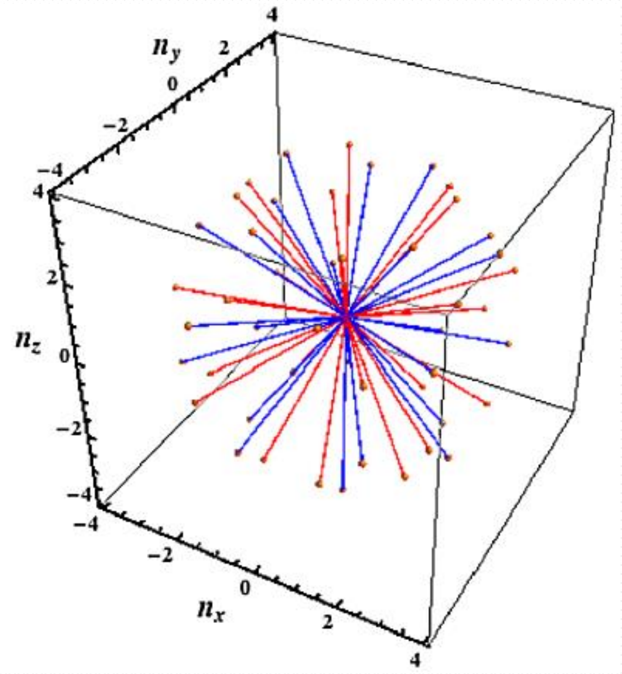
For the High Partial Wave scattering amplitudes, we need the spectra of various irreducible representations.

New Method: Rotation of O_h

Question: How to express the rotation elements of O_h group ?

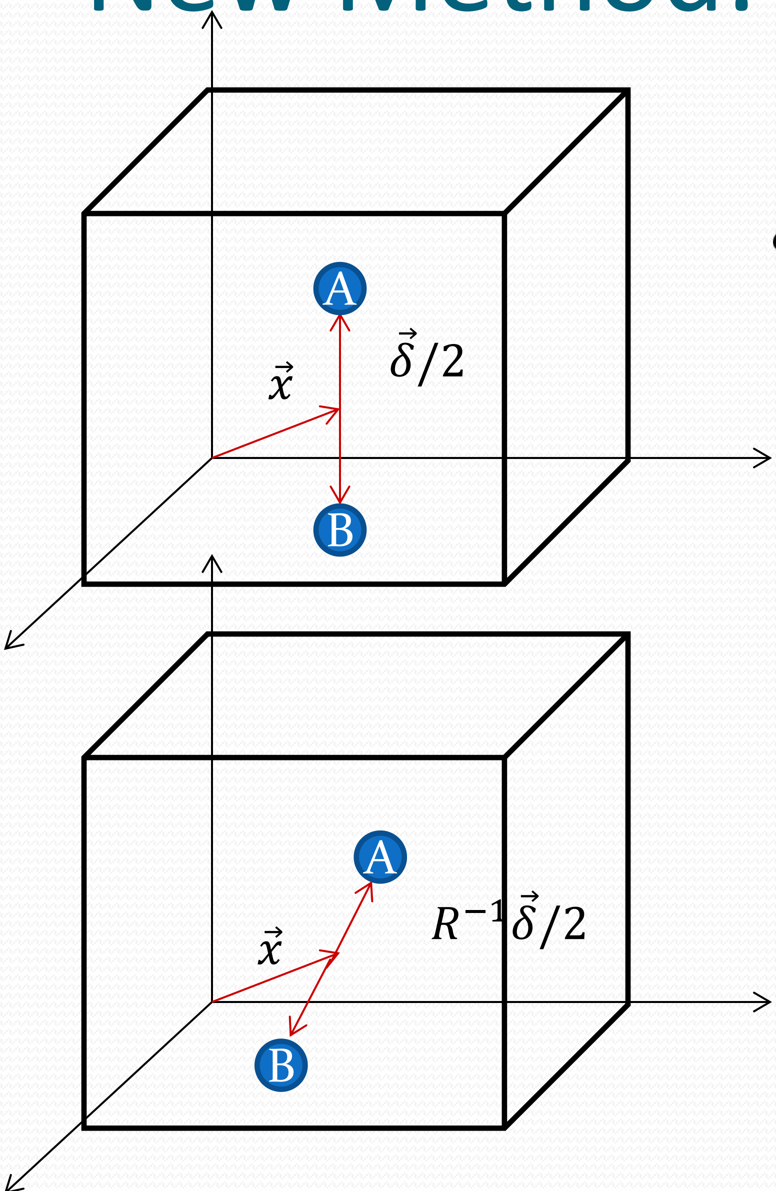
There are 48 rotation elements which can connect 48 vectors in the coordinate space.

$\hat{R}$	Rotation Axis	Rotation Angle	(x_1,x_2,x_3)
$\hat{R}_1$	Any	0^o	(x_1,x_2,x_3)
$\hat{R}_2$	(1,1,1)	-120^o	(x_2,x_3,x_1)
…	…	…	…
$\hat{R}_{24}$	(0,0,1)	-180^o	$(-x_1,-x_2,x_3)$
$\hat{R}_{25}=\hat{\pi}\hat{R}_1$	-	-	$(-x_1,-x_2,-x_3)$
$\hat{R}_{26}=\hat{\pi}\hat{R}_2$	-	-	$(-x_2,-x_3,-x_1)$
…	…	…	…
$\hat{R}_{48}=\hat{\pi}\hat{R}_{24}$	-	-	$(x_1,x_2,-x_3)$



T. Luu and M. J. Savage, PRD 83 114508(2011)

New Method: Two body Operator



$$\phi(\vec{x},\vec{\delta},t) = \chi_A(\vec{x}+\vec{\delta}/2,t)\chi_B(\vec{x}-\vec{\delta}/2,t)$$

Rotation Elements: $\hat{R} = \{\hat{R}_1, \hat{R}_2, \dots, \hat{R}_{48}\}$

$$\phi_R(\vec{x},\vec{\delta},t) = \chi_A(\vec{x}+R^{-1}\vec{\delta}/2,t)\chi_B(\vec{x}-R^{-1}\vec{\delta}/2,t)$$

There are 48 different two particles operator, ϕ_R.

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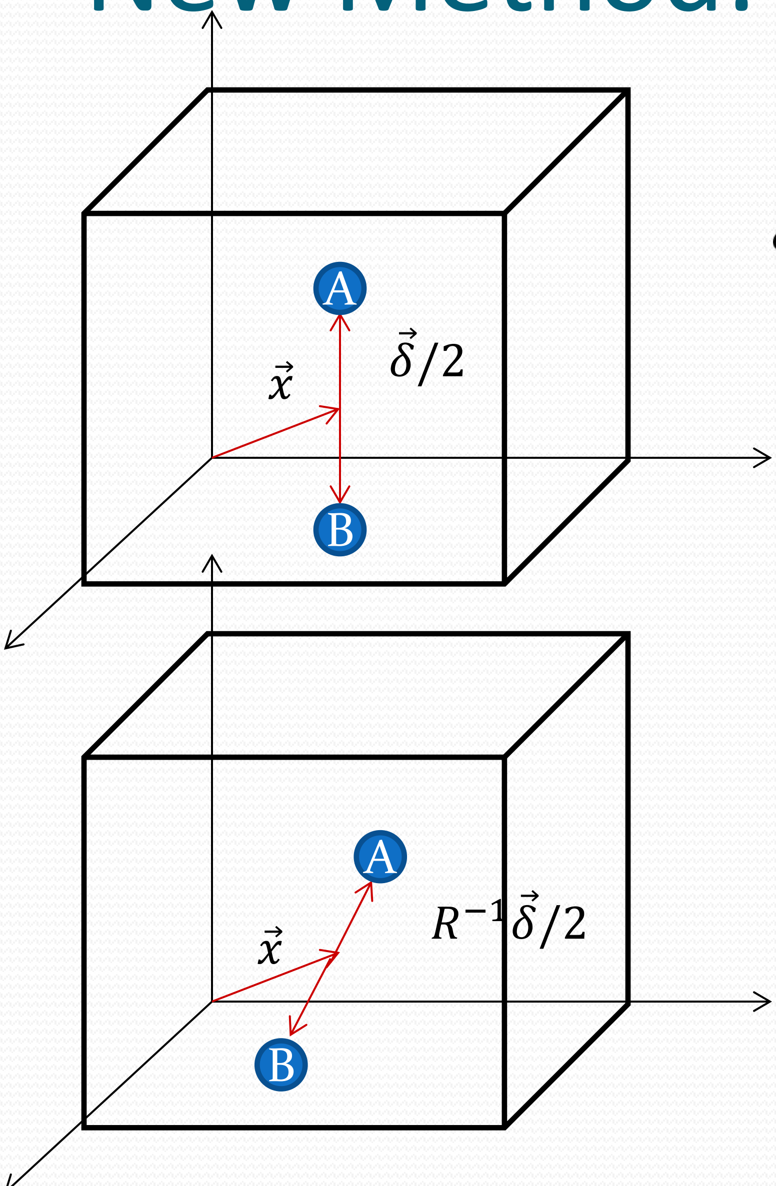

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$$\vec{\delta} = (\delta_x, \delta_y, \delta_z)$$

$$\delta_x \neq \delta_y \neq \delta_z$$

$$\vec{x} \pm \vec{\delta}/2 = \vec{n} \in \mathrm{Z}^3$$

$$\phi_R(\vec{x},\vec{\delta},t) = \chi_A(\vec{x}+R^{-1}\vec{\delta}/2,t)\chi_B(\vec{x}-R^{-1}\vec{\delta}/2,t)$$

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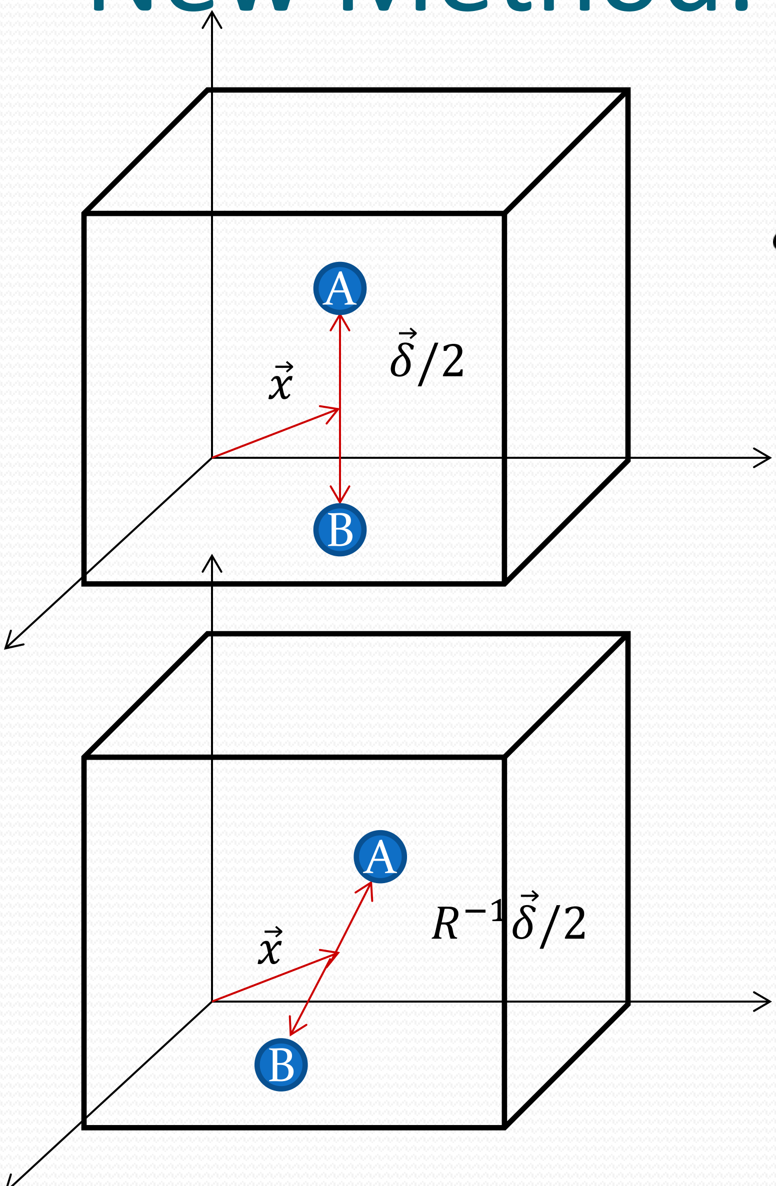

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$$\vec{x} \pm \vec{\delta}/2 = \vec{n} \in \mathrm{Z}^3$$

$$\vec{x} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \text{ and } \vec{\delta} = (1,3,5)$$

Simplifies continuum rotation symmetry

$$\phi_R(\vec{x},\vec{\delta},t) = \chi_A(\vec{x}+R^{-1}\vec{\delta}/2,t)\chi_B(\vec{x}-R^{-1}\vec{\delta}/2,t)$$

There are 48 different two particles operator, ϕ_R.

New Method: Basis State (1)

$$\phi_R{}^{+}(\vec{x},\vec{\delta},t)|\Omega\rangle = |\phi_R{}^{+}(\vec{x},\vec{\delta},t)\rangle \sim |\phi_R{}^{+}(\vec{\delta})\rangle$$

$$\hat{R}_X|\phi_R{}^{+}(\vec{\delta})\rangle = |\phi_{R_X R}{}^{+}(\vec{\delta})\rangle = \sum_{R'}|\phi_{R'}{}^{+}(\vec{\delta})\rangle\,[\bar{\Gamma}_{reg}(R_X)]_{R'R}$$

Matrix $\bar{\Gamma}_{reg}(R_X)$ is one of the matrices of representation of O_h group, and it is 48 ×48 matrix. Obviously, there are 48 matrices of $\bar{\Gamma}_{reg}$ corresponding to 48 rotation elements of O_h group. The representation matrices $\bar{\Gamma}_{reg}$ use all the elements of the group as multipliers, and the dimension of it equal to the order of the group. This representation is called "regular representation".

Regular representation $\bar{\Gamma}_{reg}$ is a reducible representation, it can be reduced to several irreducible representations through similarity transformation:

$$S^{-1}\bar{\Gamma}_{reg}(R_X)\,S = A_1{}^{+} \oplus A_2{}^{+} \oplus 2E^{+} \oplus 3T_1{}^{+} \oplus 3T_2{}^{+} \oplus A_1{}^{-} \oplus A_2{}^{-} \oplus 2E^{-} \oplus 3T_1{}^{-} \oplus 3T_2{}^{-}$$

New Method: Basis State (2)

$$\hat{R}_X|\phi_R{}^+(\vec{\delta})\rangle = |\phi_{R_X R}{}^+(\vec{\delta})\rangle = \sum_{R'}|\phi_{R'}{}^+(\vec{\delta})\rangle\,[\bar{\Gamma}_{reg}(R_X)]_{R'R}$$

$$S^{-1}\bar{\Gamma}_{reg}\,S \;=\; A_1{}^+ \oplus A_2{}^+ \oplus 2E^+ \oplus 3T_1{}^+ \oplus 3T_2{}^+ \oplus A_1{}^- \oplus A_2{}^- \oplus 2E^- \oplus 3T_1{}^- \oplus 3T_2{}^-$$

$$48 = 1 + 1 + 2\times 2 + 3\times 3 + 3\times 3 + 1 + 1 + 2\times 2 + 3\times 3 + 3\times 3$$

e.g.

i: 1, 2	i: 1, 2, 3
Γ: E	Γ: T_2
n: 1, 2	n: 1, 2, 3

This block-diagonal matrices will lead to a new set of basis states: $|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\rangle$

New Method: Basis State (3)

$$\hat{R}_X|\phi_R{}^+(\vec{\delta})\rangle = \left|\phi_{R_X R}{}^+(\vec{\delta})\right\rangle = \sum_{R'}|\phi_{R'}{}^+(\vec{\delta})\rangle\,[\bar{\Gamma}_{reg}(R_X)]_{R'R}$$

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$$\hat{R}_X\left|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\right\rangle = \sum_{i',\Gamma',n'}\left|\Phi_{i',\Gamma',n'}{}^+(\vec{\delta})\right\rangle[S^{-1}\,\bar{\Gamma}_{reg}(R_X)\,S]_{i'\Gamma'n',\,i\,\Gamma\,n}$$

$$|\phi_R{}^+(\vec{\delta})\rangle \longrightarrow \left|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\right\rangle$$

$$\left|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\right\rangle = \sum_R|\phi_R{}^+(\vec{\delta})\rangle[S]_{R,\,i\,\Gamma\,n}$$

New Method: Correlation Function(1)

The correlation function is:

$$G_{R,R'}(\vec{p}=0,\vec{y},\,\vec{x},\,\vec{\delta},\,t) = \sum_{\vec{y}-\vec{x}\in Z^3} e^{\vec{p}.(\vec{y}-\vec{x})}\langle\Omega|\phi_R(\vec{y},\,\vec{\delta},\,t)\phi^{\dagger}_{R'}(\vec{x},\,\vec{\delta},\,0)|\Omega\rangle$$

48×48

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$$\sim \sum_{\vec{y}-\vec{x}\in Z^3} \langle \phi^\dagger_R(\vec{y}, \vec{\delta}, 0) | e^{-\hat{H}t} | \phi^\dagger_{R'}(\vec{x}, \vec{\delta}, 0) \rangle \sim \sum \langle \phi^\dagger(R^{-1}\vec{\delta}) | e^{-\hat{H}t} | \phi^\dagger(R'^{-1}\vec{\delta}) \rangle$$

48×48

$$[\hat{R}_x, \hat{H}] = 0$$

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$$[\hat{R}_x,\hat{H}]=0$$

E, π
$8C_{3,}$ $8C_3\pi$
$3C_{2,}$ $3C_2\pi$
$6C_{4,}$ $6C_4\pi$
$6C'_{2,}$ $6C'_2\pi$

$$\sum\langle\phi^\dagger(R\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle$$

$$=\sum\langle\phi^\dagger(RR'\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(R'\vec{\delta})\rangle$$

$$=\sum\langle\phi^\dagger(R'R\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(R'\vec{\delta})\rangle$$

$$=\sum\langle\phi^\dagger(R'RR'^{-1}\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle$$

The operators in the same class has the same value of correlation function

New Method: Correlation Function(1)

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$$\sim \sum_{\vec{y}-\vec{x}\in Z^3}\langle\phi^\dagger_R(\vec{y},\,\vec{\delta},\,0)|e^{-\hat{H}t}|\phi^\dagger_{R'}(\vec{x},\,\vec{\delta},\,0)\rangle \sim \sum\langle\phi^\dagger(R^{-1}\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(R'^{-1}\vec{\delta})\rangle$$

48×48

↓

10

$$[\hat{R}_x,\hat{H}]=0$$

E, π
$8C_{3}$, $8C_3\pi$
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$$\sum\langle\phi^\dagger(R\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle$$

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New Method: Correlation Function(2)

The correlation function is:

$$\sum_{\vec{y}-\vec{z}\in \mathrm{Z}^3} e^{\vec{p}.(\vec{y}-\vec{x})} \langle \Omega | \Phi_{i,\Gamma,n}(\vec{y},\, \vec{\delta},\, t) \Phi^{\dagger}_{i',\Gamma',n'}(\vec{x},\, \vec{\delta},\, 0) | \Omega \rangle$$

$$|\Phi_{i,\Gamma,n}{}^{+}(\vec{\delta})\rangle = \sum_R |\phi_R{}^{+}(\vec{\delta})\rangle [S]_{R,\ i\,\Gamma\, n}$$

$$\sum \langle \phi^{\dagger}(R\vec{\delta}) | e^{-\hat{H}t} | \phi^{\dagger}(\vec{\delta}) \rangle = \sum \langle \phi^{\dagger}(R'RR'^{-1}\vec{\delta}) | e^{-\hat{H}t} | \phi^{\dagger}(\vec{\delta}) \rangle$$

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$$\sum_{\vec{y}-\vec{z}\in Z^3} e^{\vec{p}.(\vec{y}-\vec{x})} \langle\Omega|\Phi_{i,\Gamma,n}(\vec{y},\, \vec{\delta},\, t)\Phi^{\dagger}_{i',\Gamma',n'}(\vec{x},\, \vec{\delta},\, 0)|\Omega\rangle$$

$$= \delta_{i,i'}\delta_{\Gamma,\Gamma'}\delta_{n,n'} \sum_{\vec{y}-\vec{x}\in Z^3} e^{\vec{p}.(\vec{y}-\vec{x})} \frac{h}{l_\Gamma} \sum_R \chi_\Gamma(R)\langle\Omega|\phi_R(\vec{y},\, \vec{\delta},\, t)\phi^{\dagger}(\vec{x},\, \vec{\delta},\, 0)|\Omega\rangle$$

$$|\Phi_{i,\Gamma,n}{}^{+}(\vec{\delta})\rangle = \sum_R |\phi_R{}^{+}(\vec{\delta})\rangle [S]_{R,\ i\,\Gamma\, n}$$

$$\sum \langle\phi^\dagger(R\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle = \sum \langle\phi^\dagger(R'RR'^{-1}\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle$$

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Diagonal and Just rely on Γ

Character Number of Γ

h and l_Γ are the orders of Oh group and irreducible representation Γ.

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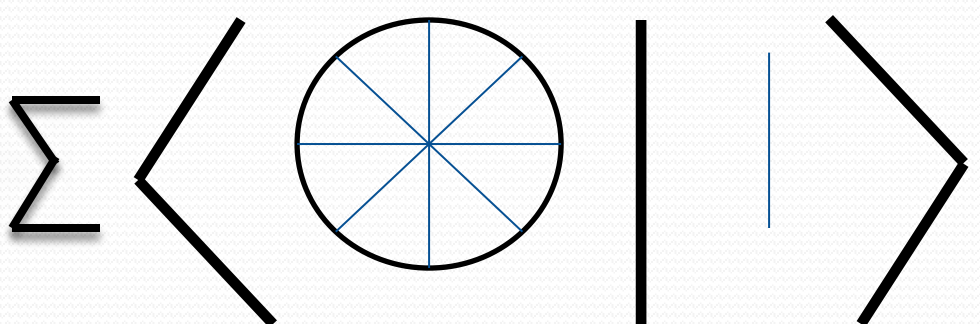
Diagonal and Just rely on Γ

Character Number of Γ

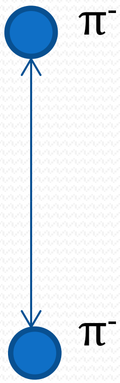
"Shell" Sink

Two source locations as $\vec{x}+\frac{\vec{\delta}}{2}$ and $\vec{x}-\frac{\vec{\delta}}{2}$

h and l_Γ are the orders of Oh group and irreducible representation Γ.



Pion-Pion with I=2: Correlation function



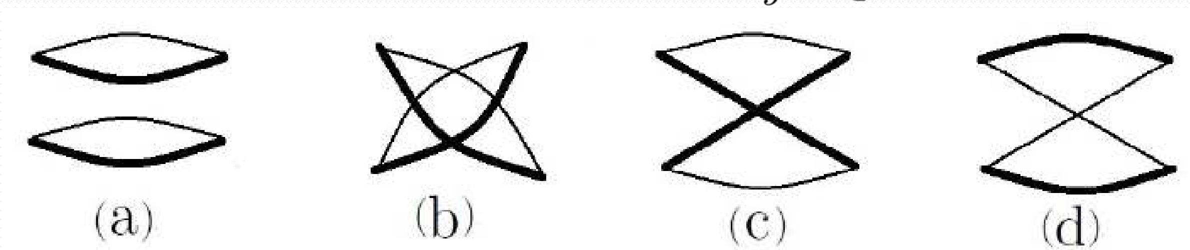
$$\chi_{\pi^-}(\vec{x},t) = u(\vec{x},t)\gamma_5\bar{d}(\vec{x},t)$$

$$\phi_R(\vec{x},\vec{\delta},t) = \chi_{\pi^-}(\vec{x}+R^{-1}\vec{\delta}/2,t)\chi_{\pi^-}(\vec{x}-R^{-1}\vec{\delta}/2,t)$$

L = 0^+, 2^+, 4^+ , …, and here we neglect L > 4

	E, π	$8C_3$, $8C_3\pi$	$3C_2$, $3C_2\pi$	$6C_4$, $6C_4\pi$	$6C'_2$, $6C'_2\pi$	L
A_1^+	1	1	1	1	1	0^+, 4^+
E^+	2	-1	2	0	0	2^+, 4^+
T_1^+	3	0	-1	-1	1	4^+
T_2^+	3	0	-1	1	-1	2^+, 4^+

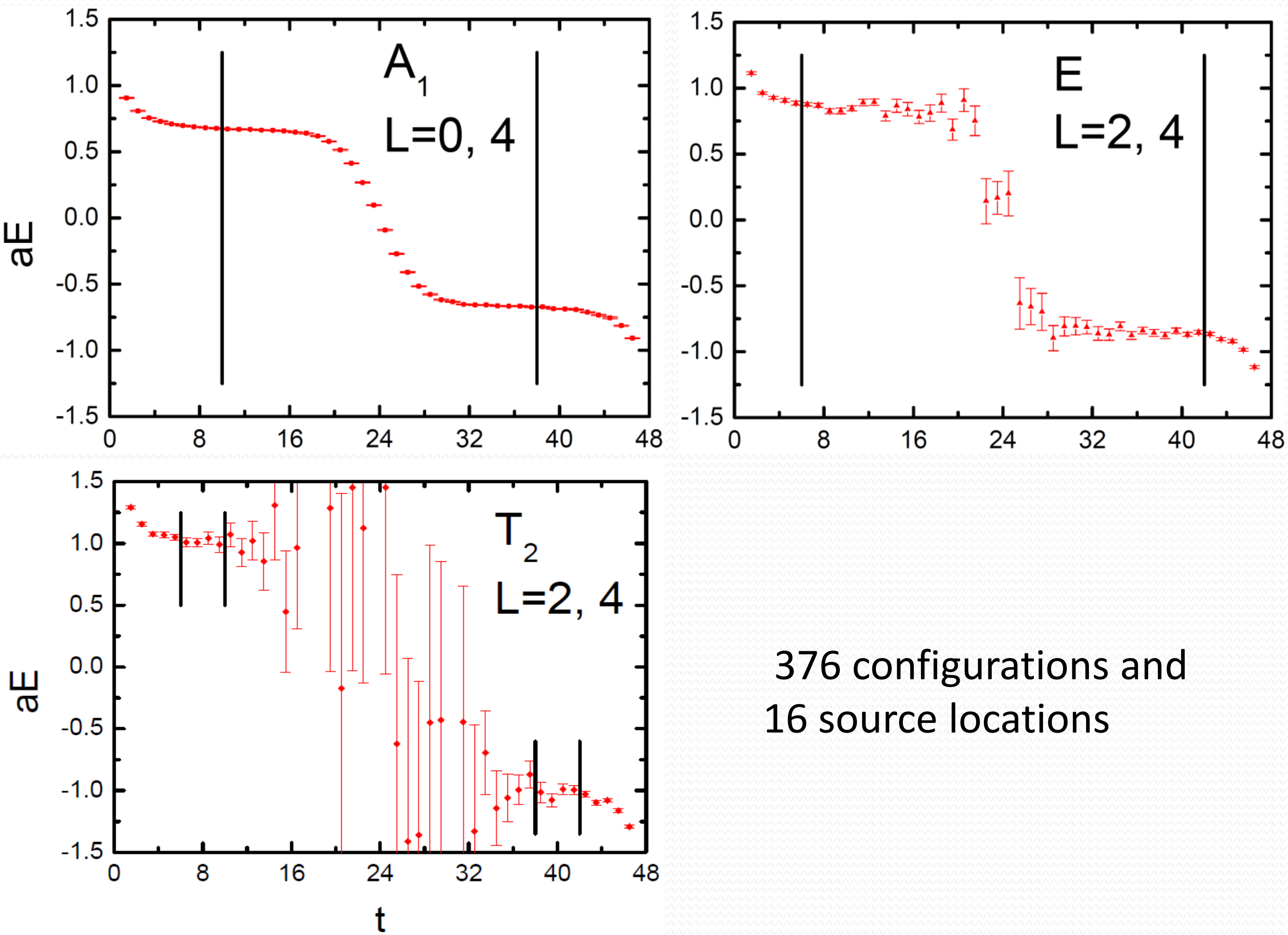
$$G_\Gamma(\vec{y},\vec{x},\vec{\delta},t) = \sum_{R\in O_h}\chi_\Gamma(R)\sum_{\vec{y}-\vec{x}\in Z^3}\langle\Omega|\phi_R(\vec{y},\vec{\delta},t)\phi^\dagger(\vec{x},\vec{\delta},0)|\Omega\rangle$$



Thick lines: $\bar{u}u$
Thin lines: $\bar{d}d$

Pion-Pion with I=2: Spectra

The result of L = $24^3 \times 48$ and a = 0.07 fm, m_π= 900 MeV



$$p_\Gamma = \frac{1}{2}\sqrt{E_\Gamma^2 - 4m_\pi^2}$$

$$\frac{p_{A_1}L}{2\pi} = 0.2455(36)$$
~ 0

$$\frac{p_E L}{2\pi} = 1.047(13)$$
~ 1

$$\frac{p_{T_2}L}{2\pi} = 1.457(23)$$
$\sim \sqrt{2}$

376 configurations and
16 source locations

Pion-Pion with I=2: Lüscher Equation

$$\det\begin{pmatrix} ctg\delta_0(p_{A_1}) + M_{11}^{A_1}(p_{A_1}) & M_{12}^{A_1}(p_{A_1}) \\ M_{12}^{A_1}(p_{A_1}) & ctg\delta_4(p_{A_1}) + M_{22}^{A_1}(p_{A_1}) \end{pmatrix} = 0$$

$$\det\begin{pmatrix} ctg\delta_2(p_E) + M_{11}^{E}(p_E) & M_{12}^{E}(p_E) \\ M_{12}^{E}(p_E) & ctg\delta_4(p_E) + M_{22}^{E}(p_E) \end{pmatrix} = 0$$

$$\det\begin{pmatrix} ctg\delta_2(p_{T_2}) + M_{11}^{T_2}(p_{T_2}) & M_{12}^{T_2}(p_{T_2}) \\ M_{12}^{T_2}(p_{T_2}) & ctg\delta_4(p_{T_2}) + M_{22}^{T_2}(p_{T_2}) \end{pmatrix} = 0$$

$$M_{ij}^{\Gamma}(p) = \sum_s C_{i,j,s}^{\Gamma} H_{s0}\left(\frac{pL}{2\pi}\right)$$

$$H_{s0}(q) = \frac{Z_{s0}(1;q)}{\sqrt{(2s+1)\pi^3}q^{s+1}}$$

Γ	(i,j)	(s, $C_{i,j,s}^{\Gamma}$)
A_1^+	(1,1)	(0,1)
	(1,2)	$(4,\frac{6\sqrt{21}}{7})$
	(2,2)	(0,1), $(4,\frac{324}{143})$, $(6,\frac{80}{11})$, $(8,\frac{560}{143})$
E^+	(1,1)	(0,1), $(4,\frac{18}{7})$
	(1,2)	$(4,-\frac{120\sqrt{3}}{77})$,$(6,-\frac{30\sqrt{3}}{11})$
	(2,2)	$(4,\frac{324}{1001})$,$(6,-\frac{64}{11})$, $(8,\frac{392}{143})$
T_1^+	(1,1)	(0,1), $(4,-\frac{12}{7})$
	(1,2)	$(4,-\frac{60\sqrt{3}}{77})$,$(6,\frac{40\sqrt{3}}{11})$
	(2,2)	(0,1), $(4,-\frac{162}{77})$, $(6,\frac{20}{11})$

But only 3 equations, but 6 different phase shifts, $\delta_0(p_{A_1}), \delta_4(p_{A_1}), \delta_2(p_E), \delta_4(p_E), \delta_2(p_{T_2}), \delta_4(p_{T_2})$.

M. Lüscher NPB 354 531 (1991)

Pion-Pion with I=2: Toy Model

By fitting experimental data up to p = 600 MeV, for L=0, 2, 4 partial wave, the phase shifts can be expressed as

$$p^{2L+1}\mathrm{Cot}\delta_l(p) = \frac{1}{a_L} + \frac{1}{2} r_L p^2$$

$$a_0 = -0.8GeV^{-1}, \quad r_0 = +2.5GeV^{-1}$$

$$a_2 = -2.4GeV^{-5}, \quad r_2 \equiv 0$$

$$a_4 = -5.0GeV^{-9}, \quad r_4 \equiv 0$$

W. Hoogland, ect NPB 126 106 (1977)
D. H. Cohen, ect PRD 7 661 (1973)
A. Zieminski, ect NPB 69 502 (1974)
N. Durusa, ect PLB 45 517 (1973)
J. J. Dudek, ect PRD 86 034031(2012)

$$\det\begin{pmatrix} cot\delta_0(p_{A_1}) + M_{11}^{A_1}(p_{A_1}) & M_{12}^{A_1}(p_{A_1}) \\ M_{12}^{A_1}(p_{A_1}) & cot\delta_4(p_{A_1}) + M_{22}^{A_1}(p_{A_1}) \end{pmatrix} = 0$$

$$\det\begin{pmatrix} cot\delta_2(p_E) + M_{11}^{E}(p_E) & M_{12}^{E}(p_E) \\ M_{12}^{E}(p_E) & cot\delta_4(p_E) + M_{22}^{E}(p_E) \end{pmatrix} = 0$$

$$\det\begin{pmatrix} cot\delta_2(p_{T_2}) + M_{11}^{T_2}(p_{T_2}) & M_{12}^{T_2}(p_{T_2}) \\ M_{12}^{T_2}(p_{T_2}) & cot\delta_4(p_{T_2}) + M_{22}^{T_2}(p_{T_2}) \end{pmatrix} = 0$$

Pion-Pion with I=2: Toy Model

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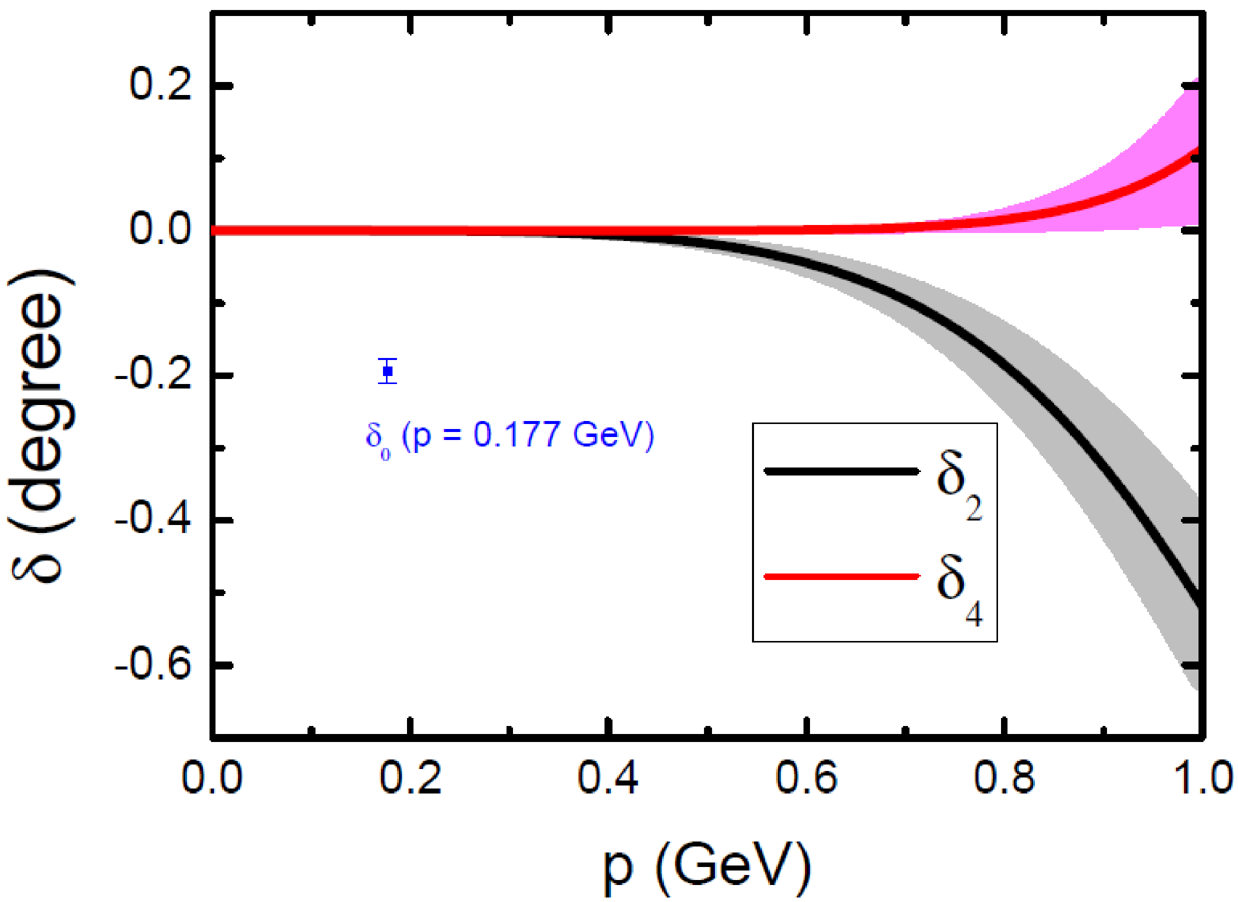
$$\det\begin{pmatrix} cot\delta_0(p_{A_1}) + M_{11}^{A_1}(p_{A_1}) & M_{12}^{A_1}(p_{A_1}) \\ M_{12}^{A_1}(p_{A_1}) & \frac{1}{a_4 p_{A_1}^9} + M_{22}^{A_1}(p_{A_1}) \end{pmatrix} = 0$$

$$\det\begin{pmatrix} \frac{1}{a_2 p_E^5} + M_{11}^{E}(p_E) & M_{12}^{E}(p_E) \\ M_{12}^{E}(p_E) & \frac{1}{a_4 p_E^9} + M_{22}^{E}(p_E) \end{pmatrix} = 0$$

$$\det\begin{pmatrix} \frac{1}{a_2 p_{T_2}^5} + M_{11}^{T_2}(p_{T_2}) & M_{12}^{T_2}(p_{T_2}) \\ M_{12}^{T_2}(p_{T_2}) & \frac{1}{a_4 p_{T_2}^9} + M_{22}^{T_2}(p_{T_2}) \end{pmatrix} = 0$$

$\delta_0(p_{A_1}), \quad a_2, \quad a_4$

Pion-Pion with I=2: Phase Shift



$$p^{2L+1}\mathrm{Cot}\delta_l(p) = \frac{1}{a_L}$$

$$a_2 = -0.571 \pm 0.155\,\mathrm{GeV}^{-5}$$

$$a_4 = 0.114 \pm 0.100\,\mathrm{GeV}^{-9}$$

$$\delta(p_{A_1} = 0.177\,\mathrm{GeV}\,)$$

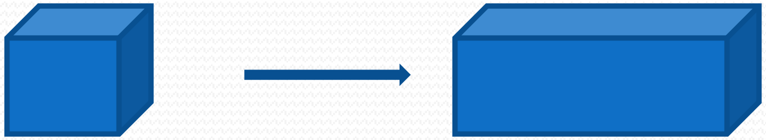
$$= -0.190 \pm 0.017$$

Summary

- In the coordinate space, we develop a new method to extract spectra of irreducible representation in the finite volume. It just cost two source inversions per measurement.
- We apply this method for ππ system with I=2 in the rest frame. It successfully generates signals of ground state of A1, E, and T2 representation. Then we can get phase shifts through Lüscher equation and toy model.

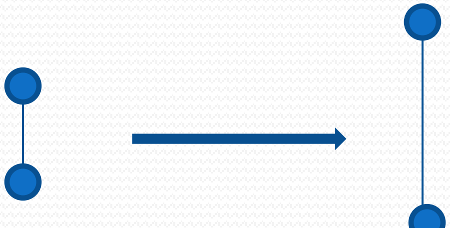
Outlook

- This method is also useful for boost frame. It just replace little group for boost frame instead of O_h group.



- For the excited states in the each irreducible representation, we need to develop more two particle operators and use variational method.

Operators basis distinguished by different $\vec{\delta}$.



Thanks for attention

New Method: Basis State (2)

$$\phi_R{}^{+}(\vec{x},\vec{\delta},t)|\Omega\rangle = |\phi_R{}^{+}(\vec{x},\vec{\delta},t)\rangle \sim |\phi_R{}^{+}(\vec{\delta})\rangle$$

$$\hat{R}_X|\phi_R{}^{+}(\vec{\delta})\rangle = \left|\phi_{R_X R}{}^{+}(\vec{\delta})\right\rangle = \sum_{R'}|\phi_{R'}{}^{+}(\vec{\delta})\rangle\,[\bar{\Gamma}_{reg}(R_X)]_{R'R}$$

$\bar{\Gamma}_{reg}$ is "regular representation".

It can reduced to several irreducible representations :

$$S^{-1}\bar{\Gamma}_{reg}\,S \;=\; A_1{}^{+} \oplus A_2{}^{+} \oplus 2E^{+} \oplus 3T_1{}^{+} \oplus 3T_2{}^{+} \oplus A_1{}^{-} \oplus A_2{}^{-} \oplus 2E^{-} \oplus 3T_1{}^{-} \oplus 3T_2{}^{-}$$

This block-diagonal matrices will lead to a new set of basis states:

$$\left|\Phi_{i,\Gamma,n}{}^{+}(\vec{\delta})\right\rangle$$

i: 1, 2	i: 1, 2, 3
Γ: E	Γ: T_2
n: 1, 2	n: 1, 2, 3

$$\hat{R}_X\left|\Phi_{i,\Gamma,n}{}^{+}(\vec{\delta})\right\rangle = \sum_{i',\Gamma',n'}\left|\Phi_{i',\Gamma',n'}{}^{+}(\vec{\delta})\right\rangle [S^{-1}\,\bar{\Gamma}_{reg}(R_X)\,S]_{i'\Gamma'n',\,i\,\Gamma\,n}$$

New Method: Basis State (3)

$$\phi_R{}^+(\vec{x},\vec{\delta},t)|\Omega\rangle = |\phi_R{}^+(\vec{x},\vec{\delta},t)\rangle \sim |\phi_R{}^+(\vec{\delta})\rangle$$

$$\hat{R}_X|\phi_R{}^+(\vec{\delta})\rangle = \left|\phi_{R_X R}{}^+(\vec{\delta})\right\rangle = \sum_{R'}|\phi_{R'}{}^+(\vec{\delta})\rangle\,[\bar{\Gamma}_{reg}(R_X)]_{R'R}$$

$\bar{\Gamma}_{reg}$ is "regular representation".

It can reduced to several irreducible representations :

$$S^{-1}\bar{\Gamma}_{reg}\,S \;=\; A_1{}^+ \oplus A_2{}^+ \oplus 2E^+ \oplus 3T_1{}^+ \oplus 3T_2{}^+ \oplus A_1{}^- \oplus A_2{}^- \oplus 2E^- \oplus 3T_1{}^- \oplus 3T_2{}^-$$

This reduced matrix will lead new basic state:

$$\hat{R}_X\left|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\right\rangle = \sum_{i',\Gamma',n'}\left|\Phi_{i',\Gamma',n'}{}^+(\vec{\delta})\right\rangle[S^{-1}\,\bar{\Gamma}_{reg}(R_X)\,S]_{i'\Gamma'n',\,i\,\Gamma\,n}$$

$$|\phi_R{}^+(\vec{\delta})\rangle \longrightarrow \left|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\right\rangle$$

$$\left|\Phi_{i,\Gamma,n}{}^+(\vec{\delta})\right\rangle = \sum_R|\phi_R{}^+(\vec{\delta})\rangle[S]_{R,\,i\,\Gamma\,n}$$

New Method: Basis State (4)

Summary, now we have two sets of basis states, related by the transformation matrix S.

$$|\Phi_{i,\Gamma,n}{}^{+}(\vec{\delta})\rangle = \sum_R |\phi_R{}^{+}(\vec{\delta})\rangle [S]_{R,\ i\,\Gamma\,n}$$

The representation matrix of these states are satisfied:

$$\hat{R}_X |\phi_R{}^{+}(\vec{\delta})\rangle \quad = \sum_{R'} |\phi_{R'}{}^{+}(\vec{\delta})\rangle\, [\bar{\Gamma}_{reg}(R_X)]_{R'R}$$

$$\hat{R}_X\, |\Phi_{i,\Gamma,n}{}^{+}(\vec{\delta})\rangle = \sum_{i',\Gamma',n'} |\Phi_{i',\Gamma',n'}{}^{+}(\vec{\delta})\rangle\, [S^{-1}\,\bar{\Gamma}_{reg}(R_X)\, S]_{i'\Gamma'n',\ i\,\Gamma\,n}$$

$$S^{-1}\bar{\Gamma}_{reg}\, S \;=\; A_1{}^{+} \oplus A_2{}^{+} \oplus 2E^{+} \oplus 3T_1{}^{+} \oplus 3T_2{}^{+} \oplus A_1{}^{-} \oplus A_2{}^{-} \oplus 2E^{-} \oplus 3T_1{}^{-} \oplus 3T_2{}^{-}$$

We can make matrix S as an unitary matrix, $S^{+} = S^{-1}$.
Then the normalization is satisfied:

$$\langle \phi_R^{\dagger}(\vec{\delta}) | \phi_{R'}^{\dagger}(\vec{\delta}) \rangle = \delta_{R,R'}$$

$$\langle \Phi_{i,\Gamma,n}^{\dagger}(\vec{\delta}) | \Phi_{i',\Gamma',n'}^{\dagger}(\vec{\delta}) \rangle = \delta_{i,i'}\delta_{\Gamma,\Gamma'}\delta_{n,n'}$$

New Method: Correlation Function(1)

The correlation function is:

$$G_{R,R'}(\vec{p}=0,\vec{y},\vec{x},\vec{\delta},t) = \sum_{\vec{y}-\vec{x}\in Z^3} e^{\vec{p}.(\vec{y}-\vec{x})}\langle\Omega|\phi_R(\vec{y},\vec{\delta},t)\phi^\dagger_{R'}(\vec{x},\vec{\delta},0)|\Omega\rangle$$

$$\sim \sum_{\vec{y}-\vec{x}\in Z^3}\langle\phi^\dagger_R(\vec{y},\vec{\delta},0)|e^{-\hat{H}t}|\phi^\dagger_{R'}(\vec{x},\vec{\delta},0)\rangle \sim \sum\langle\phi^\dagger(R^{-1}\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(R'^{-1}\vec{\delta})\rangle$$

48×48 → 10

$$\sum\langle\phi^\dagger(R\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle = \sum\langle\phi^\dagger(R'RR'^{-1}\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(\vec{\delta})\rangle$$

$$= \sum\langle\phi^\dagger(RR'\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(R'\vec{\delta})\rangle = \sum\langle\phi^\dagger(R'R\vec{\delta})|e^{-\hat{H}t}|\phi^\dagger(R'\vec{\delta})\rangle$$