Quantum Simulation of High-Energy Physics Models

J. Ignacio Cirac

MPQ
Max-Planck-Institut für Quantenoptik

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Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

1. INTRODUCTION

On the program it says this is a keynote speech—and I don't know what a keynote speech is. I do not intend in any way to suggest what should be in this meeting as a keynote of the subjects or anything like that. I have my own things to say and to talk about and there's no implication that

QUANTUM SIMULATION

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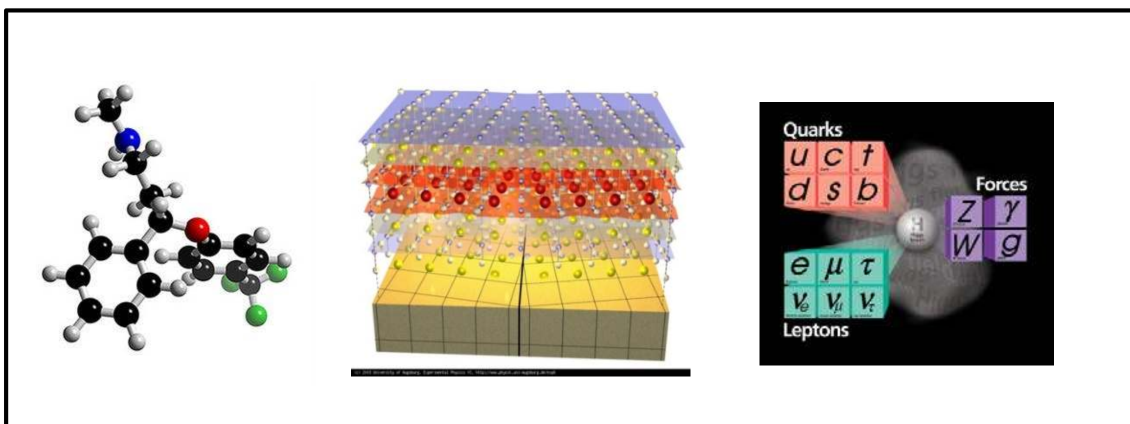
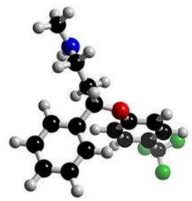
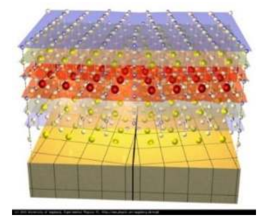
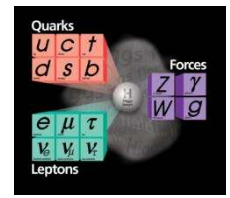


Computer time / memory scales exponentially with number of particles

QUANTUM MANY-BODY SYSTEMS
QUANTUM INFORMATION







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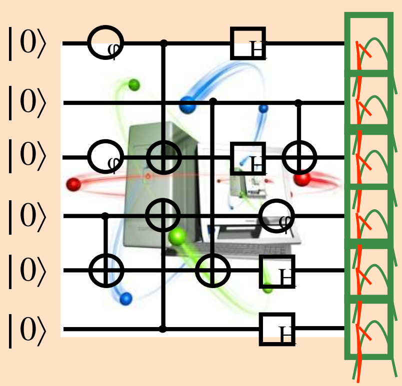

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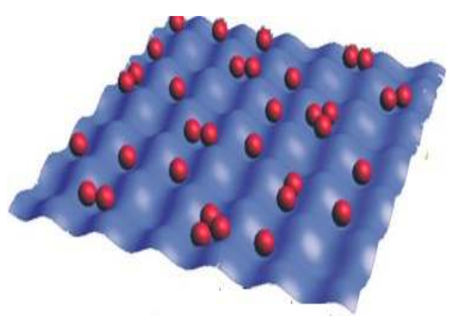
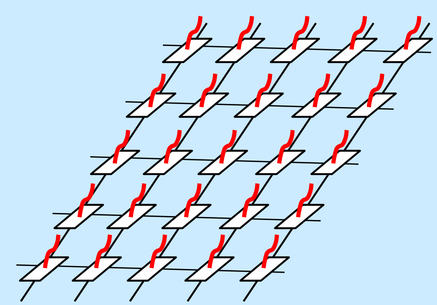
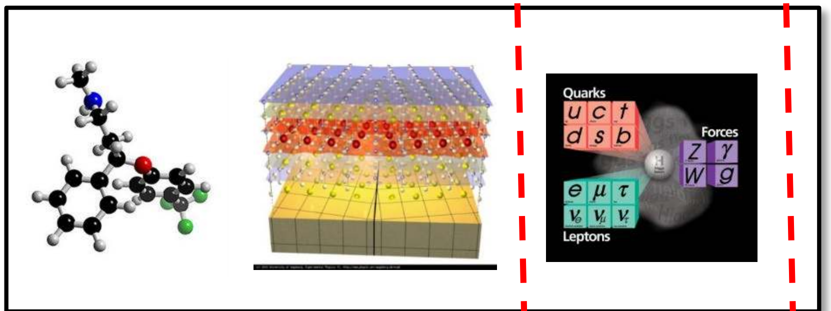


TENSOR NETWORKS



QUANTUM MANY-BODY SYSTEMS
QUANTUM INFORMATION



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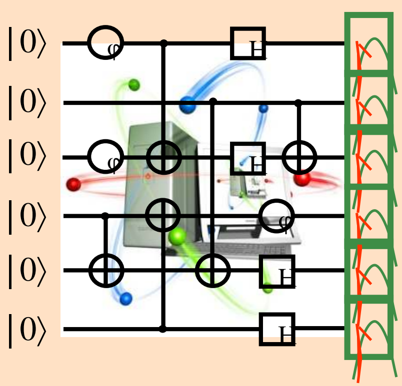
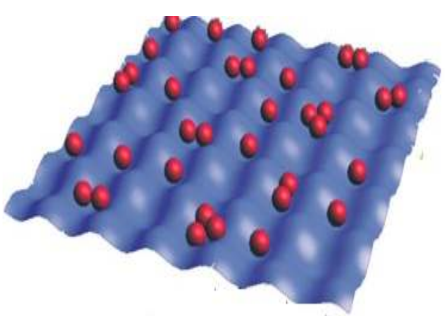
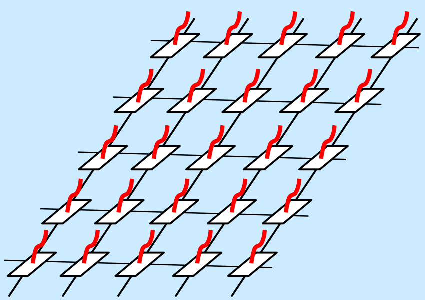


QUANTUM SIMULATION



TENSOR NETWORKS



1. QUANTUM SIMULATION OF HEP MODELS

COLLABORATORS

J. Pachos (Leeds)
B. Reznik (Tel-Aviv)

References:
PRL **105**, 19403 (2010)
PRL **107**, 275301 (2011)
PRL 1**09**, 125302 (2012)
PRL **110**, 125304 (2013)
PRL **118**, 70501 (2017)

See also: Dalmonte, Wiese, Zoller
Lewenstein et al
….

QUANTUM SIMULATORS

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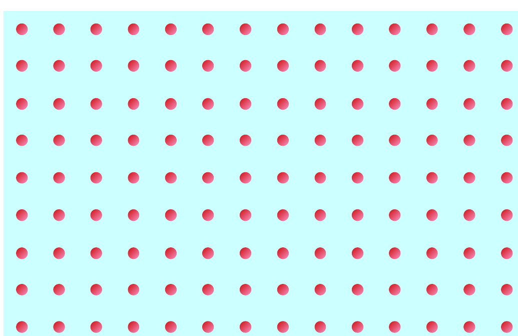
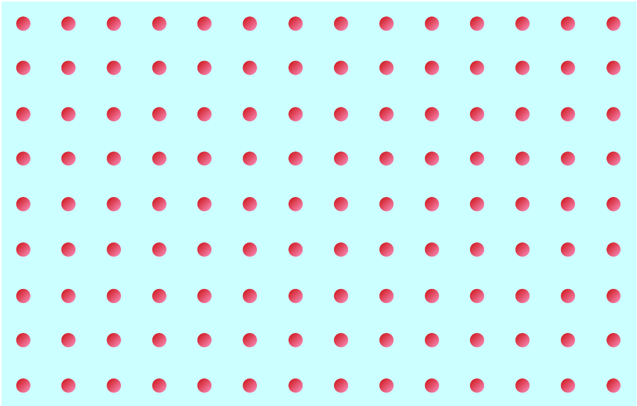
be understood very well in analyzing the situation. And I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy. Thank you.



$$c_1\,|000...0\rangle + c_2\,|000...1\rangle + ... + c_{2^N}\,|111...1\rangle$$

QUANTUM SIMULATORS

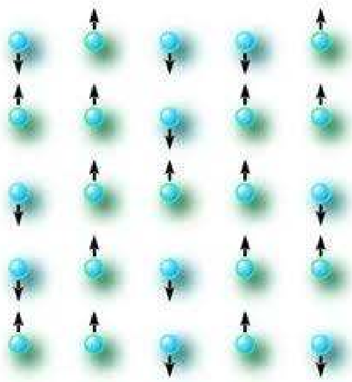
MODEL



Model Hamiltonian

$$H = \ldots$$

QUANTUM SIMULATOR

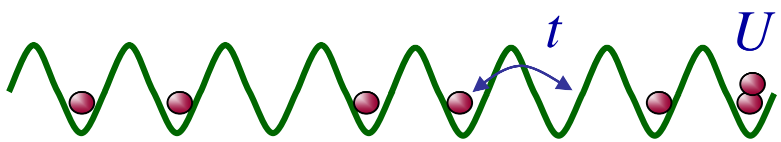


Model Hamiltonian

$$H = \ldots$$

COLD ATOMS IN OPTICAL LATTICES

- Laser standing waves: dipole-trapping

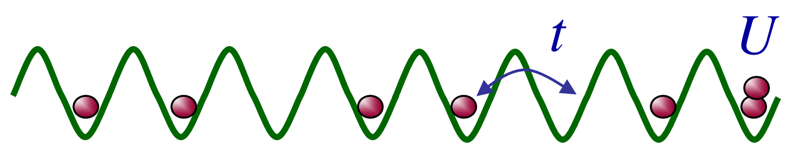


Lattice theory: Bose/Fermi-Hubbard model

$$H = -t\sum_n \left(a_n^\dagger a_{n+1} + h.c \right) + U \sum_n a_n^{\dagger 2} a_n^2$$

COLD ATOMS IN OPTICAL LATTICES

- Laser standing waves: dipole-trapping



Lattice theory: Bose/Fermi-Hubbard model

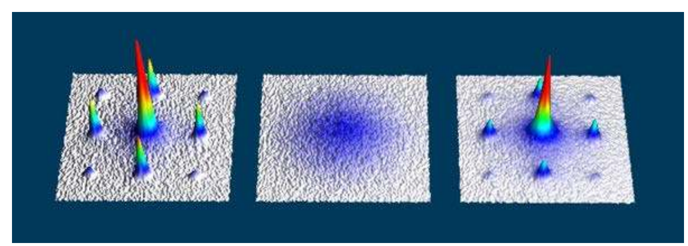
$$H = -t\sum_n \left(a_n^\dagger a_{n+1} + h.c\right) + U\sum_n a_n^{\dagger 2} a_n^2$$

articles

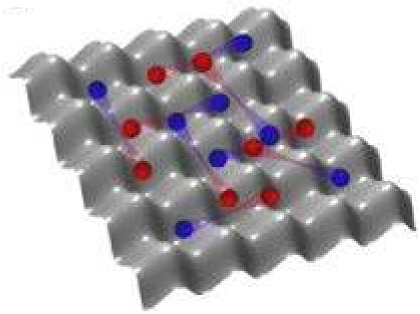
Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

* Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany
† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland



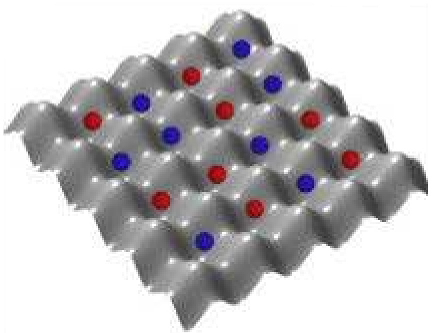
HUBBARD MODELS



electric conductivity

$$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(t_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a_{m,\sigma'} + h.c \right) + \sum_{\substack{n \\ \sigma,\sigma'}} U_{\sigma,\sigma'} a^{\dagger}_{n,\sigma} a^{\dagger}_{n,\sigma'} a_{n,\sigma'} a_{n,\sigma}$$

SPIN MODELS



magnetism

$$H = -\sum_{\substack{<n,m> \\ \sigma,\sigma'}} \left(J_x S^x_n S^x_m + J_y S^y_n S^y_m + J_z S^z_n S^z_m \right) + \sum_{\substack{n \\ \sigma,\sigma'}} B_n S^z_n$$

- Matter + Gauge Fields

- Relativistic theory

- Gauge invariant, Gauss law

- Hamiltonian formulation

- Matter + Gauge Fields

  Use bosonic and fermionic atoms

- Relativistic theory

  Use a lattice (recover Lorenz invariance in the continuum)

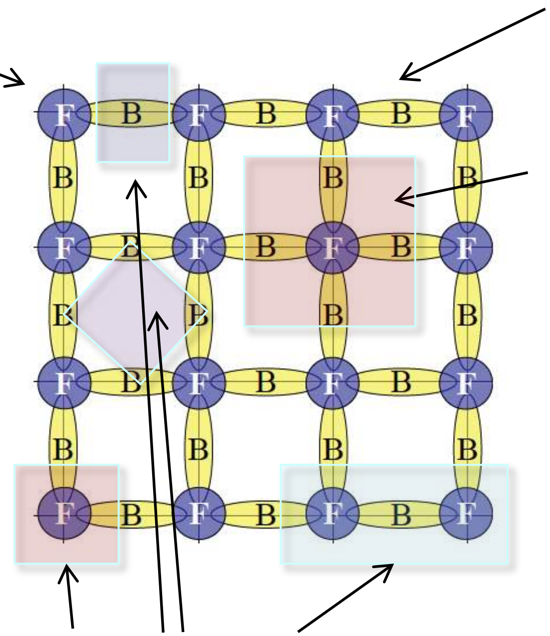
- Gauge invariant, Gauss law

  Impose energy constraints
  Encode gauge symmetry in an atomic symmetry

- Hamiltonian formulation

Matter (Fermions): can move

Gauge fields (Bosons): Static



Gauge symmetry

Hamiltonian: $H = H_M + H_{KS} + H_{\mathrm{int}}$

J. Kogut and L. Susskind, Phys. Rev. D **11**, 395 (1975).
J. B. Kogut, Rev. Mod. Phys. **51**, 659 (1979).
J. B. Kogut, Rev. Mod. Phys. **55**, 775 (1983).

QUANTUM SIMULATION
COLD ATOMS

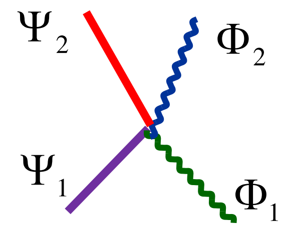
- Hamiltonian:

$$H = \int \Psi^{\dagger}_{\sigma}\left(-\nabla^{2} + V(r)\right)\Psi_{\sigma} + u\int \Phi^{\dagger}_{\mu}\Phi_{\sigma'}\Psi^{\dagger}_{\sigma}\Psi_{\sigma'} + v\int \Phi^{\dagger}_{\sigma}\Phi^{\dagger}_{\sigma'}\Phi_{\sigma'}\Phi_{\sigma} + \ldots$$

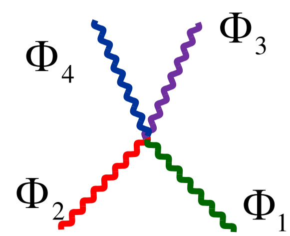
Lattice
- Fermions
- Bosons

(may depend on internal state)

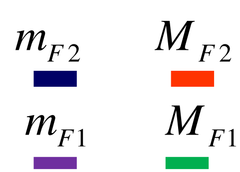
Fermion hopping coupling gauge field

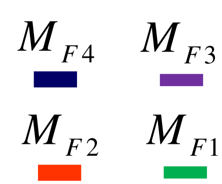


Electric field



- Conservation angular momentum:





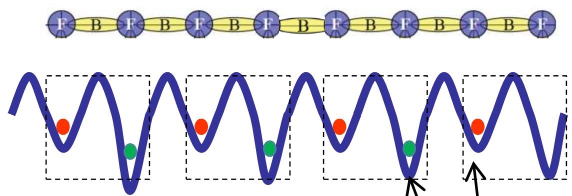
$$m_{F1} + M_{F1} = m_{F2} + M_{F2} \qquad M_{F1} + M_{F2} = M_{F3} + M_{F4}$$

Challenge: find how to tune V, u, v

QUANTUM SIMULATION

SCHWINGER MODEL 1+1

- Fermions:



$d_n \quad c_{n+1}$

internal states

2M

$$\{c_n, c_n^\dagger\} = \{d_n, d_n^\dagger\} = 1$$

$$H_M = M \sum_n (-1)^n \psi_n^\dagger \psi_n$$

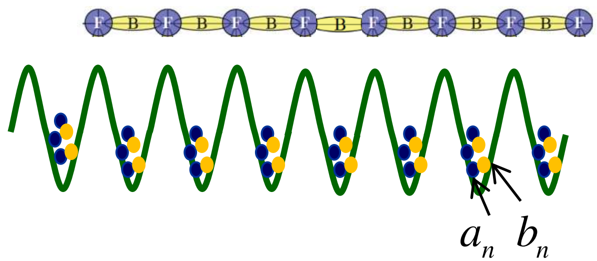
$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right) \Psi_\sigma + u \int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v \int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

Staggered Fermions:

L. Susskind, Phys. Rev. D **16**, 3031 (1977).
G. 't Hooft, Nucl. Phys. B **75**, 461 (1974)

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Bosons:



internal states

$$[a_n, a_n^\dagger] = [b_n, b_n^\dagger] = 1$$

Schwinger rep:

$$L_+ = a^\dagger b$$

$$L_z = \frac{1}{2}(N_a - N_b)$$

$$\ell = \frac{1}{2}(N_a + N_b)$$

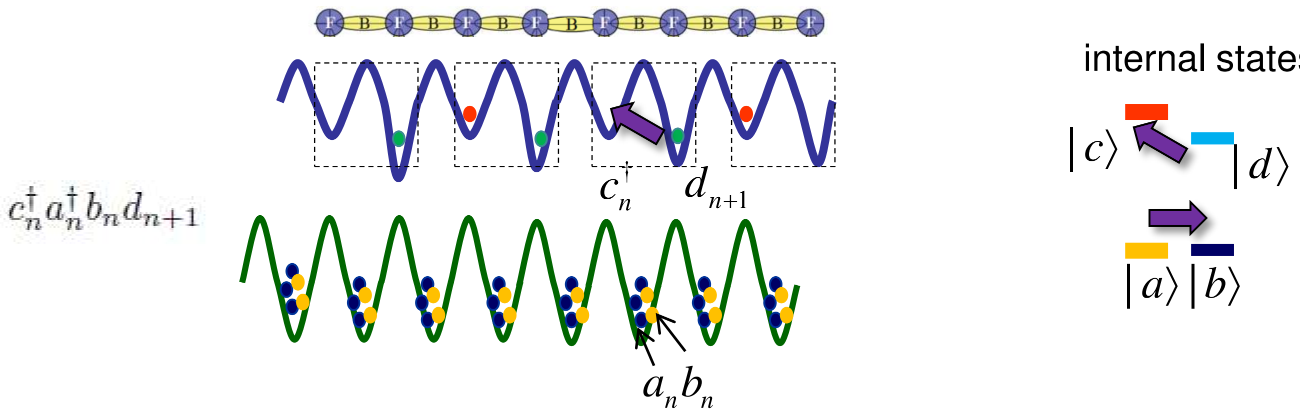
$$H_E = \frac{g^2}{2}\sum_n L_{z,n}^2$$

$$= \frac{g^2}{8}\sum_n \left(N_{a,n}^2 + N_{b,n}^2 - 2N_{a,n}N_{b,n}\right)$$

$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

conserves angular momentum

- Interactions:



$$H = \int \Psi_\sigma^\dagger \left(-\nabla^2 + V(r)\right)\Psi_\sigma + u\int \Phi_\mu^\dagger \Phi_{\sigma'} \Psi_\sigma^\dagger \Psi_{\sigma'} + v\int \Phi_\sigma^\dagger \Phi_{\sigma'}^\dagger \Phi_{\sigma'} \Phi_\sigma + \ldots$$

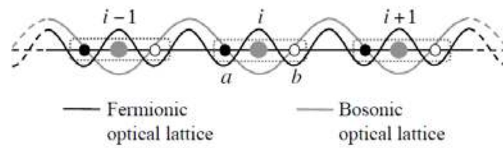
conserves angular momentum

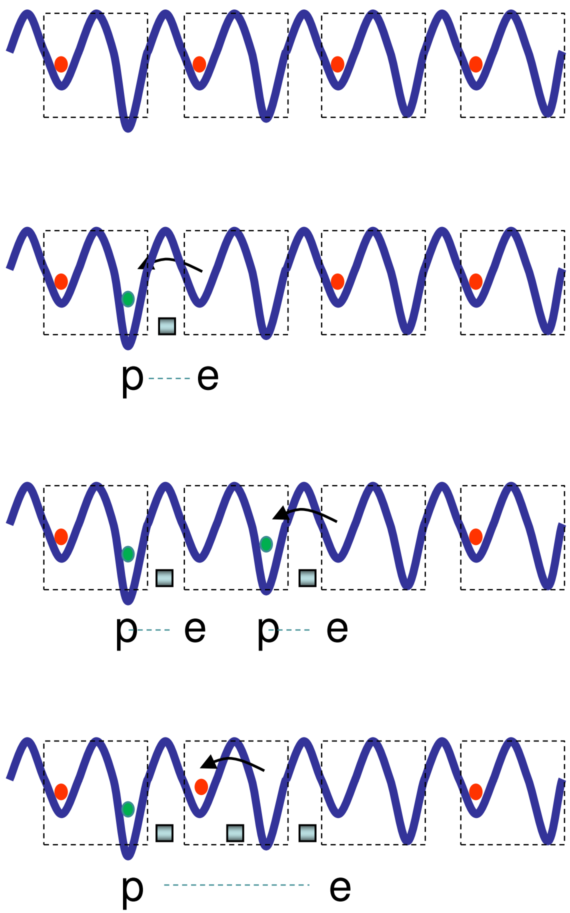
$$H_{int} = \frac{\epsilon}{\sqrt{\ell(\ell+1)}} \sum_n \left(\psi_n^\dagger L_{+,n} \psi_{n+1} + h.c.\right) \quad {}_{n+1}$$

$$\frac{\epsilon}{\sqrt{\ell(\ell+1)}} \sum_n \left(\psi_n^\dagger L_{+,n} \psi_{n+1} + h.c.\right)$$

QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Physical processes:

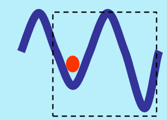




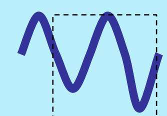
non-interacting vacuum

TABLE

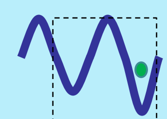
$|0\rangle_e |0\rangle_p$



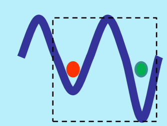
$|1\rangle_e |0\rangle_p$



$|1\rangle_e |1\rangle_p$

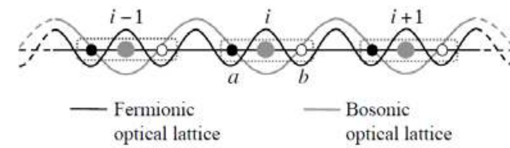


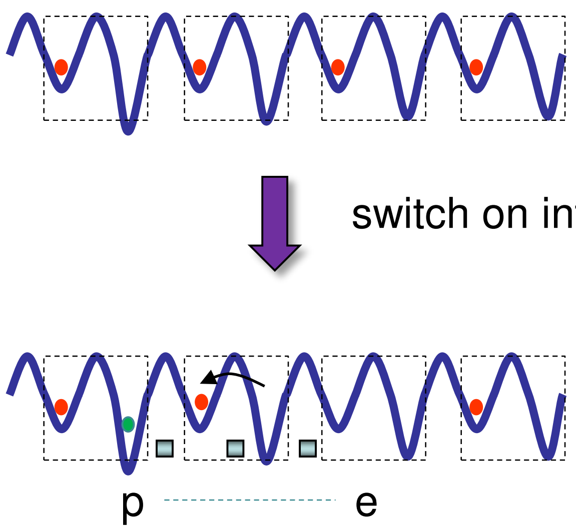
$|0\rangle_e |1\rangle_p$



QUANTUM SIMULATION
SCHWINGER MODEL 1+1

- Preparation:

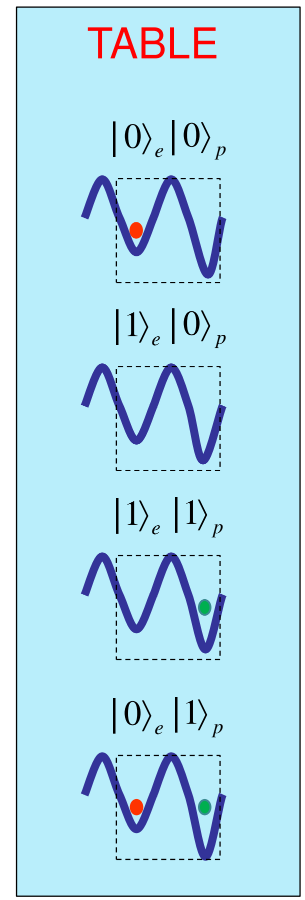




non-interacting vacuum

switch on interactions

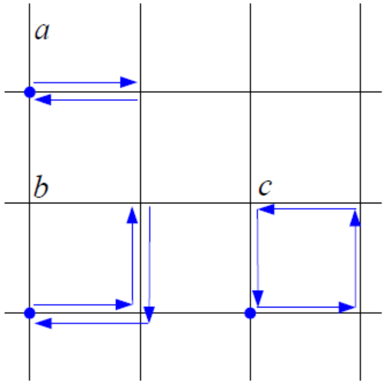
interacting vacuum



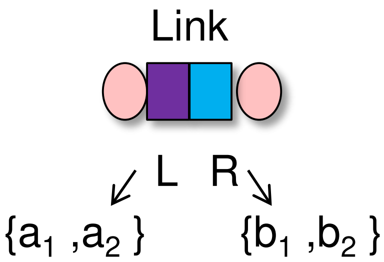
- Confinement
- Excitations: vector + scalar
- Time-dependent phenomena
- First experiments: few bosonic atoms

- Plaquette interactions:

$$H_B = -\frac{2\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\left(U_{\mathbf{n},1}U_{\mathbf{n}+\hat{1},2}U^{\dagger}_{\mathbf{n}+\hat{2},1}U^{\dagger}_{\mathbf{n},2} + h.c.\right) =$$
$$-\frac{4\epsilon^4}{\lambda^3}\sum_{\mathbf{n}}\cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{1},2} - \phi_{\mathbf{n}+\hat{2},1} - \phi_{\mathbf{n},2}\right)$$



- Non-abelian:



bosonic modes

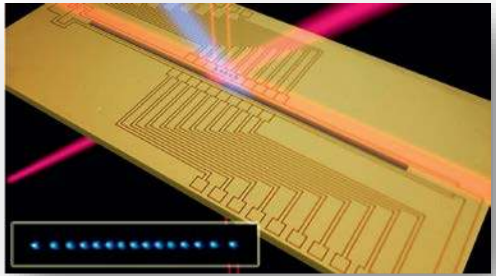
QUANTUM SIMULATIONS
EXPERIMENTAL CONSIDERATIONS

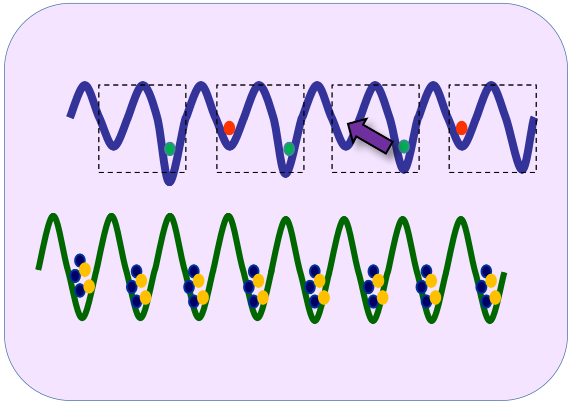
LETTER

doi:10.1038/nature18318

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez[1*], Christine A. Muschik[2,3*], Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]





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PAPER

Implementing quantum electrodynamics with ultracold atomic systems

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2 Department of Physics, Harvard University, Cambridge, Massachusetts 02138, United States of America
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2. TENSOR NETWORK DESCRIPTONS OF HEP MODELS





COLLABORATORS

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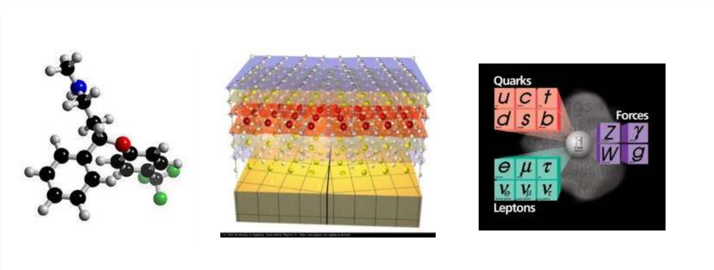
K. Cichy (Frankfurt)

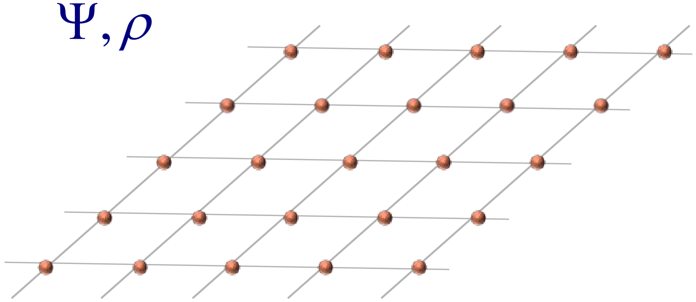
References: JHEP **11**, 158 (2013)
PRD **92**, 034519 (2015)
JHEP **07**, 130 (2015)
PRL **118**, 07161 (2017)
Ann Phys **363**, 385 (2015)
Ann. Phys **374**, 84 (2016)

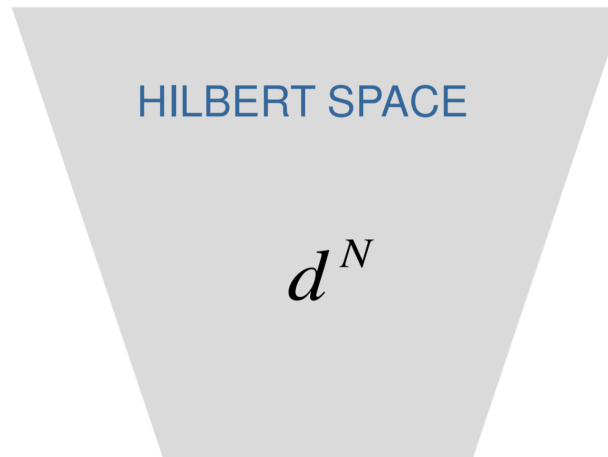
See also: Tagliacozzo, Lewenstein et al
Montangero et al
Verstraete et al
Meurice et al
….

QUANTUM MANY-BODY SYSTEMS



Ψ, ρ

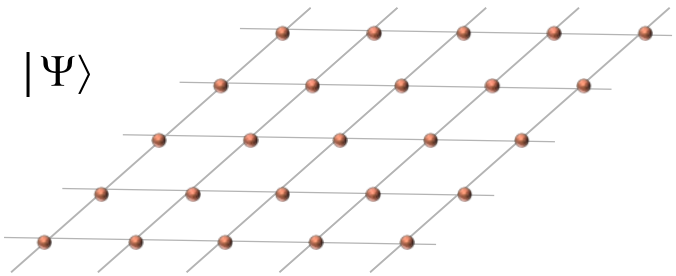




$$| \Psi \rangle = \sum c_{i_1, i_2, \ldots, i_N} | i_1, i_2, \ldots, i_N \rangle$$

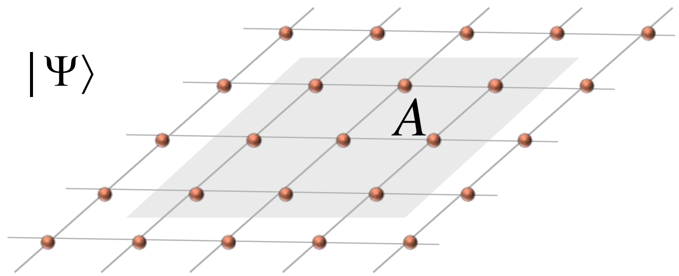
MANY-BODY QUANTUM SYSTEM

$|\Psi\rangle$



How much entangled?

MANY-BODY QUANTUM SYSTEM



How much entangled?

- Lattice in any physical dimension and geometry
- Local Hamiltonians: $H = \sum_n h_n$
- Thermal equilibrium: $T = 0$

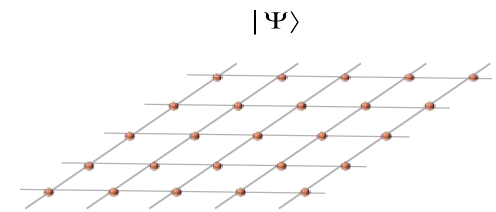
➡ Very little!

$E(\Psi) \sim |\partial A|$

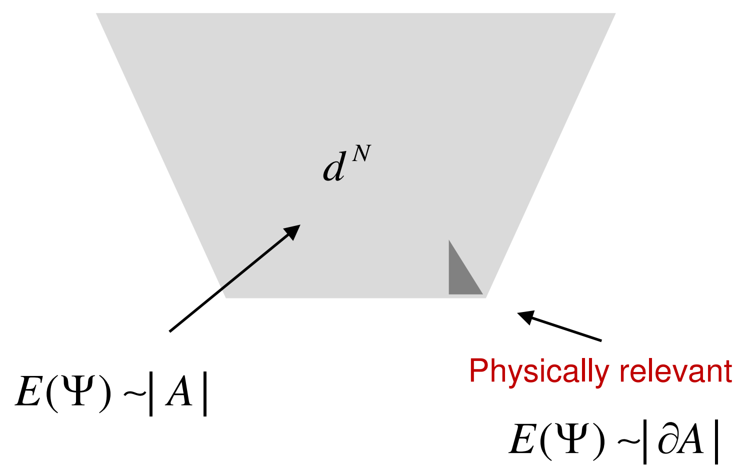
Area law

ENTANGLEMENT
MANY-BODY PHYSICS

- What do we learn?



EXPONENTIAL HILBERT SPACE



It should be possible to find an efficient description of physical states:
Number of parameters should scale polynomically with N

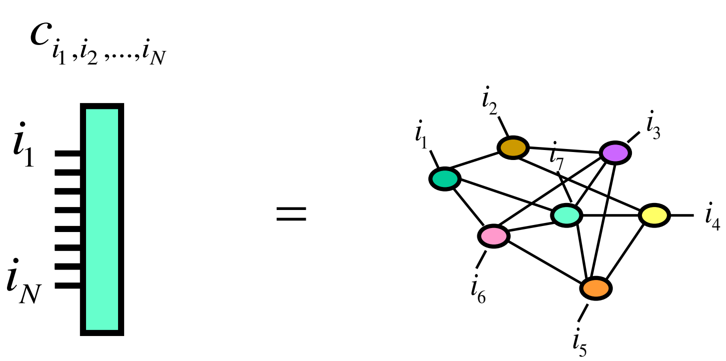
TENSORS NETWORKS

MANY-BODY QUANTUM STATES:

$$|\Psi\rangle = \sum c_{i_1,i_2,\ldots,i_N} \, |i_1,i_2,\ldots,i_N\rangle$$

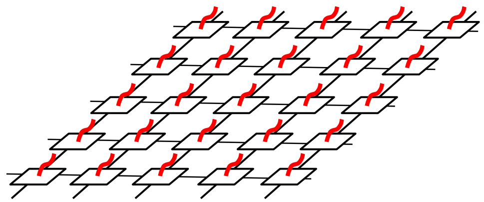
$$\sum_{\alpha,\beta,\ldots} A^{i_1}_{\alpha,\beta,\gamma,\delta} B^{i_2}_{\gamma,\varepsilon,\eta,\mu} C^{i_3}_{\eta,\sigma,\tau,\zeta} \cdots$$

Graphically

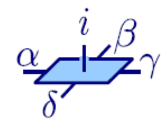


MPS, PEPS, MERA, …

PROJECTED ENTANGLED-PAIR STATES (PEPS):



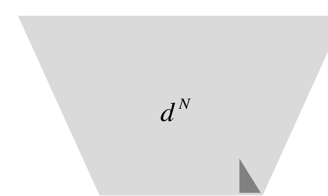
$A^{i}_{\alpha\beta\gamma\delta}$



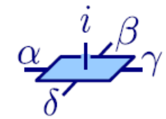
- Contracted according to the geometry of interactions
- Whole information in N tensors
- It applies to bosons/fermions and pure/mixed states
- Efficient description:

PEPS	Full State
NdD^{z}	d^{N}

The bond dimension, D, scales as poly(N)



TENSOR NETWORKS
ALGORITHMS

Variational computation with respect to $A^i_{\alpha\beta\gamma\delta}$ 

- Ground states:

  In 1+1 dimensions, it is related to DMRG

- Excitations:

- Dynamics: $|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$

  Time-dependent variational methods

- Finite temperature: $\rho = e^{-H/kT}$

+ No sign problem
  Gauge symmetry can be easily incorporated

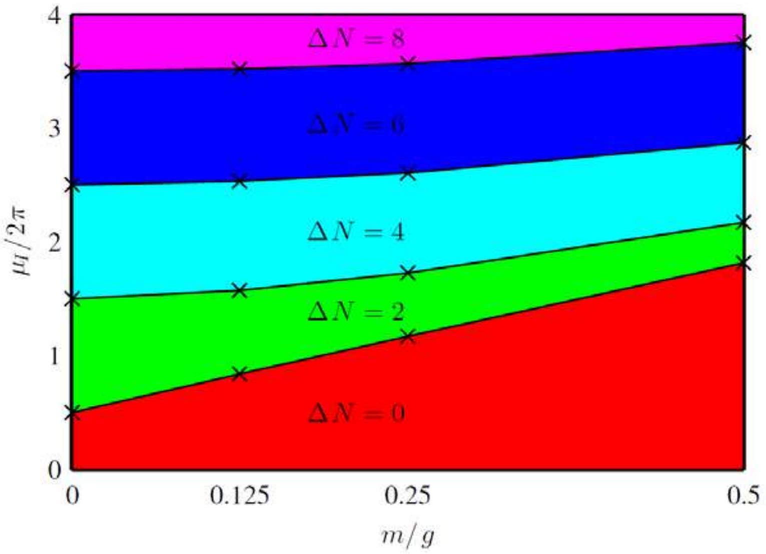
− Bad-scaling in higher dimensions D^3, D^{10}, D^{16}

RESULTS 1+1 D

SCHWINGER MODEL at T=0

$$H = -\frac{i}{2a}\sum_{n,f}(\phi^{\dagger}_{n,f}e^{i\theta_n}\phi_{n+1,f} - h.c.) + \sum_{n,f}\left(m_f(-1)^n + \nu_f\right)\phi^{\dagger}_{n,f}\phi_{n,f} + \frac{ag^2}{2}\sum_n L_n^2$$

PHASE DIAGRAM



Isospin: $\Delta N = N_0 - N_1$

Consant volume: $L = N\sqrt{2a}\,/\,g$

Chemical potential: $\mu \prec (\nu_0 - \nu_1)$

MASS SPECTRUM

	Scalar binding energy		
m/g	MPS with OBC	SCE result [36]	exact
0	1.1279(12)	1.11(3)	1.12838
0.125	1.2155(28)	1.22(2)	-
0.25	1.2239(22)	1.24(3)	-
0.5	1.1998(17)	1.20(3)	-

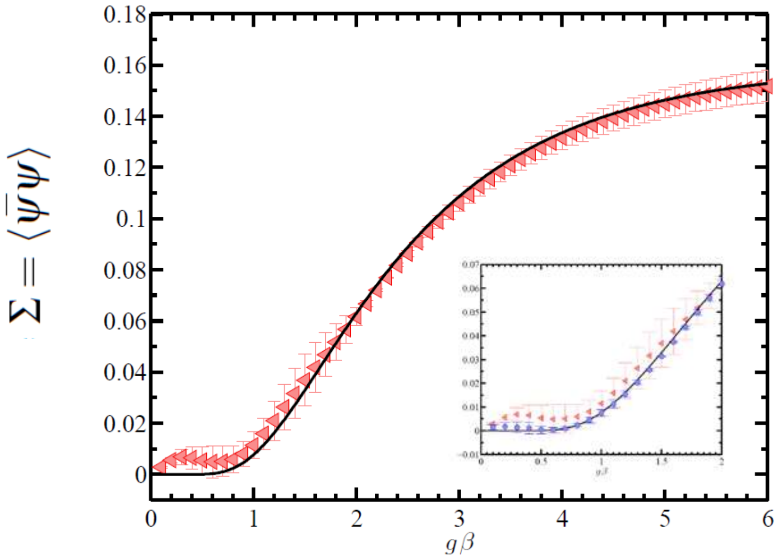
	Vector binding energy		
m/g	MPS with OBC	DMRG result [20]	exact
0	0.56421(9)	0.5642(2)	0.5641895
0.125	0.53953(5)	0.53950(7)	-
0.25	0.51922(5)	0.51918(5)	-
0.5	0.48749(3)	0.48747(2)	-

RESULTS 1+1 D

SCHWINGER MODEL at finite T

$$H = -\frac{i}{2a}\sum_{n,f}(\phi^{\dagger}_{n,f}e^{i\theta_n}\phi_{n+1,f} - h.c.) + \sum_{n,f}\left(m_f(-1)^n + \nu_f\right)\phi^{\dagger}_{n,f}\phi_{n,f} + \frac{ag^2}{2}\sum_n L_n^2$$

CHIRAL CONDENSATE



RESULTS 1+1 D
NON-ABELIAN SU(2) MODEL

$$H = \varepsilon \sum_n (\phi_n^\dagger U_n \phi_{n+1} + h.c.) + m \sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{g^2}{2} \sum_n J_n^2$$

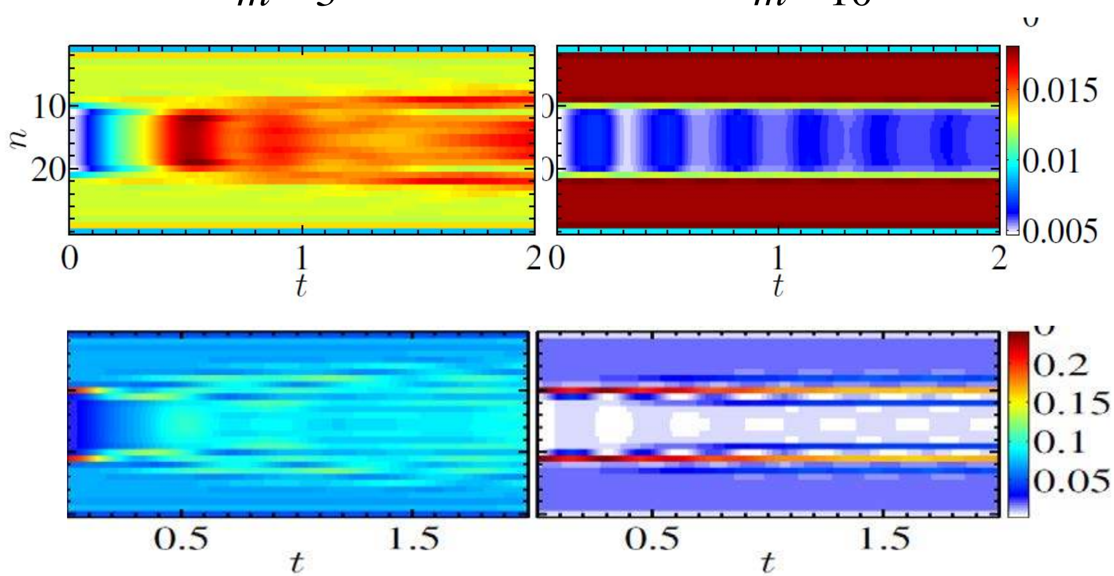
STRING BREAKING

TOTAL CHARGE: $Q_n^2 = \frac{1}{4}(n_1 - n_0)^2$

$m = 3$ $\quad$ $m = 10$

Static charges:

Dynamic charges:



RESULTS 2+1 D
TOY MODELS

- Parametrize all PEPS with D=3 and all symmetries
  - Staggered fermions
  - Rotation symmetry
  - Translationally invariant
  - Gauge invariant

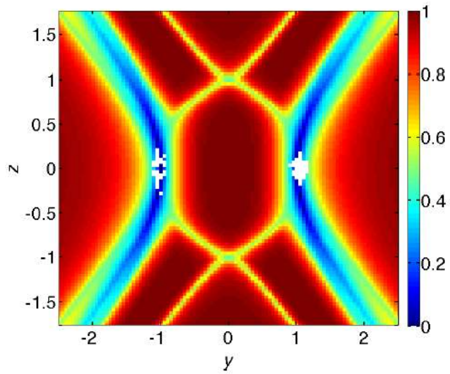
  Five-parameter family

- All are ground state of a local Hamiltonian

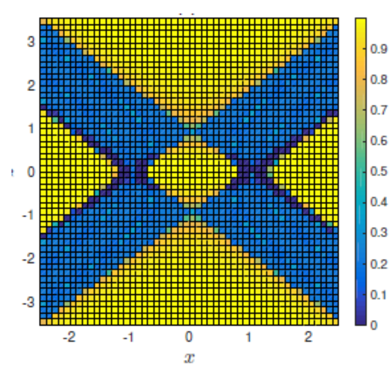
- Phase diagram in a cylinder $\infty \times 10$ sites
  - Gap of the transfer matrix
  - Loop operators: confinement

RESULTS 2+1 D
TOY MODELS

U(1) model



SU(2) model



SUMMARY & OUTLOOK

- Quantum information:

  Complementary methods to investigate QMBS

- Quantum simulation:
  - It is possible in principle. In practice, difficult.
  - First experiments are taking place.
  - Cross-fertilization between research areas.

- Tensor networks:
  - Still in their „infancy“
  - Challenge: higher dimensions
  - Combination with other methods (Monte-Carlo)
  - New perspective to study many-body quantum systems.

MPQ

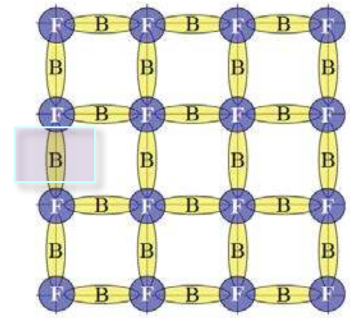
MPQ

QUANTUM SIMULATIONS
EXPERIMENTAL CONSIDERATIONS

- Cold bosons in optical lattices
  - Mott insulator – superfluid transition
  - Exchange interaction (2nd order perturbation theory)
  - Dynamics
  - Anderson-Higgs mechanism in 2D
- Cold fermions in optical lattices
  - Mott insulator in 2D
- Cold fermions and bosons in optical lattices
  - Mean-field dynamics
- Techniques
  - Tuning of interactions: Magnetic/optical Feschbach resonances
  - Lattice geometry
  - Time of flight measurements
  - Single-site addressing: initializaton
  - Single-site measurement
- Challanges: temperature, decoherence, control …

HEP LATTICE MODELS
HAMILTONIAN FORMULATION

- Example: compact-QED



  - Gauge group: U(1)
  - Bosonic operators in links

$$U_{\mathbf{n},k} = e^{i\phi_{\mathbf{n},k}} \qquad [E_{\mathbf{n},k}, \phi_{\mathbf{m},l}] = -i\delta_{\mathbf{nm}}\delta_{kl}$$

$$E_{\mathbf{n},k}$$

angular momentum ($E_{\mathbf{n},k}$) — angle ($\phi_{\mathbf{m},l}$)

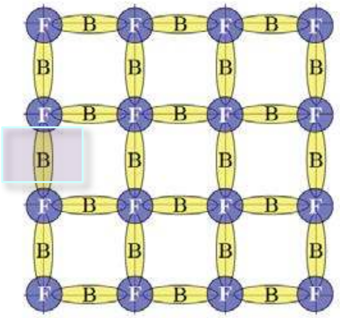
$$E_{n,k}\,|m\rangle = m\,|m\rangle$$

$$U_{\mathbf{n},k}\,|m\rangle = e^{i\phi_{\mathbf{n},k}}\,|m\rangle = |m+1\rangle$$

  - E takes integer values (→ compact)

HEP LATTICE MODELS
HAMILTONIAN FORMULATION

- Example: compact-QED
  - Gauge group: U(1)
  - Bosonic operators in links



$$U_{\mathbf{n},k} = e^{i\phi_{\mathbf{n},k}}$$

$$E_{\mathbf{n},k}$$

  - Gauss law: $G_{\mathbf{n}} = \sum_k \left(E_{\mathbf{n},k} - E_{\mathbf{n}-\hat{\mathbf{k}},k}\right) - Q_{\mathbf{n}}$
  - Hamiltonians:

$$H_M = \sum_{\mathbf{n}} M_{\mathbf{n}} \psi^\dagger_{\mathbf{n}} \psi_{\mathbf{n}}$$

$$H_{int} = \epsilon \sum_{\mathbf{n},k} \left(\psi^\dagger_{\mathbf{n}} e^{i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}+\hat{\mathbf{k}}} + \psi^\dagger_{\mathbf{n}+\hat{\mathbf{k}}} e^{-i\phi_{\mathbf{n},k}} \psi_{\mathbf{n}}\right)$$

$$H_{KS} = \frac{g^2}{2} \sum_{\mathbf{n},k} E^2_{\mathbf{n},k} - \frac{1}{g^2} \sum_{\mathbf{n}} \cos\left(\phi_{\mathbf{n},1} + \phi_{\mathbf{n}+\hat{\mathbf{1}},2} - \phi_{\mathbf{n}+\hat{\mathbf{2}},1} - \phi_{\mathbf{n},2}\right)$$