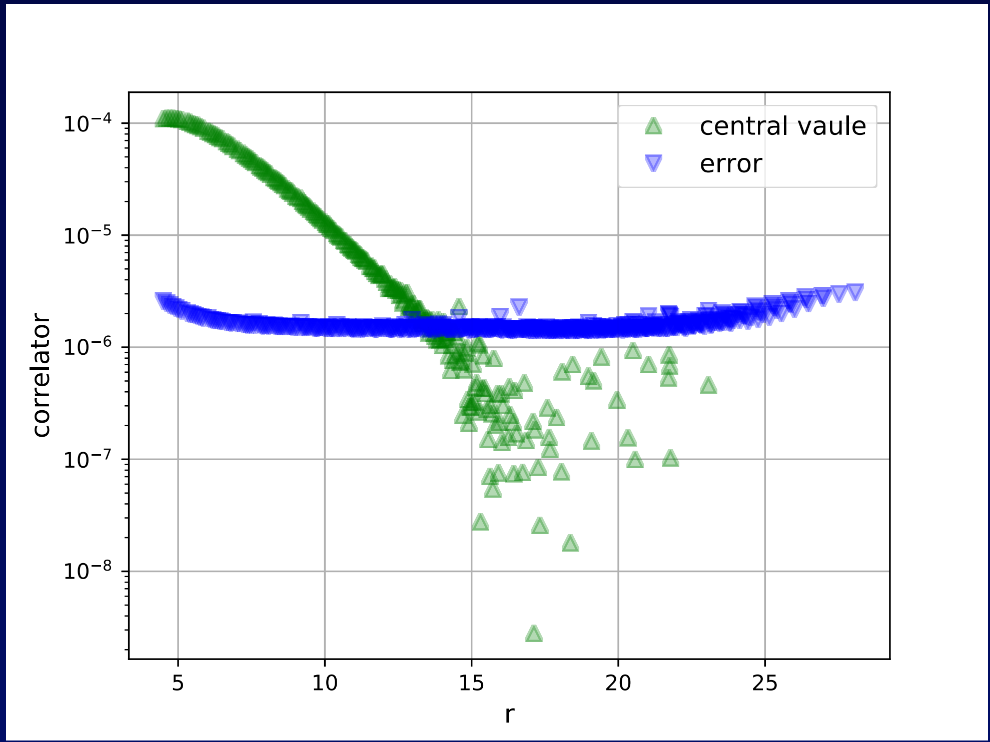
Variance Reduction via Cluster Decomposition and nEDM from θ Term

- Noise problem for large volume
- Cluster decomposition principle
- Variance reduction for disconnected three- point correlators and glueball mass
- CP-violating phase α^1 in the nucleon
- nEDM from the θ term from overlap fermion

KFL, Jian Liang, and Yi-Bo Yang,
arXiv:1705.06358

Granada, June 21, 2017

Noise problem with large volume



- Single falls off exponentially, but noise remains constant ⟹ sign problem, such as in glueball mass.
- nEDM with the θ term is noisy for large volume, because the topological charge fluctuation as $(V)^{1/2}$.

Cluster Decomposition Principle

H. Araki, K. Hepp, and D. Ruelle, Helv. Phys. Acta 35, 164 (1962);
S. Weinberg, Quantum Theory of Fields,Vol 1, pp. 169

$$\left|\langle 0 | O_1(x) O_2(y) | 0 \rangle\right|_s \leq A r^{-3/2} e^{-Mr}, \quad \text{r = x-y (space like)},$$

O_1 and O_2 are color-singlet operators,

Asympotic behavior of a boson propagator $K_1(r)/r$.

This point-to-point relation should hold for color-singlet operators separated by large Euclidean distance.

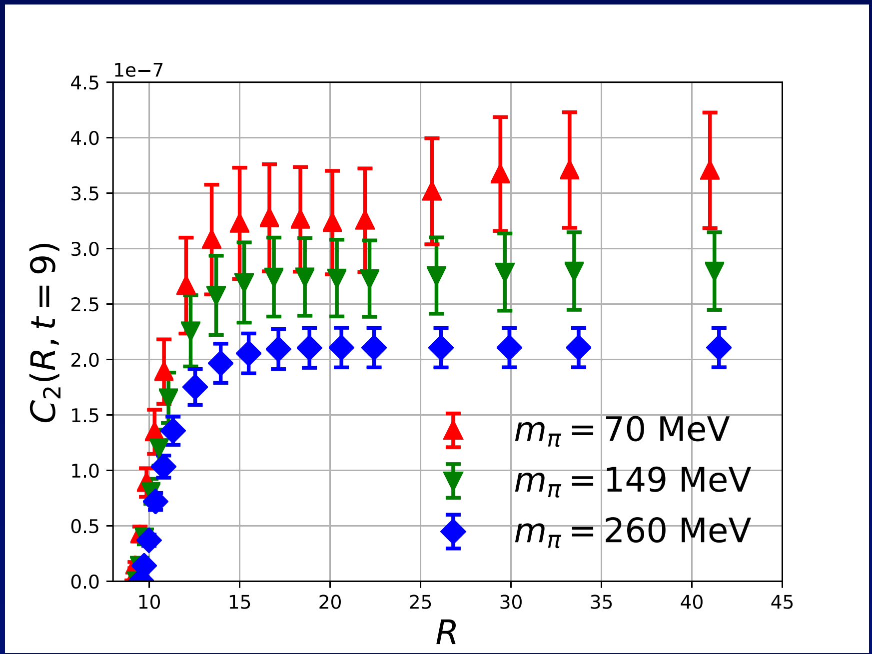
Check CDP with lattice data

Consider two-point function for the nucleon

$$C_N(R,t) = \frac{1}{V}\left\langle \sum_{\vec{x}} \sum_{r<R} O_N(\vec{x}+\vec{r}\,',t)\bar{O}_N(\vec{x},0) \right\rangle, \quad r = \sqrt{|\vec{r}\,'|^2 + t^2}$$

$$I(A,M) = \int_0^R d^3r r^{-3/2} e^{-Mr} = 4\pi\left(\frac{\sqrt{\pi}\,\mathrm{erf}(\sqrt{MR})}{2M^{3/2}} - \frac{\sqrt{R}e^{-MR}}{M} \right)$$

This saturates the integral (99.5%) at R_s=8/M



$C_N(R,t)$ with overlap valence on RBC 48^3 x 96 lattice at physical pion mass.

R_s=8/M corresponds to R ~ 17.

Disconnected Insertions

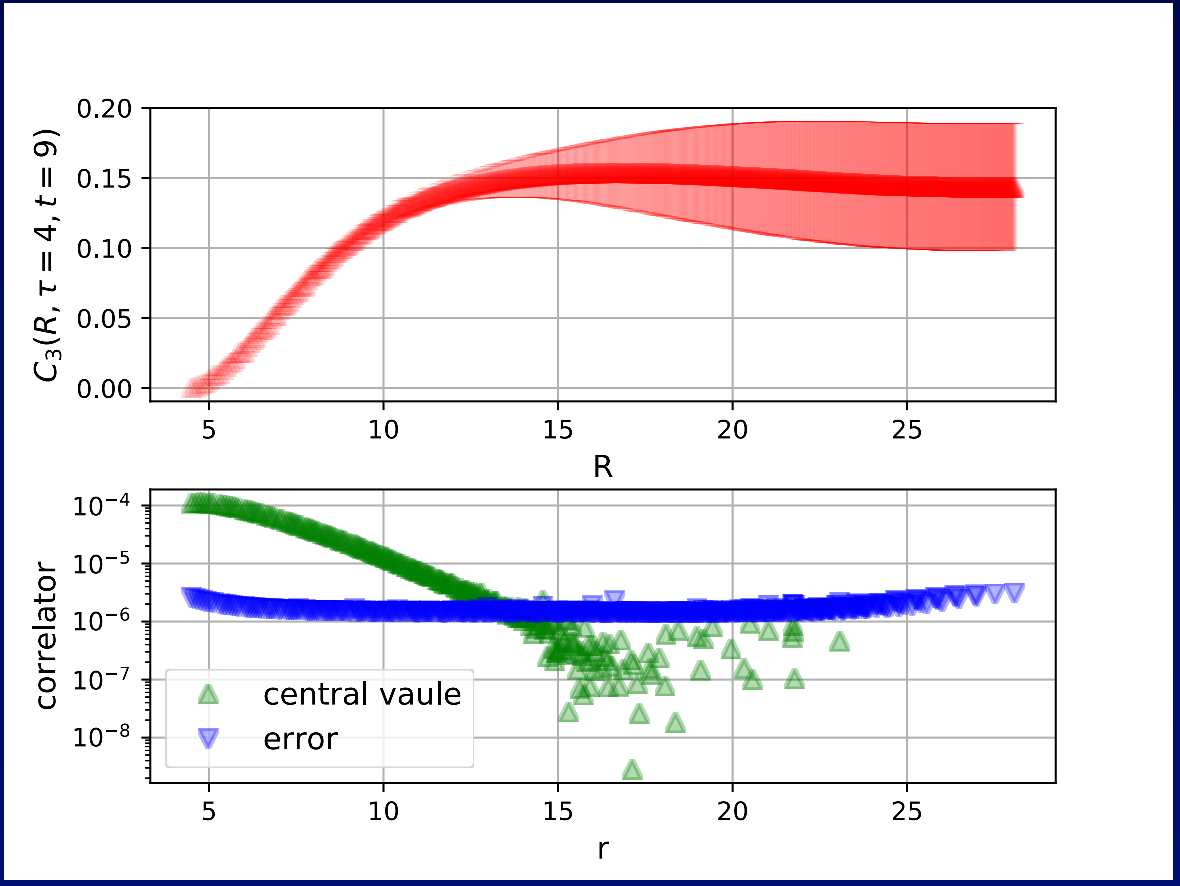
- Variance of disconnected insertion
- Vacuum insertion

$$\mathrm{Var}(R,t) = \frac{1}{V^2}\sum_{\vec{x}}\left(\left\langle \sum_{r_1<R} O_1(\vec{r}_1{}',t)\sum_{r_2<R} O_1^\dagger(\vec{x}+\vec{r}_2{}',t)\right\rangle \times \left\langle O_2(0,0)O_2^\dagger(\vec{x},0)\right\rangle\right) + \ldots$$

- $\mathrm{Var}(R_{max},t) = 1$, but $\mathrm{Var}(R_s,t) = V_s/V$.
- Gains singal to noise ratio: $S/N(R_s,t)/S/N(L,t) = (V/V_s)^{1/2}$
- Fast Fourier transform to calculate the truncated sum in relative coordinates ~ V log V operations.

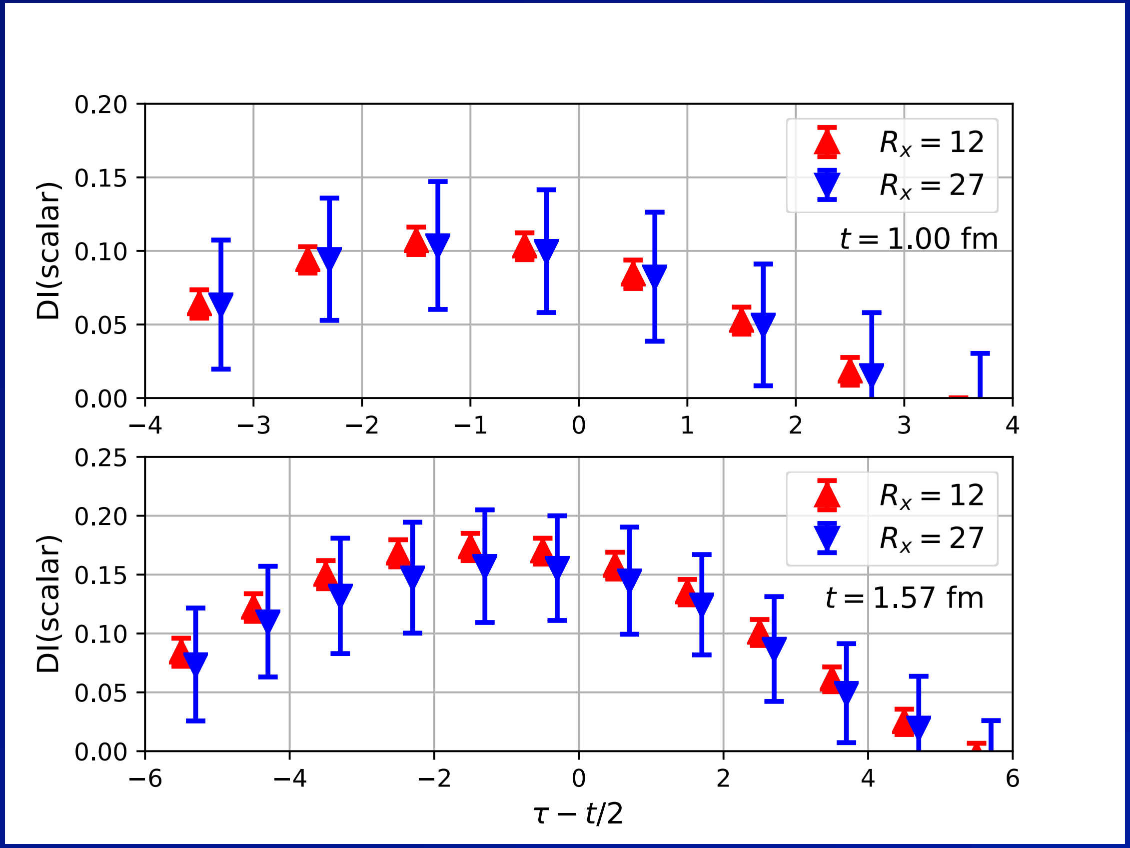
Strangeness in the Nucleon

$$C_3(R,\tau,t) = \left\langle \sum_{\vec{x}} \sum_{r<R} O_N(\vec{x},t) S(\vec{x}+\vec{r}\,',\tau) \bar{O}_N(\text{grid},0) \right\rangle, \quad \text{r} = \sqrt{(\vec{r}_x - \vec{x})^2 + (\tau - t)^2}$$



32^3 x 64 (32ID) RBC (4.6 fm, m_π=170 MeV)

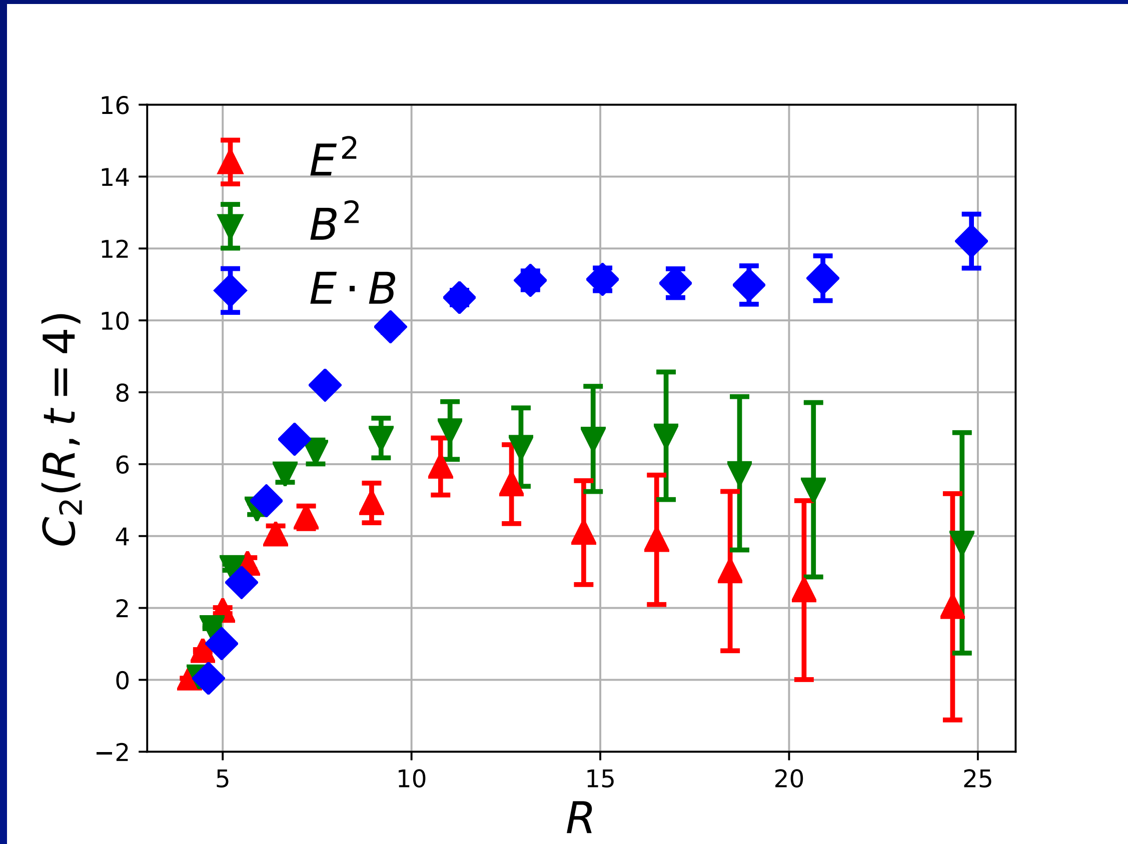
Strangeness in the Nucleon
-- two source-sink separation



Errors can be reduced by ≥ 4

32^3 x 64 (32ID) RBC (4.6 fm, m_π=170 MeV)

Glueball Masses on 48^3 x 96 lattice (L= 5.5 fm)



Scalar: cutoff at R=9 reduces error by ~ 4 which is ≈ $(25/9)^{3/2}$

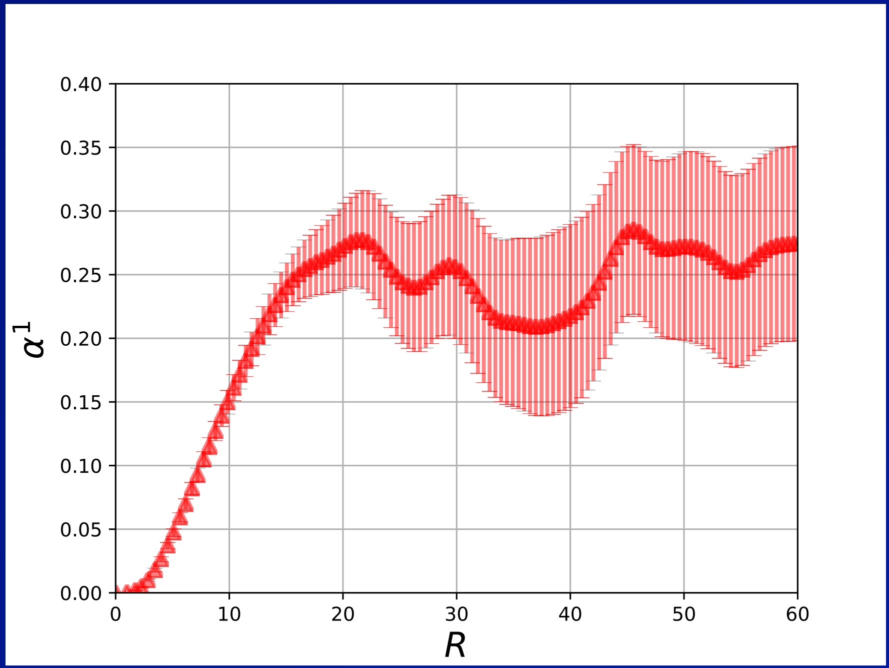
Pseudoscalar: cutoff at R=11 reduces error by ~ 3 which is ≈ $(25/11)^{3/2}$

CP violation angle in the nucleon

$$C_{3Q}(R,t) = \left\langle \sum_{\vec{x}} O_N(\vec{x},t)\bar{O}_N(\text{grid},0)Q \right\rangle$$

$$= \left\langle \sum_{\vec{x}} O_N(\vec{x},t)\bar{O}_N(\text{grid},0) \sum_{y(r\le R)} q(y) \right\rangle, \quad \text{r} = \sqrt{(\vec{y}-\vec{x})^2 + (t_y - t)^2}$$

$$\alpha^1 = \frac{Tr(C_{3Q}(t)\gamma_5)}{Tr(C_2(t)\Gamma_e)}$$

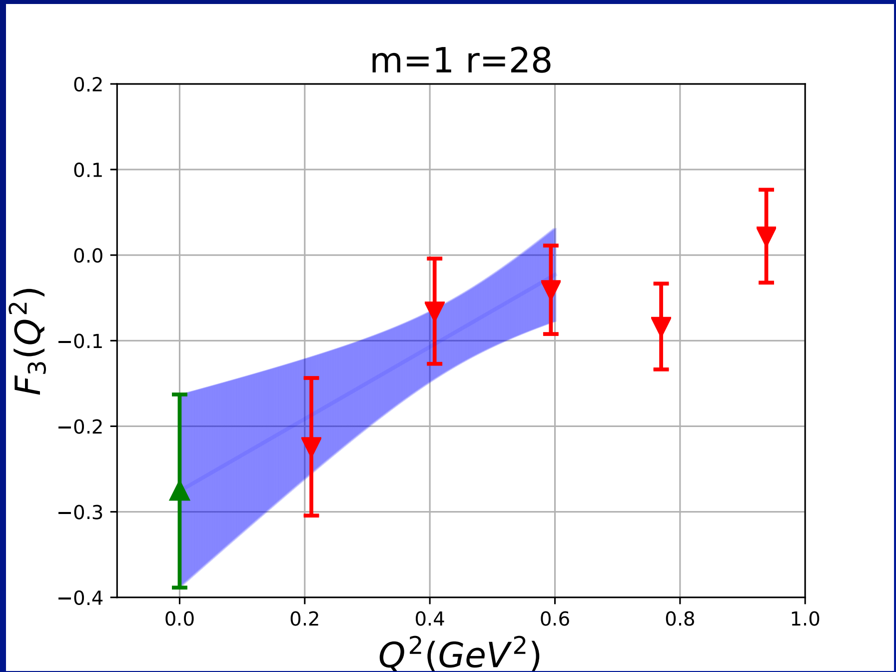


48^3 x 96 RBC lattice (a = 0.114 fm, m_π = 139 MeV),
m_π(valence) = 280 MeV, r (cut off) = 16, S/N increases by ~ 3.6

Form factors of $F_3'(Q^2)$

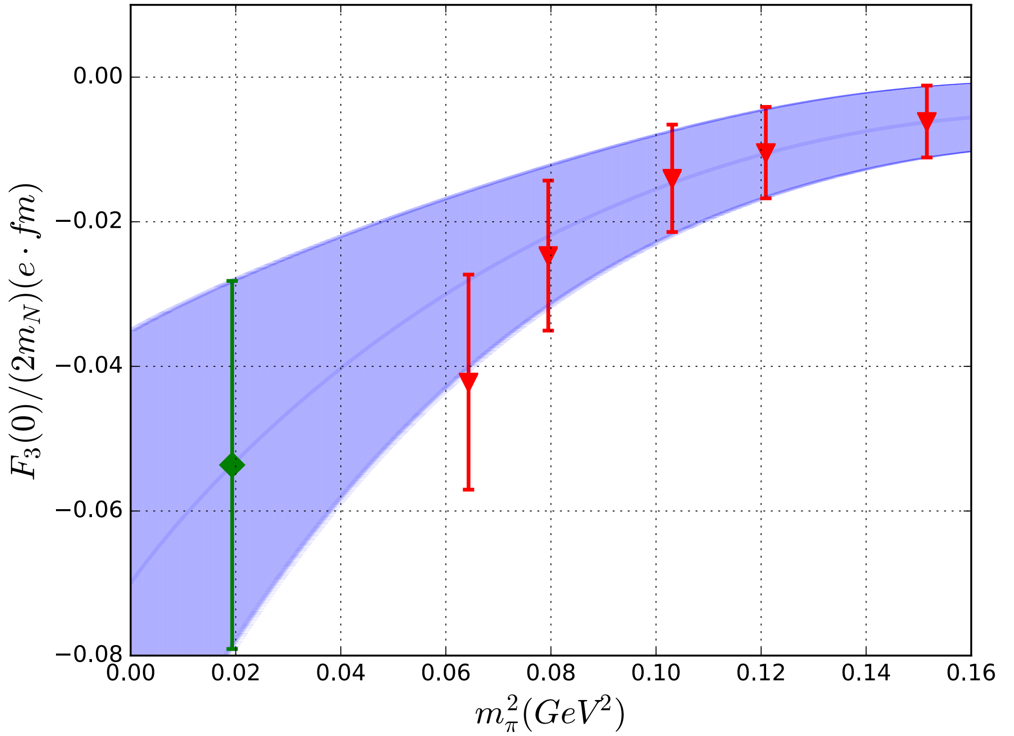
$$F_3^{'}(Q^2) = F_3(Q^2) + 2\alpha^1 F_2(Q^2)$$

M.Abramczyk et al., 1701.07792



24^3 x 64 RBC lattice (a = 0.114 fm, m_π = 330 MeV), m_π(valence) = 280 MeV, r (cut off) = 28

Neutron EDM d_n



24^3 x 64 RBC lattice (a = 0.114 fm, m_π = 330 MeV), m_π(valence) = 280 MeV, r (cut off) = 28

d_n = - 0.054(25) e fm [d_n = - 0.014(7) e fm at m_π = 330 MeV]

Summary

- Utilizing the idea of Cluster Decomposition Principle, the large volume dependence of the variance can be overcome for the disconnected insertion.

- Examples are given for glueball mass, strangeness content in the nucleon, and nEDM. One can gain a factor of 3 to 4 for lattices with size of 4.5 – 5.5 fm.

- For connected insertions, the gain is limited.