Tensor form factor for the $D \to \pi(K)$ transitions with Twisted Mass fermions

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Motivation

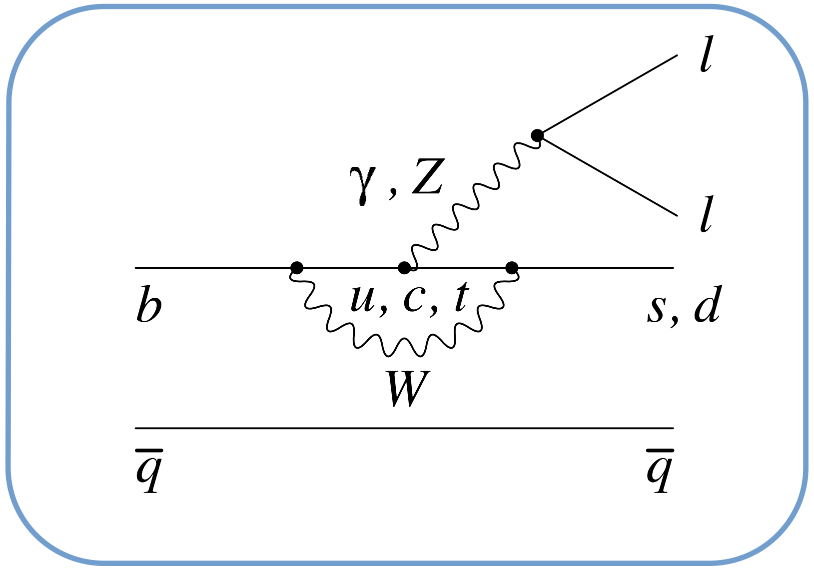
In [*arXiv:1706.03017*] we found evidence of the breaking of Lorentz Symmetry due to hypercubic effects in the vector and scalar form factors of both the $D \to \pi$ and $D \to K$ semileptonic transitions (see Giorgio Salerno's talk - *Lattice 2017*)

These effects are expected to be relevant when we move to B-physics and have to be under control

Motivations for the analysis of the tensor form factor:

- Test the method for the subtraction of hypercubic effects used for the vector and scalar form factor on a quantity with a different Lorentz structure
- complete the set of operators relevant for the transition between two PS-mesons

Motivation



Phenomenological application of the tensor form factor:

f_T may enter as BSM contribution both in semileptonic decays and in rare decays like $c \to u\, \ell^+\ell^-$ ($b \to s\, \ell^+\ell^-$) heavily suppressed in the Standard Model

this analysis can be used as the triggering point for the *ratio method* to move to B-physics

Simulation Details

Details of the ensembles used in this N_f =2+1+1 analysis

ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_s$	$a\mu_c$	M_π(MeV)	M_K(MeV)	M_D(MeV)	L(fm)	$M_\pi L$
A30.32	1.90	$32^3 \times 64$	0.0030	{0.0180,	{0.21256,	275	569	2015	2.84	3.96
A40.32			0.0040	0.0220,	0.25000,	315	578	2018		4.53
A50.32			0.0050	0.0260}	0.29404}	351	578	2018		5.04
A40.24		$24^3 \times 48$	0.0040			324	584	2024	2.13	3.49
A60.24			0.0060			386	599	2022		4.17
A80.24			0.0080			444	619	2037		4.79
A100.24			0.0100			495	639	2042		5.34
B25.32	1.95	$32^3 \times 64$	0.0025	{0.0155,	{0.18705,	258	545	1950	2.61	3.42
B35.32			0.0035	0.0190,	0.22000,	302	556	1944		3.99
B55.32			0.0055	0.0225}	0.25875}	375	578	1959		4.96
B75.32			0.0075			436	600	1965		5.77
B85.24		$24^3 \times 48$	0.0085			467	611	1974	1.96	4.63
D15.48	2.10	$48^3 \times 96$	0.0015	{0.0123,	{0.14454,	220	526	1928	2.97	3.31
D20.48			0.0020	0.0150,	0.17000,	254	533	1933		3.83
D30.48			0.0030	0.0177}	0.19995}	308	547	1939		4.65

Three values of the lattice spacing: 0.06 *fm* ÷ 0.09 *fm*

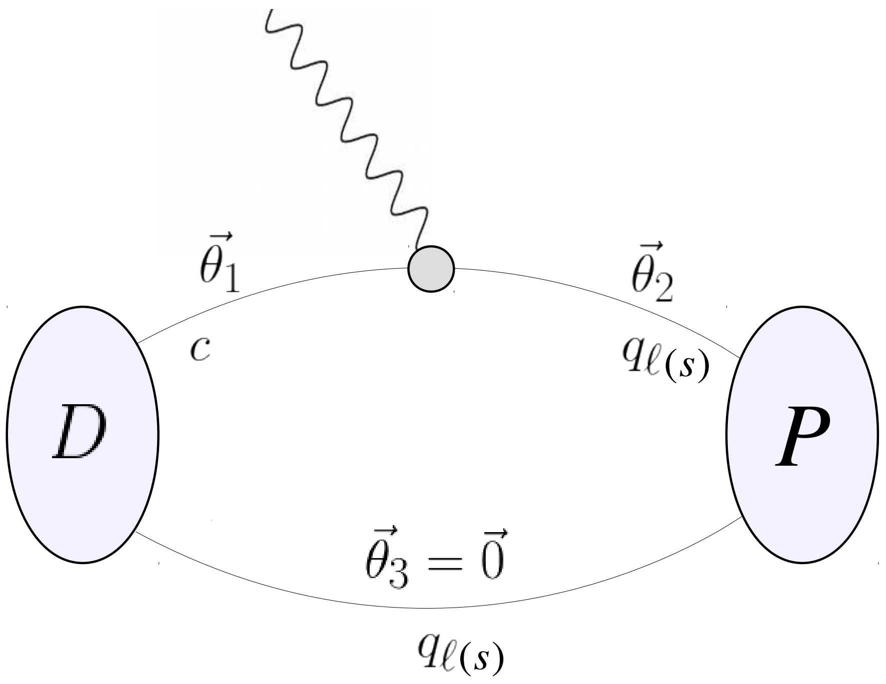
Different volumes: 2 *fm* ÷ 3 *fm*

Pion masses in range 220 ÷ 440 *MeV*

Lattice Spacings	
a(β = 1.90)	0.0885(36)fm
a(β = 1.95)	0.0815(30)fm
a(β = 2.10)	0.0619(18)fm

Simulation Details

To inject momenta we used non-periodic boundary conditions



β	V/a^4	θ
1.90	$32^3 \times 64$	$0.0, \quad \pm 0.200, \quad \pm 0.467, \quad \pm 0.867$
	$24^3 \times 48$	$0.0, \quad \pm 0.150, \quad \pm 0.350, \quad \pm 0.650$
1.95	$32^3 \times 64$	$0.0, \quad \pm 0.183, \quad \pm 0.427, \quad \pm 0.794$
	$24^3 \times 48$	$0.0, \quad \pm 0.138, \quad \pm 0.321, \quad \pm 0.596$
2.10	$48^3 \times 96$	$0.0, \quad \pm 0.212, \quad \pm 0.493, \quad \pm 0.916$

$$\vec{p}_D = \frac{2\pi}{L}\vec{\theta}_1 \qquad \vec{p}_P = \frac{2\pi}{L}\vec{\theta}_2$$

$$\vec{\theta} = \theta(1,1,1)$$

$$p \quad = (0, \pm 151, \pm 353, \pm 656)\,\mathrm{MeV}$$

Momentum range up to *0 ÷ 650 MeV*

Both the D and the π(K) mesons can be either moving or at rest

Matrix Elements Extraction

The matrix element T_i can be extracted for each ensemble and for different values of q^2 fitting the time dependence of ratios of 3-points and 2-points correlation functions

$$R(t,\vec{p}_D,\vec{p}_P) = 4E_D E_P Z_T \frac{C_{T_i}^{DP}(t,t',\vec{p}_D,\vec{p}_P)\,C_{T_i}^{PD}(t,t',\vec{p}_D,\vec{p}_P)}{\tilde{C}_2^D(t',\vec{p}_D)\,\tilde{C}_2^P(t',\vec{p}_P)} \xrightarrow[t\gg a \quad (t'-t)\gg a]{} \left|\langle P|\hat{T}_i|D\rangle\right|^2$$

$$C_2(t,\vec{p}) = \xrightarrow[t\gg a]{} \frac{Z}{2E_0}\left(e^{-E_0 t} + e^{-E_0(T-t)}\right)$$

$$\tilde{C}_2(t,\vec{p}) = \frac{1}{2}\left[C_2(t,\vec{p}) + \sqrt{C_2^D(t,\vec{p})^2 - C_2^D(T/2,\vec{p})^2}\right] \xrightarrow[t\gg a]{} \frac{Z}{2E_0} e^{-E_0 t}$$

$$T_i \equiv T_{0i}$$
$$\hat{T}_i = Z_T T_i = Z_T \bar{c}\sigma_{0i} q$$

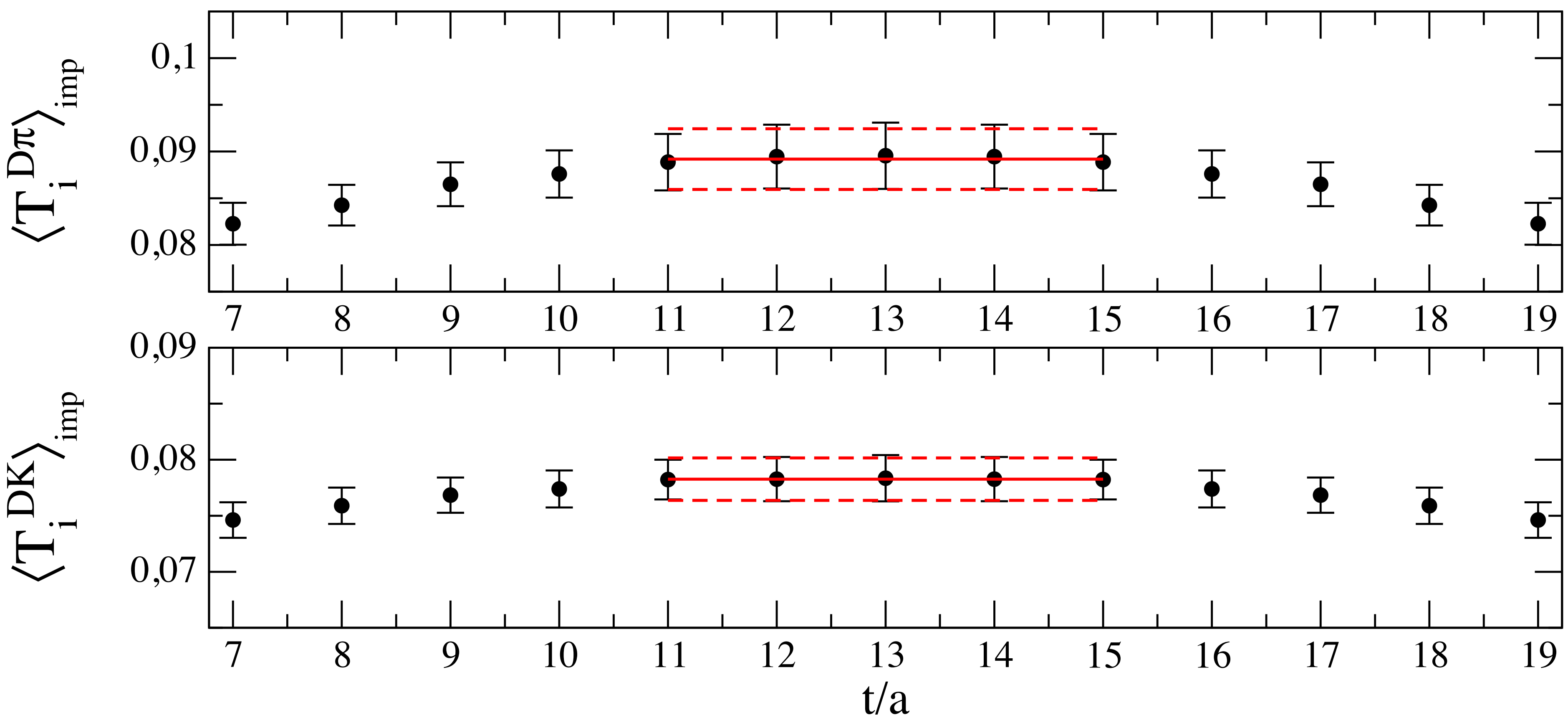
β	$Z_T^{\overline{MS}}(2\,\mathrm{GeV})(M_1)$	$Z_T^{\overline{MS}}(2\,\mathrm{GeV})(M_2)$
1.90	0.711(5)	0.700(3)
1.95	0.724(4)	0.711(2)
2.10	0.774(4)	0.767(2)

The values of the Z_T renormalization constant have been determined non-perturbatively in ***[Nucl. Phys. B887 (2014)]***

Matrix Elements Plateaux

Example of the time dependence of the ratio R and the extraction of the matrix elements

Ensemble D20.48



p_D=-151 MeV
p_π=151 MeV
M_π=254 MeV
M_D=1640 MeV

p_D=-151 MeV
p_π=151 MeV
M_K=516 MeV
M_D=1640 MeV

fit intervals:

β	V	[2-pts]$_{(\ell\ell,\ell s)}$	[2-pts]$_{(\ell c)}$	t'	[3-pts]
1.90	$32^3 \times 64$	[12, 31]	[8, 16]	18	[7, 11]
	$24^3 \times 48$	[12, 23]	[8, 17]	18	[7, 11]
1.95	$32^3 \times 64$	[13, 31]	[9, 18]	20	[8, 12]
	$24^3 \times 48$	[13, 23]	[9, 18]	20	[8, 12]
2.10	$48^3 \times 96$	[18, 40]	[12, 24]	26	[11, 15]

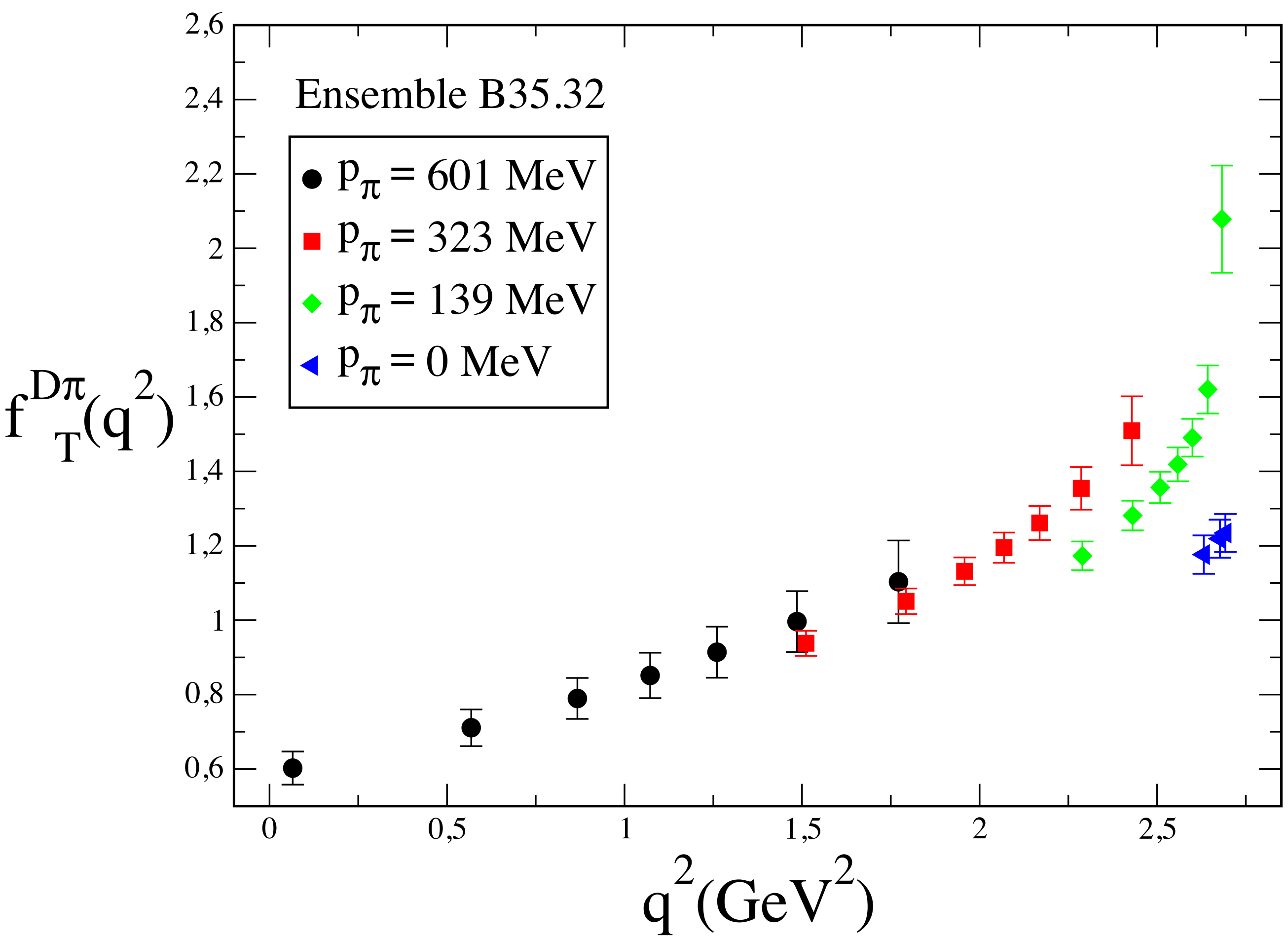
the tensor form factor can be determined from T_i

$$\left\langle T_i^{DP} \right\rangle = \frac{2}{M_D + M_P}\left[E_P p_D^i - p_P^i E_D \right] f_T\left(q^2\right)$$

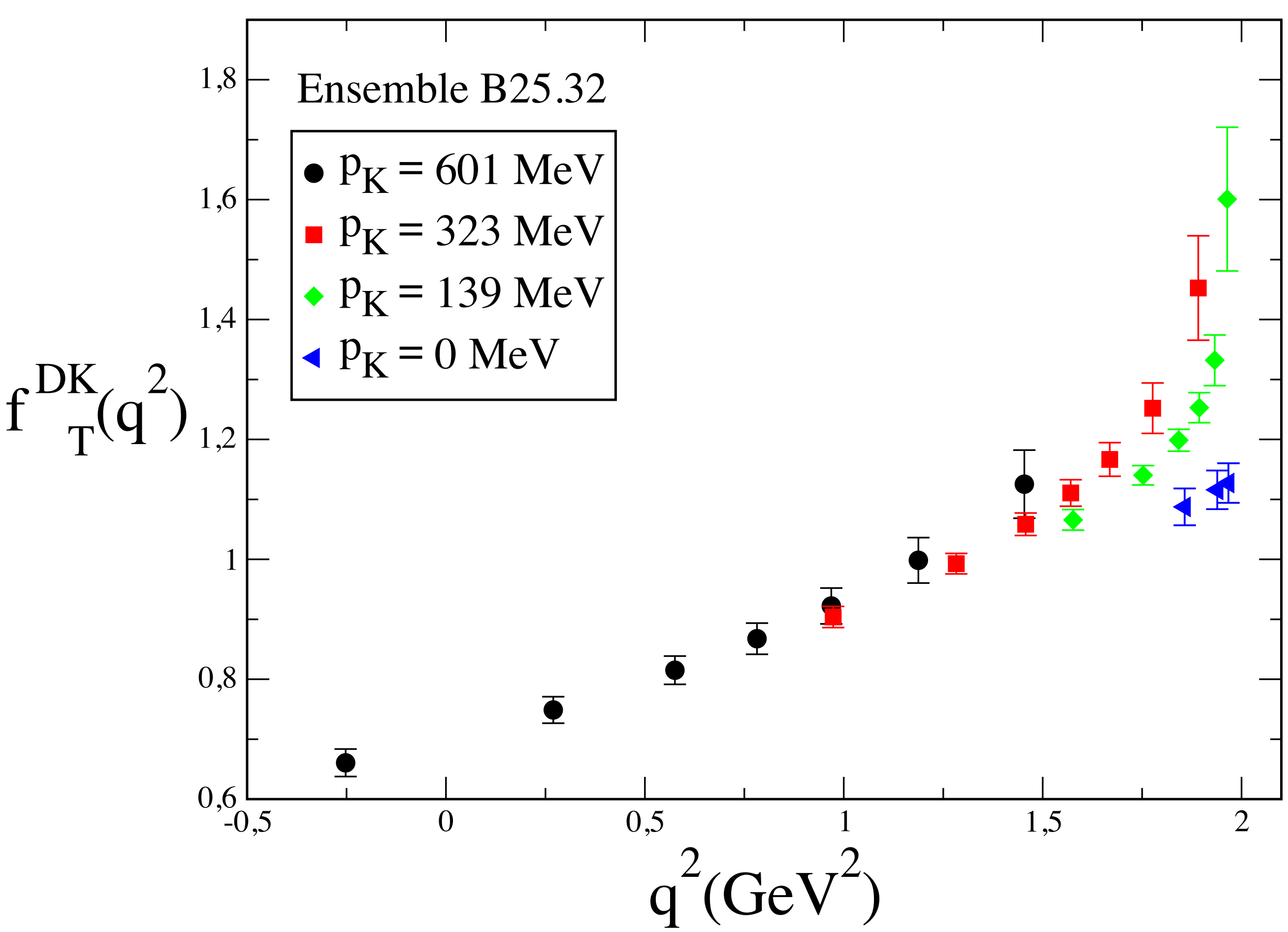
Evidence of hypercubic effects

Hypercubic effects: breaking of the Lorentz invariance

M_π=302 MeV $M_D \simeq M_D^{phys}$

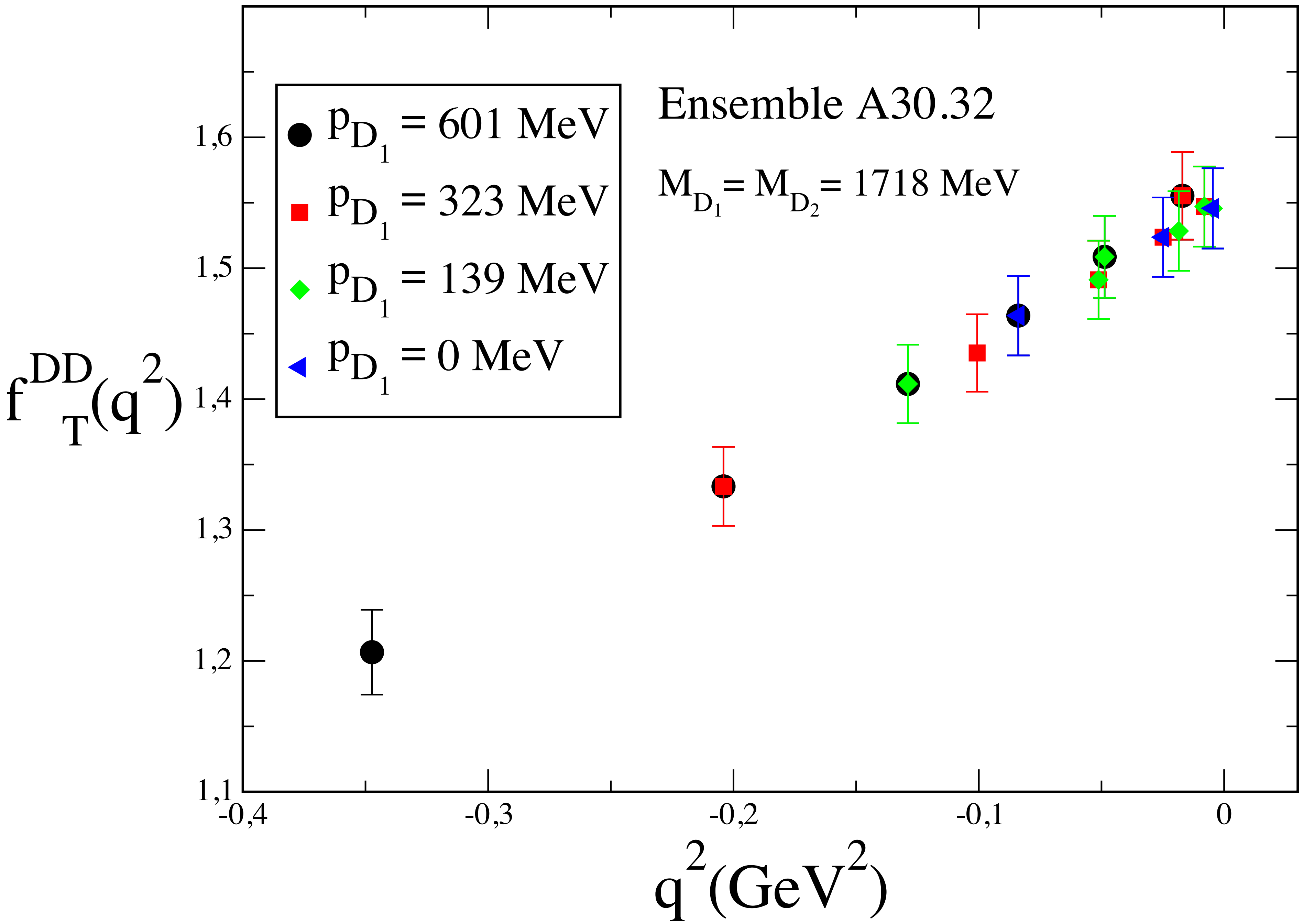


M_π=258 MeV $M_K \simeq M_K^{phys}$ $M_D \simeq M_D^{phys}$



Elastic form factor

The elastic form factor shows no evidence of hypercubic effects



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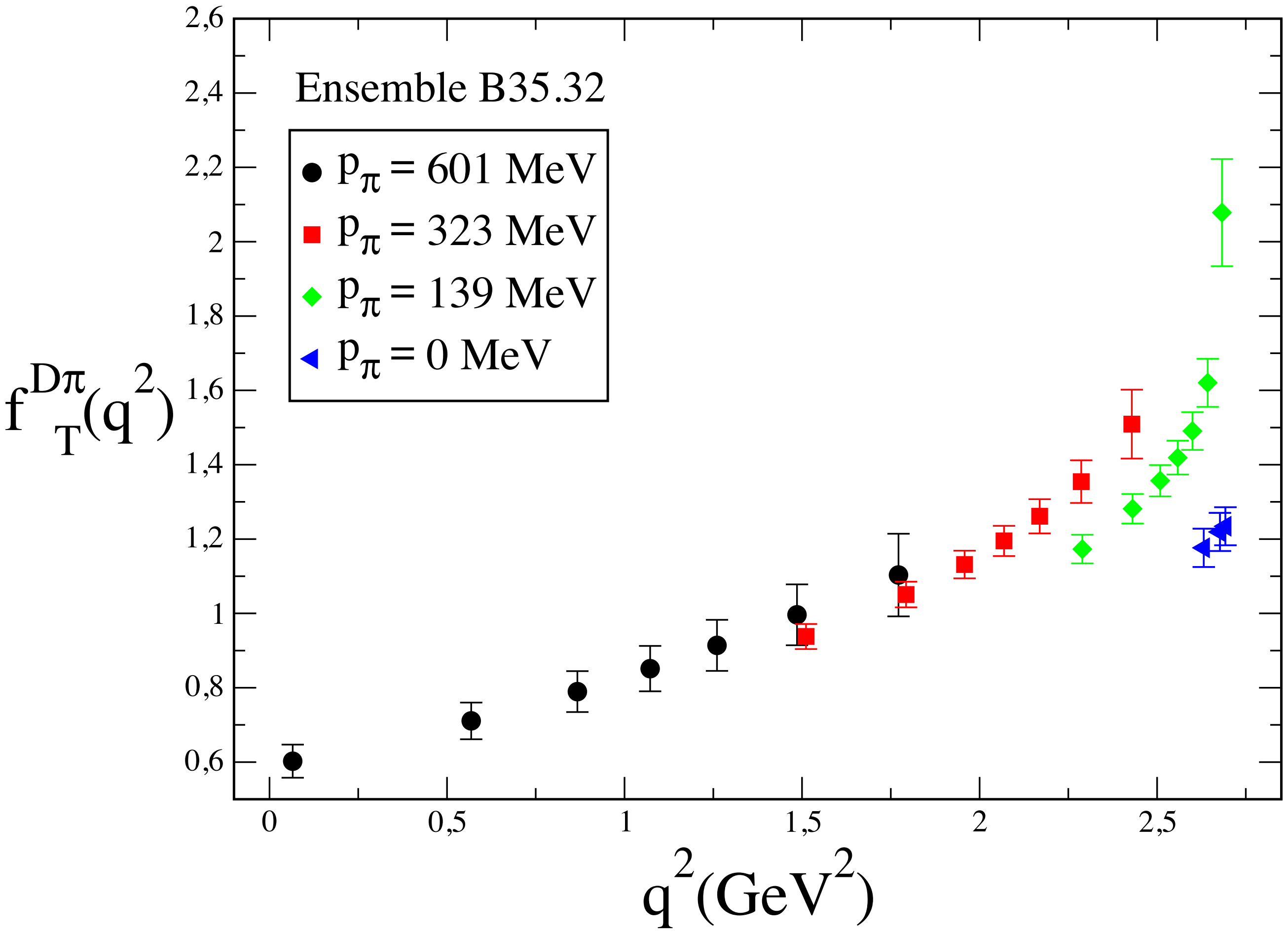
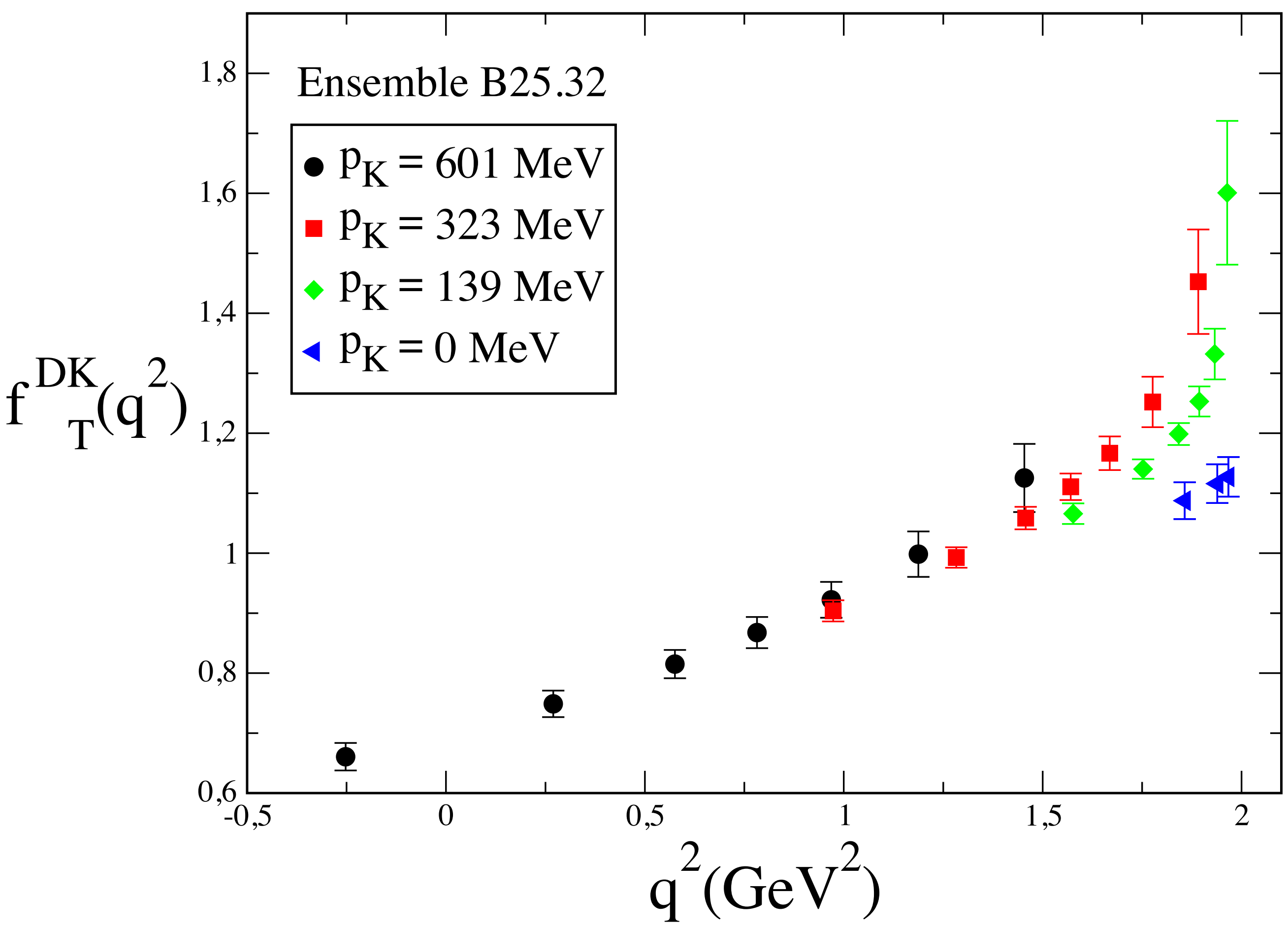
M_π=302 MeV $M_D \simeq M_D^{phys}$



M_π=258 MeV $M_K \simeq M_K^{phys}$ $M_D \simeq M_D^{phys}$



- f_T is not function of q^2 only we need to estimate and subtract these effects
- this subtraction cannot be done at the level of a single ensemble (with the present data)

Global Fit

Global Fit

Global fit studying simultaneously a^2 m_l and q^2 dependencies to extract the physical f_T form factor

ingredients:

- the T_i matrix elements are decomposed in a Lorentz-covariant part + a Lorentz-breaking Hypercubic part proportional to a^2
- Lorentz-breaking part must transform properly under hypercubic rotation
- Lorentz-breaking part will contain additional form factors
- The physical form factors will be described with modified z-expansion

Global Fit

Hypercubic effects

T_i matrix elements decompositions

$$\langle P(p_P)|T_i|D(p_D)\rangle = \langle T_i\rangle_{\text{Lor}} + \langle T_i\rangle_{\text{Hyp}}$$

$$\langle T_i\rangle_{\text{Lor}} = \frac{2}{M_D+M_P}\left[p_P^4 p_D^i - p_P^i p_D^4\right] f_T\left(q^2\right) \qquad p^4 = ip^0$$

$$\langle T_i\rangle_{\text{Hyp}} = a^2 \frac{2}{M_D+M_P}\left\{\left[\left(p_P^4\right)^3 p_D^i - \left(p_P^i\right)^3 p_D^4\right] H_1 + \left[p_P^4\left(p_D^i\right)^3 - p_P^i\left(p_D^4\right)^3\right] H_2\right\}$$

- $(M_D+M_P)^{-1}$ is conventionally inserted to make f_T dimensionless
- $\langle T_i\rangle_{\text{Hyp}}$ is the most general structure up to $O(a^2)$ that transforms properly under hypercubic rotations
- H_1 e H_2 are assumed to depend only on q^2, M_D and M_P

$$H_i(z) = d_0^i + d_1^i z$$

Global Fit

Modified z expansion

Bourrely Caprini and Lellouch [Phys.Rev. D79 (2009) 013008]

$$f_T^{D\to\pi(K)}(q^2) = \frac{f_T^{D\to\pi(K)}(0) + C(a^2)(z - z_0)\left(1 + \frac{z+z_0}{2}\right)}{1 - q^2 / M_T^2}$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = (M_D + M_P)^2$$

$$t_0 = (M_D + M_P)\left(\sqrt{M_D} - \sqrt{M_P}\right)^2$$

for C we adopted a simple linear dependence in a^2

M_T is left as a free parameter

Global Fit

Chiral extrapolation

Ansatz 1

$$f_T^{D\to\pi(K)}(0) = \left(M_D + M_{\pi(K)}\right) F \left[1 + A\,\xi_\ell \log \xi_\ell + b_1\,\xi_\ell + b_2\,a^2\right]$$

Ansatz 2

$$f_T^{D\to\pi(K)}(0) = \left(M_D + M_{\pi(K)}\right) F \left[1 + b_1\,\xi_\ell + b_2\,a^2\right]$$

$$\xi_l = \frac{2B\,m_l}{16\pi^2 f^2}$$

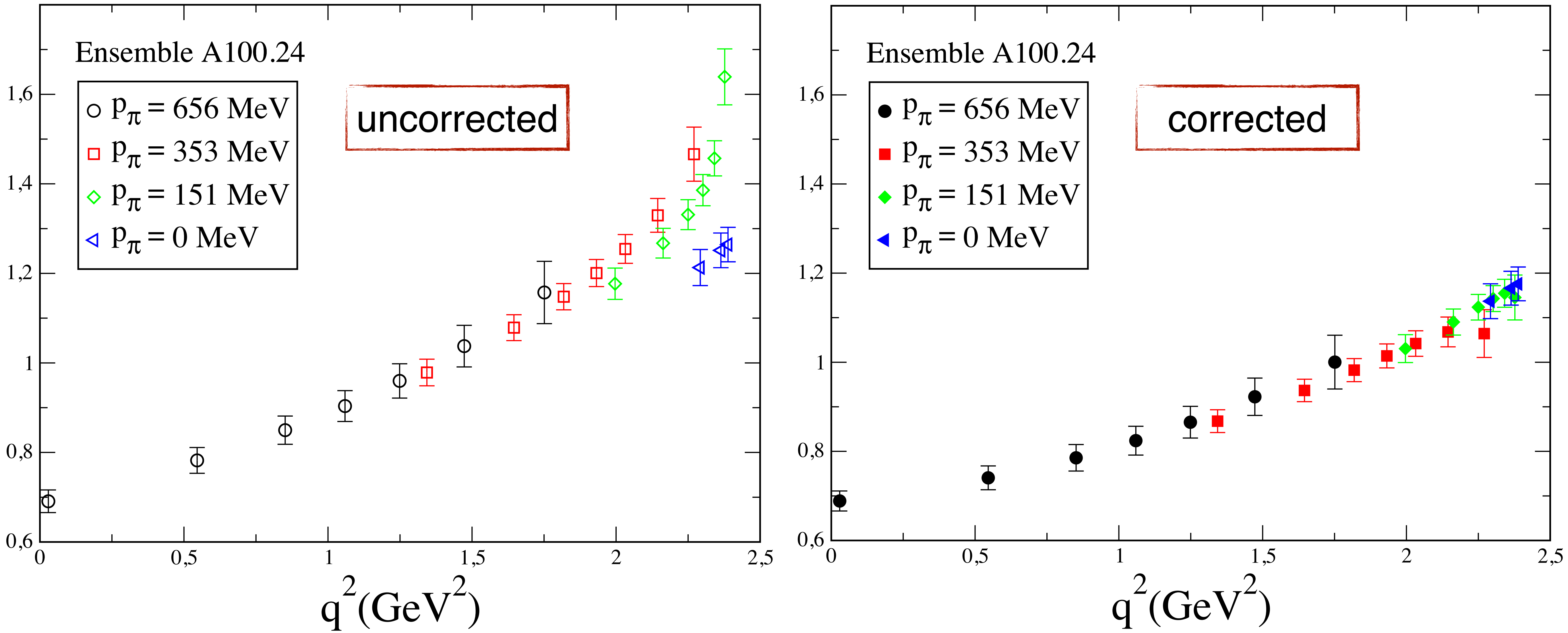
F, A, b_1 and b_2 are left as a free parameter

Results

$$f_T^{D\to\pi}(q^2)$$

Subtraction of the hypercubic effects in the D→π tensor form factor and restored q^2 dependence

M_π=495 MeV $M_D \simeq M_D^{phys}$

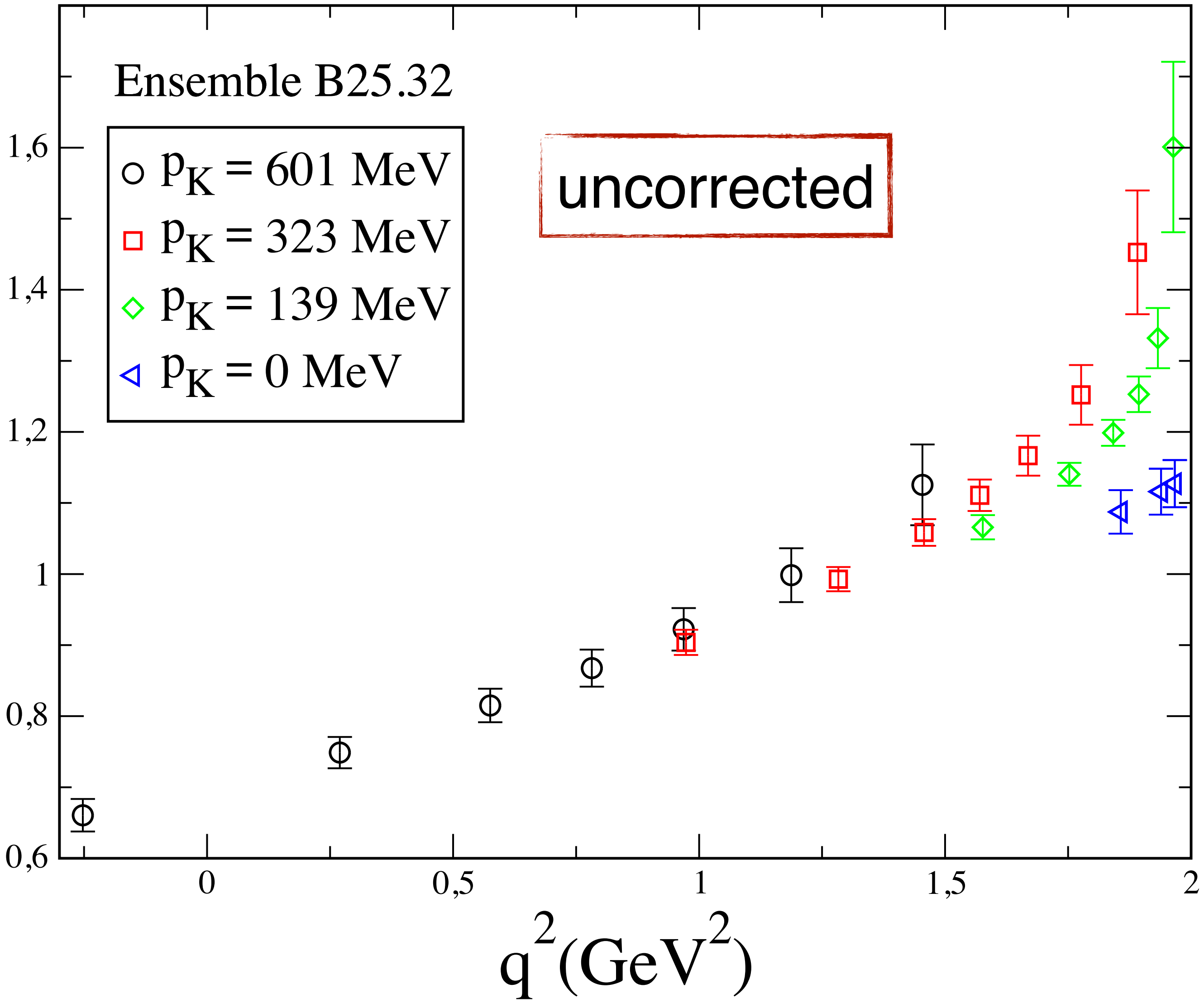


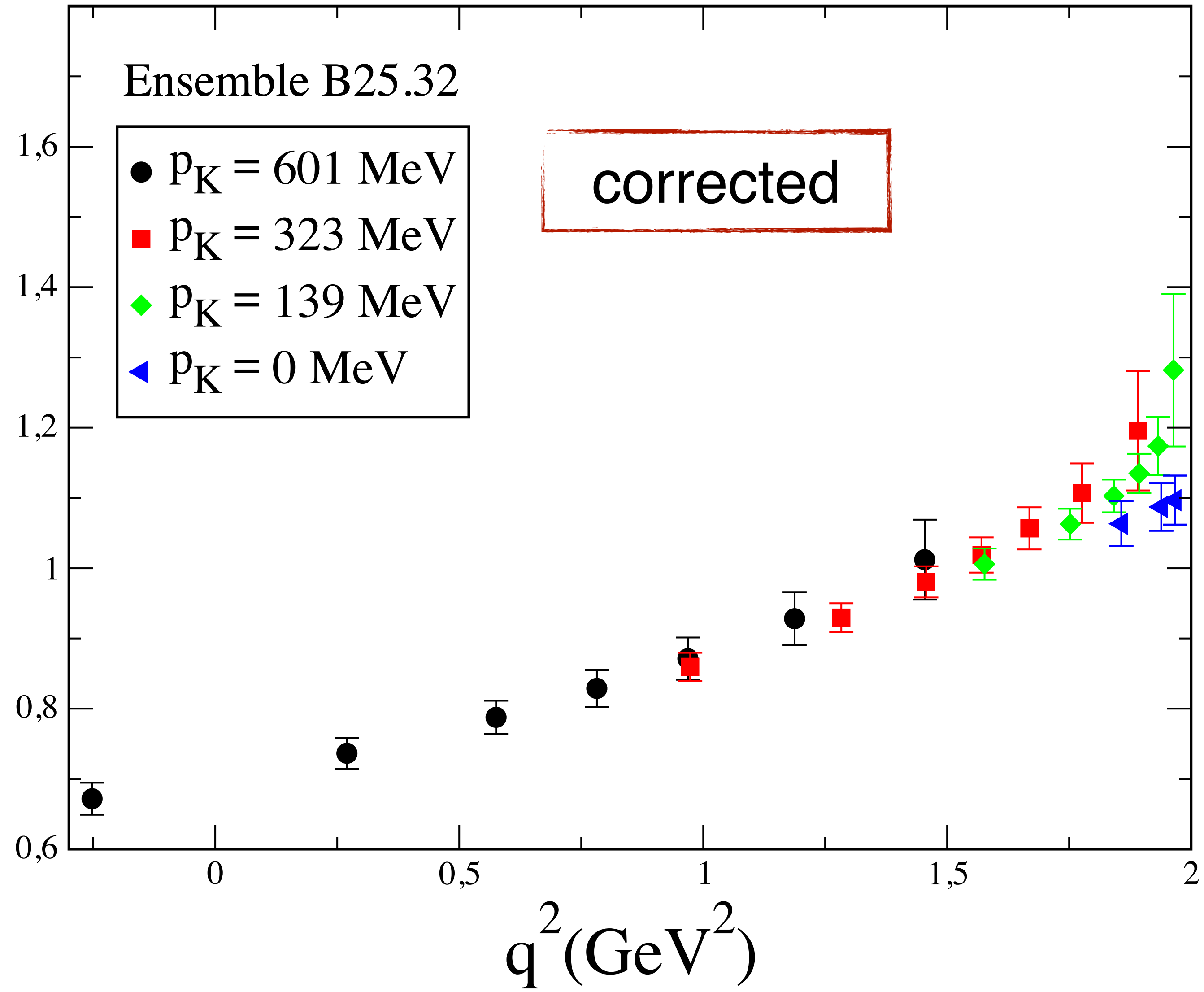
Results

$$f_T^{D\to K}(q^2)$$

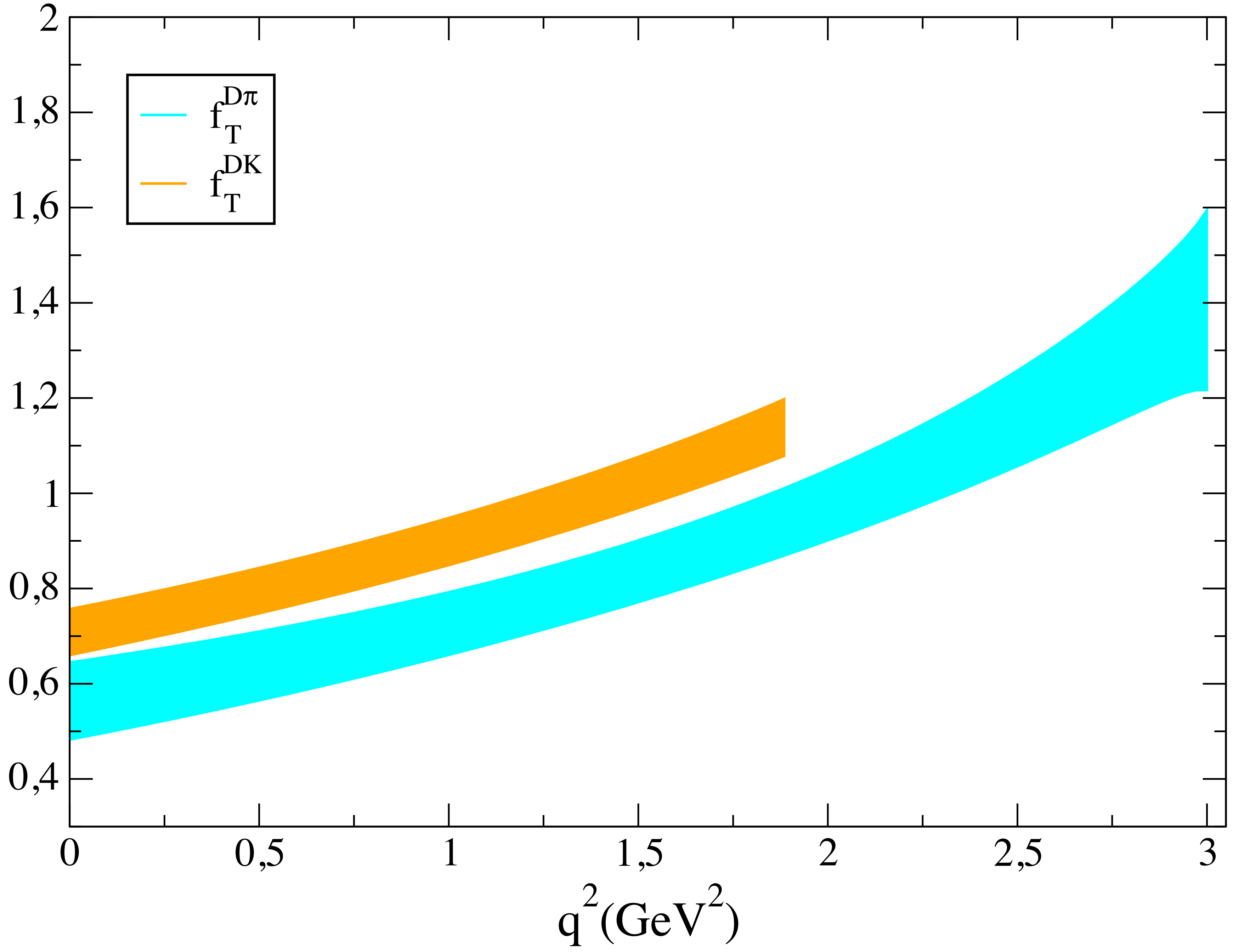
Subtraction of the hypercubic effects in the D→K tensor form factor and restored q^2 dependence

M_π=258 MeV $M_K \simeq M_K^{phys}$ $M_D \simeq M_D^{phys}$

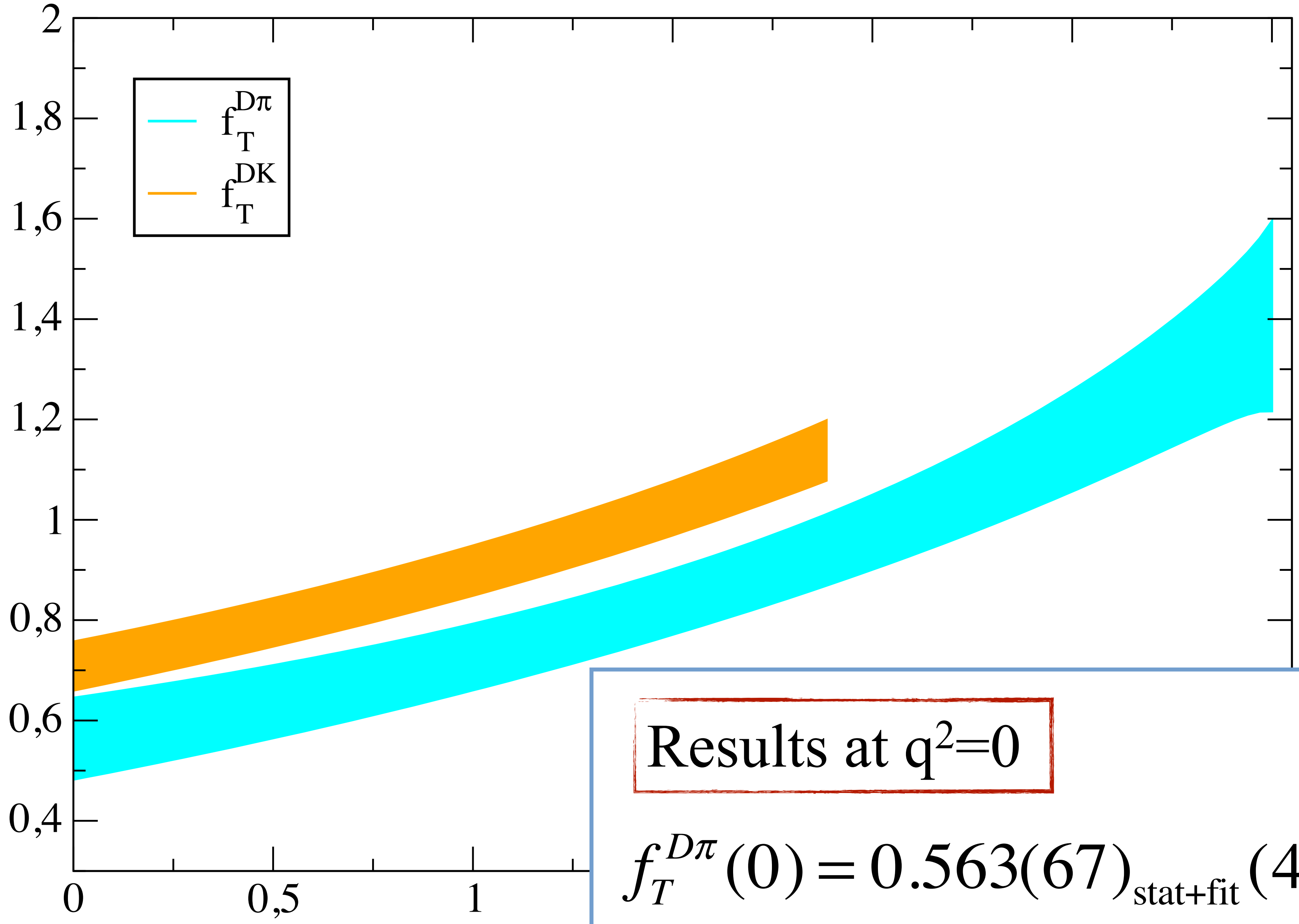




Results



Results

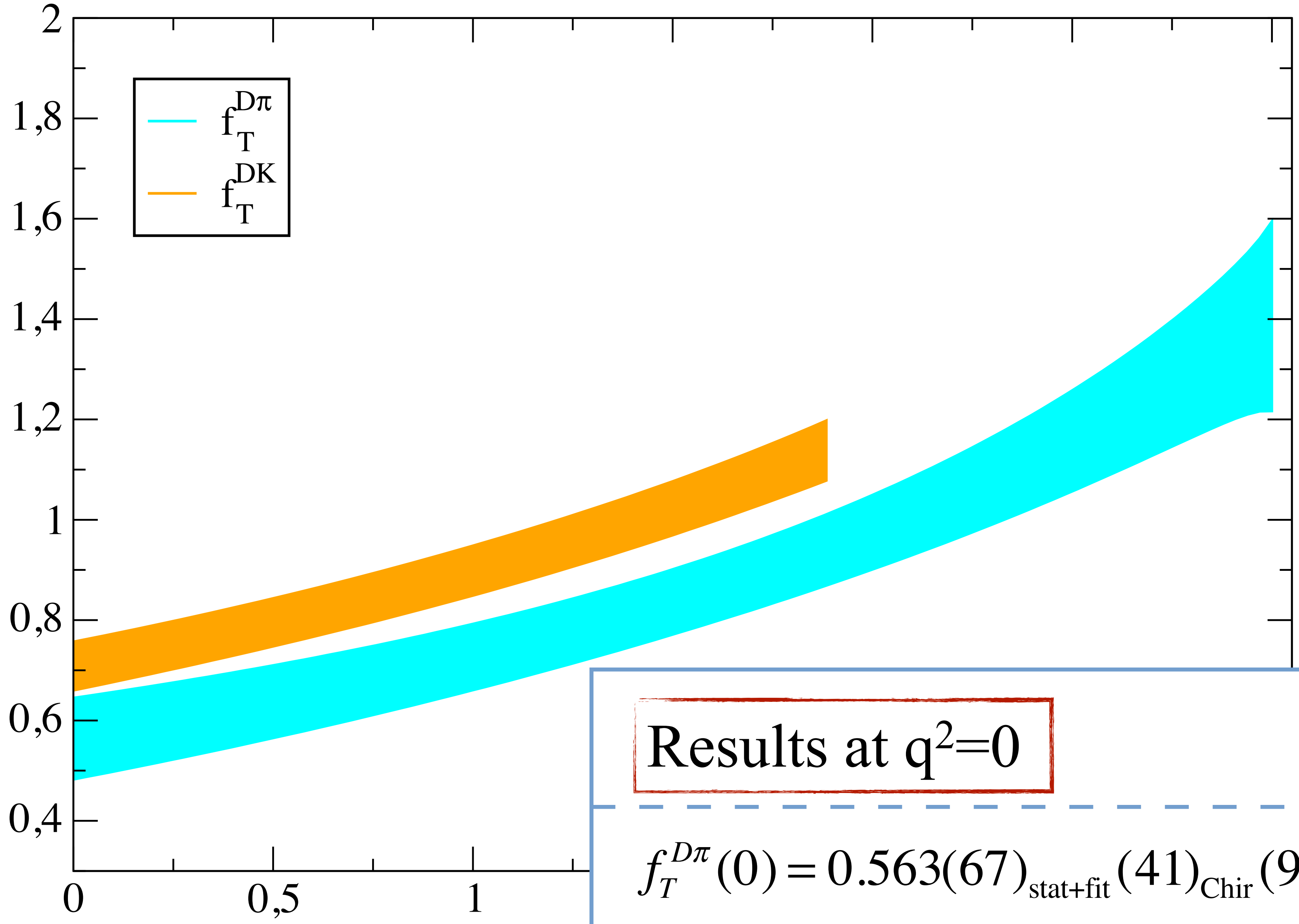


Results at q^2=0

$$f_T^{D\pi}(0) = 0.563(67)_{\text{stat+fit}}(42)_{\text{Syst}} = 0.563(80)$$

$$f_T^{DK}(0) = 0.706(45)_{\text{stat+fit}}(14)_{\text{Syst}} = 0.706(47)$$

Results

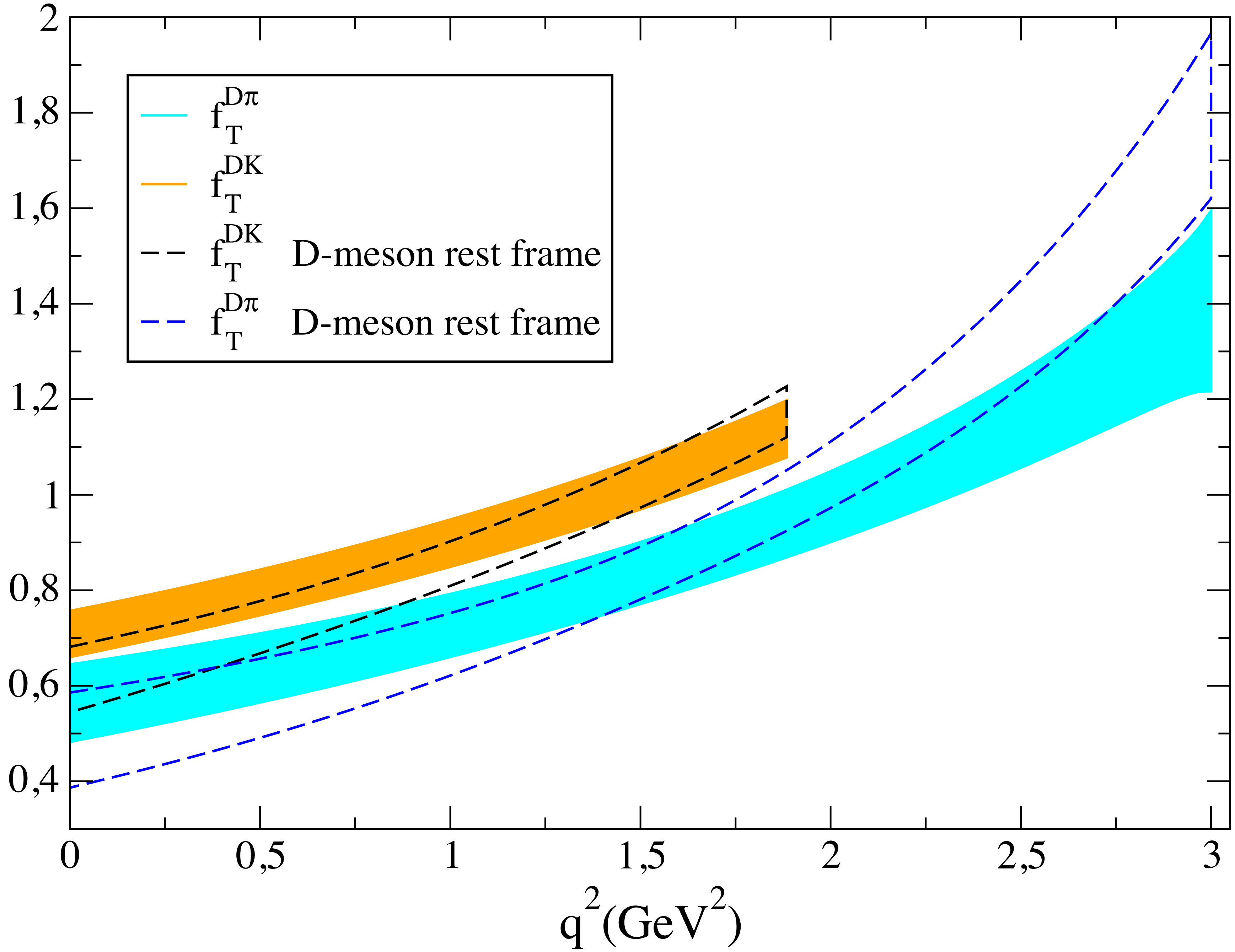


Results at q^2=0 $d.o.f. \approx 360$

$f_T^{D\pi}(0) = 0.563(67)_{\text{stat+fit}}(41)_{\text{Chir}}(9)_{\text{disc}}$ $\chi^2 / d.o.f. = 0.8$

$f_T^{DK}(0) = 0.706(45)_{\text{stat+fit}}(13)_{\text{Chir}}(3)_{\text{disc}}$ $\chi^2 / d.o.f. = 0.5$

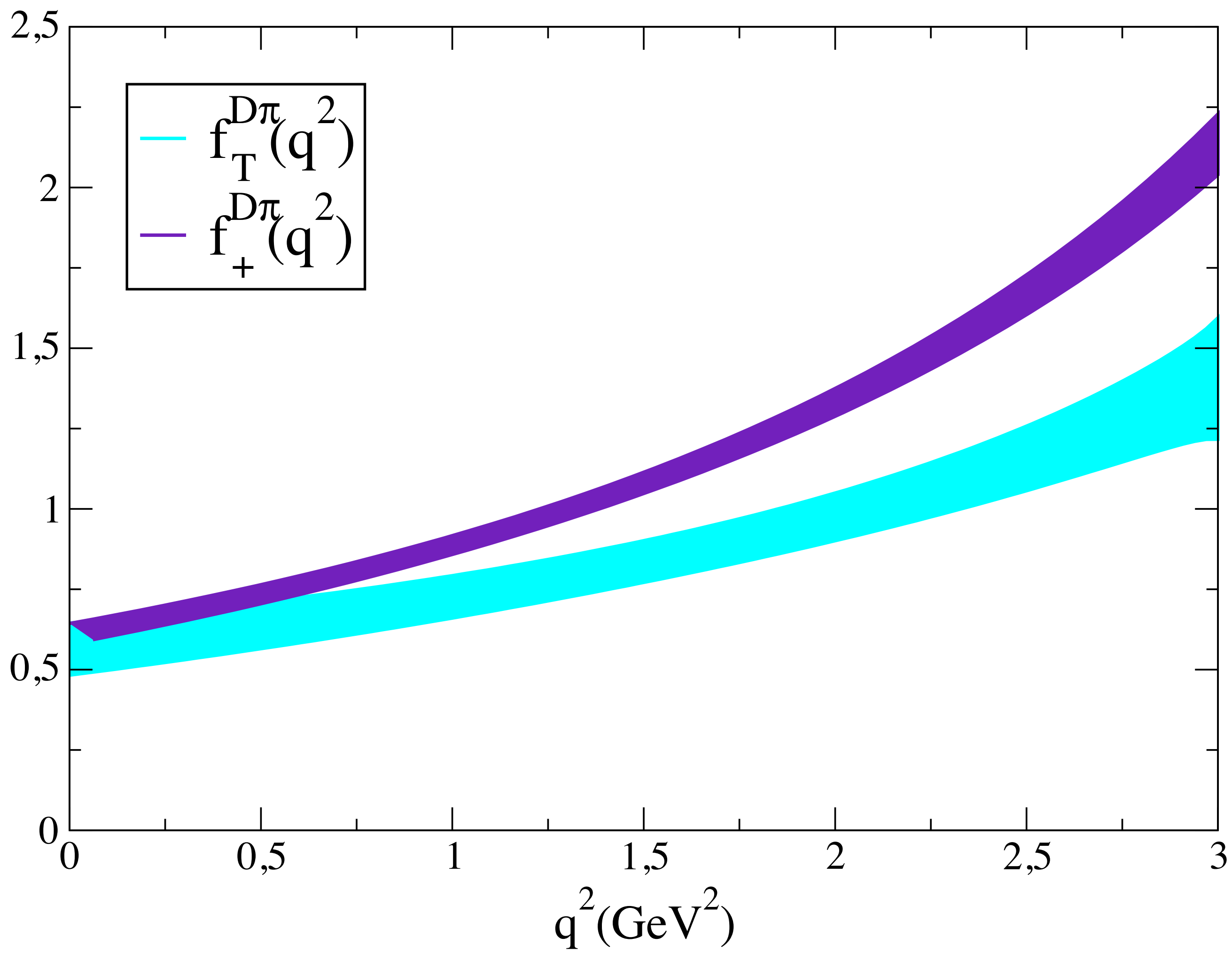
Results



Comparison with an analysis using only D at rest and without any subtraction of hypercubic effects

f_T vs f_+

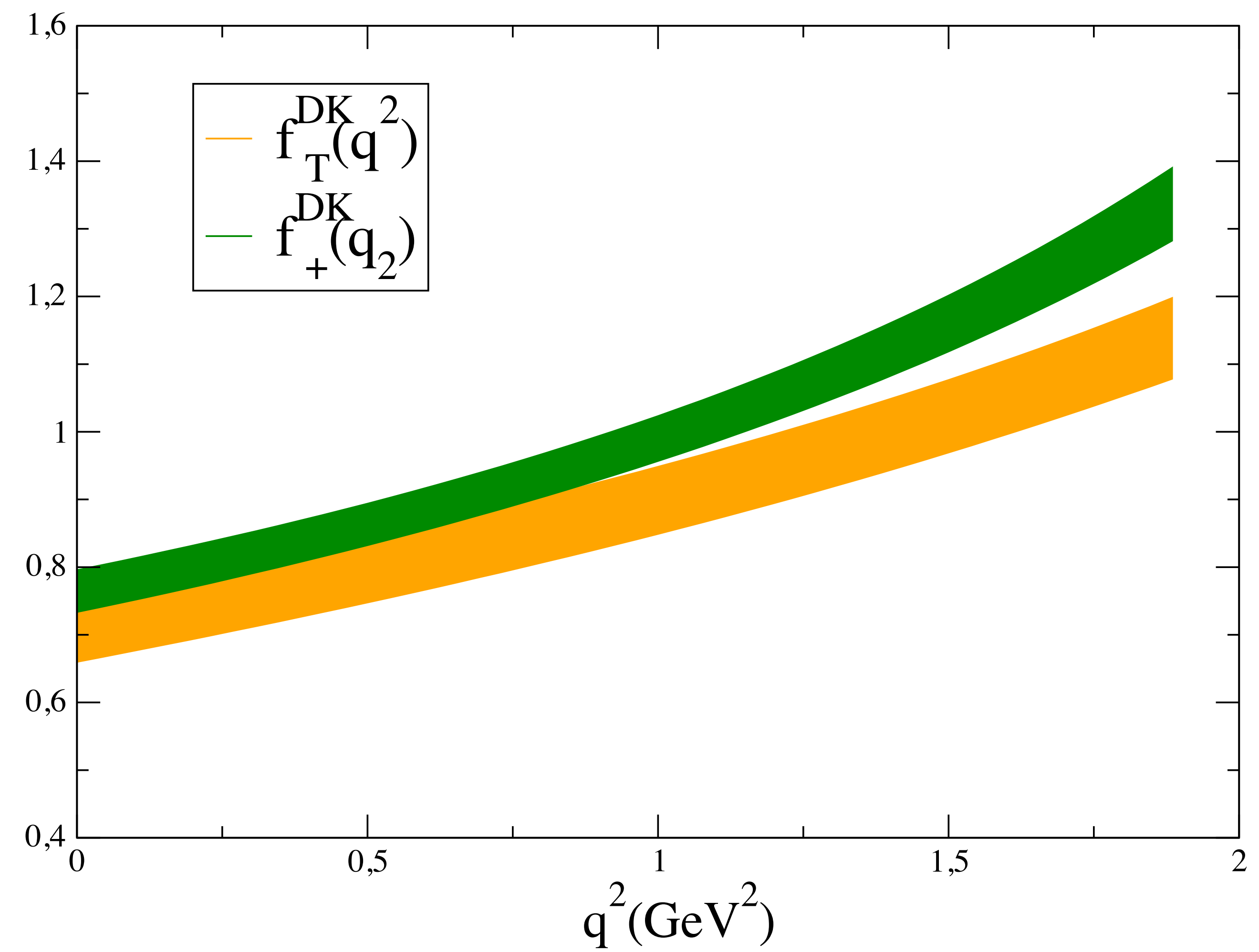
$D \to \pi$



$$f_T^{D\pi}(0) = 0.563(80)$$

$$f_+^{D\pi}(0) = 0.612(35)$$

$D \to K$



$$f_T^{DK}(0) = 0.706(47)$$

$$f_+^{DK}(0) = 0.765(31)$$

Conclusions

We have presented preliminary results for the $D \to \pi$ and $D \to K$ tensor form factor f_T in all the physical q^2-range using ETMC gauge ensembles with $N_f=2+1+1$

Similarly to the vector case, data showed evidence of Lorentz Symmetry breaking due to hypercubic effects

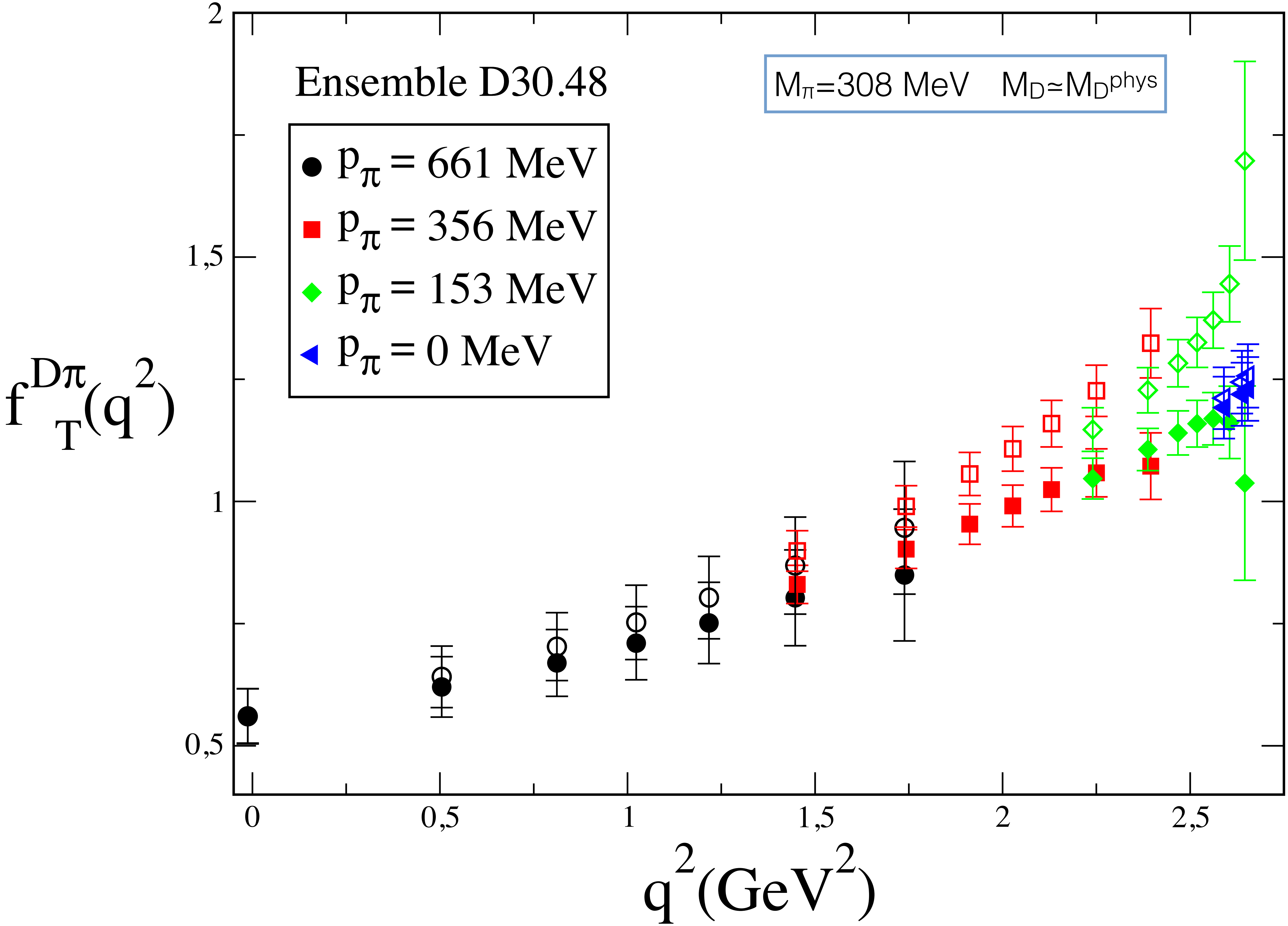
The strategy to describe and subtract these hypercubic effects introduced for the vector current matrix elements was applied to the tensor case and restored the physical q^2 dependence of the two form factors

As the parent meson mass grows we expect these effects to be more and more relevant so their understanding will be crucial for moving to B physics.

Thank you for the attention

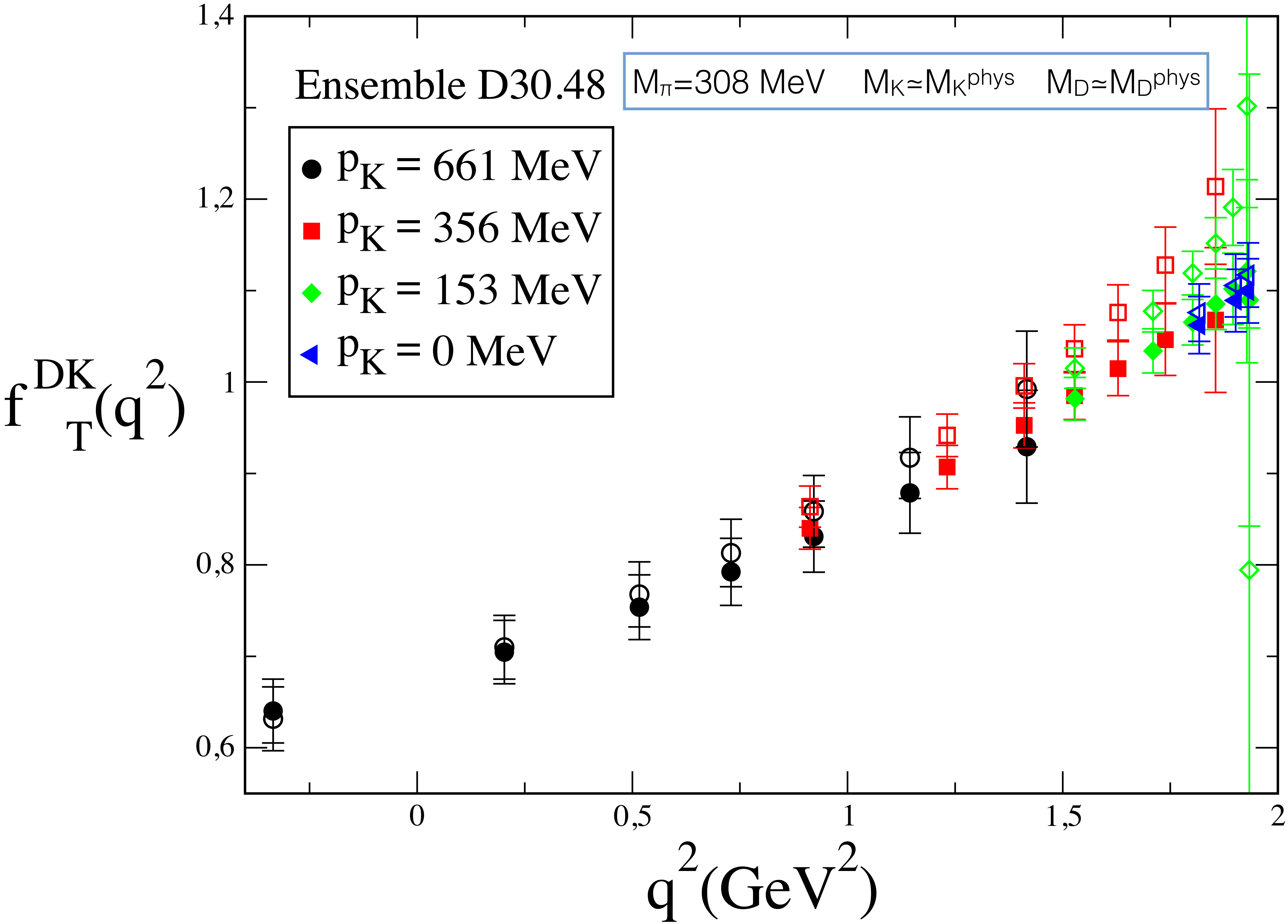
Backup slides

Results



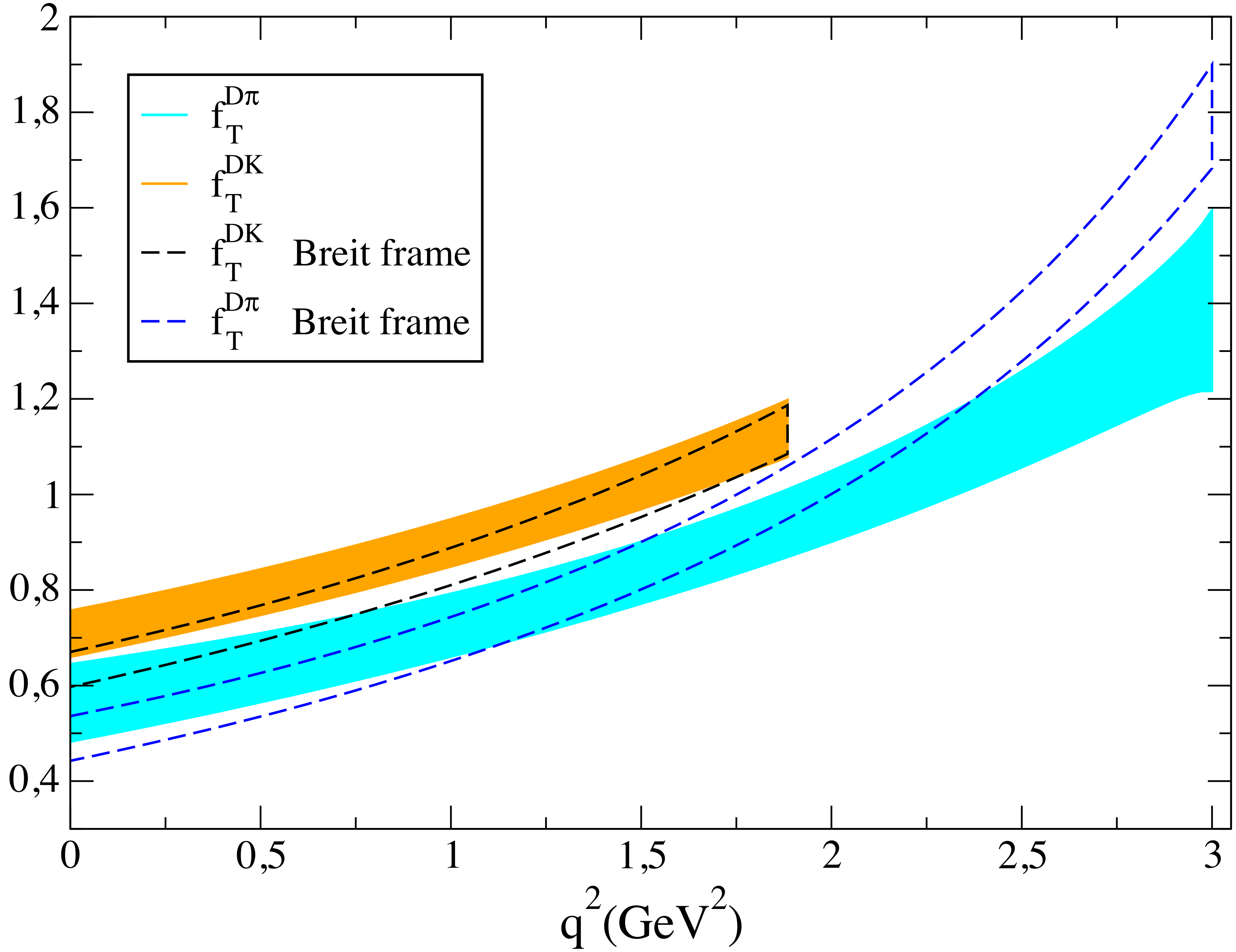
Ensemble D30.48
M_π=308 MeV $M_D \simeq M_D^{phys}$
p_π = 661 MeV
p_π = 356 MeV
p_π = 153 MeV
p_π = 0 MeV
$f_T^{D\pi}(q^2)$
$q^2 (GeV^2)$
2
1,5
1
0,5
0
0,5
1
1,5
2
2,5

Results



Ensemble D30.48
Mπ=308 MeV MK≃MKphys MD≃MDphys
pK = 661 MeV
pK = 356 MeV
pK = 153 MeV
pK = 0 MeV
1,4
1,2
1
0,8
0,6
0
0,5
1
1,5
2
f DK T(q 2)
q 2(GeV 2)

Outline



$f_T^{D\pi}$
f_T^{DK}
f_T^{DK} Breit frame
$f_T^{D\pi}$ Breit frame
2
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0
0,5
1
1,5
2
2,5
3
$q^2(GeV^2)$

Simulation Details

Something on the action:

- Wilson Twisted Mass action at maximal twist with $N_f=2+1+1$ sea quarks
- Osterwalder-Seiler valence quark action
- Iwasaki gluon action