ITEP Lattice

Magnetic moments and polarizabilities in lattice Quantum Chromodynamics

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21.06.2017

Introduction

In an external abelian constant magnetic field we calculate

- the energies of the ground states of vector $\rho^{\pm}$ and $K^{*\pm}$ mesons;
- g-factor of the vector mesons;
- their magnetic dipole polarizabilities and magnetic hyperpolarizabilities.

Motivation

g-factor (magnetic moment)

- characterizes the gyromagnetic ratio of an hadron;
- reveal the contribution of the strong interactions.

The external magnetic field of an hadronic scale (~ 0.3 GeV^2) can be used as the probe of QCD properties.

Motivation

Magnetic polarizability and hyperpolarizability

- are the fundamental quantities describing the spin interactions of quarks and the ability to form instantaneous dipoles;
- characterize the distribution of quark currents inside a meson in an external field.

Technique for calculation of energies

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m_q}.$$

Calculate the correlation functions on the lattice:

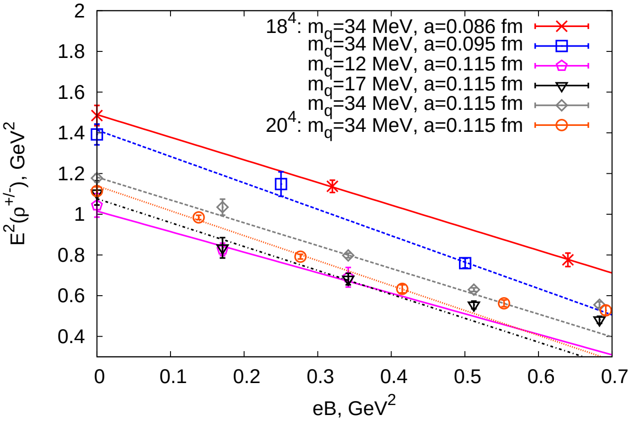
$$\langle\bar\psi\Gamma_1\psi\bar\psi\Gamma_2\psi\rangle_A = -tr[\Gamma_1 D^{-1}(x,y)\Gamma_2 D^{-1}(y,x)]+$$

$$+tr[\Gamma_1 D^{-1}(x,x)]tr[\Gamma_2 D^{-1}(y,y)],$$

$x = (\mathbf{n}a, n_t a),\ y = (\mathbf{n}'a, n_t' a), \mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3)|n_i = 0, 1, ..., N-1\}$

Magnetic moment of the $\rho^{\pm}$ meson

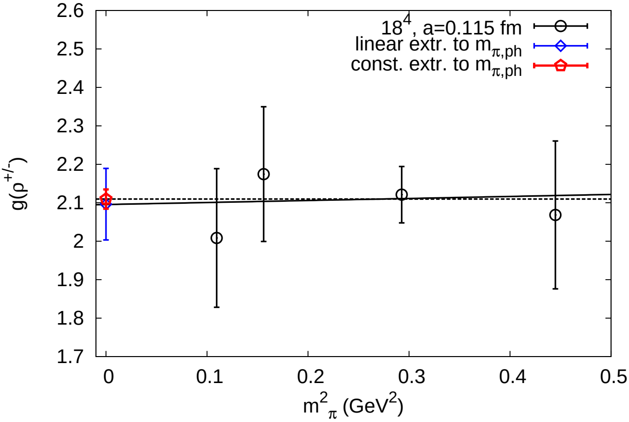
$$E^2 = |qB| - g s_z qB + m^2$$



Magnetic moment of the $\rho^{\pm}$ meson

V	m_q(MeV)	m_π(MeV)	a(fm)	g-factor	χ^2/n	eB (GeV^2)
18^4	11.99	331 ± 7	0.115	2.01 ± 0.18	0.826	$[0, 0.35]$
18^4	17.13	395 ± 6	0.115	2.17 ± 0.18	0.969	$[0, 0.35]$
18^4	34.26	541 ± 3	0.115	2.12 ± 0.07	1.159	$[0, 0.35]$
18^4	51.39	667 ± 3	0.115	2.07 ± 0.19	1.695	$[0, 0.35]$
18^4		135 (con)	0.115	2.11 ± 0.03	0.418	$[0, 0.35]$
18^4		135 (lin)	0.115	2.10 ± 0.09	0.509	$[0, 0.35]$
18^4	34.26	625 ± 21	0.084	2.11 ± 0.01	0.153	$[0, 0.70]$
18^4	34.26	596 ± 12	0.095	2.30 ± 0.12	1.094	$[0, 0.55]$
18^4	34.26	572 ± 16	0.105	2.05 ± 0.03	0.644	$[0, 0.45]$
20^4	34.26	535 ± 4	0.115	2.22 ± 0.08	1.398	$[0, 0.45]$

Extrapolation to the π^0 physical mass



Extrapolation by a linear function: $g = 2.10 \pm 0.09$
Extrapolation by a constant function: $g = 2.110 \pm 0.025$

Comparison with the other results

BaBar cross section data for the reaction $e^+e^- \to \pi^+\pi^- 2\pi^0$:
$g_\rho^{exp} = 2.1 \pm 0.5$,
D.G. Gudino, G.T.Sanchez, Int.J.Mod.Phys.A 30, 1550114 (2015).

2+1 lattice QCD: $g_\rho = 2.21 \pm 0.08$,
B. Owen et.al., Phys. Rev. D 91, 074503 (2015).

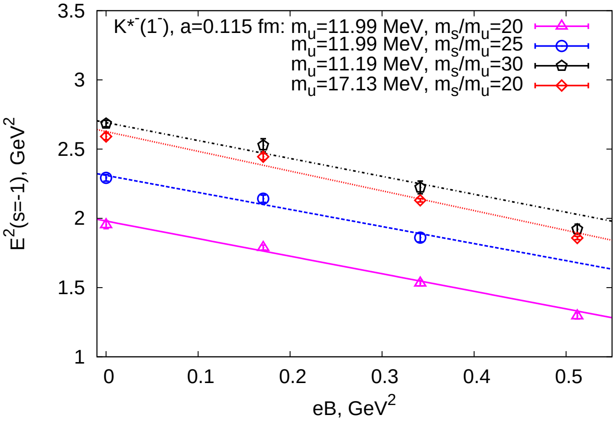
QCD sum rules: $g_\rho = 2.4 \pm 0.4$,
T.M. Aliev et.al., Phys. Lett. B 678, 470 (2009).

covariant quark model: $g_\rho = 2.14$,
J.P.B.C. de Melo and T. Frederico, Phys. Rev. C 55, 2043 (1997).

gravitation theory: $g = 2$.
O. V. Teryaev, Front. Phys. (Beijing) 11, 111207 (2016).

The magnetic moment of $K^{*\pm}$ meson

$$E^2 = |qB| - g s_z qB + m^2$$

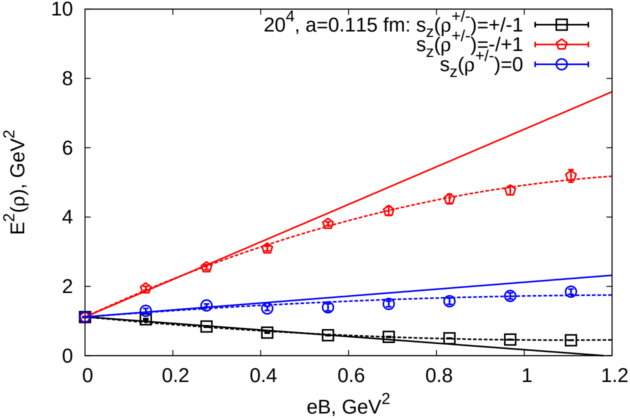


m_u(MeV)	m_s/m_u	g-factor	χ^2/d.o.f.	fit, eB (GeV^2)
11.99	20	2.27 ± 0.18	1.845	$[0, 0.35]$
11.99	25	2.23 ± 0.23	1.986	$[0, 0.35]$
11.99	30	2.29 ± 0.19	1.366	$[0, 0.35]$

QCD sum rules: $g_{K^*} = 2.0 \pm 0.4$,
T.M. Aliev et.al., Phys. Lett. B 678, 470 (2009).

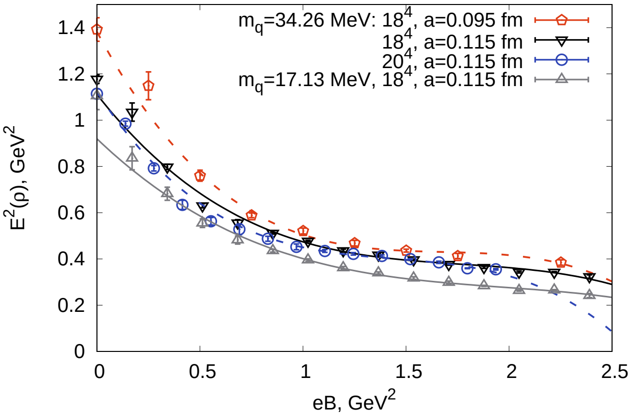
gravitation theory: $g_{K^*} = 2$,
O.V.Teryaev, Front. Phys. 11, 111207 (2016).

The magnetic polarizabilities of the $\rho^{\pm}$ meson



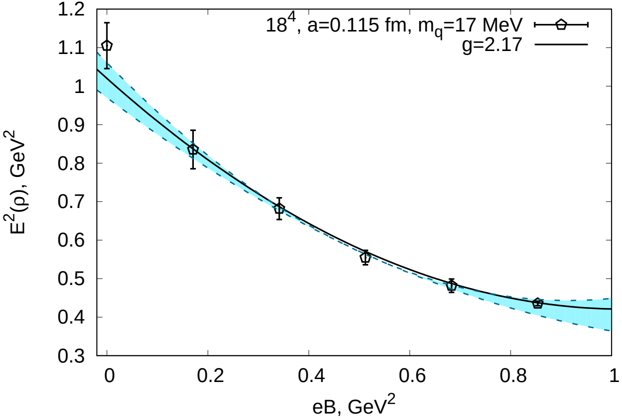
$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2$$

$\rho^{\pm}(s_z = \pm 1)$, 4-param. fit



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2 - 4\pi m \beta_m^{h1} (qB)^3$$

Fixed g-factor, 2-param. fit



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2, \quad g = 2.17 \pm 0.18$$

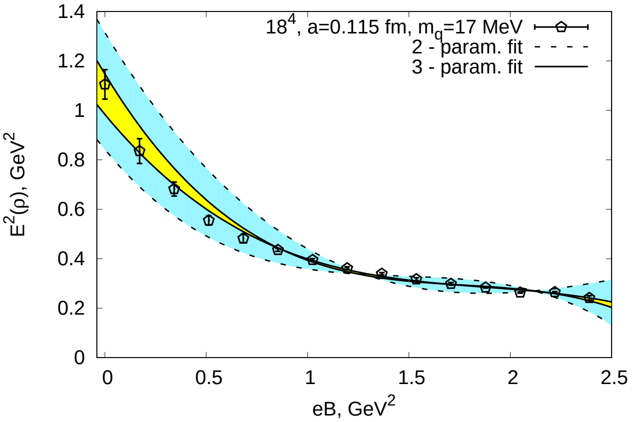
β_m for $|s_z| = 1$ from the 2-param. fit

Fixed g,

$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2$$

V	m_q (MeV)	a (fm)	β_m (GeV^{-3})	χ^2/n	fit, eB (GeV^2)
18^4	34.26	0.095	$-0.025^{+0.016}_{-0.014}$	1.656	[0, 1]
18^4	34.26	0.095	$-0.036^{+0.007}_{-0.006}$	1.904	[0, 1.3]
18^4	34.26	0.115	$-0.037^{+0.006}_{-0.005}$	2.774	[0, 1.1]
20^4	34.26	0.115	$-0.042^{+0.008}_{-0.008}$	2.274	[0, 1]
18^4	17.13	0.115	$-0.045^{+0.011}_{-0.012}$	0.823	[0, 1]

Fixed g-factor, 2– and 3-param. fit



$$E^2 = |qB| - g s_z qB + m^2 - 4\pi m \beta_m (qB)^2 - 4\pi m \beta_m^{h1} (qB)^3, \quad g = 2.17 \pm 0.18$$

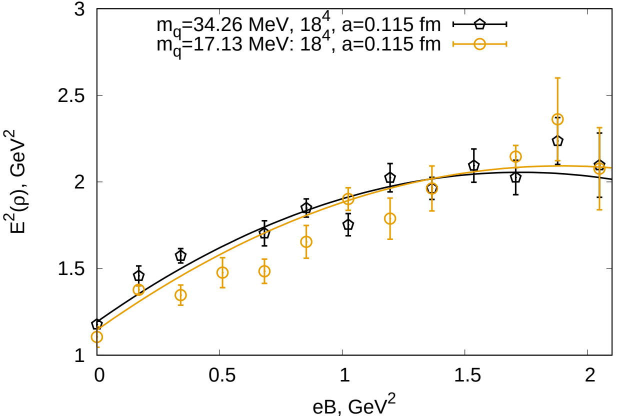
β_m and β_m^{1h} for $\rho^{\pm}(s_z = \pm)$ from the 3-param. fit

Fixed g,

$$E^2 = |qB| - gs_z qB + m^2 - 4\pi m\beta_m (qB)^2 - 4\pi m\beta_m^{h1} (qB)^3$$

V	m_q (MeV)	a (fm)	β_m (GeV^{-3})	β_m^{1h} (GeV^{-5})	χ^2/n	eB
18^4	34.26	0.095	$-0.050^{+0.009}_{-0.008}$	$0.009^{+0.002}_{-0.003}$	1.965	$[0, 2.5]$
18^4	34.26	0.115	$-0.045^{+0.005}_{-0.005}$	$0.009^{+0.001}_{-0.001}$	2.787	$[0, 2.5]$
20^4	34.26	0.115	$-0.058^{+0.008}_{-0.008}$	$0.013^{+0.002}_{-0.003}$	2.697	$[0, 2.0]$
18^4	17.13	0.115	$-0.047^{+0.009}_{-0.009}$	$0.009^{+0.002}_{-0.002}$	2.255	$[0, 2.5]$

$\rho^{\pm}(s=0)$



$$E^2 = |qB| + m^2 - 4\pi m \beta_m (qB)^2$$

$$\beta_m \sim 0.02 \div 0.05 \text{ GeV}^{-3}$$

Conclusions

1. We calculate the g-factor of the $\rho^{\pm}$ and $K^{*\pm}$ mesons,
2. obtain the magnetic dipole polarizability of the $\rho^{\pm}$ meson for the $|s_z| = 1$,
3. estimate the magnetic dipole polarizability of the $\rho^{\pm}$ meson for the $s_z = 0$,
4. calculate the hyperpolarizability of the $\rho^{\pm}$ for the $s_z = \pm 1$.