# Semileptonic $B_c$ decays from highly improved staggered quarks and NRQCD

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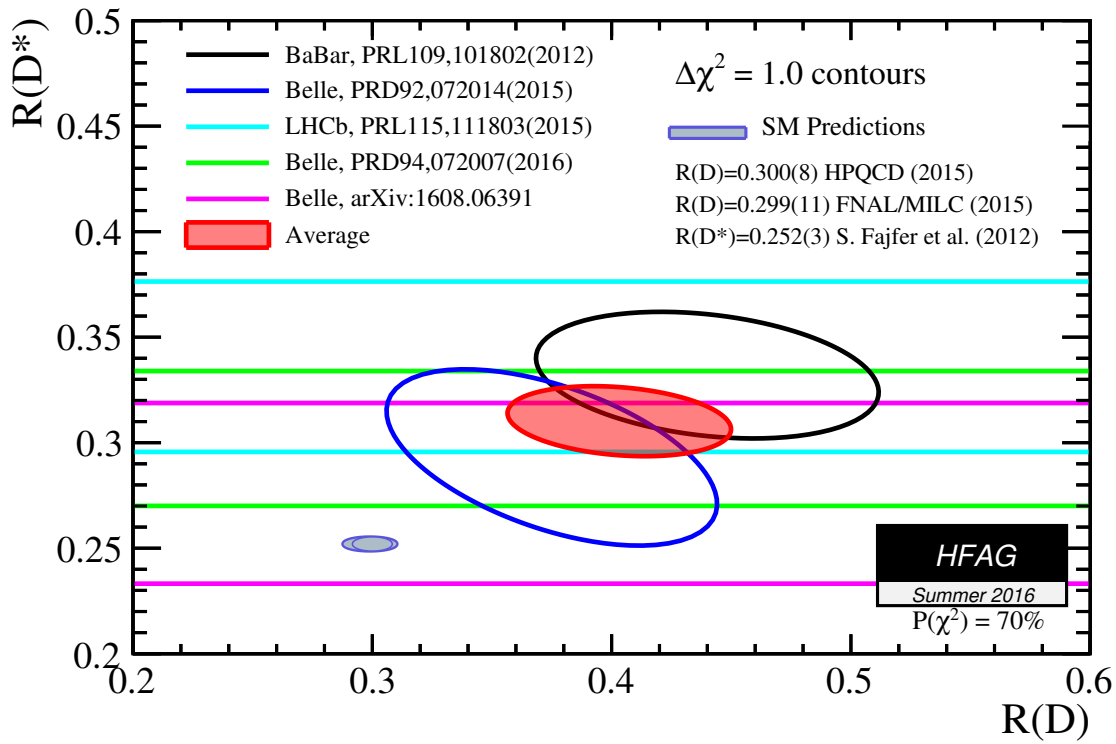
# Intro & Motivation

Persistent discrepancy w/ SM in $b \to c$ semileptonic observables

$$R(B \to D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\,\tau\nu)}{\mathcal{B}(B \to D^{(*)}\,l\nu)}\,, \quad l = \mu,\, e\,.$$



# Intro & Motivation

$b \to c$ transition $R$-ratios need to be tested in other channels:

- $B_c \to J/\psi\, \mu\, \nu$ measured by LHCb. [1401.6932]
- $\tau$ channel measurement from LHCb expected soon,
  $\to R(B_c \to J/\psi)$.

Reliable SM predictions are needed.

# Intro & Motivation

- Treatment of $c$ and especially $b$ quarks challenging in lattice simulations due to lattice artifacts which grow as $(am_q)^n$.
- We use two complementary approaches:
  - Highly improved relativistic action at small $a$, extrapolate $m_h \to m_b$.
  - Improved non-relativistic formalism (NRQCD) at $m_b$.
- First study:
  - $B_c \to \eta_c$
  - $B_c \to J/\psi$ [accessible at LHCb]
- Better understand syst. $b \to c$ NRQCD currents used in $B \to D$, $B \to D^*$.

# Outline

1. Intro & Motivation.

2. Calculation Framework.
   - HISQ action.
   - Improved NRQCD.

3. Semileptonic Decays.
   - Correlation functions.
   - $B_c \to \eta_c$ and results.
   - $B_c \to J/\psi$ and results.

4. Discussion & Future Work.

# DiRAC II computing

Computations carried out on the Darwin cluster at Cambridge.

Includes:

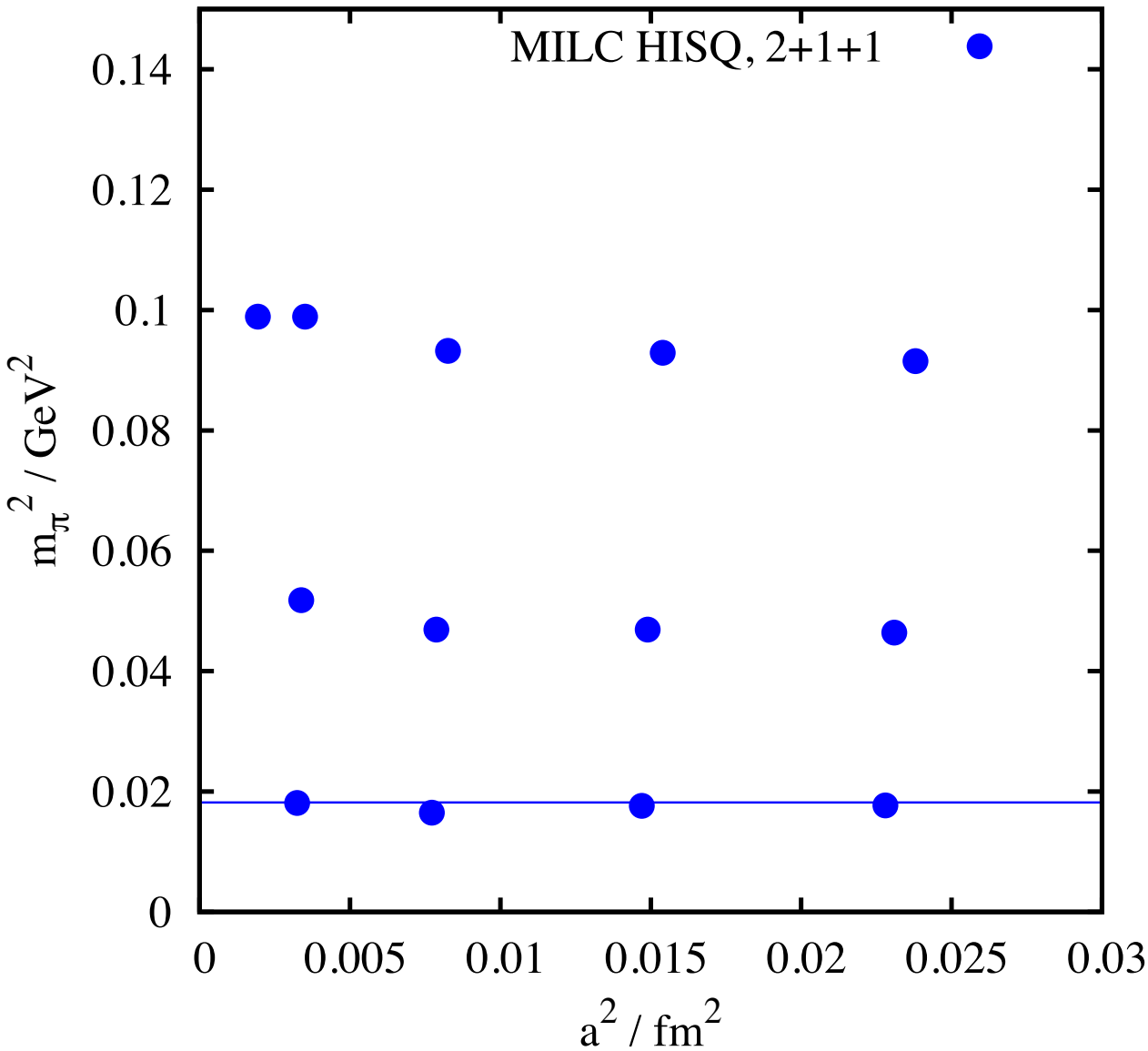
- 9600 Intel Sandy Bridge cores
- 2.6 GHz, 4 GB RAM/core
- 2 PB storage

Part of STFC's HPC facility for theoretical particle physics and astronomy.

# MILC ensembles [1004.0342, 1212.4768]

- HISQ fermion action.
- Symanzik-improved gauge action, takes into account $\mathcal{O}(N_f \alpha_s a^2)$ effects of HISQ quarks in sea. [0812.0503]
- Multiple lattice spacings down to $\sim 0.045$ fm.
- Effects of $u/d$, $s$, and $c$ quarks in the sea.
- Multiple light-quark input parameters down to physical pion mass.
  - Chiral fits.
  - Reduce statistical errors.

# MILC ensemble parameters



# NRQCD

Heavy quark propagators are calculated using a non-relativistic formalism.

Improved Non-relativistic QCD action

- Accurate through $\mathcal{O}(\alpha_s v^4)$.
- Discretisation corrections through $\mathcal{O}(\alpha_s v^2 a^2 p^2)$.
- $v^2 \sim 0.1$ bottomonium, $\sim 0.3$ charmonium.
- $am > 1 \rightarrow b$ quarks on $a = 0.15 - 0.06$ fm (down to $m_b/2$ on $a = 0.15$ fm).

Propagators constructed via an evolution equation,
$G(\mathbf{x}, t+a) = e^{-aH_{\text{eff}}} G(\mathbf{x}, t)$ .

# NRQCD

$$
\begin{aligned}
aH_{\mathrm{NRQCD}} &= aH_0 + a\delta H \\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b} \\
a\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} (\nabla \cdot \mathbf{E} - \mathbf{E} \cdot \nabla) \\
&\quad - c_3 \frac{1}{8(am_b)^2} \sigma \cdot (\nabla \times \mathbf{E} - \mathbf{E} \times \nabla) \\
&\quad - c_4 \frac{1}{2am_b} \sigma \cdot \mathbf{B} + c_5 \frac{\Delta^{(4)}}{24am_b} \\
&\quad - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}
\end{aligned}
$$

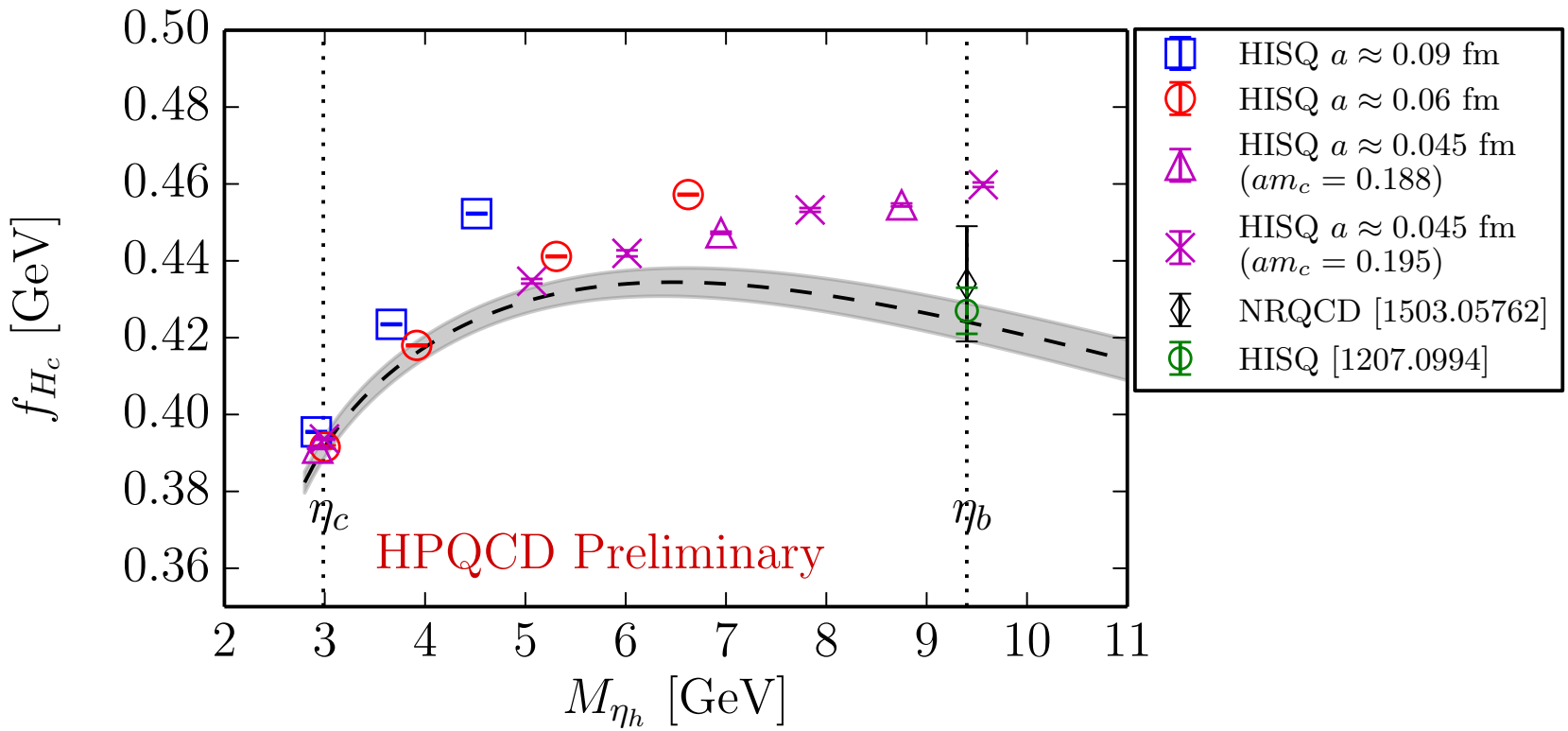
# General strategy

Strategy pursued by HPQCD collaboration:

- Staggered quarks $\rightarrow$ small $a$, physical pions, multiple lattice spacings..
- Highly improved action
  $\rightarrow$ discretisation effects under control at $m_c$
  $\rightarrow$ reduced taste-splittings
  $\rightarrow$ physical point ensembles with dynamical $u/d$, $s$, and $c$ quarks.
- Compute heavy quarks using (improved) NRQCD.

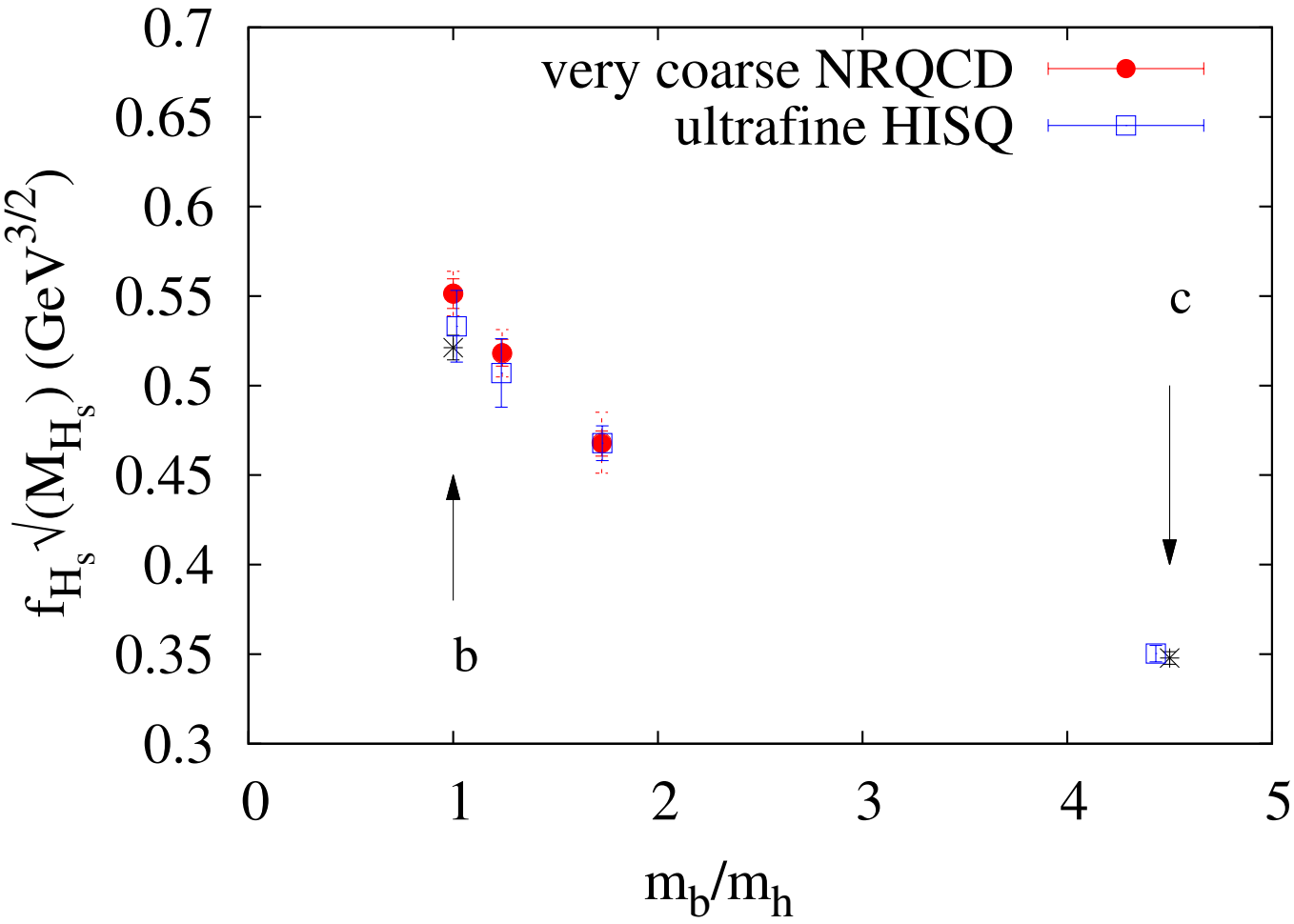
These two approaches are complementary. Ideally there is a range of overlap in applicability to check the approaches are mutually consistent.

# $f_{H_c}$ from HISQ.

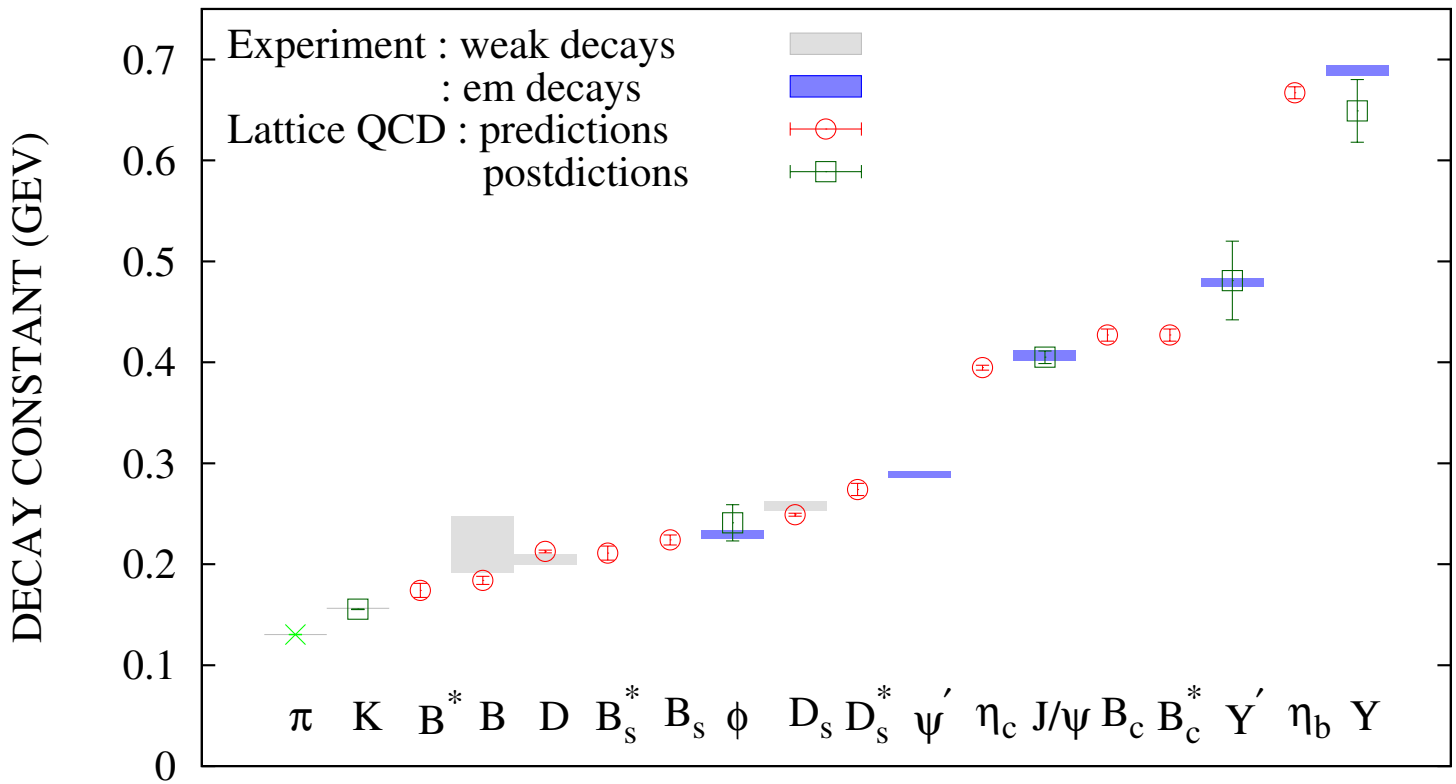


# From $b$ to $c$.

[1302.2644], [1008.4018]



# Decay constants – summary plot.



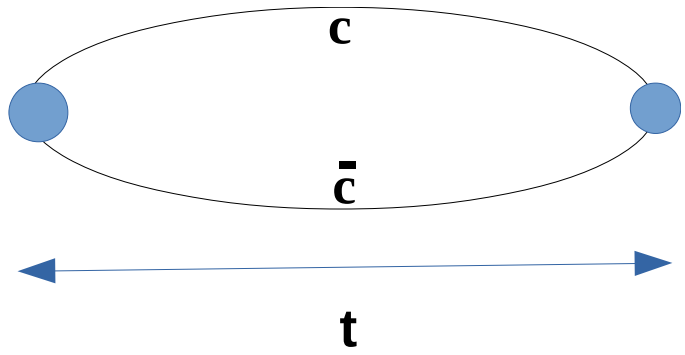
# $B_c$ semileptonic decays

# Semileptonic decays

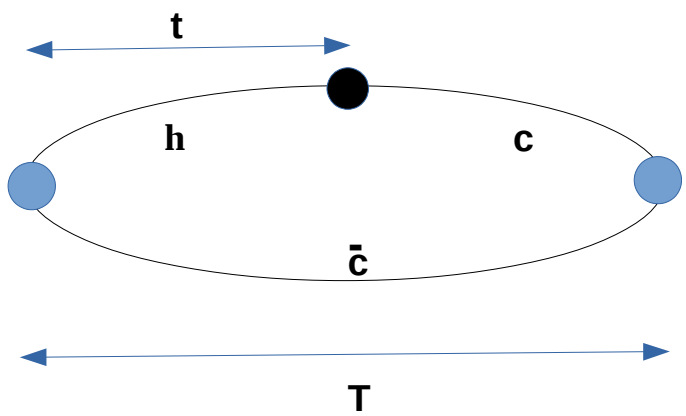
- Study of $B_c \to \eta_c$, $B_c \to J/\psi$ decay matrix elements.
- We work in the frame where the $B_c$ is at rest.
- The form factors which parametrise the matrix elements are functions of $q^2$, where $q$ is the four-momentum transferred to the leptons.
  - $q^2_{\text{max}} = (M - m)^2$, zero recoil of decay hadron.
  - $q^2 = 0$, maximum recoil of decay hadron.
- Matrix elements are determined by simultaneous fitting of three-point and two-point functions.

# Semileptonic decays – meson correlators

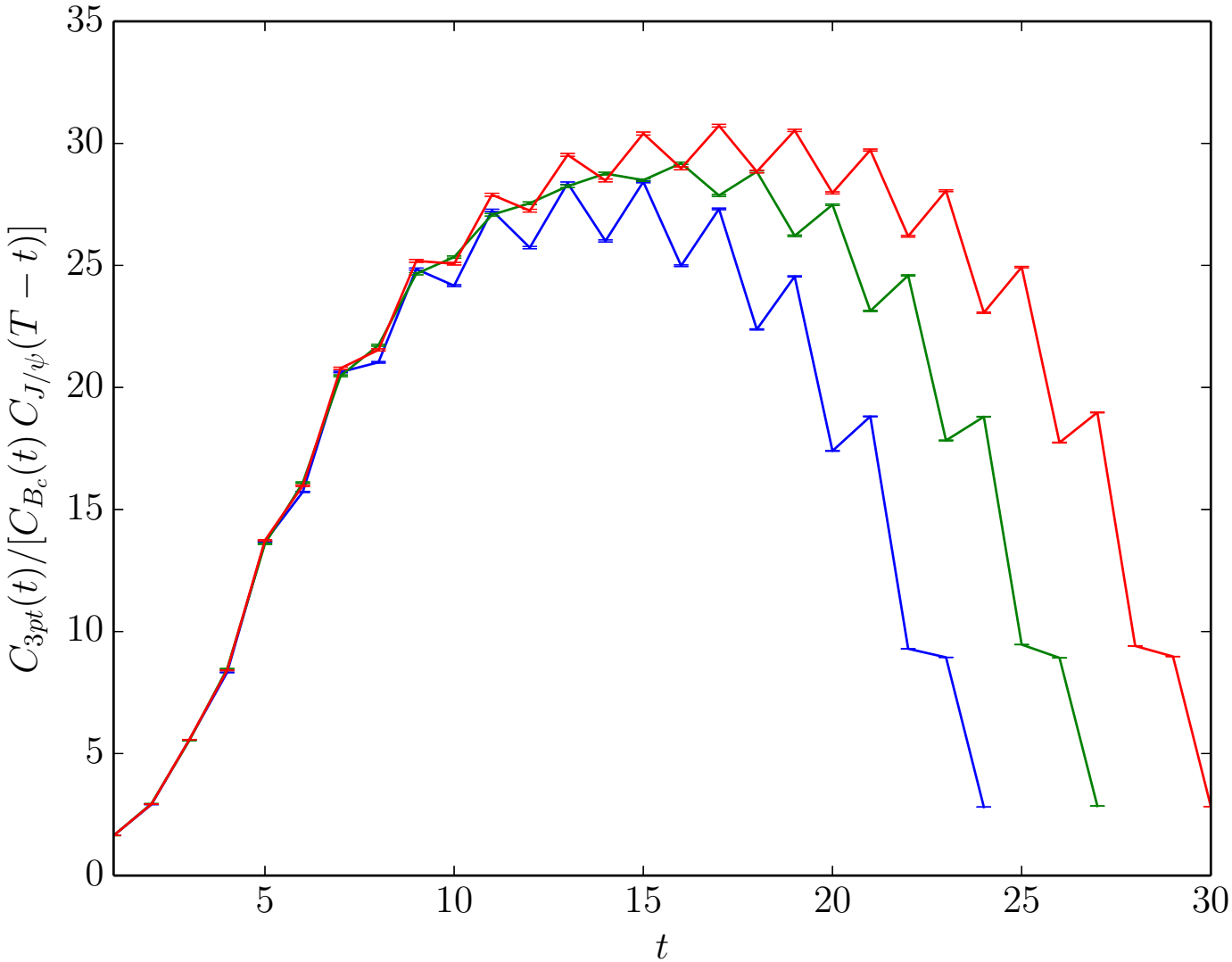
Two-point functions:




Three-point functions:



# Semileptonic decays



# $B_c \to \eta_c$

$$Z\langle \eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\,\frac{M^2-m^2}{q^2}\,q^\mu\,,$$

From PCVC,

$$\langle \eta_c(p)|S|B_c(P)\rangle = \frac{M^2-m^2}{m_{b0}-m_{c0}} f_0(q^2)$$

Find $Z$ by calculating both matrix elements at $q^2_{\rm max}$.

# $B_c \to \eta_c$

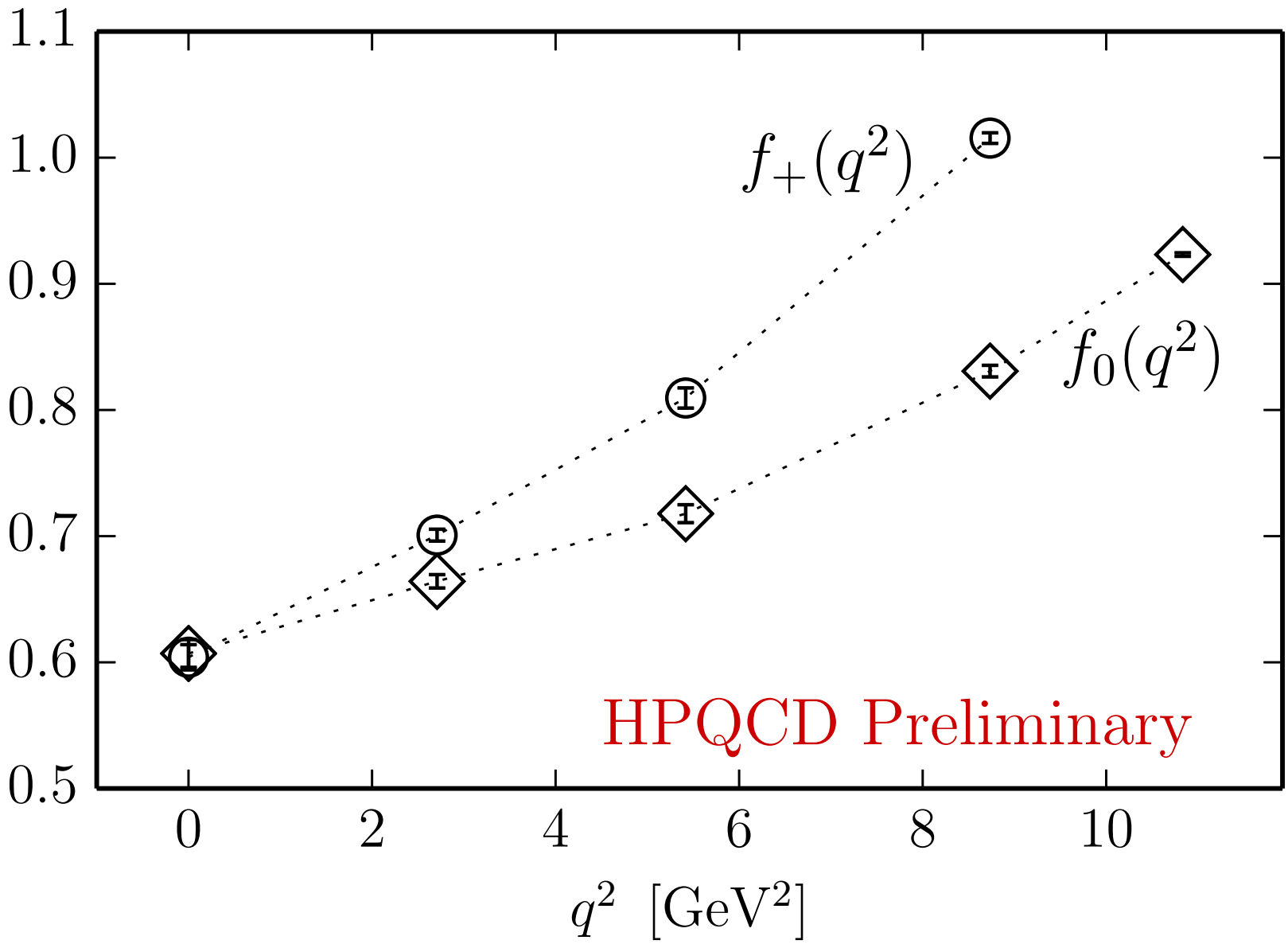
$f_0$ and $f_+$ are determined in the NRQCD formalism from matrix elements of the vector current $\langle V_\mu^{\rm nrqcd} \rangle$, where

$$V_0^{\rm nrqcd} = (1 + \alpha_s z_0^{(0)}) \left[ V_0^{(0)} + (1 + \alpha_s z_0^{(1)}) V_0^{(1)} + \alpha_s z_0^{(2)} V_0^{(2)} \right]$$

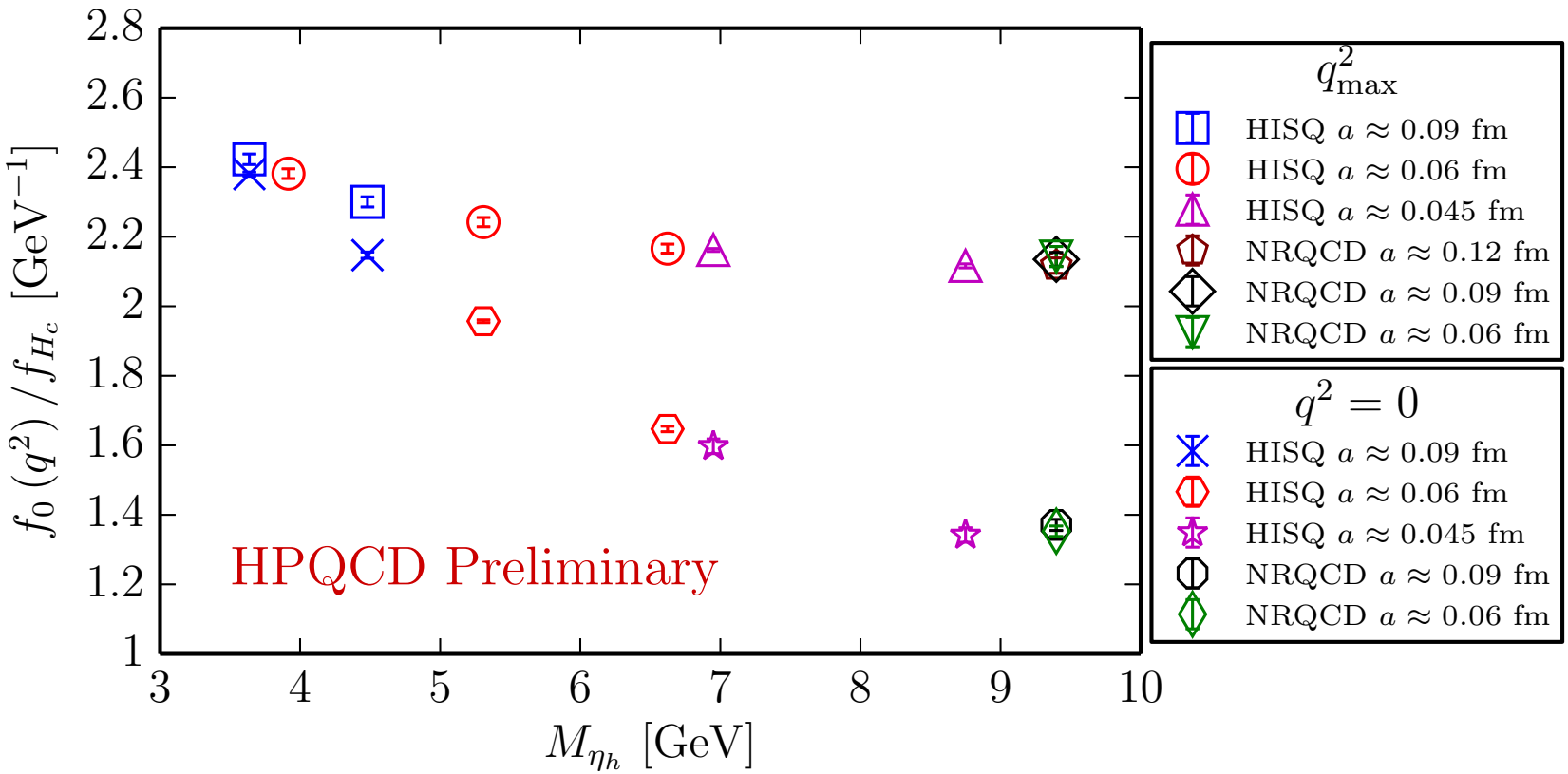
$$V_k^{\rm nrqcd} = (1 + \alpha_s z_k^{(0)}) \Big[ V_k^{(0)} + (1 + \alpha_s z_k^{(1)}) V_k^{(1)} + \alpha_s z_k^{(2)} V_k^{(2)} + \alpha_s z_k^{(3)} V_k^{(3)} + \alpha_s z_k^{(4)} V_k^{(4)} \Big] .$$

One goal of the present work is to constrain the coefficients entering $V_\mu^{\rm nrqcd}$ using fully relativistic HISQ data.

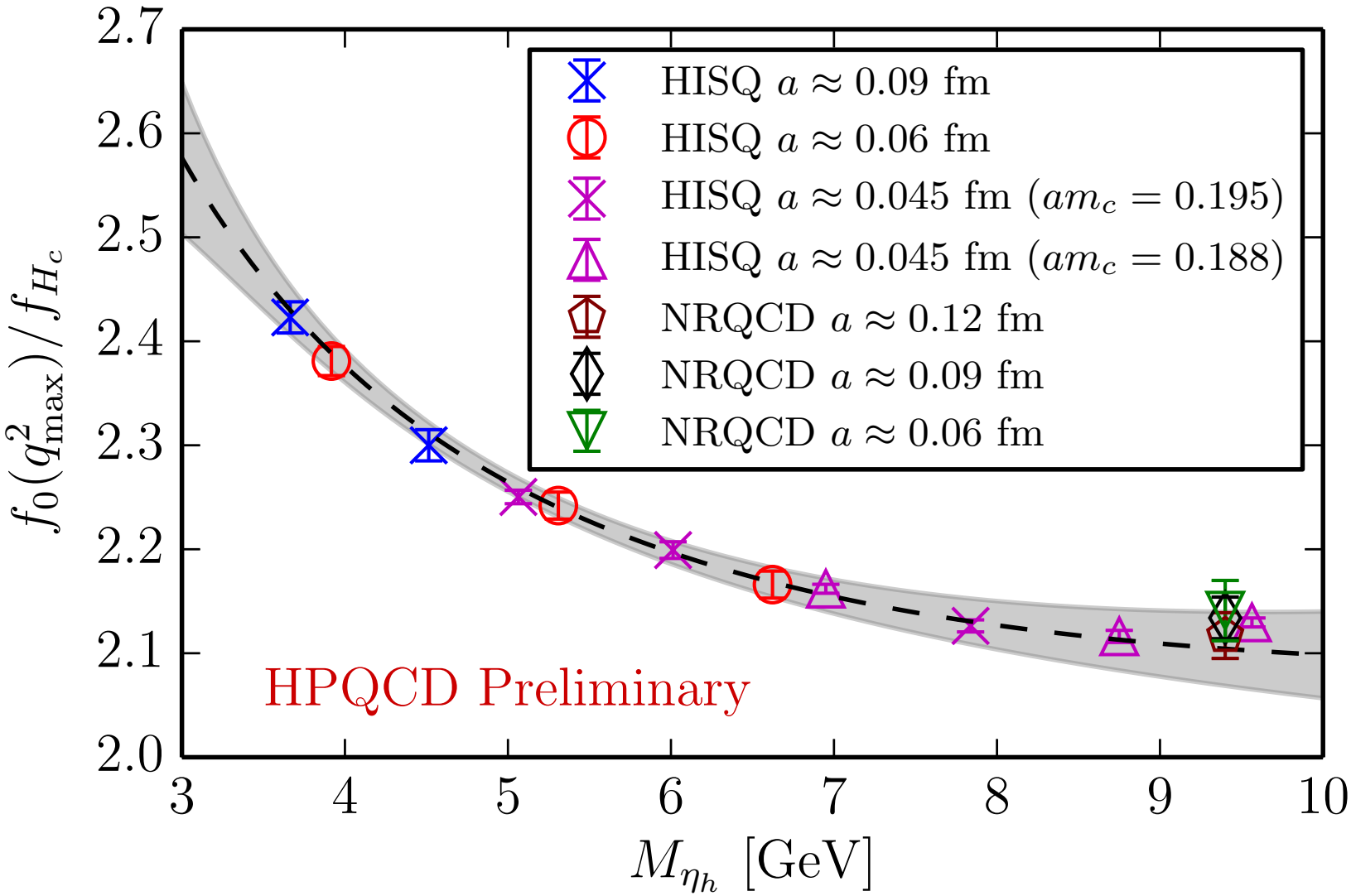
# NRQCD form factors.



# $f_0(q^2)/f_{H_c}$ from HISQ.



# $f_0(q^2_{\text{max}})/f_{H_c}$ from HISQ.



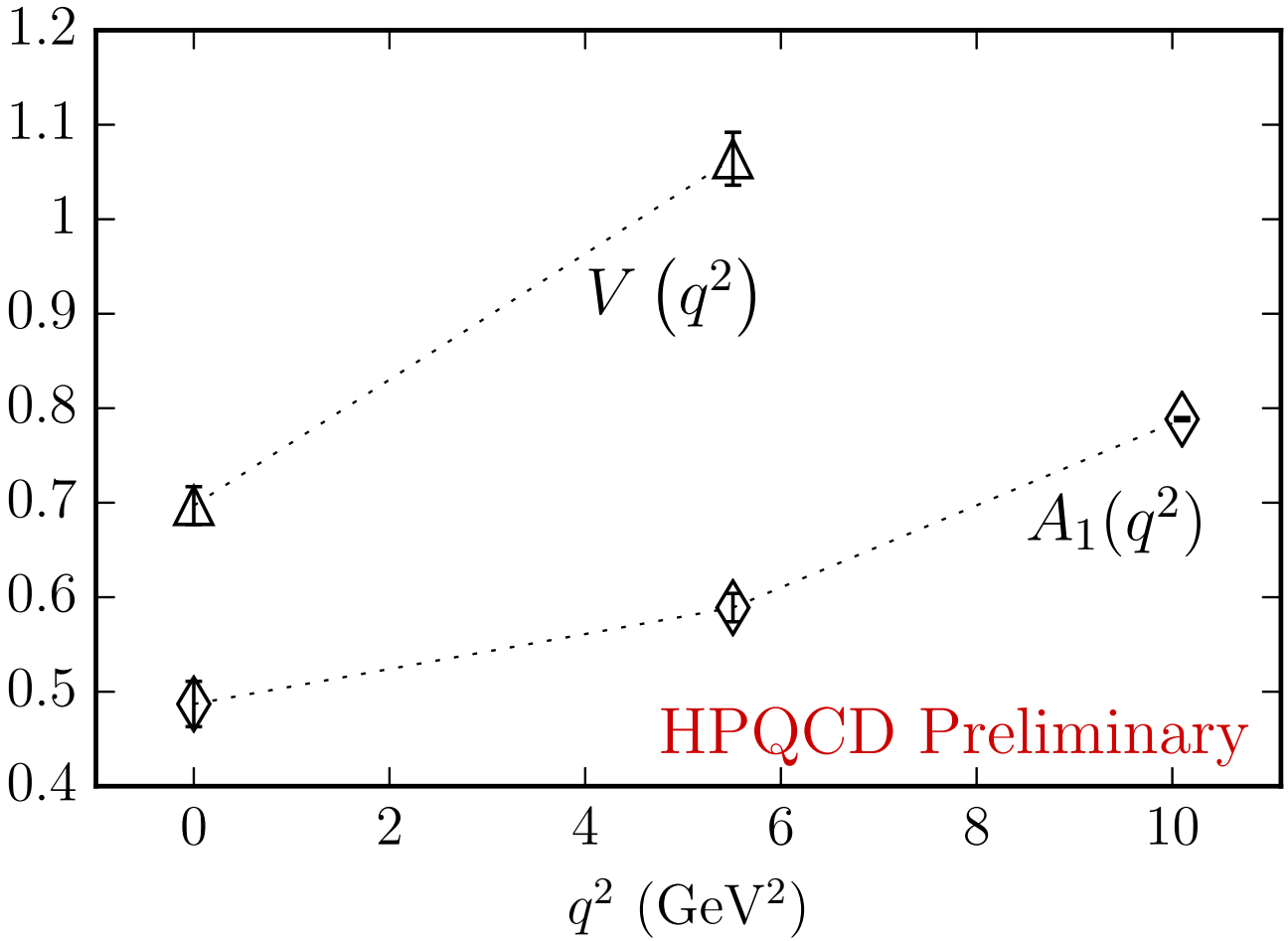
# $B_c \to J/\psi$

$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle =$$

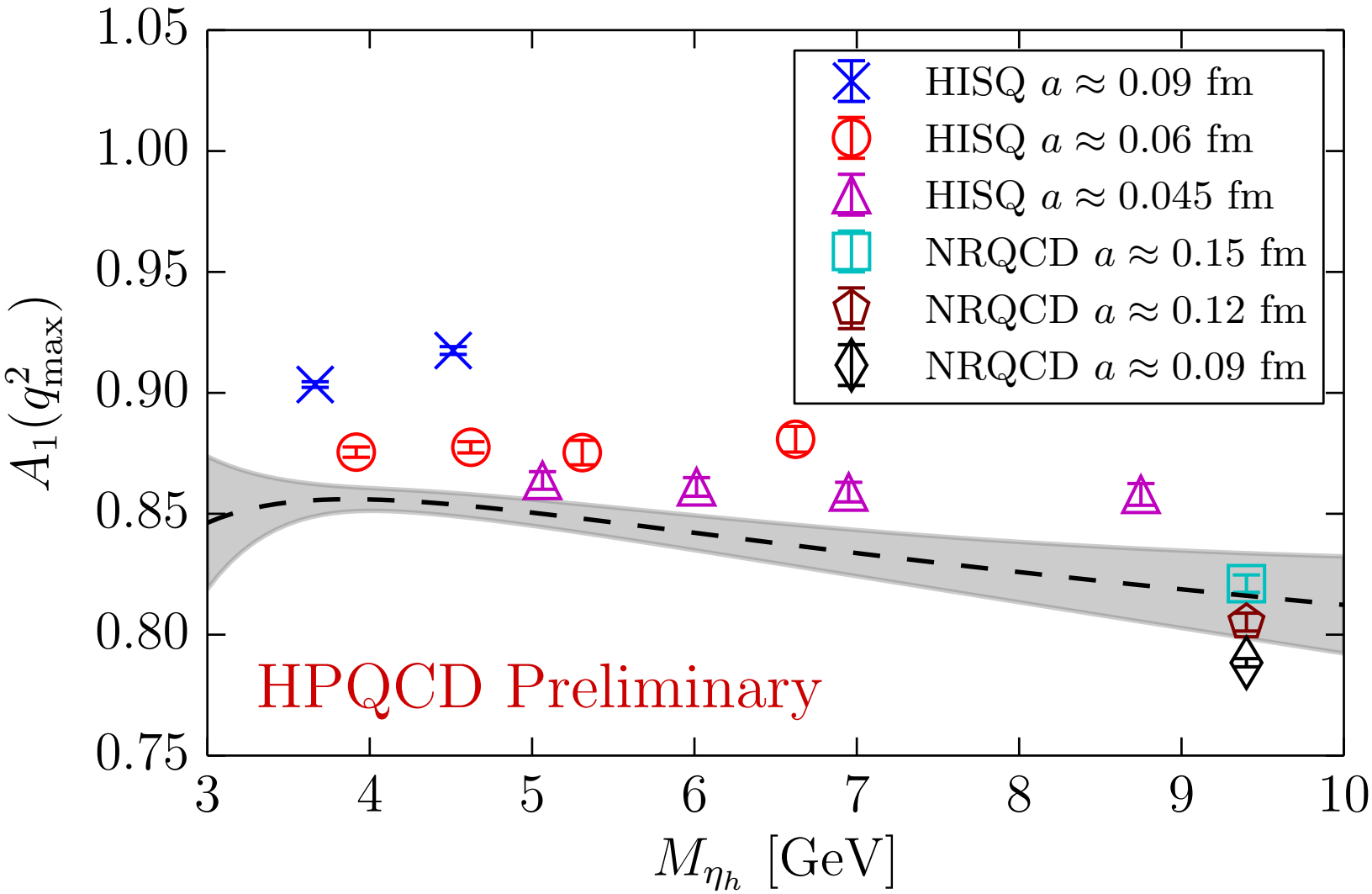
$$\frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon^*_\nu p_\rho P_\sigma V(q^2) - (M+m)\varepsilon^{*\mu}A_1(q^2) +$$

$$\frac{\varepsilon^*\cdot q}{M+m}(p+P)^\mu A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2)$$

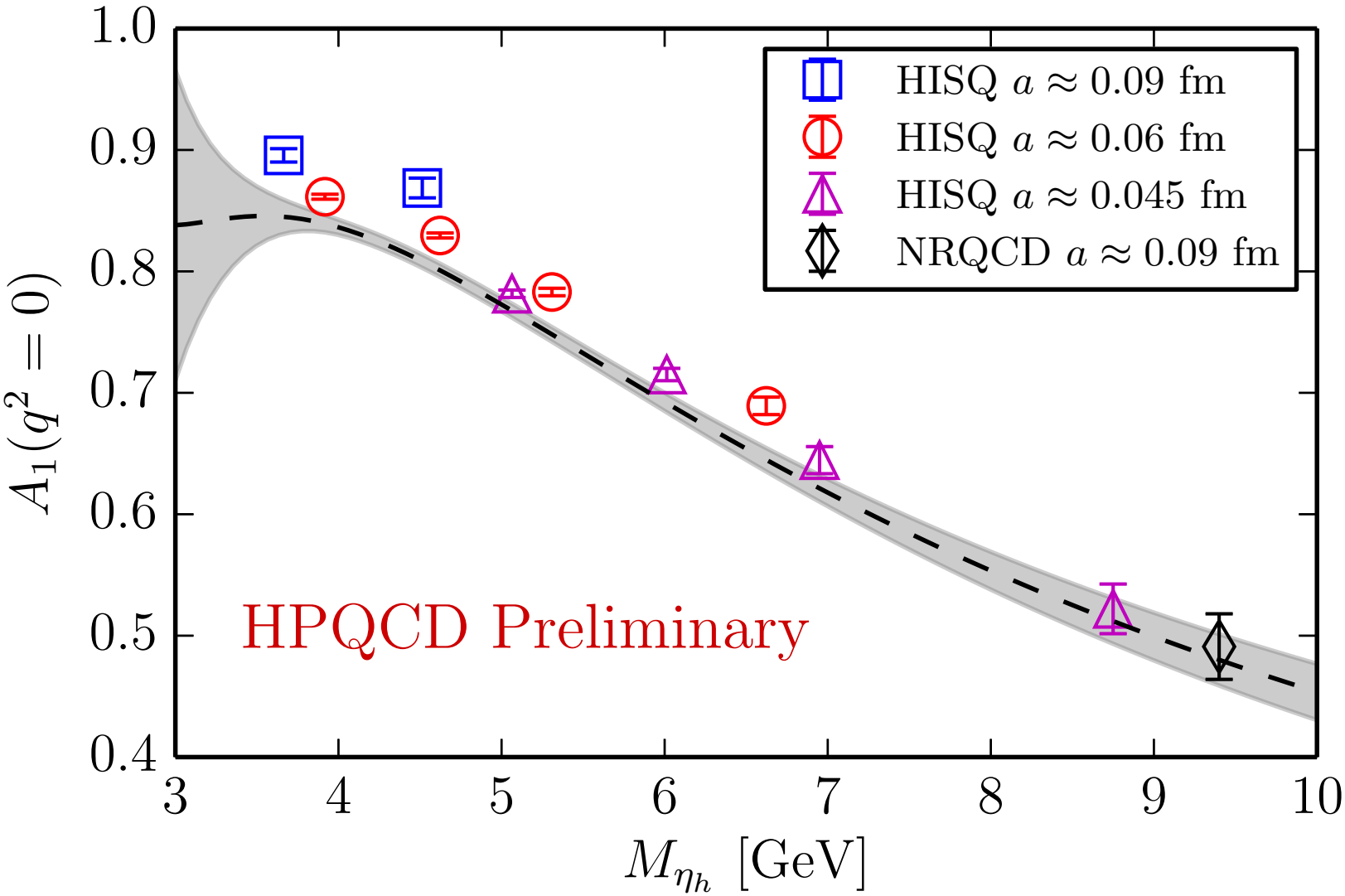
# $B_c \to J/\psi$.


1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
2
4
6
8
10
$q^2$ (GeV$^2$)
$V\,(q^2)$
$A_1(q^2)$
HPQCD Preliminary

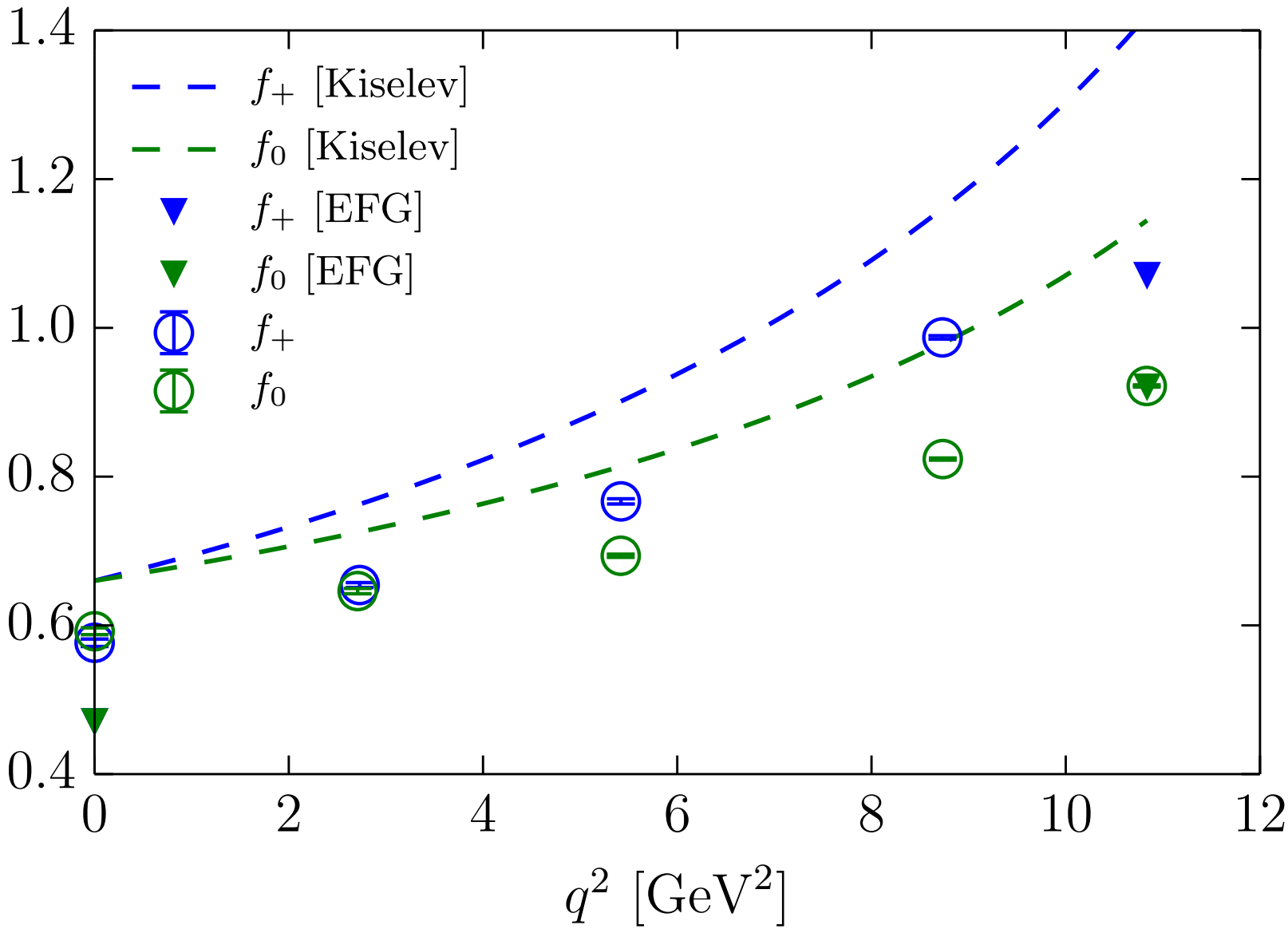
# $A_1(q^2_{\text{max}})$ from HISQ.



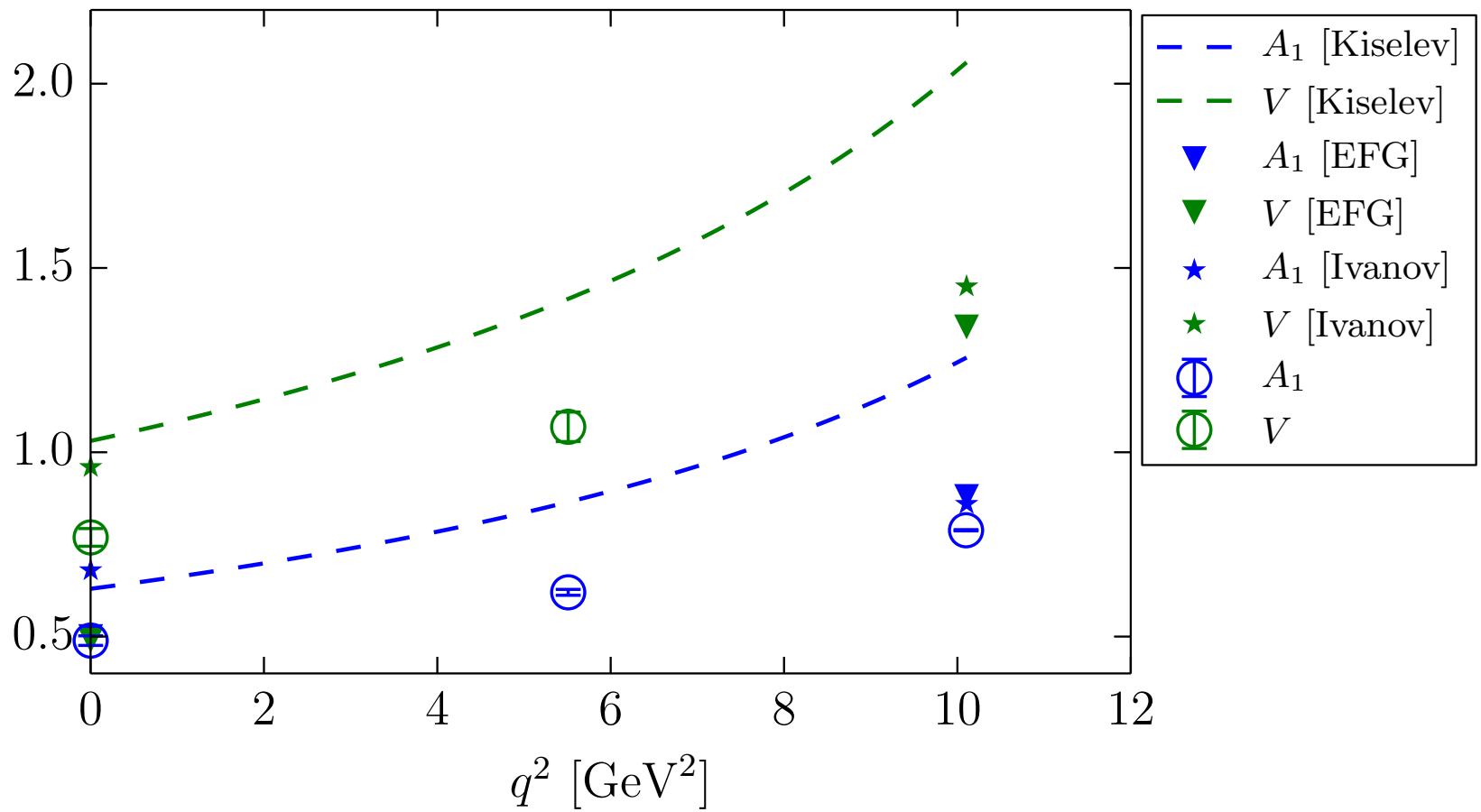
# $A_1(q^2 = 0)$ from HISQ.



# Comparisons $B_c \to \eta_c$ [hep-ph/0211021,0306306]



# Comparisons $B_c \to J/\psi$ [hep-ph/0007169,0211021,0306306]



# Summary

- A promising approach to study of $b \to c$ transitions:
  - Lattice NRQCD with HISQ quarks, plus
  - Fully relativistic formulation, extrapolate $m_h$ to $m_b$.
- Proof-of-principle demonstrated for $f_0$.
  - Controlled calculation over full $q^2$ range.
  - Good agreement seen with NRQCD results.
- Outputs:
  - $B_c$ to $J/\Psi \to$ SM prediction for $R(B_c \to J/\psi)$.
  - Improved understanding of NRQCD currents feeds into additional calculations ($B$ to $D$, $B$ to $D^*$, ...).

Thank you!