Large N scaling and factorization in $\mathrm{SU}(N)$ Yang-Mills gauge theory

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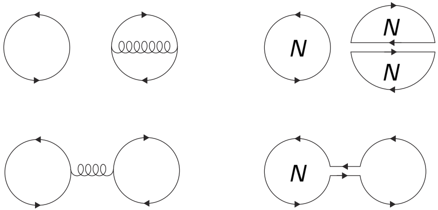
Factorization in large N

In the 't Hooft large N limit of $\mathrm{SU}(N)$ Yang-Mills gauge theory ($N \to \infty$, $\lambda = g^2 N$ fixed)

$$\langle AB \rangle = \langle A \rangle \langle B \rangle + \mathrm{O}(1/N^2)$$

Follows from the large N counting rules order by order in perturbation theory.



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Factorization beyond perturbation theory:

- Makeenko-Migdal equations on the lattice [Makeenko and Migdal (1979)].
- Analogy to classical mechanics [Yaffe (1982)].
- Proof in the strong coupling regime [Jafarov (2016), Chatterjee (2016)].

Why are we interested?

Factorization plays a crucial role in the large N volume reduction which allows to simulate the infinite N theory in a single site lattice model

[Eguchi and Kawai (1982), Gonzalez-Arroyo and Okawa (1983,2010)].

Can we test this non-perturbatively?

Objective

Our goal

Use the lattice to:

1. test the property of factorization
2. check the ‘t Hooft scaling with very high accuracy

Observables: smooth Wilson loops defined through the Yang-Mills gradient flow

$$W^t(R,R) = \frac{1}{N}\mathrm{Tr}\, U^t(R\times R)$$

We can use these observables to test for factorization and large N scaling at finite lattice spacing and in the continuum

$$a\Lambda_L = \left(\frac{48\pi^2}{11\lambda}\right)^{51/121} \exp\left(-\frac{24\pi^2}{11\lambda}\right)\left(1+\mathrm{O}\left(\lambda\right)\right)$$

The Yang-Mills gradient flow

To test factorization, we look at the large N limit of

$$\langle G_W \rangle = \frac{\langle W^2 \rangle - \langle W \rangle^2}{\langle W \rangle^2}$$

In order to have precise renormalized operators with a finite continuum limit we use the Yang-Mills gradient flow

$$\frac{dB^t_\mu(x)}{dt} = D_\nu G^t_{\nu\mu}(x) \,, \quad B^{t=0}_\mu(x) = A_\mu(x)$$

Loops at positive flow time measured with a resolution given by $\sqrt{8t}$ and are free from perimeter and corner divergences

[Lüscher and Weisz (2011), Lohmayer and Neuberger (2012)]

The reference scale t_0

We define a scale for $\mathrm{SU}(N)$ based on the gradient flow coupling

$$\bar{\lambda}_\infty(\mu) = \frac{128\pi^2}{3}\left(\frac{N}{N^2-1}\right) t^2 \left\langle e^t \right\rangle |_{\mu=1/\sqrt{8t}} ,$$

where $\bar{\lambda}_\infty(\mu) = N\bar{g}^2_{\overline{\mathrm{MS}}}(\mu) + c_1\, N^2\bar{g}^4_{\overline{\mathrm{MS}}}(\mu) + \cdots$, and e^t is the Yang-Mills energy density.

In $\mathrm{SU}(3)$, the reference flow time t_0 is defined implicitly by fixing a value of the coupling such that [Lüscher (2010)]

$$t^2\langle e^t\rangle_{t=t_0} = 0.3$$

By looking at the N dependence of $\bar{\lambda}_\infty(\mu)$, for $\mathrm{SU}(N)$ we define the scale t_0 as: [Cè, MGV, Giusti and Schaefer (2016)]

$$t^2\langle e^t\rangle_{t=t_0} = 0.1125\,\frac{(N^2-1)}{N}$$

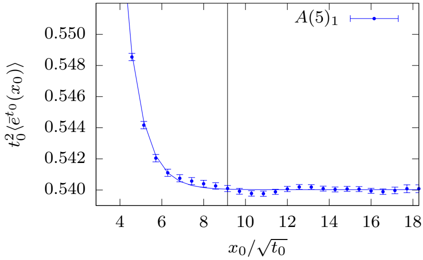
Simulations

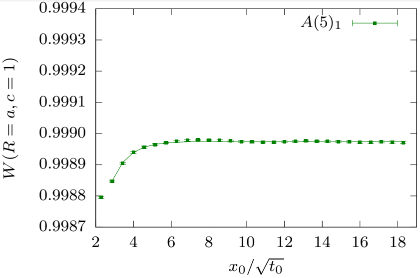
- Gauge groups with $N = 3, 4, 5, 6, 8$.
- Lattice spacing $a \approx 0.1$ fm down to $a \approx 0.05$ fm.
- Open boundary conditions in the time direction.
- $L \approx 1.6$ fm, and additional simulations at $L \approx 2.4$ fm to check for finite volume effects.
- Around 300 independent measurements for all the ensembles.

$$W(R,c) = \left\langle W^{ct_0}(R,R) \right\rangle = \frac{a^4}{(T-2d)\,L^3} \sum_{x_0=d}^{T-d-a} \sum_{\vec{x}} \left\langle W^{ct_0}(\vec{x}, x_0, R, R) \right\rangle$$

$$W^2(R,c) = \left\langle [W^{ct_0}(R,R)]^2 \right\rangle = \frac{a^4}{(T-2d)\,L^3} \sum_{x_0=d}^{T-d-a} \sum_{\vec{x}} \left\langle [W^{ct_0}(\vec{x}, x_0, R, R)]^2 \right\rangle$$

Open boundary conditions





We fit the data to a one excited state contribution from the boundary:

$$f(x_0) = A + Be^{-mx_0}$$

Then:

$$d_e \approx 9.5\sqrt{t_0},$$

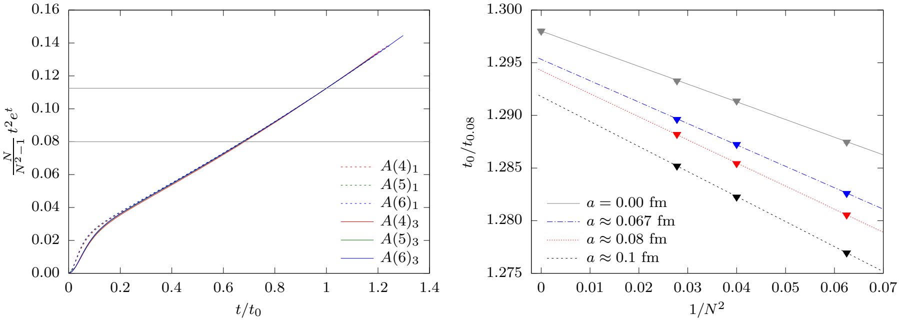
Plateau region:

$$|f(d) - A| < 0.25\sigma$$

$$d_W \approx 8.0\sqrt{t_0}$$

The reference scale t_0

Note the excellent scaling with $1/N^2$



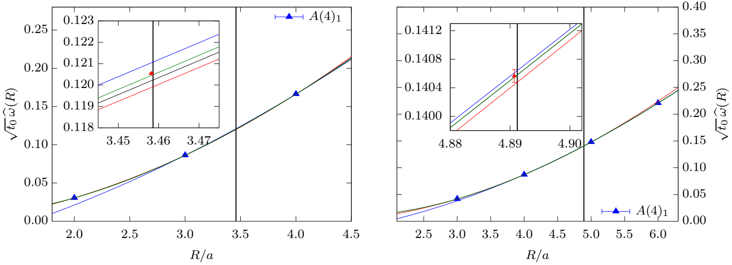
$$t^2 \left\langle e^t \right\rangle \Big|_{t=t_{0.08}} = 0.08 \frac{(N^2-1)}{N}$$

Smooth wilson loops (interpolations)

We match the loops by fixing $R_c = \sqrt{8ct_0}$, with $c = \{1/2, 1, 9/4\}$ and then interpolate to $W(c) \equiv W(R_c, c)$.

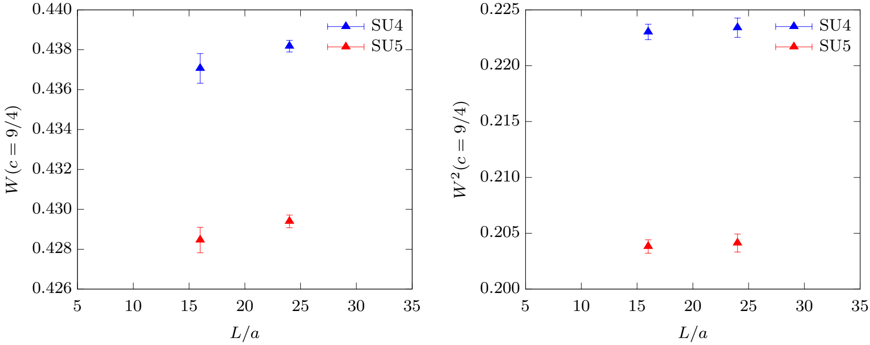
Systematics are included by using several interpolating functions.

The interpolation to R_c is performed in the variable $\hat{\omega}(R) = -\log W(R,c)/R$

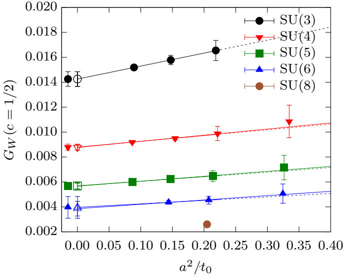


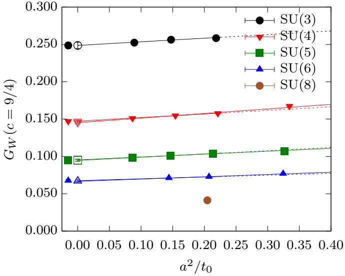
Finite volume checks

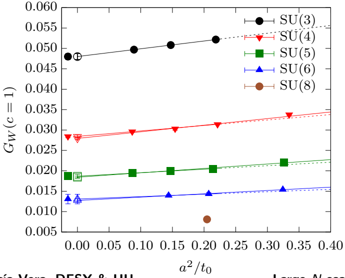
We performed additional simulations at the coarsest lattice spacing and $L/a = 24$ ($L \approx 2.4$ fm).



Continuum limits



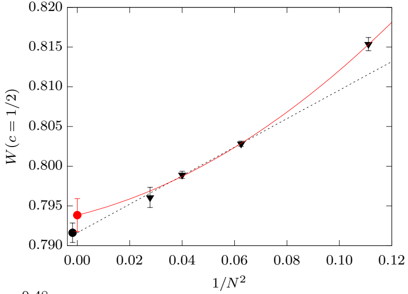


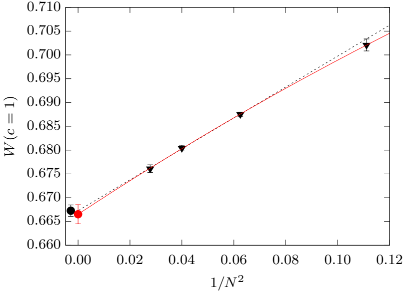


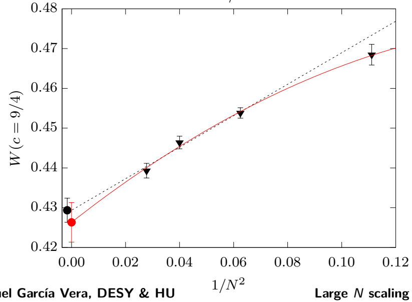
$$\langle G_W \rangle = \frac{\langle W^2 \rangle - \langle W \rangle^2}{\langle W \rangle^2}$$

The continuum limit is taken using only the finest points, i.e., those at $a^2/t_0 < 0.25$. This is compatible with other strategies.

Large N Limit (after the continuum limit)

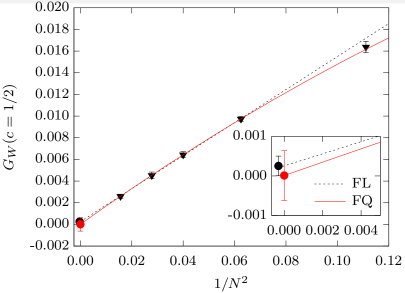


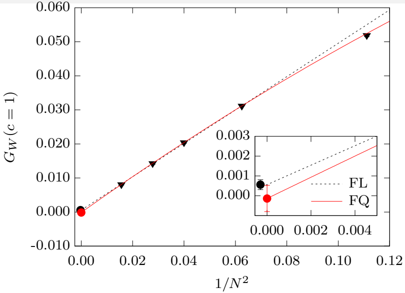


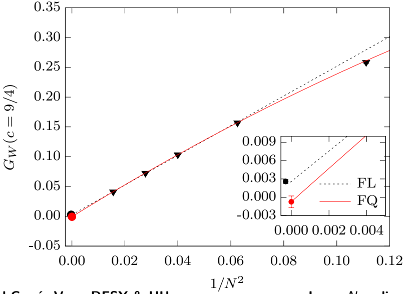


The large N limit is taken using a polynomial function in $1/N^2$. Including SU(3) requires small corrections up to $O(1/N^4)$.

Large N limit (at fixed $t_0/a^2 = 4.782$)



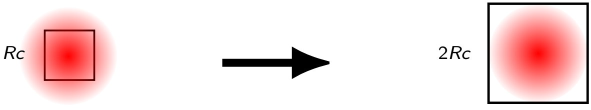




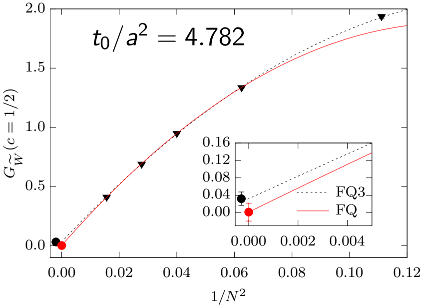
The large N limit of G_W is consistent with factorization. When looking at the data at $a \approx 0.08\,\text{fm}$ the ‘t Hooft scaling works really well up to $\mathrm{SU}(3)$ when $O(1/N^4)$ corrections are included.

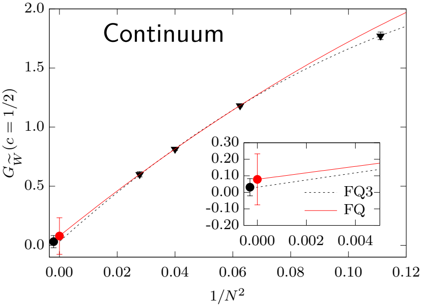
Large N limit

Consider larger loops: $\widetilde{W}(c) = W^{ct_0}(2\sqrt{8ct_0})$



Factorization works equally well:



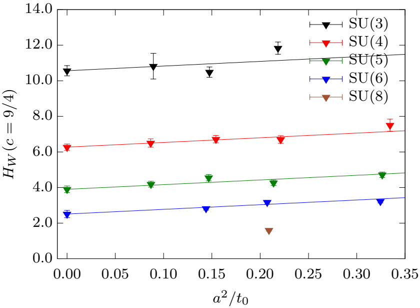


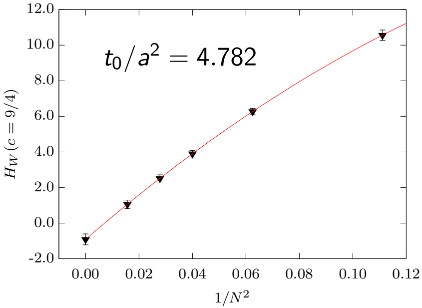
Large N limit

$$W(R,c) = \frac{a^4}{(T-2d)L^3} \sum_{x_0=d}^{T-d-a} \sum_{\vec{x}} W^{ct_0}(\vec{x}, x_0, R, R)\,,$$

$$W_s^2(R,c) = \frac{a}{(T-2d)} \sum_{x_0=d}^{T-d-a} \left(\frac{a^3}{L^3} \sum_{\vec{x}} W^{ct_0}(\vec{x}, x_0, R, R) \right)^2$$

$$H_W = \left(\frac{L^3}{t_0^{3/2}} \right) \frac{\left\langle W_s^2 \right\rangle - \langle W \rangle^2}{\langle W \rangle^2}$$





Conclusions

- For our observables, the validity of the **'t Hooft $1/N^2$ expansion is shown up to very high accuracy**.
- We have presented a **direct non-perturbative verification of factorization**.
- By using the Yang-Mills gradient flow, we are able to test factorization and the large N scaling of Wilson loops both:
  - in the continuum limit,
  - and at finite a.
- The value computed for the large N limit of G_W and H_W is compatible with factorization, and we observe the leading correction $O(1/N^2)$ to provide a good approximation up to SU(4), while small $O(1/N^4)$ corrections are required to include the SU(3) results.
- Non factorizable contributions can be large for $N = 3, 4, 5$; as observed by the fact that $G_{\widetilde{W}} = \mathrm{O}(1)$.

THANK YOU VERY MUCH FOR YOUR ATTENTION