Condensation thresholds and scattering data - a study in the relativistic Bose gas at finite density

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FWF Der Wissenschaftsfonds.

NAWI Graz Natural Sciences

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Introduction

- For many lattice field theories the sign problem poses a serious challenge for simulations density at finite chemical potential
- In some cases a dual formulation with world-lines solves the sign problem and allows one to access finite density physics with ab-initio simulations
- Here we analyze condensation as a function of chemical potential μ in the relativistic Bose gas (charged ϕ^4) field at low temperatures
- At the second condensation threshold one excites 2-particle states and it has been shown for the 2-d O(3) model that the onset of 2-particle condensation is related to scattering data of the theory:

  F. Bruckmann, C. Gattringer, T. Kloiber and T. Sulejmanpasic P.R.L.(2015)
- Here we study the connection between 2-particle condensation and scattering data for the relativistic Bose gas in 4-d

Relativistic Bose Gas standard representation

- In the conventional representation the Lagrangian of the system is:

$$S = \sum_x \left(\eta |\phi_x|^2 + \lambda |\phi_x|^4 - \sum_{\nu=1}^{4} \left[e^{\mu \delta_{\nu,\mathbf{4}}} \phi_x^* \phi_{x+\hat{\nu}} + e^{-\mu \delta_{\nu,\mathbf{4}}} \phi_x^* \phi_{x-\hat{\nu}} \right] \right)$$

$\eta = 8 + m^2$

- The partition sum is:

$$Z = \int \mathcal{D}[\phi] e^{-S} \Rightarrow \mathcal{D}[\phi] = \prod_x \int_{\mathbb{C}} \frac{d\phi_x}{2\pi}$$

- Sign problem for $\mu \neq 0$

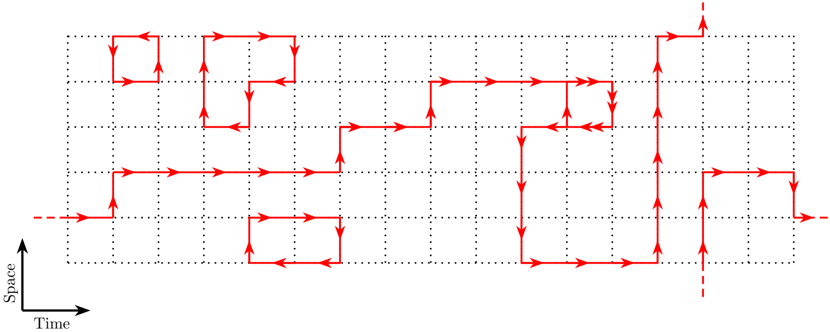
Dual formulation

- The partition function can be rewritten exactly using dual variables:

$$Z = \sum_k e^{\mu N_T \omega[k]} \, W[k] \, \prod_x \delta(\vec{\nabla}\vec{k}_x)$$

- $k_{x,\nu} \in \mathbb{Z}$ constrained flux variables
- $\vec{\nabla}\vec{k}_x = \sum_\nu \left[k_{x,\nu} - k_{x-\hat{\nu},\nu} \right] = 0$ implies flux conservation for every site x
- $\omega[k]$ temporal winding number of flux k
- $W[k]$ positive weight, comes from the integration of $e^{-\eta\phi^2-\lambda\phi^4}$ and contains auxiliary fluxes upgraded with conventional MC

Admissible flux configurations



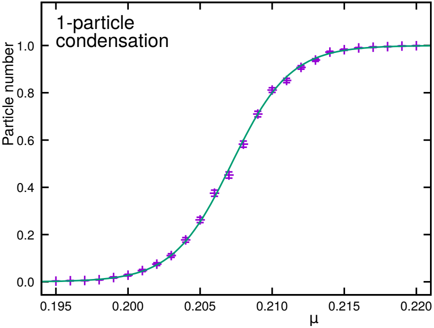
- Update with worm algorithm

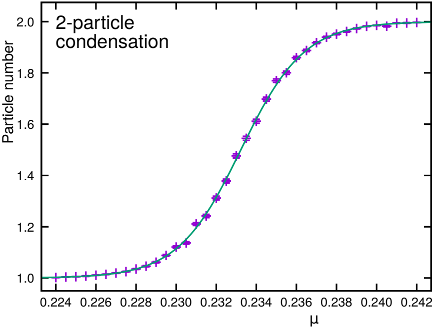
- Particle number: $N = \langle \omega[k] \rangle$

Condensation of charges

- At finite volume and $T = 0$; different particle sectors are separated by finite energy steps

- These can be identified by the values $\mu_1, \mu_2 ...$ of the chemical potential where condensation sets in for the respective sector





Analysis of the threshold

- The system is described by the grand canonical partition sum:

$$Z(\mu) = \mathrm{Tr}\left(e^{-\frac{(\hat{H}-\mu\hat{Q})}{T}}\right) = e^{-\frac{\Omega(\mu)}{T}}$$

- For $T \to 0$, every sector is dominated by the minimum of H in that sector:

$$\Omega(\mu) \xrightarrow{T\to 0} \begin{cases} E_{min}^{Q=0} = 0 & \mu \in [0, \mu_1) \\ E_{min}^{Q=1} - \mu = m - \mu & \mu \in (\mu_1, \mu_2) \\ E_{min}^{Q=2} - 2\mu = W - 2\mu & \mu \in (\mu_2, \mu_3) \end{cases}$$

Where m is the renormalized mass and W is the 2-particle energy

- The first transition occurs at:

$$0 = m - \mu_1 \;\rightarrow\; m = \mu_1$$

- The second transition occurs at:

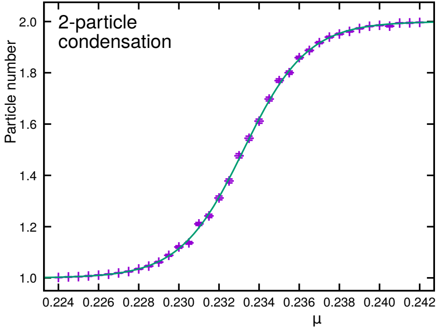
$$m - \mu_2 = W - 2\mu_2 \;\rightarrow\; W = \mu_1 + \mu_2$$

Determination of the threshold values

- Simulation parameters: $\eta = 7.44$; $\lambda = 1.0$; $L = 3, 4, \ldots, 10$; $N_T = 320, 640$

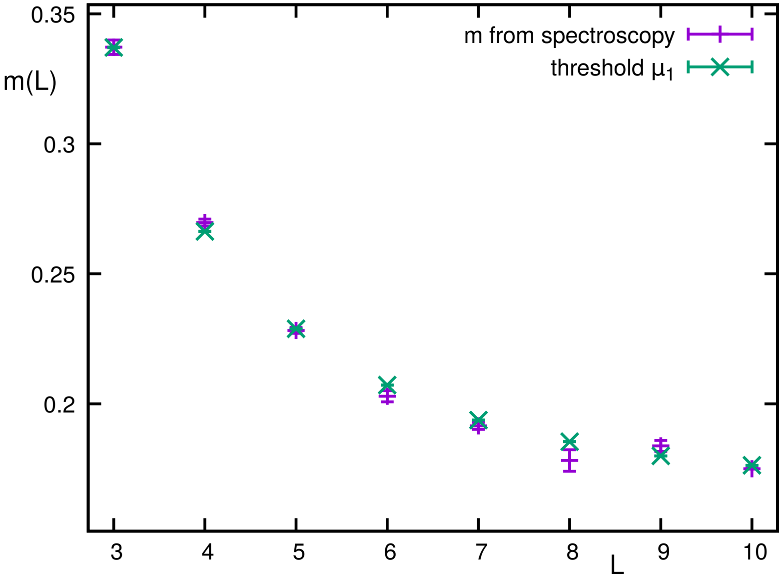
- To find the threshold chemical potentials: $\mu_1, \mu_2, \ldots,$ we fit the particle number near the threshold μ_i with the logistic function:

$$f_i(x) = \frac{1}{1+e^{-k(\mu-\mu_i)}} + (i-1)$$



- We determine the critical values μ_1 and μ_2 for different spatial extents L and thus obtain the mass m(L) and the interaction energy W(L) as a function of L

Consistency check for $\mu_1 = m$



Analysis of volume dependence

- For 1 particle in a box we use:

$$m(L) = m_0 + c\, L^{-\frac{3}{2}} e^{-m_0 x}$$

K. Rummukainen, S. Gottlieb Nuclear Physics B 450 (1995)

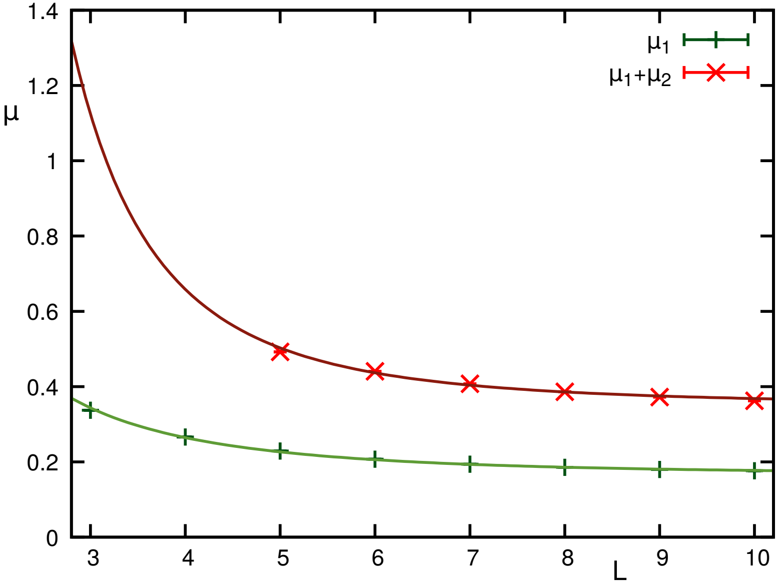
- For 2 particles in a box we use:

$$W(L) = 2m_0 - \frac{4\pi a_0}{m_0 L^3}\left\{1 + c_1 \frac{a_0}{L} + c_2 \frac{a_0^2}{L^2}\right\} + \mathcal{O}\left(\frac{1}{L^6}\right)$$

$$c1 = -2.837297 \quad , \quad c2 = 6.375183$$

Lüscher, M. Commun. Math. Phys. 104 & 105 (1986)

Analysis of volume dependence



Fit results

Very preliminary results:

- $m(L)$:

$$m_0 = 0.168 \pm 0.001$$
$$c = 1.508 \pm 0.025$$

- $W(L)$:

$$m_0 = 0.175 \pm 0.002$$
$$a_0 = -0.232 \pm 0.011$$

- Currently running:

  1. Increase statistics and improve analysis
  2. Control simulation for determination of a_0 from 4-point functions to fully establish the connection of condensation to scattering data

Conclusions

- Worldline representations solve sign problems and allows one to study finite density physics with ab-initio lattice calculations

- Using the relativistic Bose gas, we analyze condensation to the 2-particle sector and study its relation to scattering data

- We extract the volume dependence of the 1- and 2-particle thresholds and use the Lüscher formula for connecting them to the scattering length a_0

- Currently we improve statistics and the analysis for quantitatively establishing the connection of 2-particle condensation and scattering data with a conventional computation of a_0

Thank you for your attention!!!