$\mathcal{N}$=1 supersymmetric Yang-Mills theory on the lattice

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The Model

$\mathcal{N}=1$ SUSY Yang-Mills Theory

Vector supermultiplet:

- Gauge field $A^a_\mu(x)$, $\quad a = 1, \ldots, N_c^2 - 1$, "Gluon"
  Gauge group SU(N_c)
- Majorana-spinor field $\lambda^a(x)$, $\quad \overline{\lambda} = \lambda^T C$, "Gluino"
  adjoint representation: $\mathcal{D}_\mu \lambda^a = \partial_\mu \lambda^a + g\, f_{abc} A^b_\mu \lambda^c$
- (auxiliary field $D^a(x)$)

Lagrangean

$$\mathcal{L} = \int d^2\theta\, \mathrm{Tr}(W^A W_A) + \mathrm{h.c.} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} \overline{\lambda}^a \gamma_\mu (\mathcal{D}_\mu \lambda)^a + \frac{1}{2} D^a D^a$$

SUSY: (on-shell) $\quad \delta A^a_\mu = -2\,\mathrm{i}\,\overline{\lambda}^a \gamma_\mu \varepsilon\,, \quad \delta\lambda^a = -\sigma_{\mu\nu} F^a_{\mu\nu} \varepsilon$

- Simplest model with SUSY and local gauge invariance
- Part of the supersymmetrically extended standard model
- Similar to QCD

  Differences: λ: 1) Majorana, "$N_f = \frac{1}{2}$"
  2) adjoint representation of SU(N_c)
- Gluino mass term $m_{\tilde g}\, \overline{\lambda}^a \lambda^a$ breaks SUSY softly.

Motivation

Non-perturbative Problems

- Spectrum of bound states → Supermultiplets
- SUSY restoration on the lattice
- Check predictions from effective Lagrangeans (Veneziano, Yankielowicz, ...)
- and more (breaking of chiral symmetry, gluino condensate, static confinement, finite temperatures, . . .)

Spectrum of bound states

Expect colour neutral bound states of gluons and gluinos
→ Supermultiplets

Predictions from effective Lagrangeans:
Chiral supermultiplet (Veneziano, Yankielowicz)

- 0^- gluinoball $\quad$ a - η' $\quad \sim \quad \overline{\lambda}\gamma_5\lambda$
- 0^+ gluinoball $\quad$ a - f_0 $\quad \sim \quad \overline{\lambda}\lambda$
- spin $\frac{1}{2}$ gluino-glueball $\quad \sim \quad \sigma_{\mu\nu}\,\mathrm{Tr}\,(F_{\mu\nu}\lambda)$

Generalisation (Farrar, Gabadadze, Schwetz):
additional chiral supermultiplet

- 0^- glueball, $\quad 0^+$ glueball, $\quad$ gluino-glueball

possible mixing

SUSY on the Lattice

Lattice breaks SUSY. Restoration in the continuum limit?
Curci, Veneziano: use Wilson action, search for continuum limit with SUSY

$$S = -\frac{\beta}{N_c} \sum_p \mathrm{Re\,Tr}\, U_p$$

$$+\frac{1}{2}\sum_x \left\{ \overline{\lambda}^a_x \lambda^a_x - \kappa \sum_{\mu=1}^{4} \left[\overline{\lambda}^a_{x+\hat\mu} V_{ab,x\mu} (1+\gamma_\mu) \lambda^b_x + \overline{\lambda}^a_x V^t_{ab,x\mu} (1-\gamma_\mu) \lambda^b_{x+\hat\mu} \right] \right\}$$

$\beta = \frac{2N_c}{g^2}, \quad \kappa = \frac{1}{2m_0 + 8}$ hopping parameter, $\quad m_0$: bare gluino mass

$V_{ab,x\mu} = 2\,\mathrm{Tr}\,(U^\dagger_{x\mu} T_a U_{x\mu} T_b)$, $\quad$ adjoint link variables

We study gauge groups SU(2) and SU(3).

Simulation

Fermion integration $\quad \int [d\lambda]\, \mathrm{e}^{-\frac{1}{2}\overline{\lambda}Q\lambda} = \mathrm{Pf}(CQ) = \pm\sqrt{\det Q}$
Pfaffian $\sim$ effectively $N_f = \frac{1}{2}$ fermion flavours.
Include sign Pf(M) in the observables.

- Two-Step Polynomial Hybrid Monte Carlo algorithm (TS-PHMC)
  Frezzotti, Jansen; Montvay, Scholz
  very efficient, $\tau < 10$ at smallest $m_{\tilde g}$
- Rational Hybrid Monte Carlo algorithm (RHMC)

Monitoring of sign Pf(M)

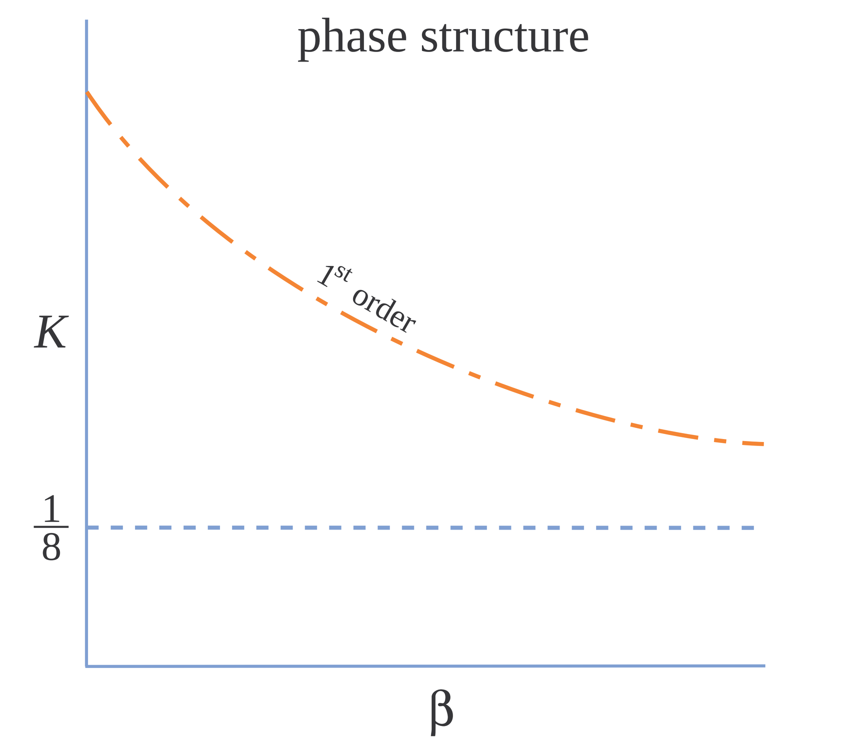
- through spectral flow
- by calculation of real negative eigenvalues of Q with Arnoldi

→ Negative Pfaffians occur in our simulations near κ_c, but rarely.

Chiral limit

Phase transition

Anomaly breaks $\quad \mathrm{U}(1)_\lambda \to Z_{2N_c}$
Spontaneous breaking of $\quad Z_{2N_c} \to Z_2$
by Gluino condensate $< \lambda\lambda > \neq 0$
↔ first order phase transition at $m_{\tilde g} = 0$



Line $\kappa = \kappa_c(\beta)$: first order phase transition at zero gluino mass.

SUSY Ward identities (Ali)

SUSY transformations on the lattice:

$$\delta U_\mu(x) = -\frac{\mathrm{i}g_0 a}{2}\left(\bar\varepsilon(x)\gamma_\mu U_\mu(x)\lambda(x) + \bar\varepsilon(x+\hat\mu)\gamma_\mu\lambda(x+\hat\mu)U_\mu(x)\right)$$
$$\delta\lambda(x) = \frac{1}{2} P^{(cl)}_{\mu\nu}(x)\sigma_{\mu\nu}\epsilon(x)$$

Renormalised SUSY Ward identities:

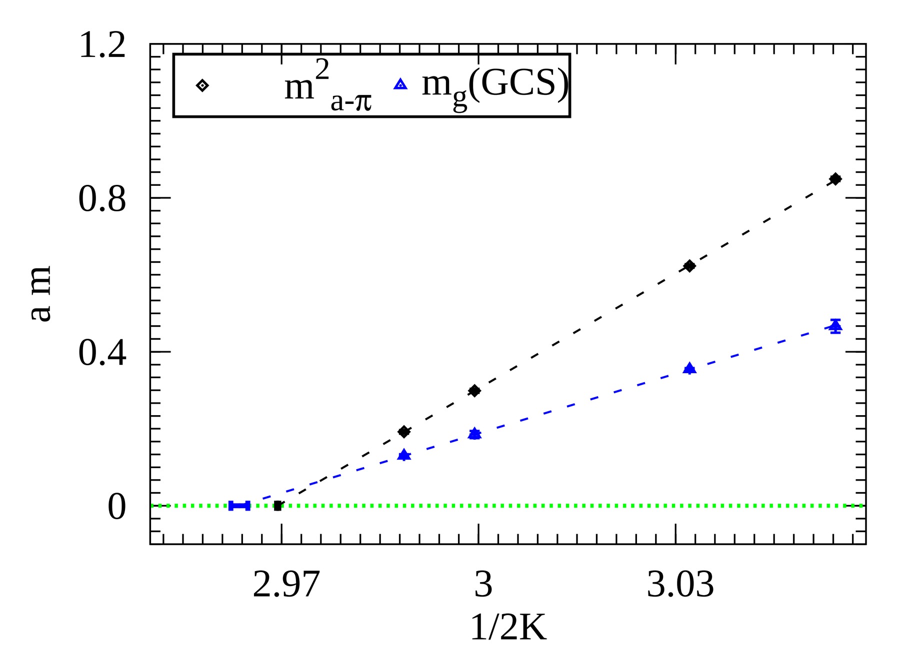
$$\left\langle (\nabla_\mu S_\mu(x)) Q(y) \right\rangle + \frac{Z_T}{Z_S}\left\langle (\nabla_\mu T_\mu(x)) Q(y) \right\rangle = \frac{m_{\tilde g}}{Z_S}\left\langle \chi(x) Q(y) \right\rangle + O(a)$$

Supercurrent $S_\mu(x) = -\sum_{\rho\nu} \sigma_{\rho\nu}\gamma_\mu \mathrm{Tr}\big(P^{(cl)}_{\rho\nu}(x)\lambda(x)\big)$,
renormalisation coefficients Z_S and Z_T
→ set of independent equations for time-slice correlation functions:

$$C^{(S,\mathcal{O})}_{\mathbf{1}}(t) + (Z_T Z_S^{-1}) C^{(T,\mathcal{O})}_{\mathbf{1}}(t) = (am_{\tilde g} Z_S^{-1}) C^{(\chi,\mathcal{O})}_{\mathbf{1}}(t)$$
$$C^{(S,\mathcal{O})}_{\gamma_0}(t) + (Z_T Z_S^{-1}) C^{(T,\mathcal{O})}_{\gamma_0}(t) = (am_{\tilde g} Z_S^{-1}) C^{(\chi,\mathcal{O})}_{\gamma_0}(t)$$

Solution, using $t = 3$ to $T/2$ by generalised least squares method with correlations ⟶ estimates for $am_{\tilde g} Z_S^{-1}$

Phase transition point for SU(3)



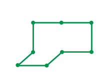
SUSY Ward identities $\qquad$ OZI-arguments, PQChPT

renormalised gluino mass

$$am_{\tilde g} Z_S^{-1} = \frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right) \qquad (am_{a-\pi})^2 \simeq A\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)$$

Particle spectrum

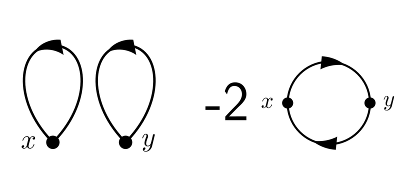
Bound states

Glueballs: $\quad 0^+, \quad 0^- \cong$ 

Gluino-glueballs, $\quad$ Spin $\frac{1}{2}$ $\quad$ Majorana: $\quad \chi_\alpha \simeq \frac{1}{2} F^a_{\mu\nu} (\sigma_{\mu\nu})_{\alpha\beta} \lambda^a_\beta$

Gluino-balls: $\quad \overline{\lambda}\gamma_5\lambda$: a - η', 0^-, $\quad \overline{\lambda}\lambda$: a - f_0, 0^+

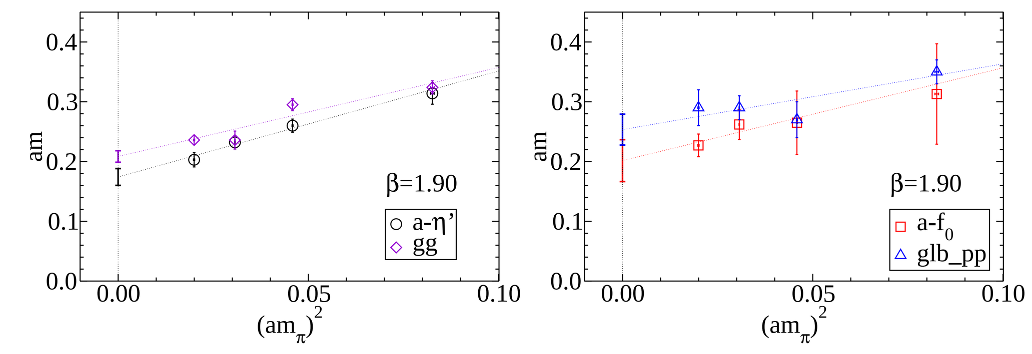
Correlators of mesons have disconnected pieces, which are numerically demanding and require sophisticated techniques.



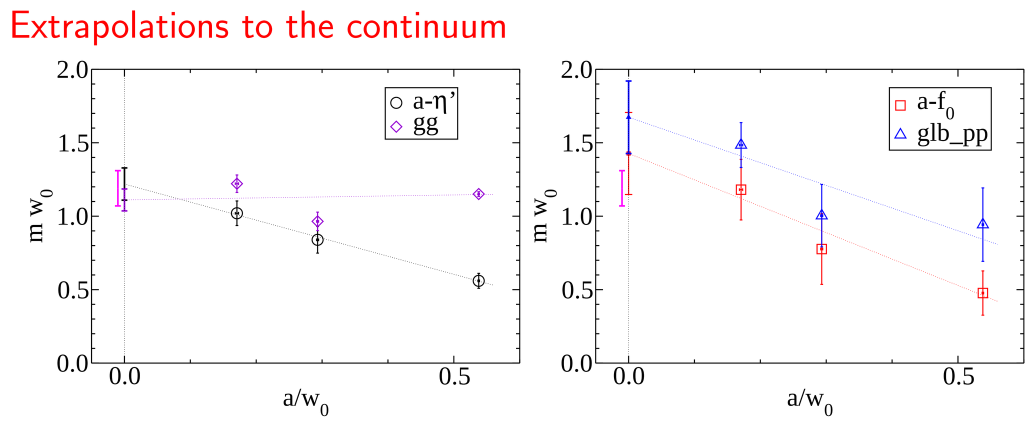
Spectrum for SU(2)

Lattices $\quad 16^3 \cdot 32, \quad 24^3 \cdot 48, \quad 32^3 \cdot 64, \qquad$ Stout links
$\beta = 1.6, \quad a \sim 0.087$ fm, $\quad L \geq 2$ fm
$\beta = 1.75, \quad a \sim 0.054$fm, $\quad L = 1.29$ fm, (1.69 fm)
$\beta = 1.9, \quad a \sim 0.036$fm, $\quad L = 1.15$ fm
(QCD units: $r_0 = 0.5$ fm)

Extrapolations to $m_{\tilde g} = 0$



Extrapolations to the continuum



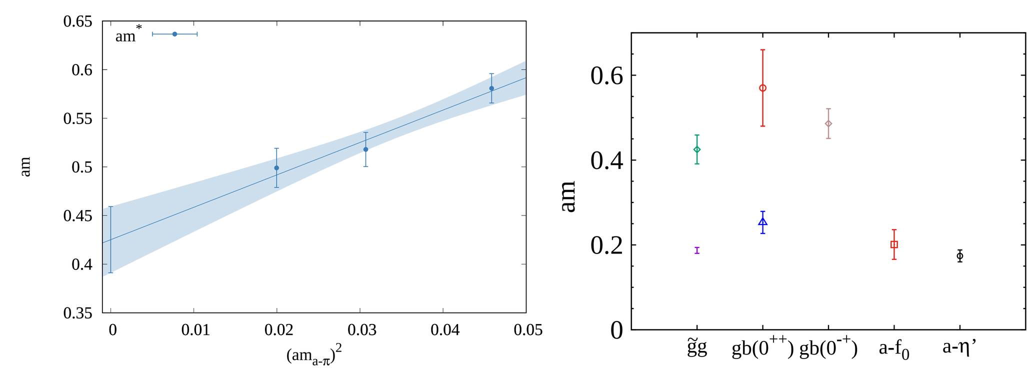
a−η′	a−f_0	$\tilde g g$	glueball 0^{++}
1.06(10)	1.25(24)	0.97(6)	1.46(22)

Bound state masses in units of GeV (QCD units)

Excited states (Kuberski, Gerber)

SU(2), $\beta = 1.9$

Gluino-glue extrapolations to $m_{\tilde g} = 0$ $\qquad$ Mass spectrum (preliminary)



Spectrum for SU(3) (Bergner,...)

Clover improved fermions
$\beta = 5.5, \quad$ Lattice $16^3 \cdot 32, \quad a \sim 0.08$ fm, $\quad L \sim 1.3$ fm
$\beta = \ldots, \quad a \sim 0.04$ fm
(QCD units: $r_0 = 0.5$ fm)

Preliminary results: see Georg Bergner's talk

Summary

Status

- Finite size effects are sufficiently small for $L > 1.2$ fm
- Efficient algorithm: TS-PHMC, RHMC
- Consistency with SUSY Ward identities
- Quantitative results about the low-energy spectrum
- Extrapolations towards vanishing gluino mass
- Extrapolation to continuum
- Results for SU(2) are consistent with the formation of degenerate supermultiplets

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