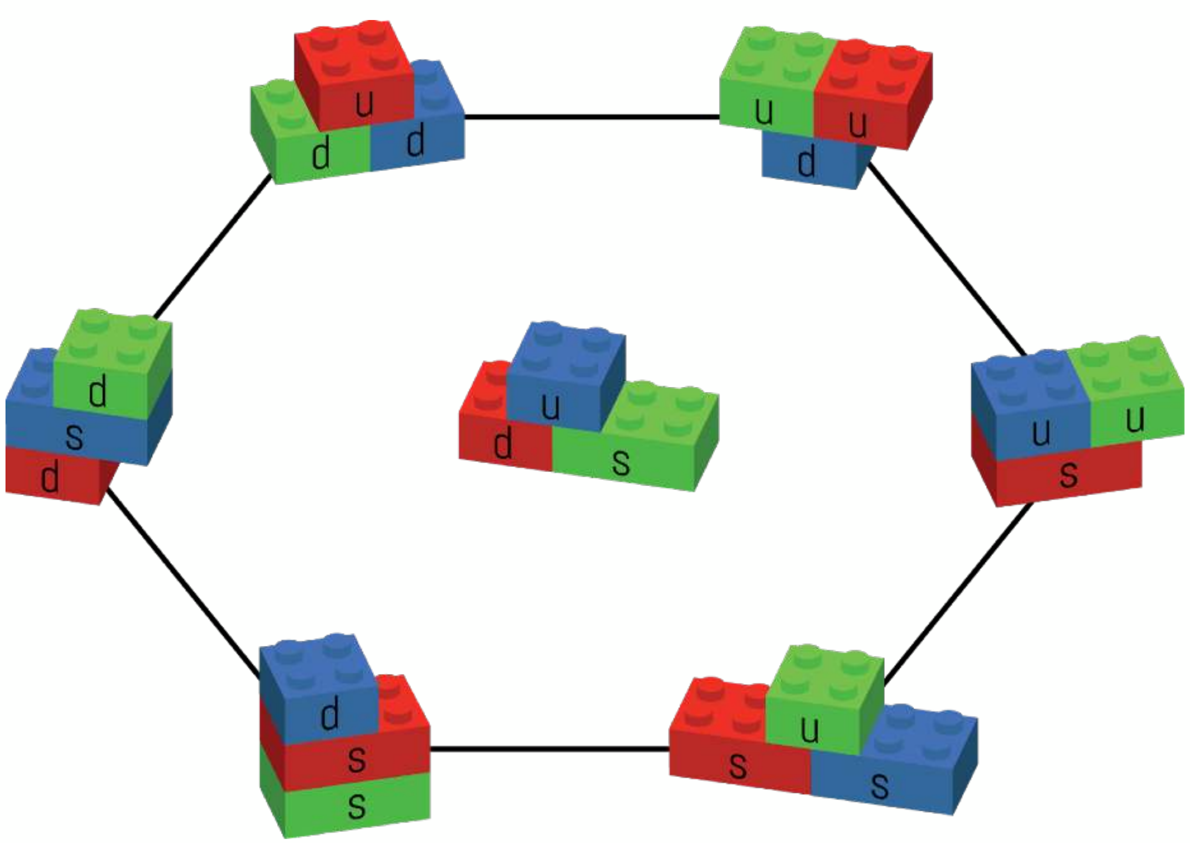
Nuclear Physics Lattice QCD

Parreño
Savage
Tiburzi
Wilhelm
Chang
Detmold
Orginos
(*NPLQCD*)
arXiv:1609.03985

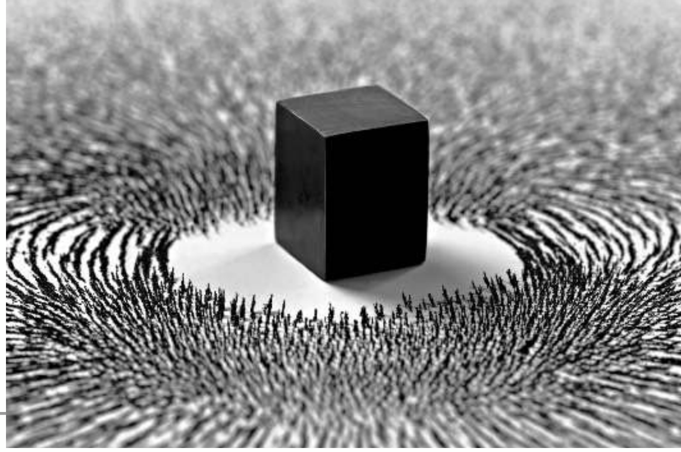
Baryon Magnetic Moments : Symmetries and Relations



B C Tiburzi
21 June 2017

The City College of New York, CUNY

Magnetic Highlights



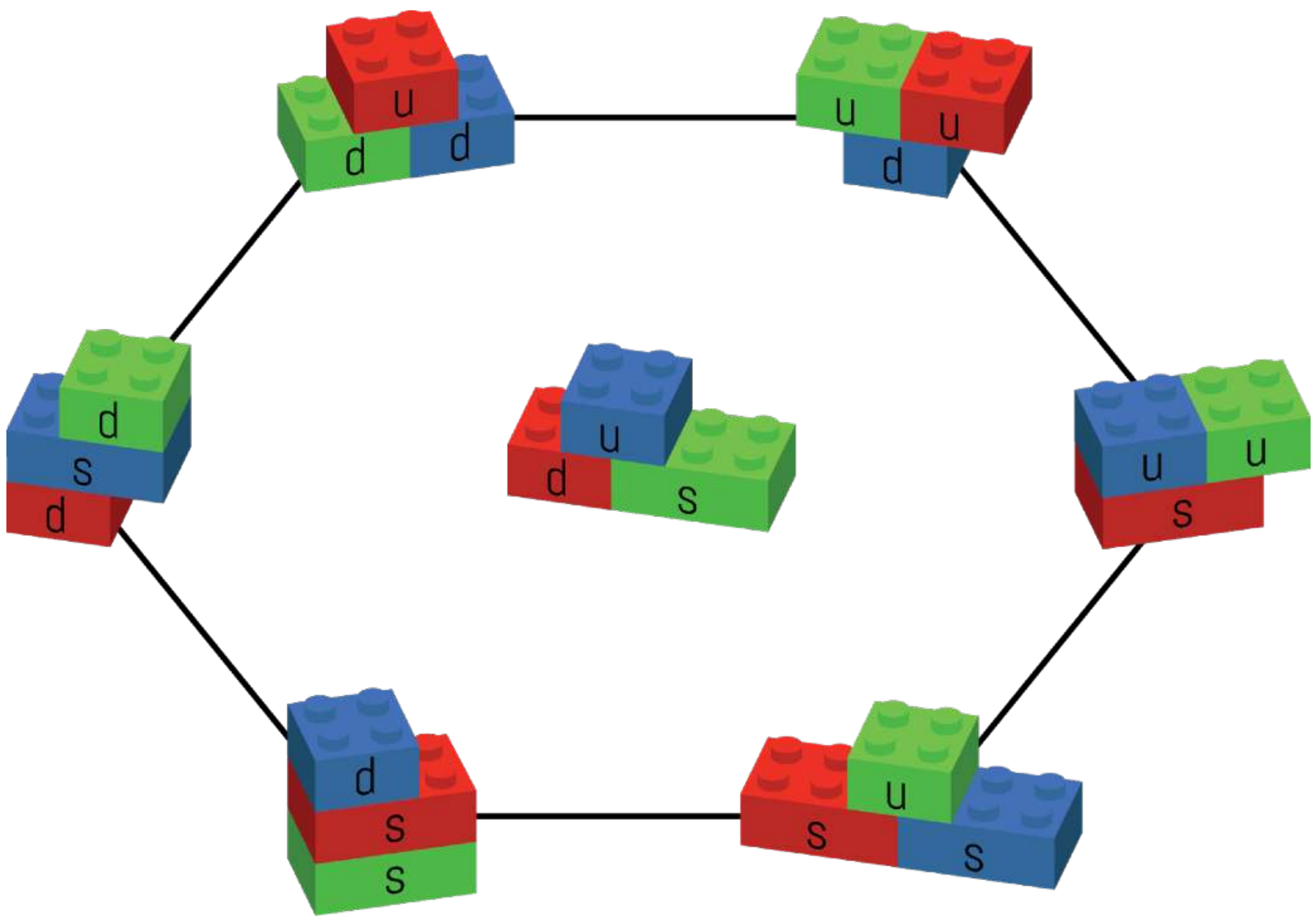
- Magnetic moments and polarizabilities of light nuclei — Phys.Rev.Lett.**113** (2014), Phys.Rev. D**92** (2015)
- Simplest nuclear reaction (M1 transition) $n + p \to d + \gamma$ — Phys.Rev.Lett.**115** (2015)
- Hints of unitary NN interactions in large magnetic fields — Phys.Rev.Lett.**116** (2016)

First computations of their kind made possible by
i) external field techniques
ii) rather large quark masses

- **This talk**: extra curricular look at octet baryon magnetic moments

Strong interactions in unphysical environments, *e.g.* $\mathbf{m_u = m_d = m_s}$

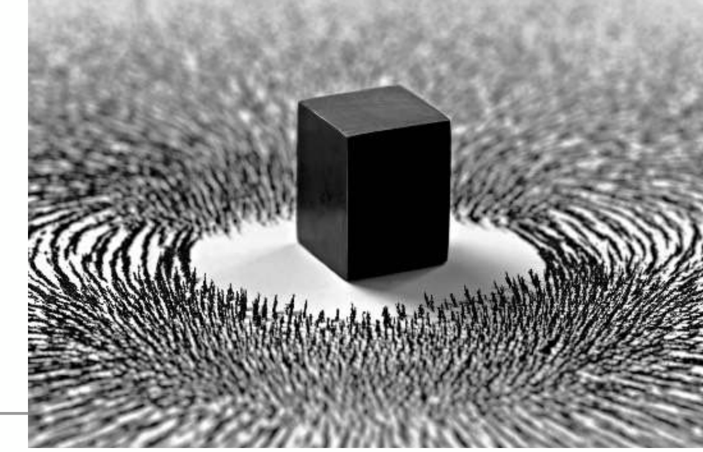
Magnetic Moments of Octet Baryons



- Brief details of calculation
- Magneton units
- Coleman-Glashow moments
- Quark model vs. large N_c
- Lambda-Sigma mixing

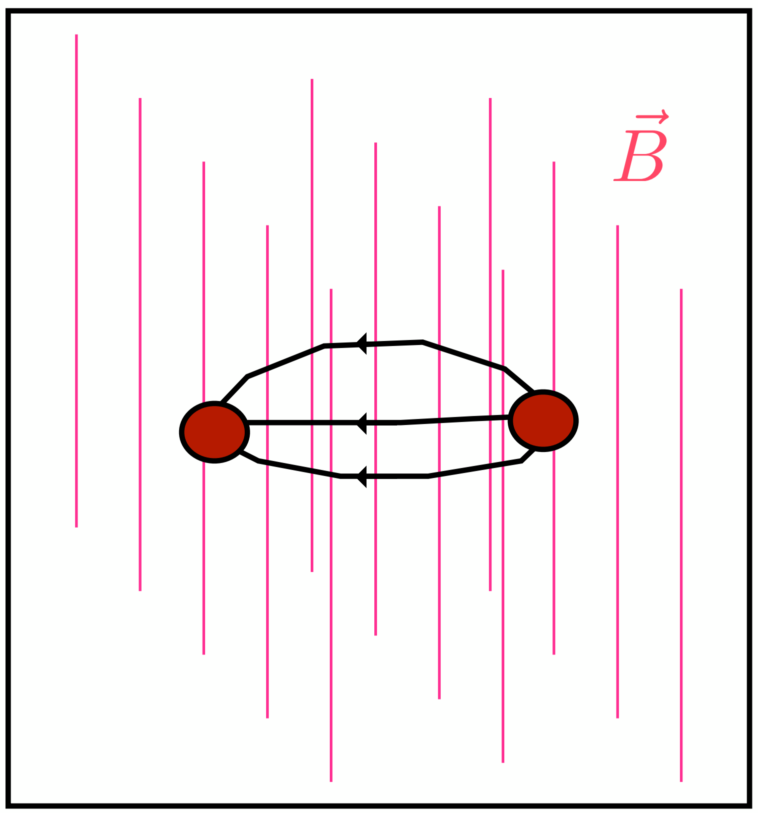
Parreño, Savage, Tiburzi, Wilhelm, Chang, Detmold, Orginos (*NPLQCD*), `arXiv:1609.03985`

Lattice QCD in Magnetic Fields



- Add classical magnetic field to QCD

$$U_\mu^{\text{e.m.}}(x) = e^{iqA_\mu(x)} \in U(1)$$
$$= e^{-iqx_2 B\delta_{\mu 1}} \, e^{+iqx_1 BN\,\delta_{\mu 2}\delta_{x_2,N-1}}$$



't Hooft flux quantization $\quad qB = \frac{2\pi}{N^2} n_\Phi$

- Compute two-point correlators in magnetic field

$$G_B(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle_B = Z(B)e^{-E(B)\,t} + \cdots$$

- Extract Zeeman splitting from double ratios

$$\Delta E = E_{+\frac{1}{2}}(B) - E_{-\frac{1}{2}}(B) = -2\mu\, B + \cdots$$

Tadpole-improved clover-fermion ensembles

	L/a	T/a	β	am_l	am_s	a[fm]	m_π[MeV]	N_{cfg}
I	32	48	6.1	−0.2450	−0.2450	0.1453(16)	806.9(8.9)	1006
~~II~~	~~48~~	~~64~~	~~6.3~~	~~−0.2050~~	~~−0.2050~~	~~0.1036(11)~~	~~766.9(8.1)~~	~~94~~
III	32	96	6.1	−0.2800	−0.2450	0.1167(16)	449.9(4.6)	544

48 scr / cfg

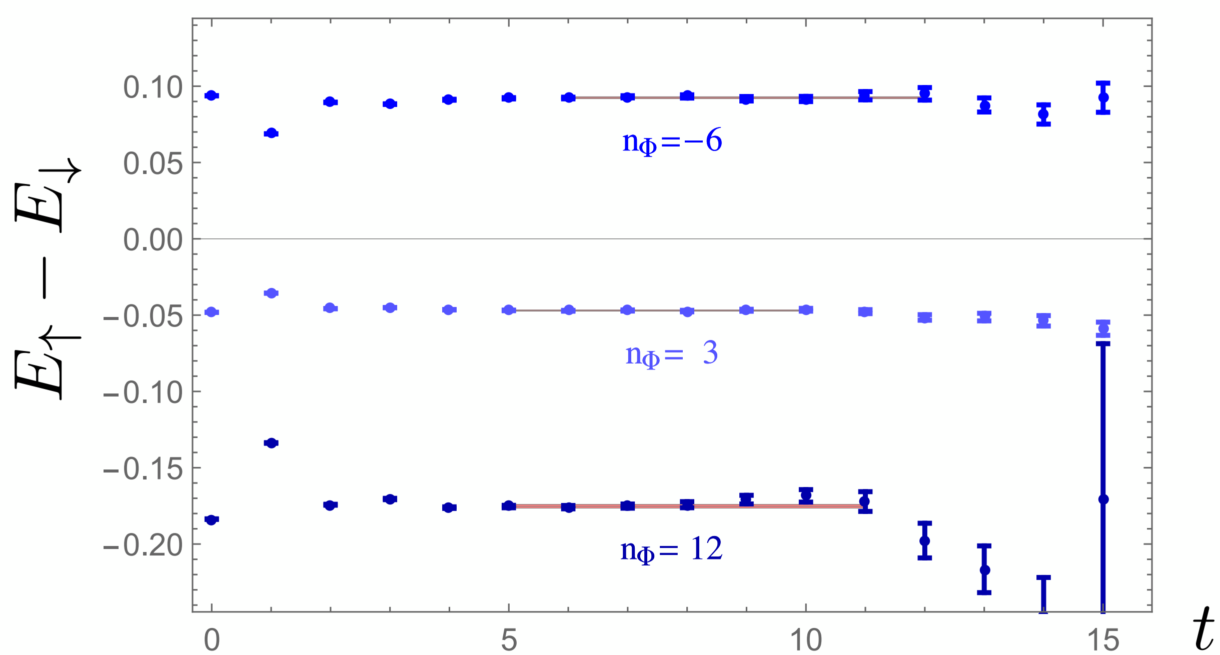
$n_\Phi = 0, +3, -6, +9 \qquad u/d, \quad s$

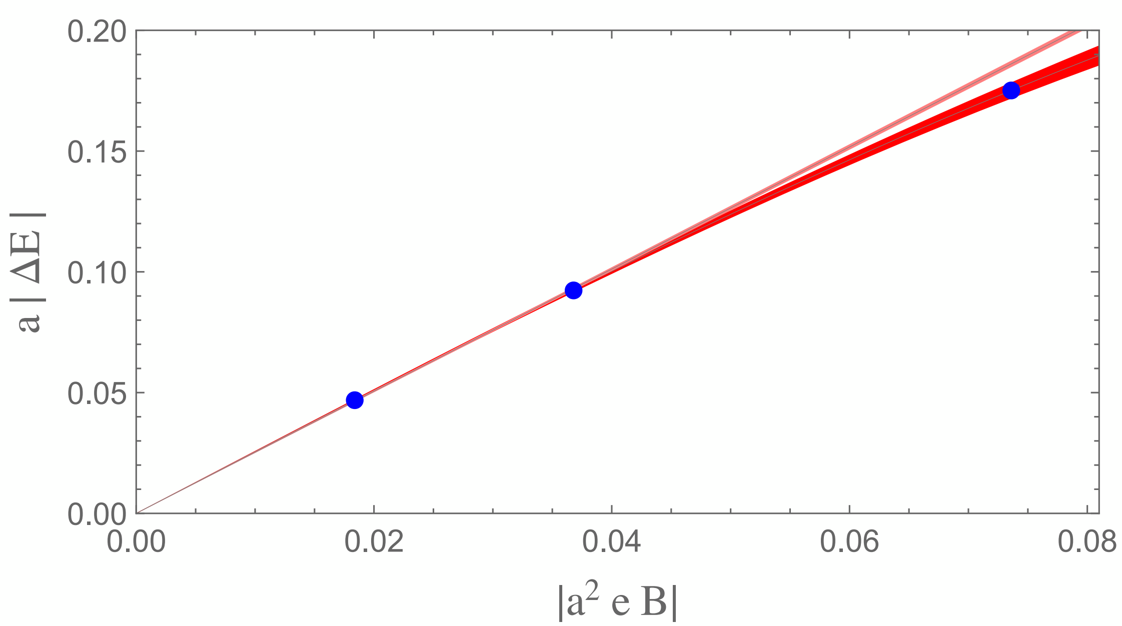
I + III = 400k measurements

Magnetic Moments of Octet Baryons

Compute Zeeman splitting using Lattice QCD + Uniform Magnetic fields

Proton $m_u = m_d = m_s$ $m_\pi \sim 800\,\texttt{MeV}$





Units!

$\mu_p = 2.560(09)(52)\ \texttt{[LatM]}$

$$\texttt{[LatM]} = \frac{e\,a}{2}$$

$$a = 0.145(2)\,\texttt{fm}$$

$\mu_p = 1.770(06)(36)(19)\ \texttt{[NM]}$

$$\texttt{[NM]} = \frac{e}{2M_N}$$

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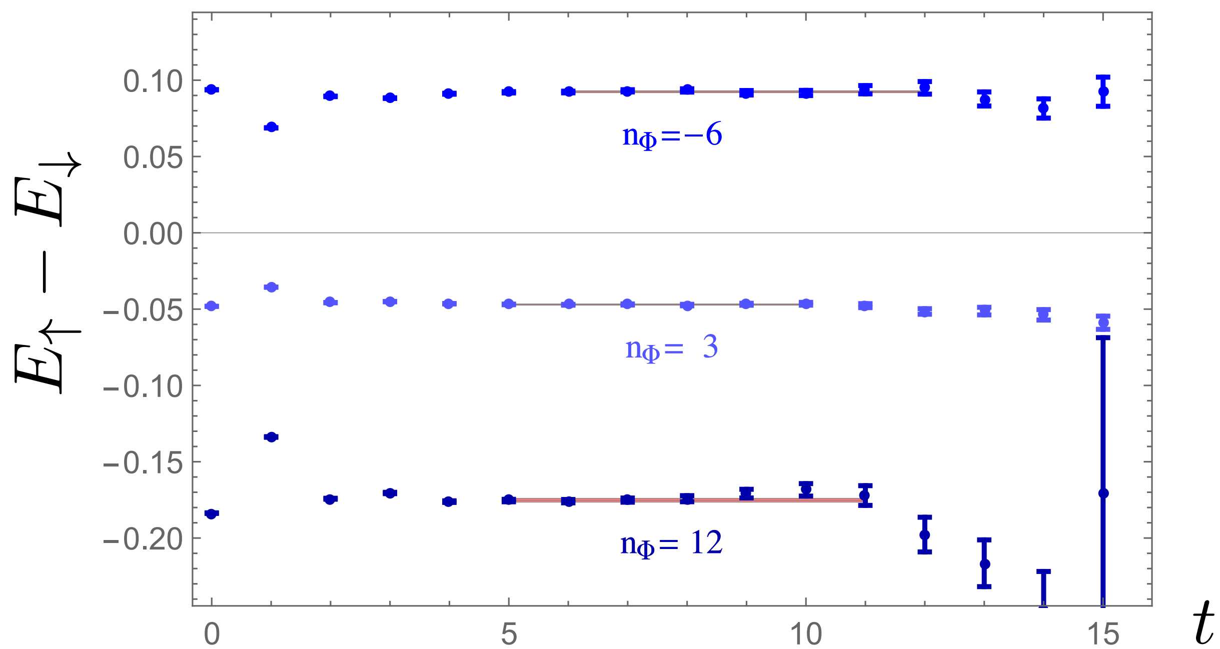
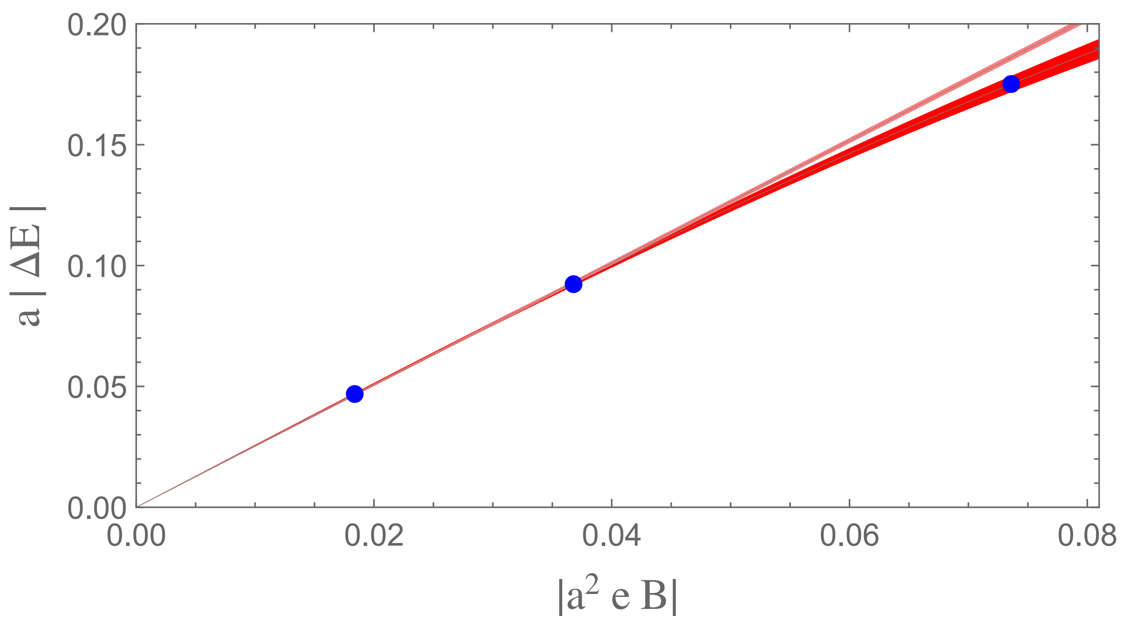


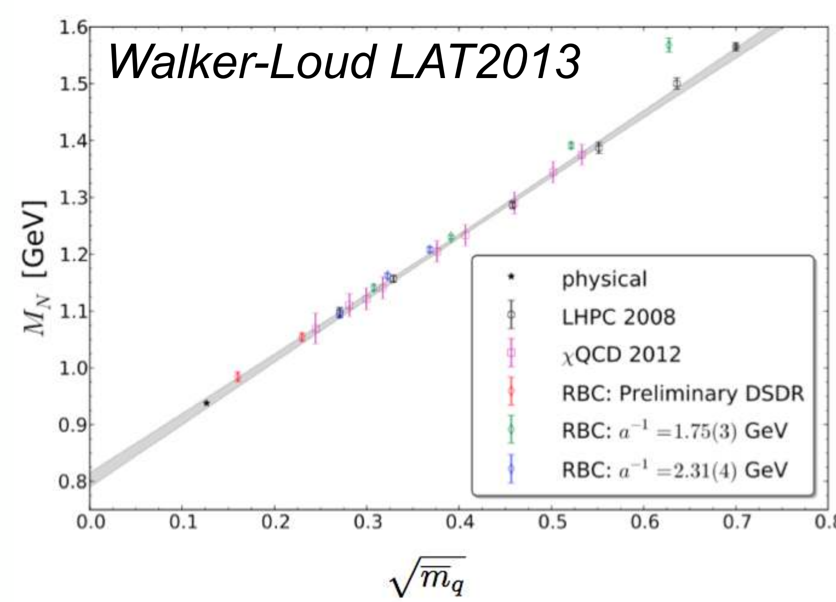


$$\mu_p = 1.770(06)(36)(19)\ \texttt{[NM]}$$

$$\texttt{[NM]} = \frac{e}{2M_N}$$

Ruler Mass Rule *(Walker-Loud, LHPC)*

$$M_N(m_\pi) = 800\ \texttt{MeV} + m_\pi \sim 1,600\,\texttt{MeV}$$

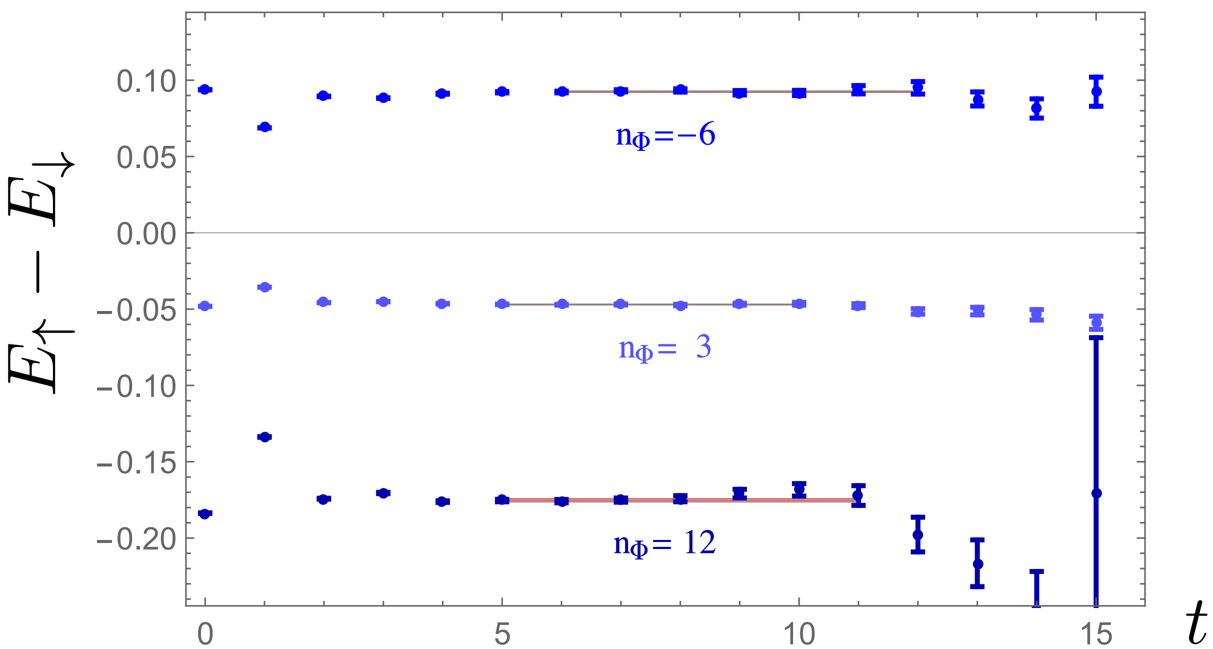


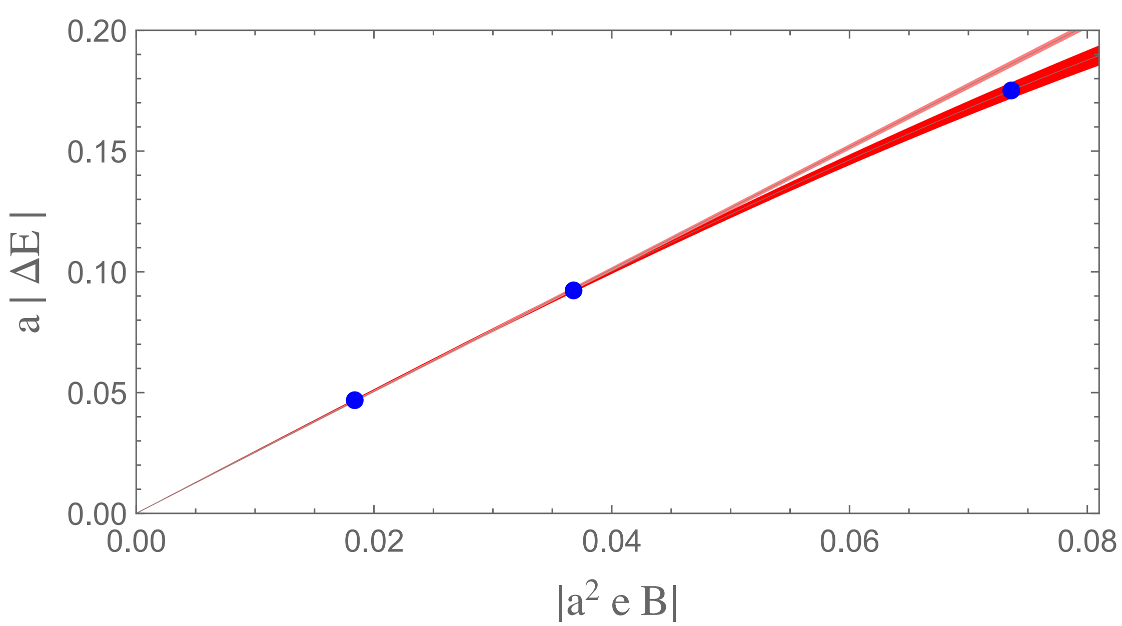
$$\texttt{[nNM]} = \frac{e}{2M_N(m_\pi)}$$

Magnetic Moments of Octet Baryons

Compute Zeeman splitting using Lattice QCD + Uniform Magnetic fields

Proton $\quad m_u = m_d = m_s \quad m_\pi \sim 800\,\texttt{MeV}$





$$\mu_p = 1.770(06)(36)(19)\ \texttt{[NM]}$$

$$\texttt{[NM]} = \frac{e}{2M_N}$$

Natural nucleon magnetons

$$\texttt{[nNM]} = \frac{e}{2M_N(m_\pi)}$$

$$\mu_p = 3.087(10)(62)\ \texttt{[nNM]}$$

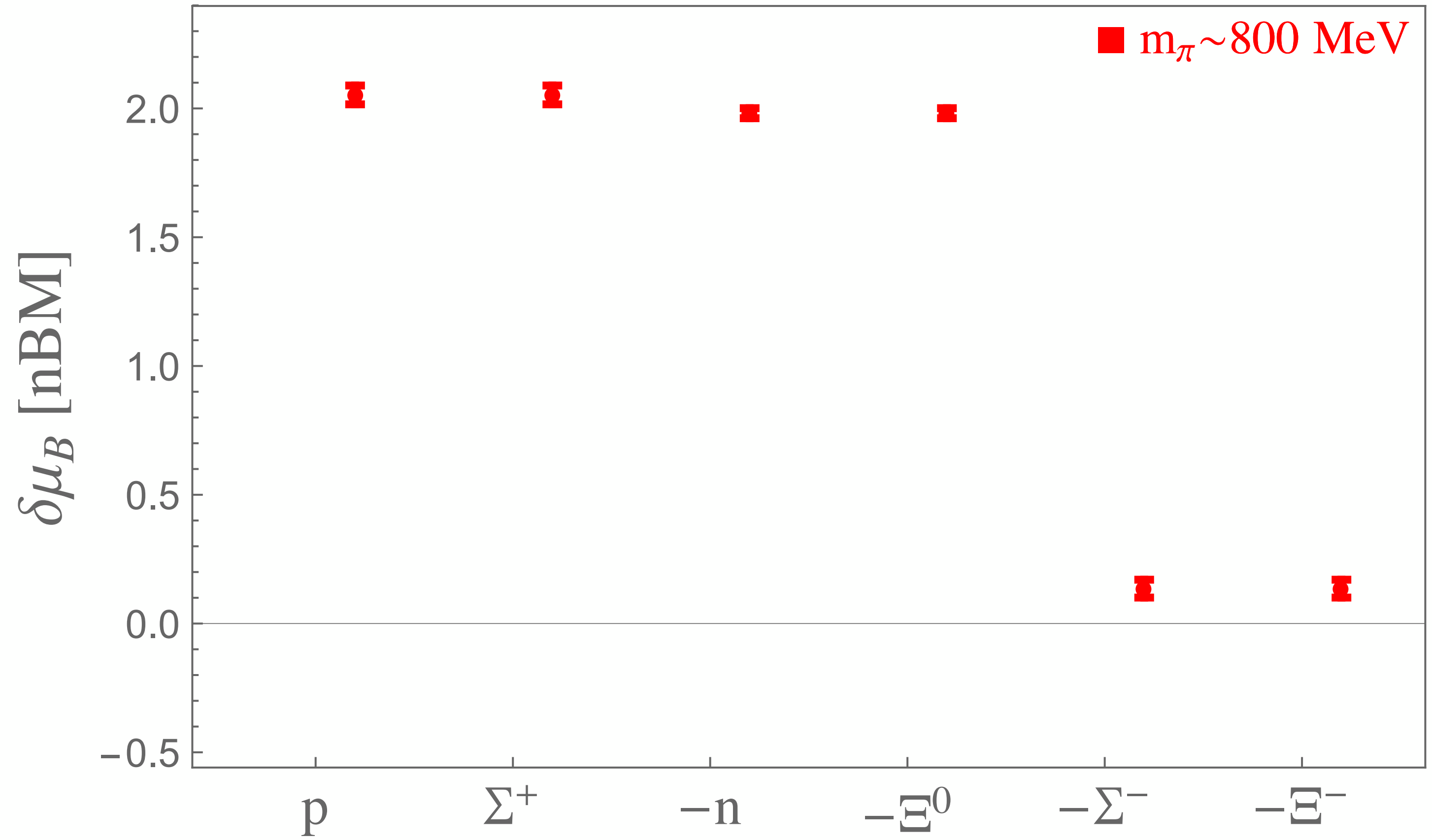
Dirac part is short-distance & guaranteed to $\mathcal{O}(a^2\Lambda_{\text{QCD}}^2)$

$$\delta\mu_p = 2.087(10)(62)\ \texttt{[nNM]}$$

$$\delta\mu_p^{\text{exp}} = 1.7929...\ \texttt{[NM]}$$

Magnetic Moments of Octet Baryons

Compute Zeeman splitting using Lattice QCD + Uniform Magnetic fields



$$m_u = m_d = m_s$$

$$U(3)_F \xrightarrow{Q} U(1)_U \times U(1)_{D+S} \times SU(2)_{U-\mathrm{spin}}$$

Natural baryon magnetons

$$\texttt{[nBM]} = \frac{e}{2M_B(m_\pi)}$$

Anomalous magnetic moments

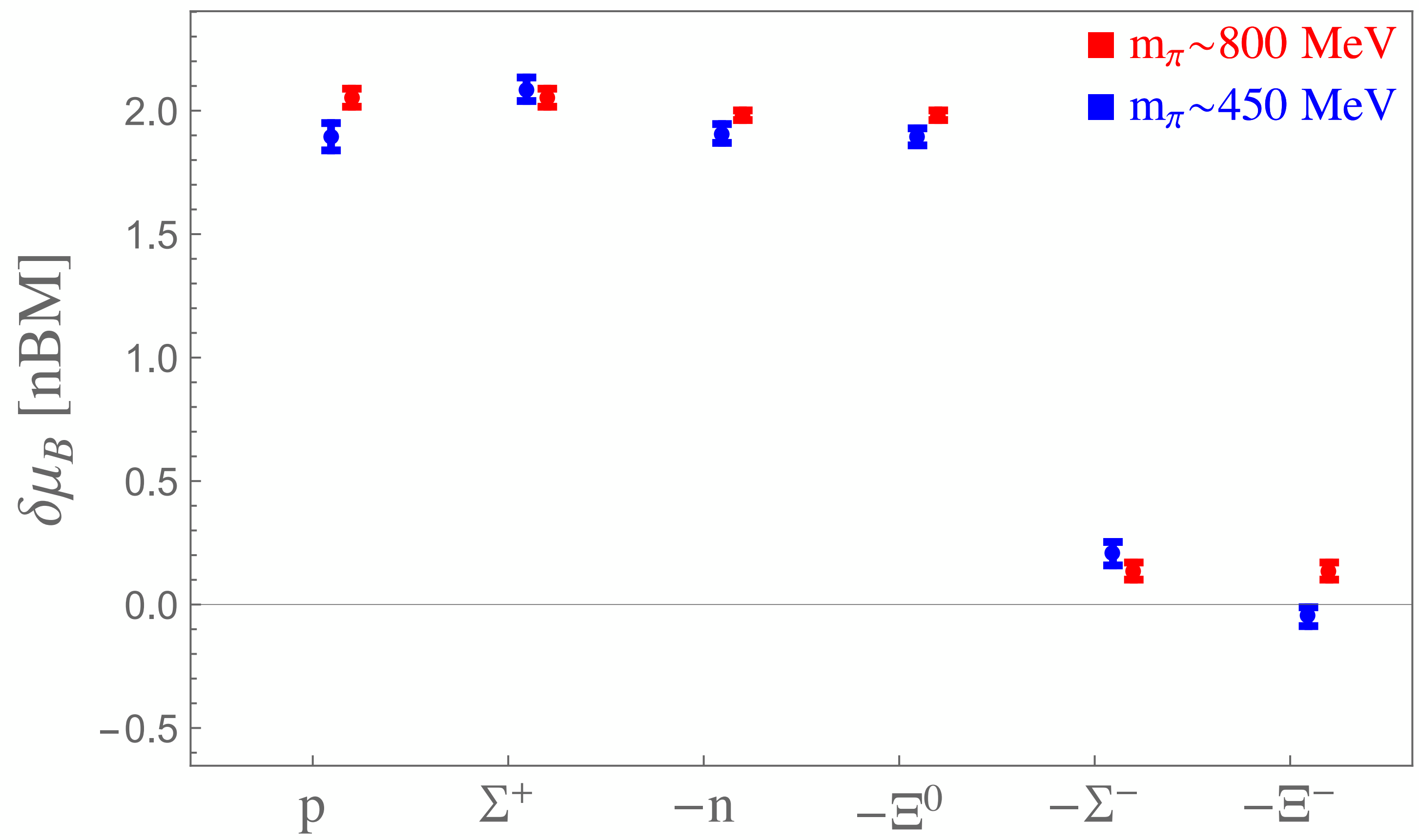
$$\delta\mu_B\ \texttt{[nBM]} = \mu_B\ \texttt{[nBM]} - Q_B$$

U-spin

$$\begin{pmatrix} d \\ s \end{pmatrix} \xrightarrow{SU(2)} U \begin{pmatrix} d \\ s \end{pmatrix}$$

Magnetic Moments of Octet Baryons

Compute Zeeman splitting using Lattice QCD + Uniform Magnetic fields



$$m_u = m_d = m_s$$

$$U(3)_F \xrightarrow{Q} U(1)_U \times U(1)_{D+S} \times SU(2)_{U-\text{spin}}$$

$$m_u = m_d < m_s$$

$$U(2)_I \times U(1)_S \xrightarrow{Q} U(1)_B \times U(1)_{I_3} \times U(1)_S \qquad m_\pi \sim 450\,\texttt{MeV}$$

Natural baryon magnetons

$$\texttt{[nBM]} = \frac{e}{2M_B(m_\pi)}$$

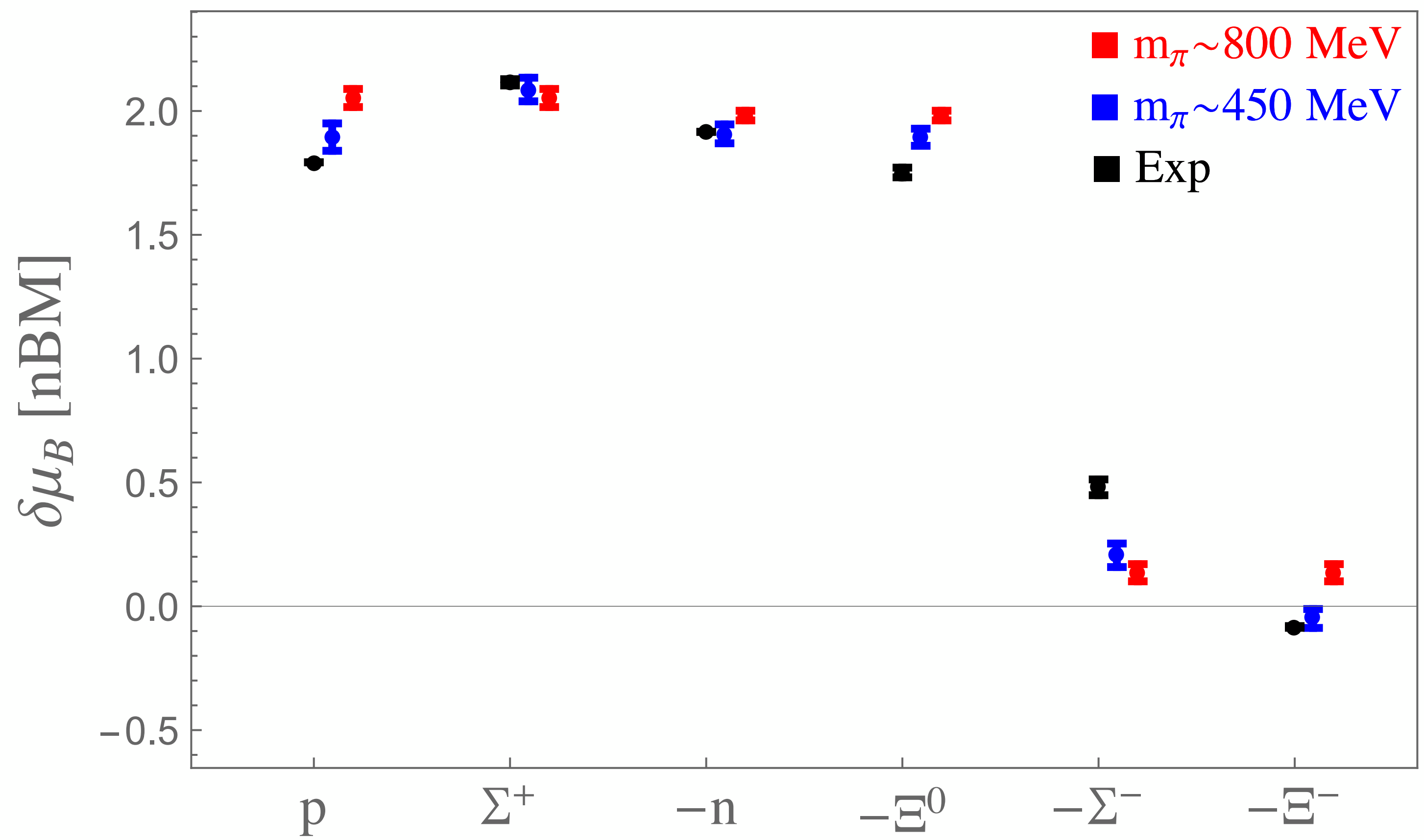
Anomalous magnetic moments

$$\delta\mu_B\,\texttt{[nBM]} = \mu_B\,\texttt{[nBM]} - Q_B$$

[Actually more complicated, our sea quarks are neutral]

Magnetic Moments of Octet Baryons

Compute Zeeman splitting using Lattice QCD + Uniform Magnetic fields



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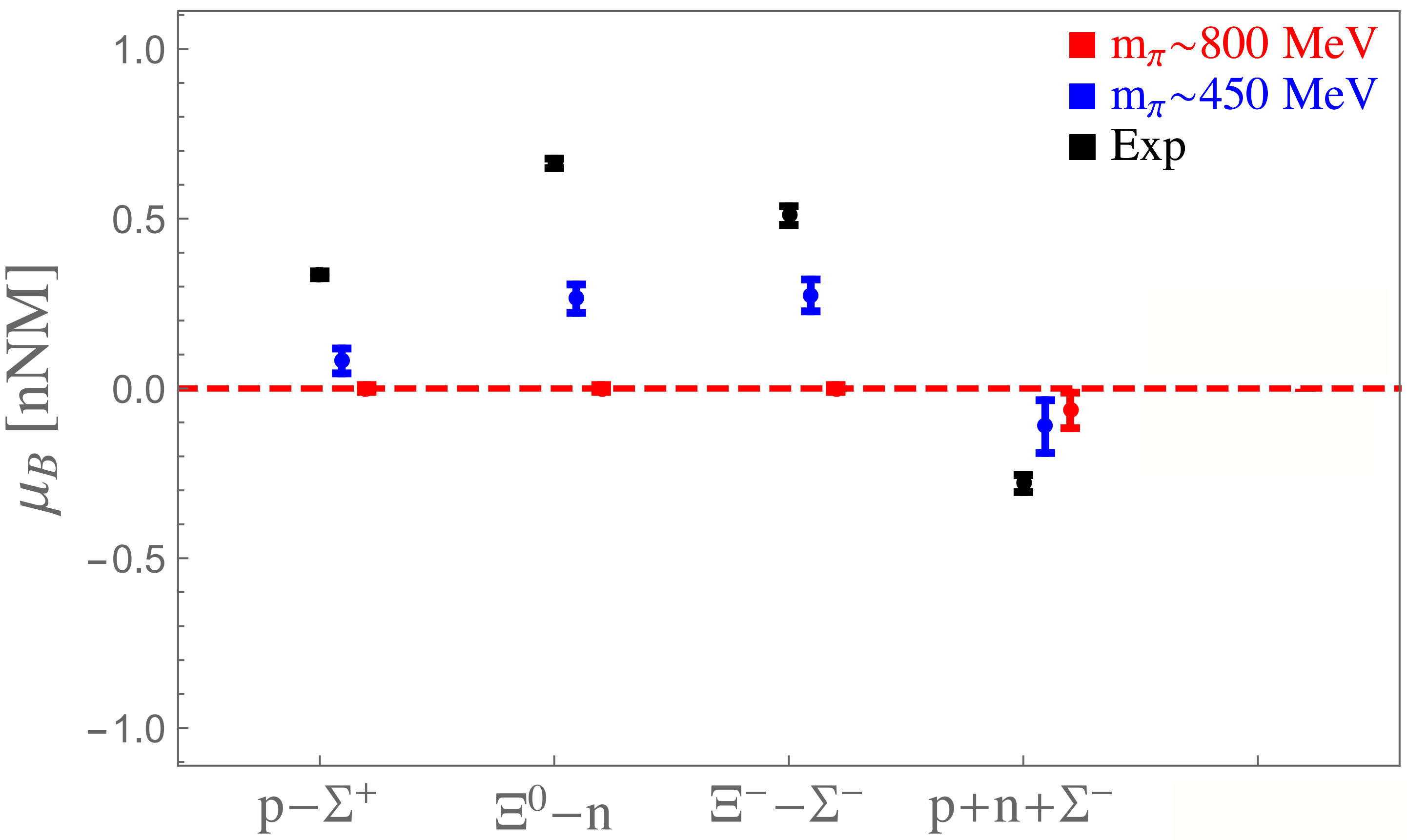
[Actually more complicated, our sea quarks are neutral]

Coleman-Glashow Relations

$$\mathcal{H} = -\frac{e\,\vec{\sigma}\cdot\vec{B}}{2M_B}\left[\mu_D\langle\overline{B}\{Q,B\}\rangle + \mu_F\langle\overline{B}[Q,B]\rangle\right]$$

$$m_u = m_d = m_s$$



$$\mu_p = \frac{1}{3}\mu_D + \mu_F$$
$$\mu_n = -\frac{2}{3}\mu_D$$
$$\mu_{\Sigma^-} = \frac{1}{3}\mu_D - \mu_F$$

$$\mu_D(m_\pi = 800\,\texttt{MeV}) = 2.958(35)$$
$$\mu_F(m_\pi = 800\,\texttt{MeV}) = 2.095(34)$$

$$\delta\mu_B = \pm 2, 0$$

Coleman-Glashow Magnetic Moments

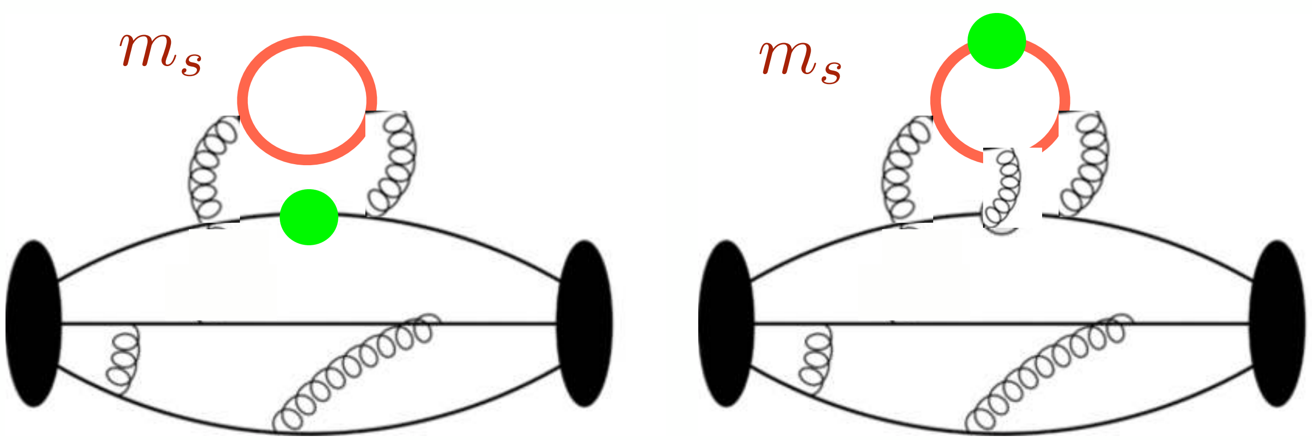
Estimate ***SU(3)*** moments away from ***SU(3)*** point?

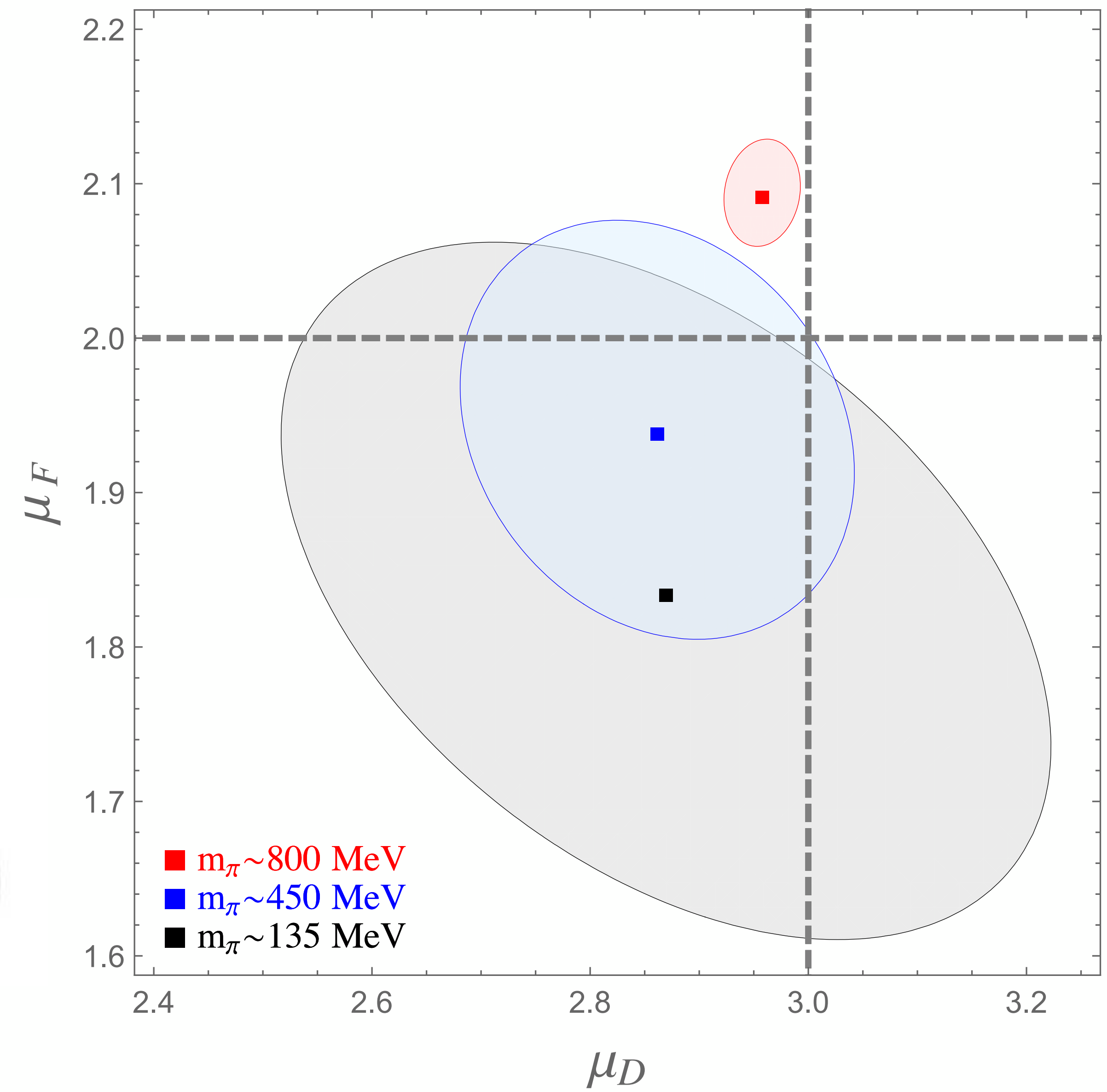
Stick with proton and neutron moments **!!!**

$$m_\pi = 450\ \texttt{[MeV]}: \qquad \frac{1}{2}\,33\% \Big/ 3 \sim 6\%$$

$$m_\pi = 135\ \texttt{[MeV]}: \qquad 33\% \Big/ 3 \sim 11\%$$

$$\Delta m_q / N_c$$





Whole number **CG** moments imply counting?

NRQM $\quad \frac{e}{2M_Q} = \texttt{[cQM]}$

$$\mu_D = \texttt{[cQM]} / \texttt{[nBM]} = M_B / M_Q$$

$\mu_D \sim +3$

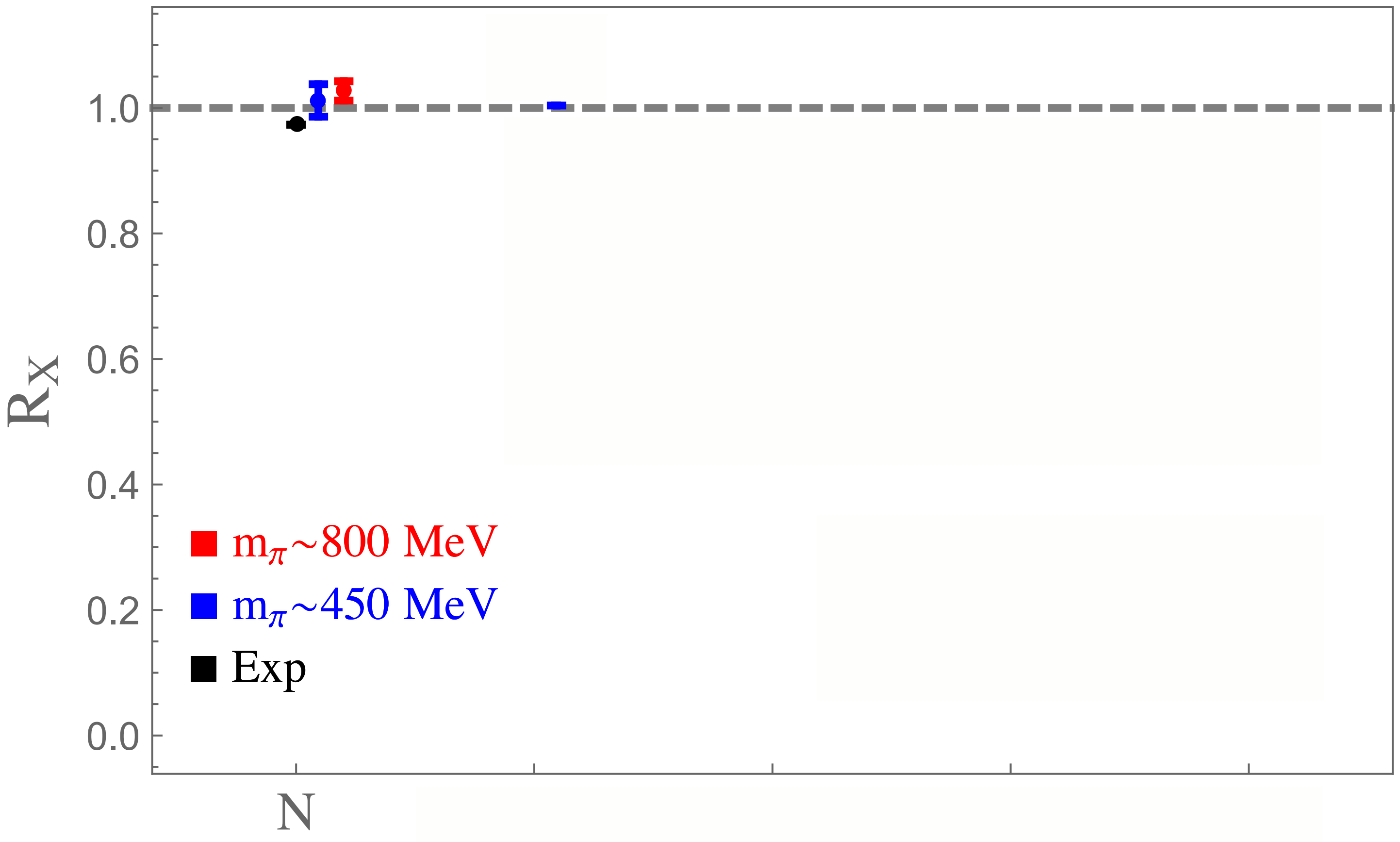
$\mu_F \sim +2$

$\delta\mu_B = \pm 2, 0$

Naïve Quark Model Moment Ratio

Grand success of ***NRQM*** is the ratio

$$1 = R_N = -\frac{2}{3}\frac{\mu_p}{\mu_n}$$



Naïve Quark Model Moment Ratios

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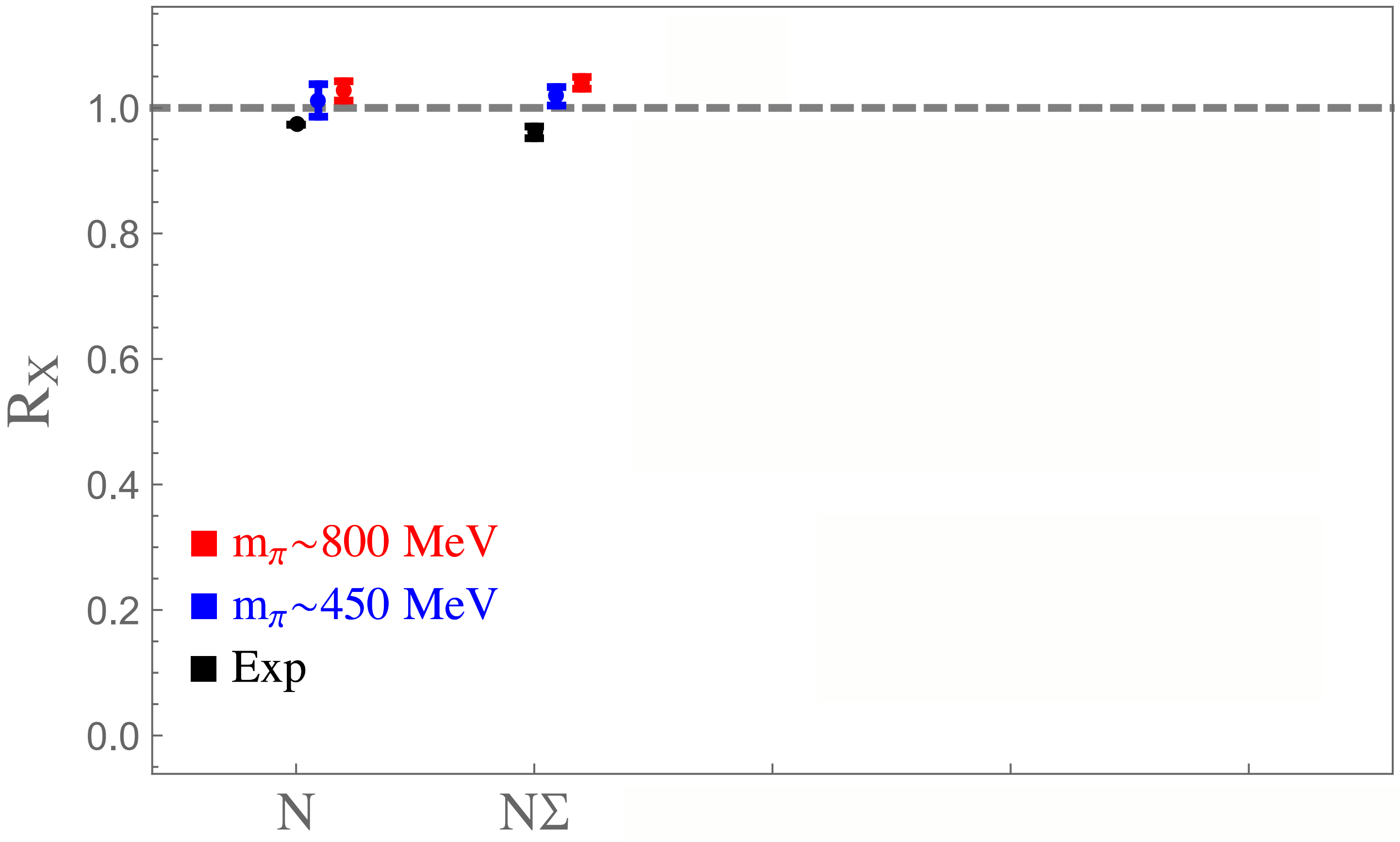
But there are other ***NRQM*** ratios too

$$1 = R_{N\Sigma} = \frac{5}{4}\frac{\Delta\mu_\Sigma}{\Delta\mu_N}$$



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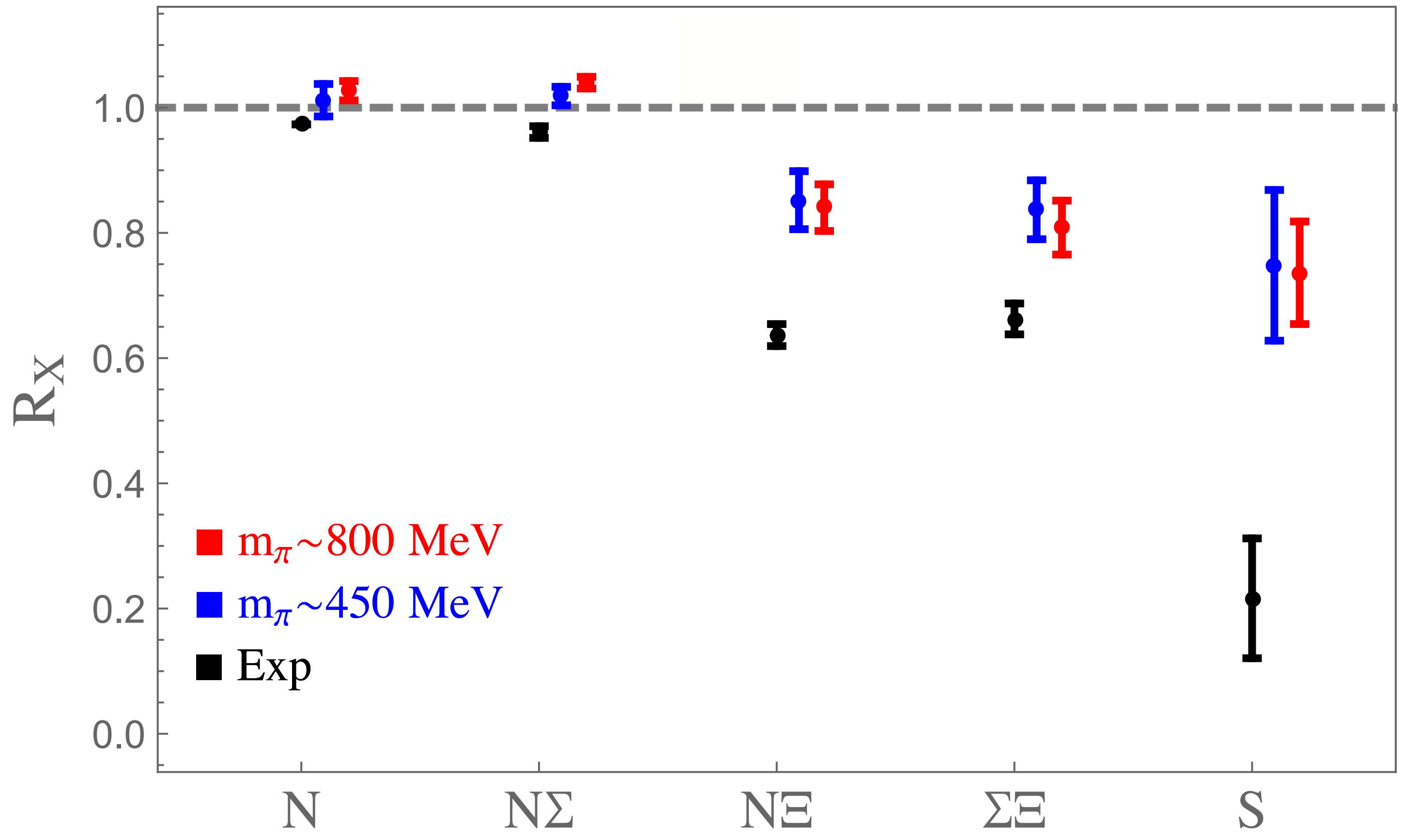
But there are other ***NRQM*** ratios too

$$1 = R_{N\Sigma} = \frac{5}{4}\frac{\Delta\mu_\Sigma}{\Delta\mu_N}$$

$$1 = R_{N\Xi} = 5\frac{\Delta\mu_\Xi}{\Delta\mu_N}$$

$$1 = R_{\Sigma\Xi} = 4\frac{\Delta\mu_\Xi}{\Delta\mu_\Sigma}$$

$$1 = R_S = -4\frac{\mu_{\Sigma^+} + 2\mu_{\Sigma^-}}{\mu_{\Xi^0} + 2\mu_{\Xi^-}}$$



Lattice **QCD** results generally agree better with ***NRQM*** than experiment

Why do some *NRQM* predictions work better than others?

Large-N_c Limit

Dashen, Jenkins, Manohar (1994)

Our calculations & nature have N_c = 3 …

$$\mathcal{R}_{S7} = \frac{5(\mu_p + \mu_n) - (\mu_{\Xi^0} + \mu_{\Xi^-})}{4(\mu_{\Sigma^+} + \mu_{\Sigma^-})}$$

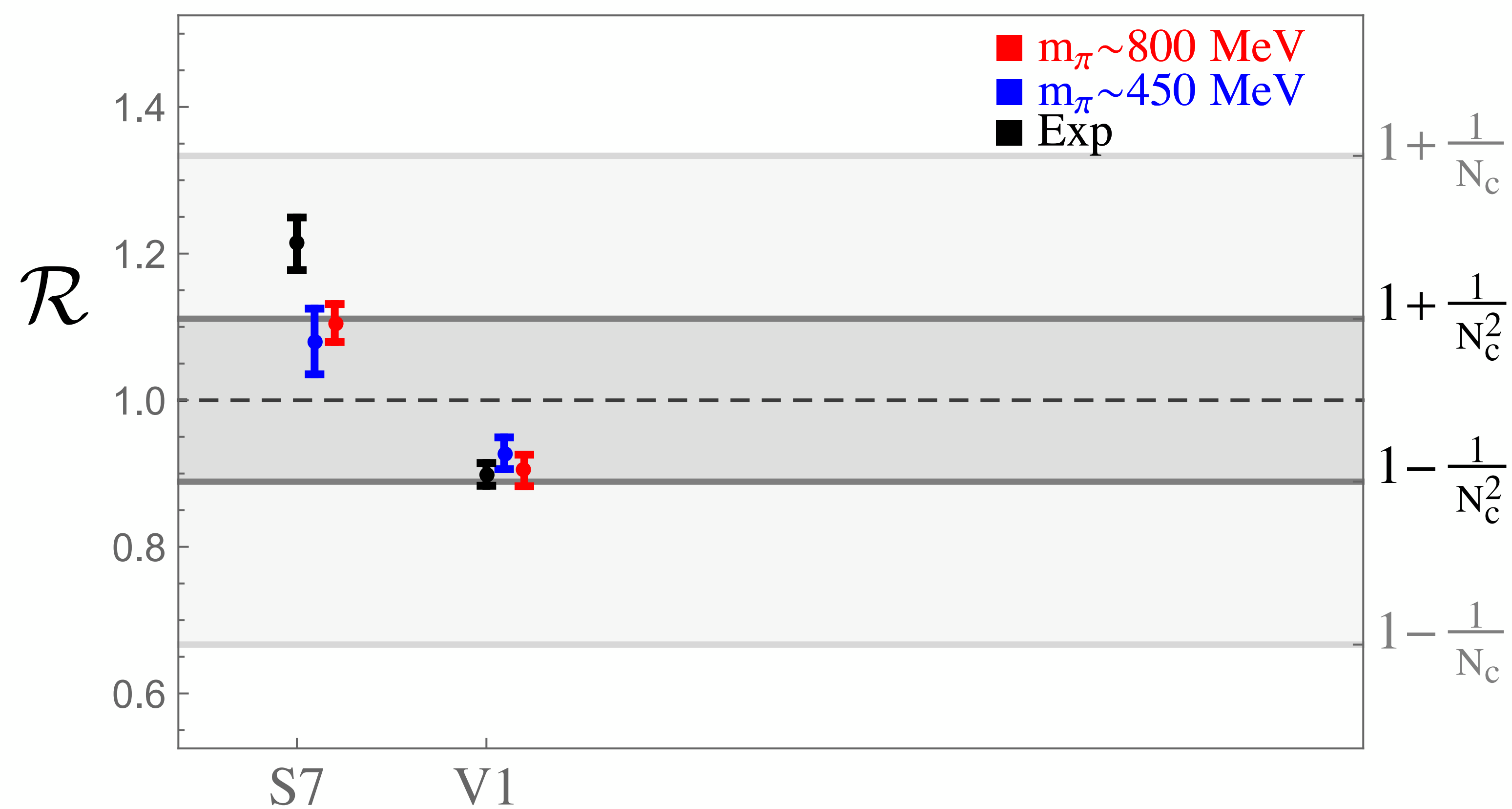
$$= 1 \quad NRQM$$

$$= 1 + \mathcal{O}(1/N_c)$$

$$\mathcal{R}_{V1} = \frac{\Delta\mu_N + 3\,\Delta\mu_\Xi}{2\,\Delta\mu_\Sigma}$$

$$= 1 \quad NRQM$$

$$= 1 + \mathcal{O}(1/N_c^2)$$



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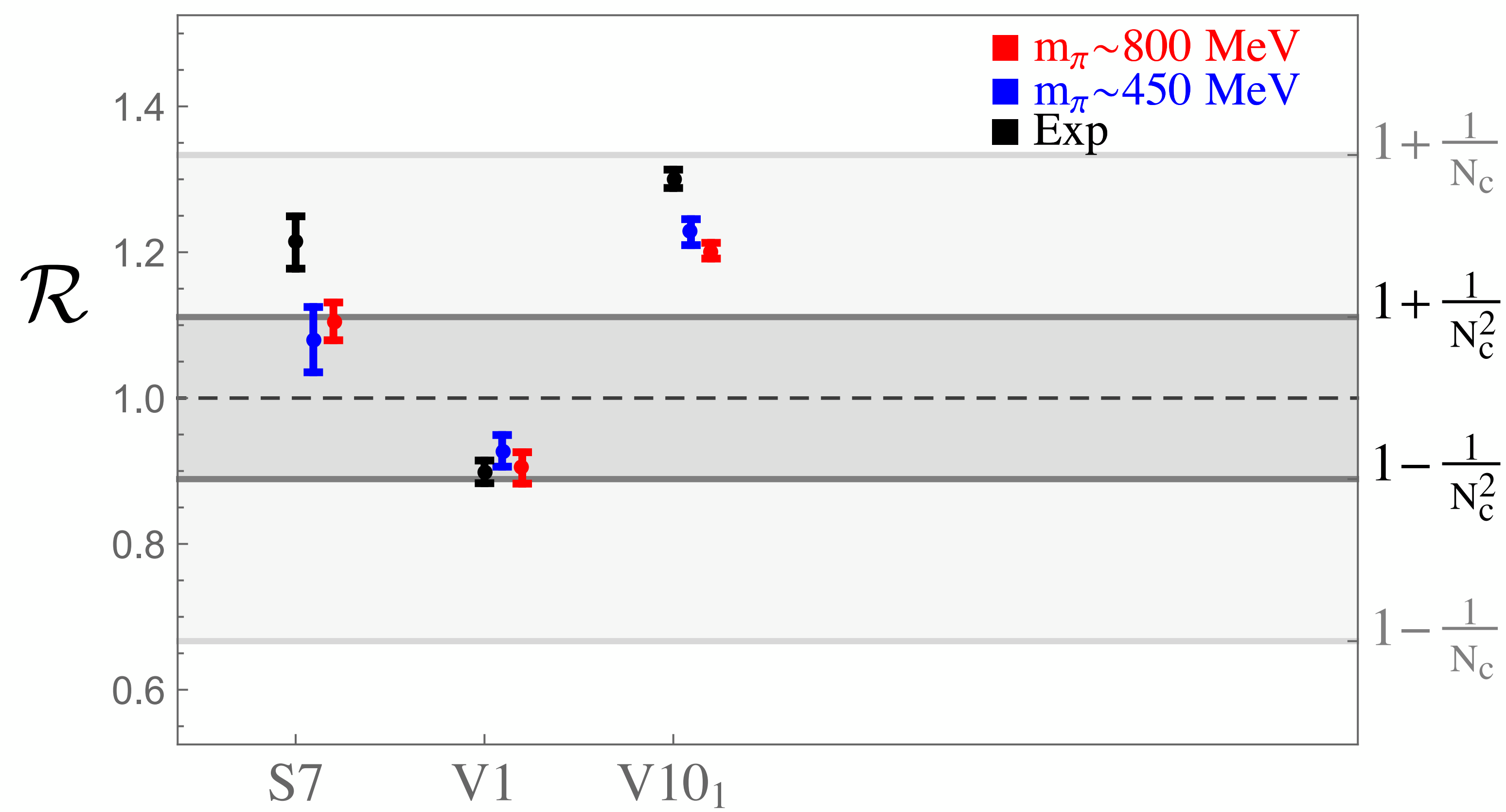
$$\mathcal{R}_{V10_1} = \frac{\Delta\mu_N}{\Delta\mu_\Sigma}$$

$$= 1.25 \quad NRQM$$

$$= 1 + \mathcal{O}(1/N_c)$$



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$$= 1.25 \quad NRQM$$

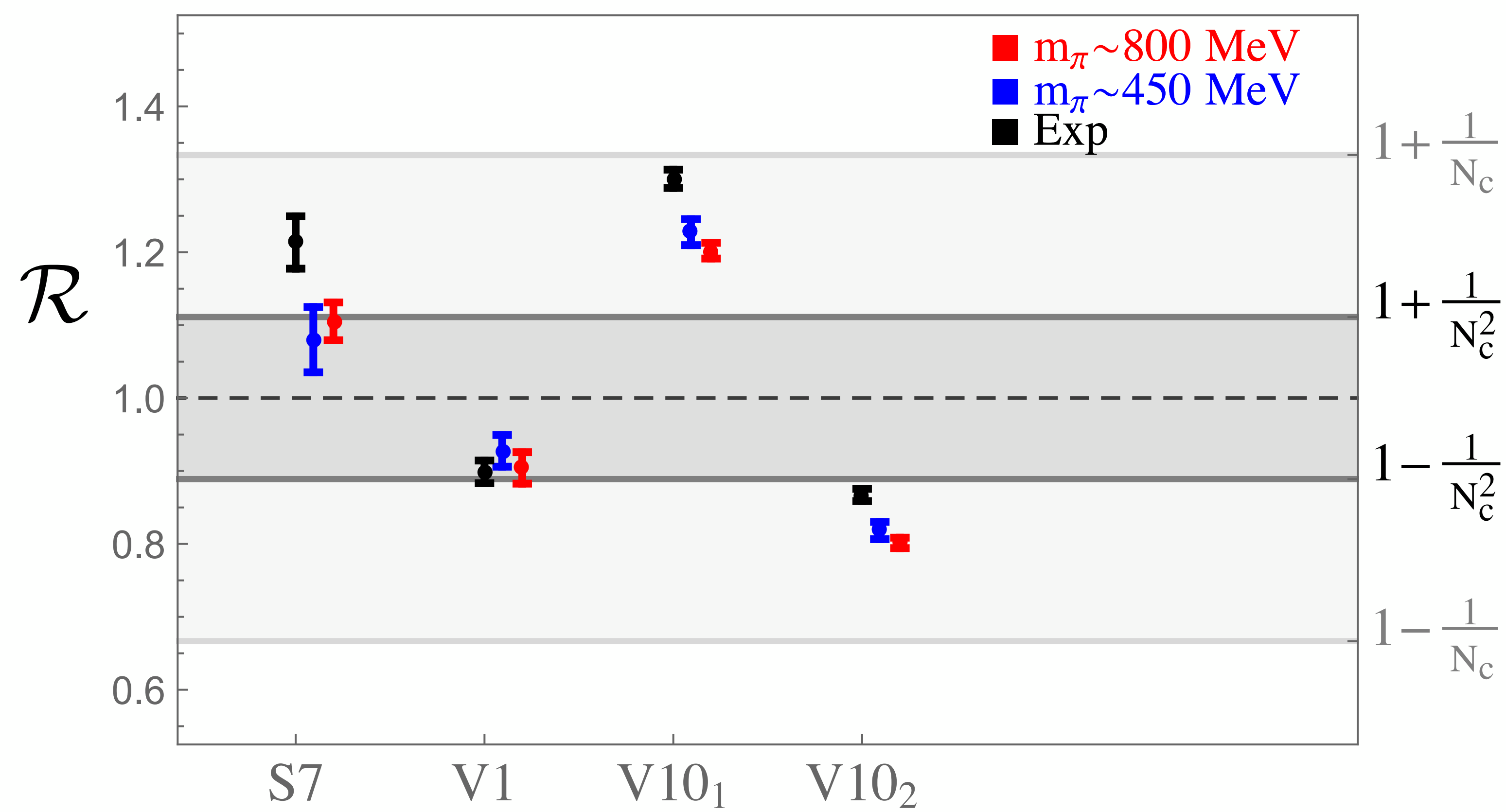
$$= 1 + \mathcal{O}(1/N_c)$$

$$\mathcal{R}_{V10_2} = (1 - 1/N_c)\,\frac{\Delta\mu_N}{\Delta\mu_\Sigma}$$

$$= 0.83 \quad NRQM$$

$$= 1 + \mathcal{O}(\Delta m_q/N_c)$$

$$= 1 + \mathcal{O}(1/N_c^2)$$



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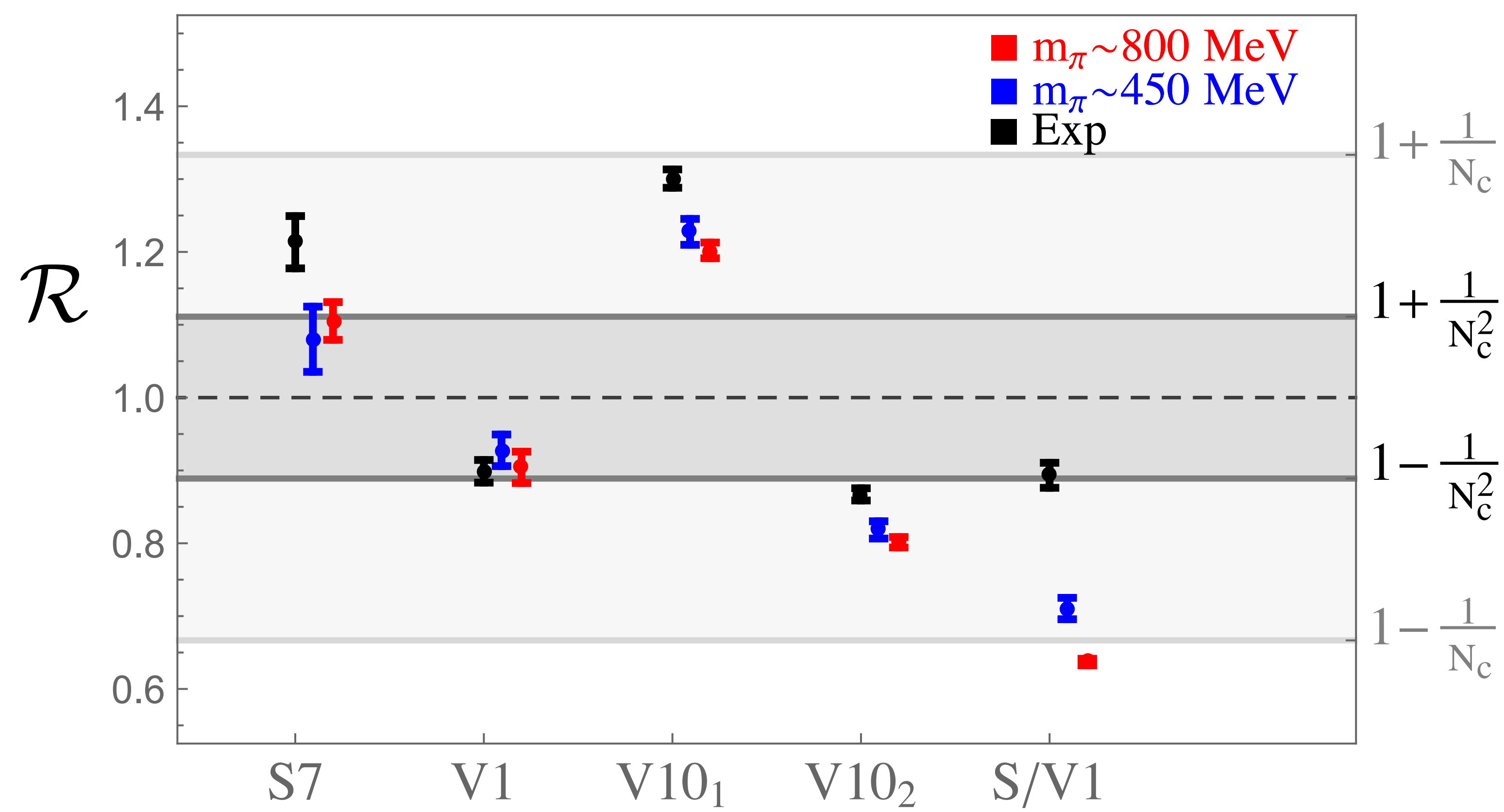

$$\mathcal{R}_{S/V1} = \frac{\frac{1}{2}(\mu_p + \mu_n) + 3(1/N_c - 2/N_c^2)\Delta\mu_N}{\mu_{\Sigma^+} + \mu_{\Sigma^-} - \frac{1}{2}(\mu_{\Xi^0} + \mu_{\Xi^-})}$$

$$= 0.62 \quad NRQM$$

$$= 1 + \mathcal{O}(\Delta m_q) + \mathcal{O}(\Delta m_q/N_c) + \mathcal{O}(1/N_c^2)$$

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Our calculations & nature have N_c = 3 …

$$\mathcal{R}_{V10_1} = \frac{\Delta\mu_N}{\Delta\mu_\Sigma}$$

$$= 1.25 \quad NRQM$$

$$= 1 + \mathcal{O}(1/N_c)$$

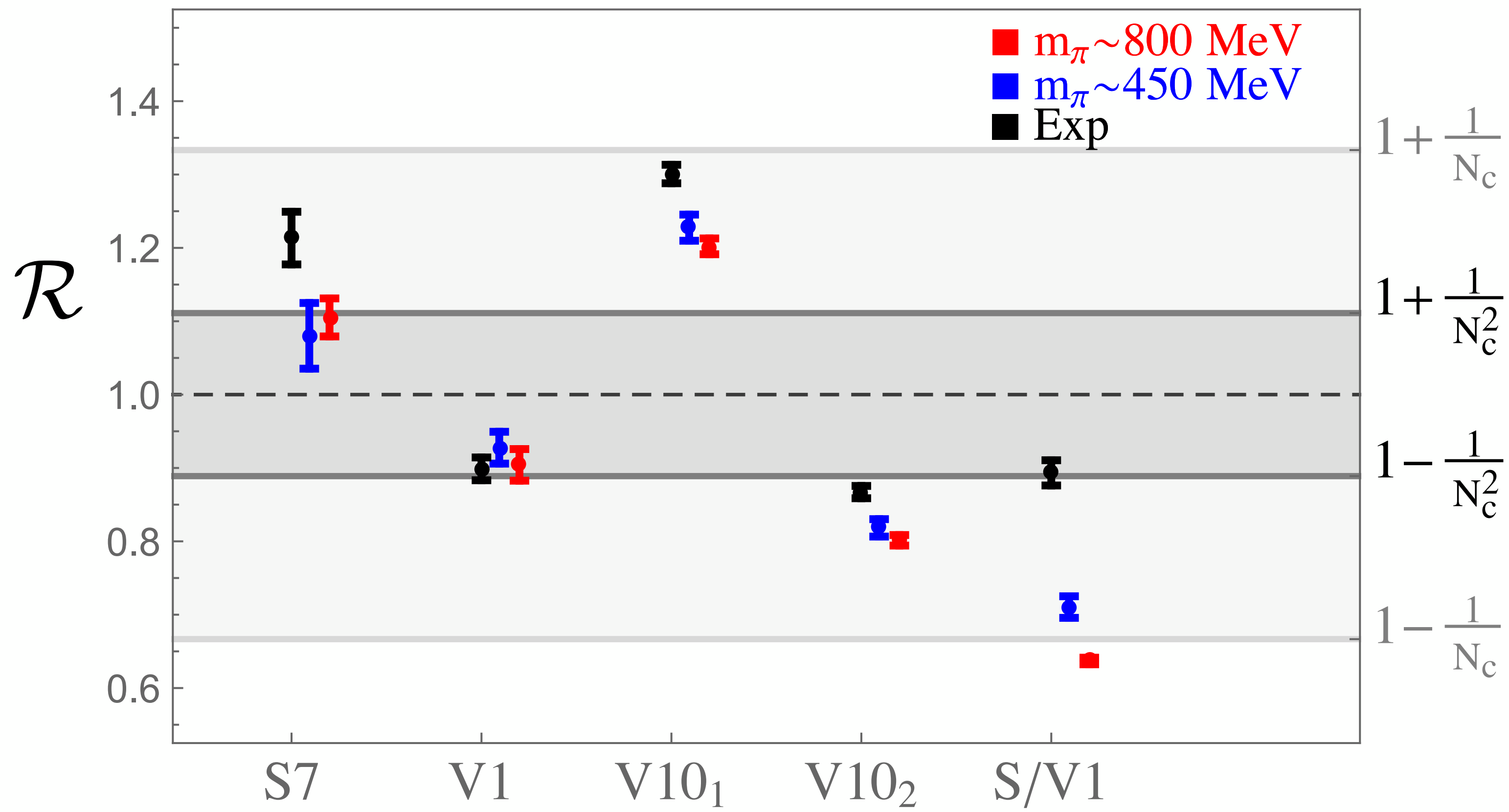
$$\mathcal{R}_{V10_2} = (1 - 1/N_c)\,\frac{\Delta\mu_N}{\Delta\mu_\Sigma}$$

$$= 0.83 \quad NRQM$$

$$= 1 + \mathcal{O}(\Delta m_q/N_c)$$

$$= 1 + \mathcal{O}(1/N_c^2)$$

Why do some large-N_c predictions work better than others?

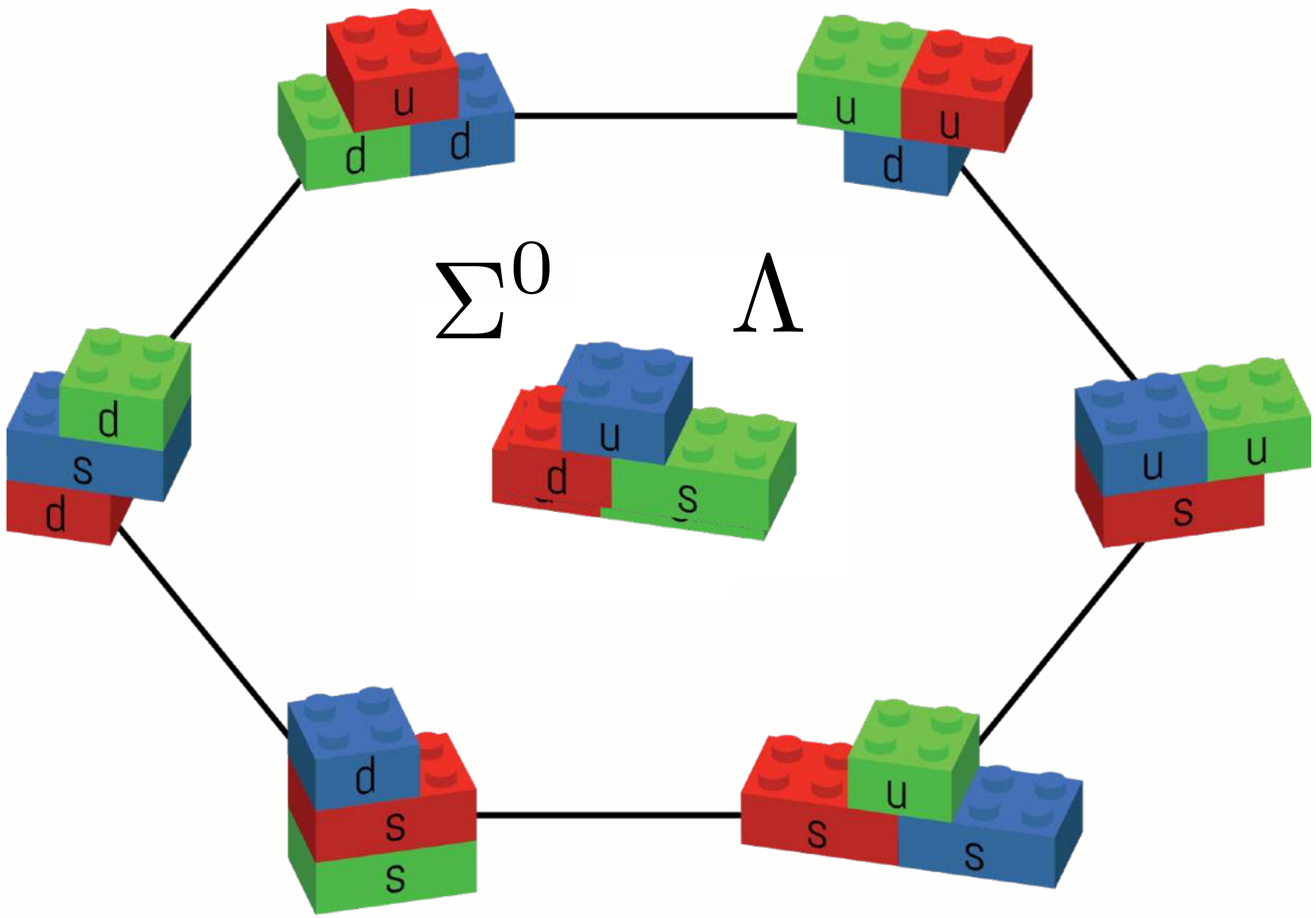


$$\mathcal{R}_{S/V1} = \frac{\frac{1}{2}(\mu_p + \mu_n) + 3(1/N_c - 2/N_c^2)\Delta\mu_N}{\mu_{\Sigma^+} + \mu_{\Sigma^-} - \frac{1}{2}(\mu_{\Xi^0} + \mu_{\Xi^-})}$$

$$= 0.62 \quad NRQM$$

$$= 1 + \mathcal{O}(\Delta m_q) + \mathcal{O}(\Delta m_q/N_c) + \mathcal{O}(1/N_c^2)$$

Magnetic Moments of Octet Baryons



$$H_{I_3=0} = \Delta_{\Lambda\Sigma} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \frac{e\,\vec{\sigma}\cdot\vec{B}}{2M_N} \begin{pmatrix} \mu_{\Sigma^0} & \mu_{\Lambda\Sigma} \\ \mu_{\Lambda\Sigma} & \mu_{\Lambda} \end{pmatrix} + \mathcal{O}(B^2)$$

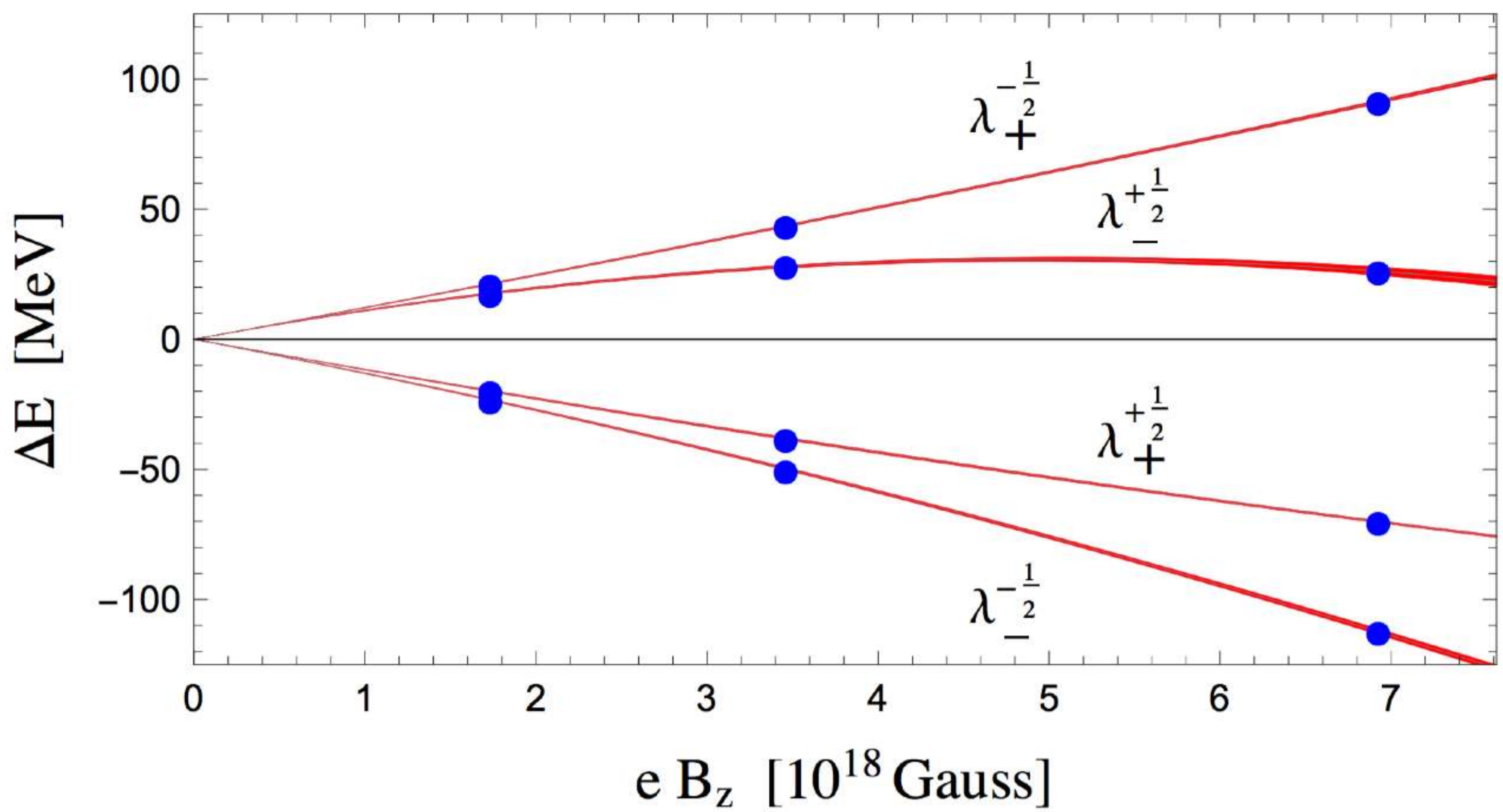
$\Sigma^0\,\Lambda$ Coupled-Channels Analysis

Diagonalize matrix of correlation functions

$$\mathbb{G}^{(s)}(t,n_\Phi) = \begin{pmatrix} G^{(s)}_{\Sigma\Sigma}(t,n_\Phi) & G^{(s)}_{\Sigma\Lambda}(t,n_\Phi) \\ G^{(s)}_{\Lambda\Sigma}(t,n_\Phi) & G^{(s)}_{\Lambda\Lambda}(t,n_\Phi) \end{pmatrix}$$

$$E^{(-\frac{1}{2})}_{\lambda_-}(B_z) = M_B + \mu_n \frac{eB_z}{2M_B} - 2\pi\beta_n B_z^2,$$
$$E^{(+\frac{1}{2})}_{\lambda_+}(B_z) = M_B + \mu_n \frac{eB_z}{2M_B} - 2\pi\left(\beta_n + \frac{4}{\sqrt{3}}\beta_{\Lambda\Sigma}\right) B_z^2,$$
$$E^{(+\frac{1}{2})}_{\lambda_-}(B_z) = M_B - \mu_n \frac{eB_z}{2M_B} - 2\pi\beta_n B_z^2,$$
$$E^{(-\frac{1}{2})}_{\lambda_+}(B_z) = M_B - \mu_n \frac{eB_z}{2M_B} - 2\pi\left(\beta_n + \frac{4}{\sqrt{3}}\beta_{\Lambda\Sigma}\right) B_z^2,$$

$$\mathbf{m_u = m_d = m_s}$$

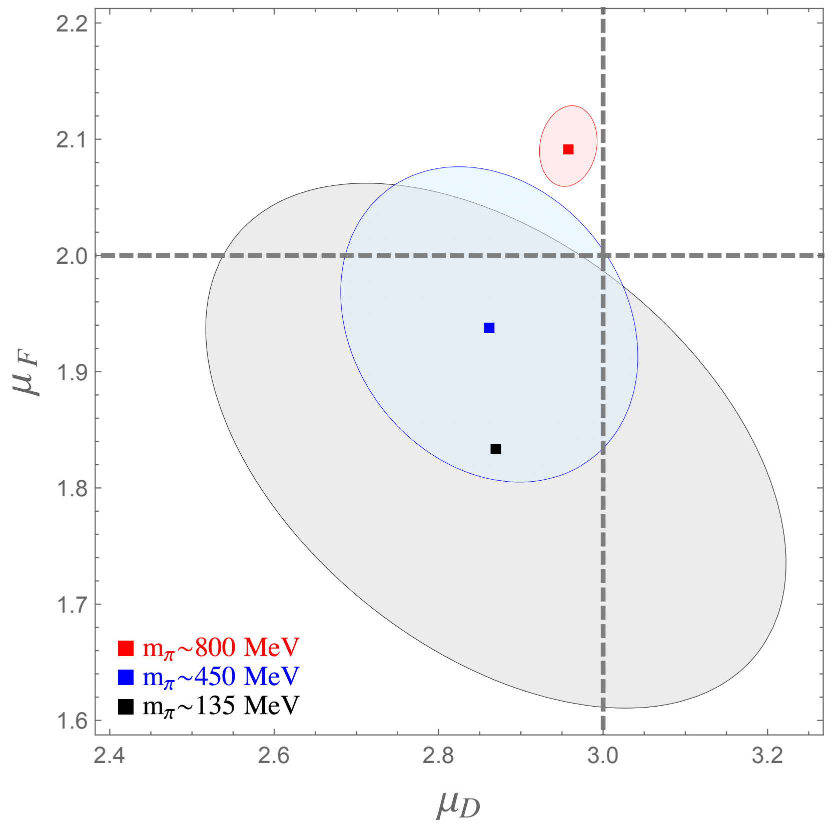


Coleman-Glashow
+ magnetic polarizability

$$\mu_{\lambda_\pm} = \mp\mu_n \sim \pm 2\ \texttt{[nBM]}$$

$$\beta_n = \quad 3.48(12)(26)(04)\,[10^{-4}\,\mathrm{fm}^3]$$
$$\beta_{\Lambda\Sigma} = -1.82(06)(12)(02)\,[10^{-4}\,\mathrm{fm}^3]$$

New Features = New Puzzles

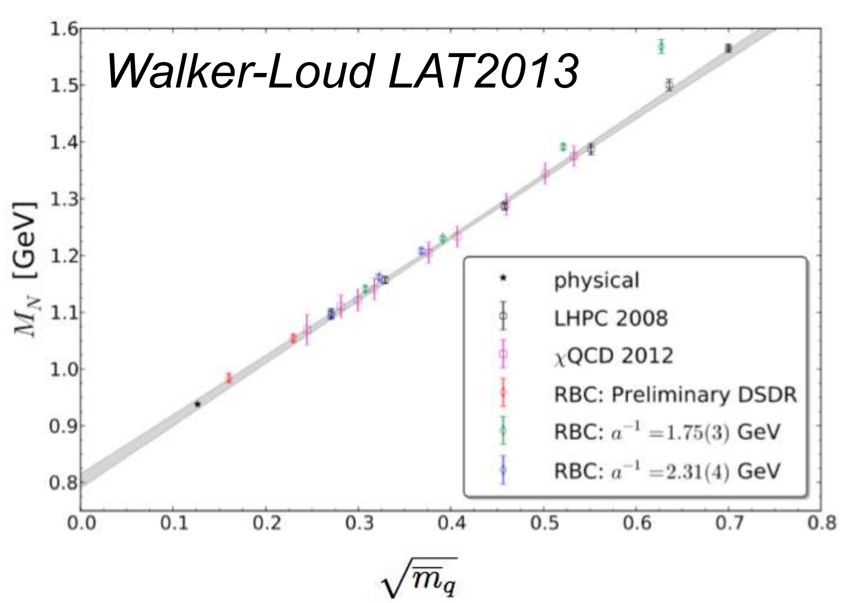


- **Mild pion-mass dependence** `[nBM]`
- **Nearness to SU(3) really nearness to SU(6)?**
- ***NRQM* explains large-*Nc* relations for *Nc* = 3?**

Need to compute octet, decuplet, and their transition moments

Need further pion masses, even light SU(3) symmetric ensembles

- **Why is *NRQM* successful @ spectrum & moments?**



?

$$g_A = \frac{5}{3}$$