

Testing a non-perturbative mechanism for elementary fermion mass generation: numerical results

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(see also talk by M. Garofalo: “Testing a non-perturbative mechanism for elementary fermion mass generation: lattice setup”)

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Overview

- **Dynamical generation of fermion masses**
 - owing to a NP mechanism triggered by a Wilson-like (naively irrelevant) chiral breaking term.
- QCD extended to a theory with *enriched symmetry* offers possibility for tackling naturalness problem.
- Simplest **Toy-model** where the mechanism can be realised:
 - $SU(N_f = 2)$ doublet of strongly ($SU(3)_c$) interacting fermions coupled to scalars via Yukawa and Wilson-like terms
 - physics depends crucially related on the phase (Wigner or NG)
 - enhanced symmetry (naturalness à la t'Hooft) leads to $\langle \Phi \rangle$ -independence of fermion masses
- The **intrinsic NP character** of the mechanism requires lattice numerical investigation of the toy model.

R. Frezzotti and G.C. Rossi, PRD 2015, [arXiv:1402.0389 [hep-lat]]

R. Frezzotti, M. Garofalo and G.C. Rossi, PRD 2016, [arXiv:1602.03684 [hep-ph]]

S. Capitani *et al.*, PoS LATTICE2016, [arXiv:1611.03997 [hep-lat]]

Theoretical setup

- Toy-model: $\text{QCD}_{N_f=2}$ + Scalar field + Wilson

$L_{\text{toy}} = L_{\text{kin}}(Q, A, \Phi) + V(\Phi) + L_Y(Q, \Phi) + L_W(Q, A, \Phi)$, with:

$$L_{\text{kin}}(Q, A, \Phi) = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{Q}_L \gamma_\mu D_\mu Q_R + \bar{Q}_R \gamma_\mu D_\mu Q_L + \frac{1}{2} \text{Tr} [\partial\Phi^\dagger \partial\Phi]$$

$$V(\Phi) = \frac{1}{2} \mu^2 \text{Tr} [\Phi^\dagger \Phi] + \frac{1}{4} \lambda \left(\text{Tr} [\Phi^\dagger \Phi] \right)^2$$

$$L_Y(Q, \Phi) = \eta (\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi Q_L)$$

$$L_W(Q, A, \Phi) = \rho \frac{b^2}{2} \left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \bar{Q}_R \overleftarrow{D}_\mu \Phi^\dagger D_\mu Q_L \right)$$

- Q : fermion $SU(2)$ doublet coupled to $SU(3)$ gauge field and to scalar field through Yukawa and Wilson terms.
- b^{-1} : UV cutoff.

Theoretical setup

- $\chi_L \times \chi_R$ transformations are symmetry of L_{toy} :

$$\begin{aligned}\chi_L &: \tilde{\chi}_L \otimes (\Phi \rightarrow \Omega_L \Phi) & \chi_R &: \tilde{\chi}_R \otimes (\Phi \rightarrow \Omega_R \Phi) \\ \tilde{\chi}_L &: Q_L \rightarrow \Omega_L Q_L, & \tilde{\chi}_R &: Q_R \rightarrow \Omega_R Q_R, \\ \bar{Q}_L &\rightarrow \bar{Q}_L \Omega_L^\dagger & \bar{Q}_R &\rightarrow \bar{Q}_R \Omega_R^\dagger \\ \Omega_L &\in SU(2)_L & \Omega_R &\in SU(2)_R\end{aligned}$$

- **Exact symmetry** $\chi \equiv \chi_L \times \chi_R$ acting on fermions and scalars \Rightarrow no power divergent mass terms.
- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations are not a symmetry for generic (non-zero) η and ρ .
- P , C , T , gauge invariance ... are symmetries.
- When the scalar potential $V(\Phi)$ has one minimum
 - ▶ $\chi_L \times \chi_R$ is realized à la Wigner.
 - ▶ No $\tilde{\chi}$ -SSB phenomenon is expected to occur.

Theoretical setup

- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations generate Schwinger-Dyson Eqs (unrenormalised).
- They get renormalised after considering the operator mixing procedure.
- **Critical Model:** $\tilde{\chi}$ -symmetry restoration occurs when the Yukawa term is compensated by the Wilson (6-d) term. This takes place (in the Wigner phase) at a certain value of the Yukawa coupling.

- In fact, for $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) get

$$\partial_\mu \langle \tilde{Z}_j \tilde{J}_\mu^{L,i}(x) O(0) \rangle = (\eta - \bar{\eta}(\eta; g_0^2, \rho, \lambda)) \langle [\bar{Q}_L \tau^i \Phi_{QR} - h.c.](x) O(0) \rangle + O(b^2)$$

(SDE renm/tion here analogous to chiral SDE renm/tion in [Bochicchio et al. NPB 1985](#))

- ▶ then **enforce** the current $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) conservation \implies
 $\eta - \bar{\eta}(\eta; g_0^2, \rho, \lambda) = 0 \quad \rightarrow \quad \eta_{cr}(g_0^2, \rho, \lambda).$

- In the Wigner phase the low-energy effective action reads

$$\Gamma_{\mu_\Phi^2 > 0}^{Wig} = \frac{1}{4}(F \cdot F) + \bar{Q} \mathcal{D} Q + (\eta - \eta_{cr})(\bar{Q}_L \Phi_{QR} + h.c.) + \frac{1}{2} \text{Tr} [\partial_\mu \Phi^\dagger \partial_\mu \Phi] + V_{\mu_\Phi^2 > 0}(\Phi)$$

- in the **critical theory** ($\tilde{\chi}$ is a **symmetry**, up to $O(b^2)$)

- ▶ Φ decoupled (up to cutoff effects) from quarks and gluons.
- ▶ no fermionic mass ($m_Q = 0$ up to $O(b^2)$).

Numerical investigation

► Determination of η_{cr} in the Wigner phase

● Lattice simulation details

- Lattice discretization, $L_{latt.}$, with exact χ -symmetry.
- Use naive fermions with symmetric covariant derivative, $\tilde{\nabla}_\mu$, throughout.
- To avoid “exceptional configurations” introduce twisted mass regulator
 $L_{latt.} + i\mu_Q \bar{Q} \gamma_5 \tau^3 Q$. (Frezzotti, Grassi, Sint and Weisz, JHEP 2001)
- Locally smeared Φ in $\bar{Q} D_{lat}[U, \Phi] Q$ for noise reduction.
- We limit our first study to the **quenched approximation** (\rightarrow it is quite certain that the mechanism under investigation survives quenching).
- Quenching: independent generation of gauge (U) and scalar (Φ) configurations.

(see also discussion and details on the lattice action in M. Garofalo's talk, this session.)

Numerical investigation

► Determination of η_{cr} in the Wigner phase

● Lattice simulation parameters

$$\beta = 5.85, L/b = 16 \text{ \& } T/b = 40$$

$$b = 0.12 \text{ fm, from } r_0 = 0.5 \text{ fm} \quad (\text{motivated from QCD, for illustration})$$

$$\lambda = 0.59, \rho = 1.96 \quad (\text{in the Wigner \& in the NG phases})$$

$$\mu_\Phi^2 \equiv \mu^2 - \mu_{cr}^2 = 0.074(1) b^{-2} > 0 \quad (\text{in the Wigner phase})$$

statistics: #configs (gauge \times scalar) $\sim 750 - 1400$

Ⓞ several values of the Yukawa coupling η (and μ_Q).

► Determination of η_{cr} in the Wigner phase

- Compute correlation function

$$C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x) \tilde{D}^{S3}(y) \rangle$$

where

$$\tilde{J}_0^{V3}(x) = \tilde{J}_0^{L3}(x) + \tilde{J}_0^{R3}(x)$$

$$\tilde{J}_0^{L/R3}(x) = \frac{1}{2} \left[\bar{Q}_{L/R}(x - \hat{0}) \gamma_0 \frac{\tau^3}{2} U_0(x - \hat{0}) Q_{L/R}(x) + \bar{Q}_{L/R}(x) \gamma_0 \frac{\tau^3}{2} U_0^\dagger(x - \hat{0}) Q_{L/R}(x - \hat{0}) \right]$$

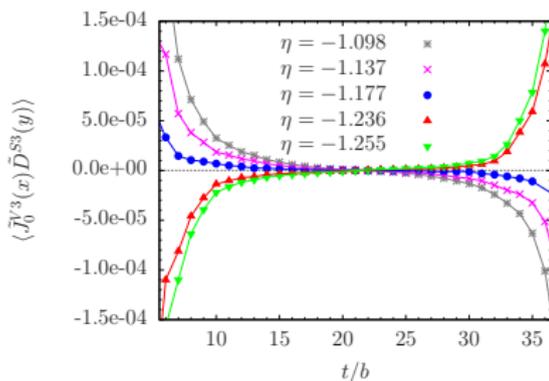
$$\tilde{D}^{S3}(y) = \bar{Q}_L(y) \left[\Phi, \frac{\tau^3}{2} \right] Q_R(y) - \bar{Q}_R(y) \left[\frac{\tau^3}{2}, \Phi^\dagger \right] Q_L(y)$$

- Renormalised Schwinger-Dyson eqs of \tilde{V}^3 -type (in the form of a would be $\tilde{\chi}$ -WTI):

$$\partial_\mu \tilde{J}_\mu^{V3} = (\eta - \eta_{cr}) \tilde{D}^{S3} + O(b^2) \quad \text{and} \quad \langle 0 | \partial_0 \tilde{J}_0^{V3} | M_S \rangle \equiv f_{M_S} M_{M_S}^2$$

► Determination of η_{cr} in the Wigner phase

In the Wigner phase at $\eta = \eta_{cr}$ the restored $\tilde{\chi}$ -symmetry is realised à la Wigner (*first example in a local setting*) and leads to vanishing correlation function $C_{\tilde{J}\tilde{D}}(x - y)$

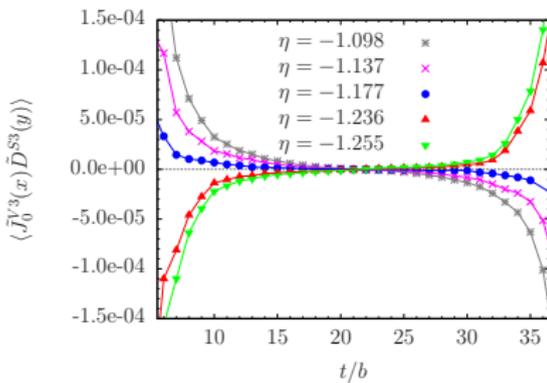


(at $a\mu_Q = 0.0224$)

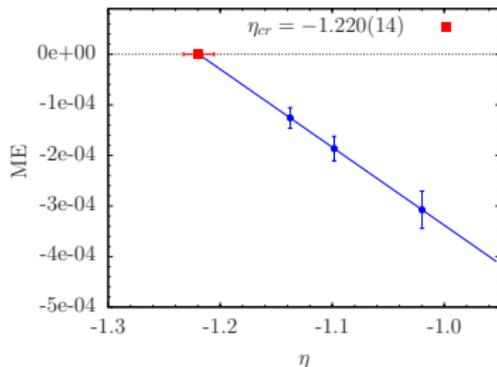
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vanishing matrix element
$$\text{ME} = \langle 0 | \tilde{J}_0^{V3} | M_S \rangle \langle M_S | \tilde{D}^{S3} | 0 \rangle \xrightarrow{\eta \rightarrow \eta_{cr}} 0$$



(at $a\mu_Q = 0.0224$)



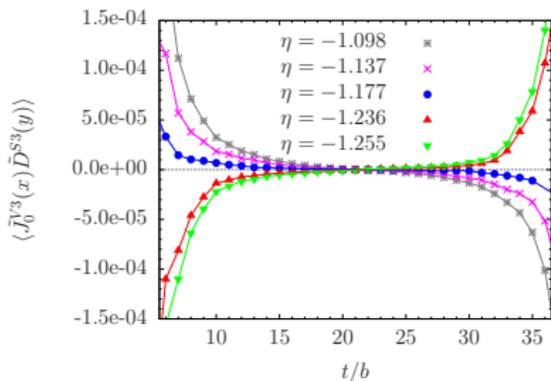
(after extrapolating to $a\mu_Q = 0$)

► Determination of η_{cr} in the Wigner phase

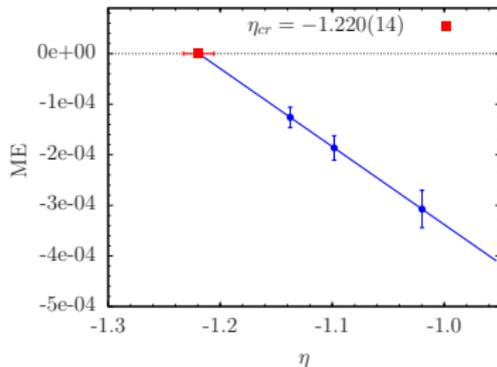
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(at $a\mu_Q = 0.0224$)



(after extrapolating to $a\mu_Q = 0$)

$$\eta_{cr} = -1.220(14) \quad (@ \beta = 5.85)$$

PRELIMINARY!

► Features and properties of the toy-model in NG-phase

- $V(\Phi)$ of mexican hat shape $\rightarrow \chi_L \times \chi_R$ realised à la NG.
- $\chi_L \times \chi_R$ spontaneously broken: $\Phi = v + \sigma + i\vec{\tau}\vec{\pi}$, $\langle \Phi \rangle = v \neq 0$
- $L_W(Q, A, \Phi) = \frac{\rho b^2}{2} \left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \text{h.c.} \right) \xrightarrow[a \leftrightarrow b]{r \leftrightarrow bv\rho} L_W^{QCD}(Q, A) = -\frac{ar}{2} (\bar{Q}_L D^2 Q_R + \text{h.c.})$
- In the *critical* theory $\eta = \eta_{cr}$:
 - the (Yukawa) mass term, $v\bar{Q}Q$, gets cancelled.
 - $\tilde{\chi}$ -breaking due to residual $O(b^2v)$ effects is expected to trigger dynamical χ SB.

⇒ Look for dynamically generated fermion mass:

- At generic η , two $\tilde{\chi}$ breaking operators:

Yukawa induced + **dynamically generated** (\leftarrow conjecture)

- $\Gamma^{NG} = \dots + (\eta - \eta_{cr})(\bar{Q}_L \langle \Phi \rangle Q_R + \text{h.c.}) + c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.})$

$$\text{where } \mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger \Phi}} = \frac{(v + \sigma)\mathbf{1} + i\vec{\tau}\vec{\varphi}}{\sqrt{v^2 + 2v\sigma + \sigma^2 + \vec{\varphi}\vec{\varphi}}} \simeq \mathbf{1} + i\frac{\vec{\tau}\vec{\varphi}}{v} + \dots$$

and $\Lambda_s \equiv$ RGI NP mass scale.

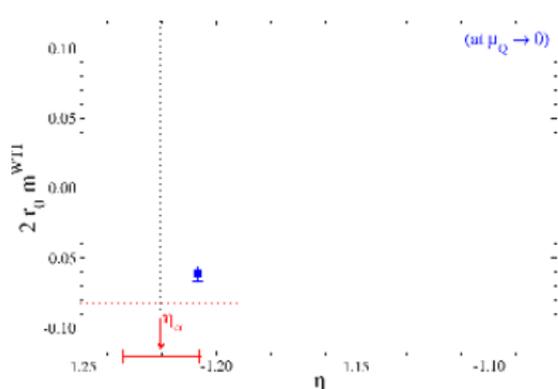
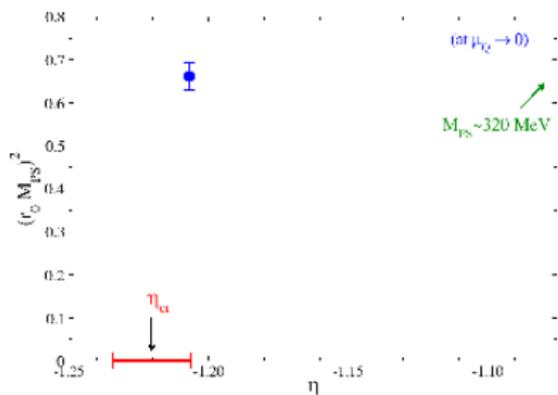
Note that (χ -inv. term): $c_1 \Lambda_s (\bar{Q}_L \mathcal{U} Q_R + \text{h.c.}) \simeq c_1 \Lambda_s \bar{Q}Q + \dots$

► Features and properties of the toy-model in NG-phase

- Work at the same lattice parameters (β , λ , ρ and volume) as in the Wigner phase
- Determine M_{PS} (from e.g. $\langle PP \rangle$ correlation function) and $m^{WTI} = \frac{\partial_0 \langle 0 | \tilde{J}_0^{A\pm} | M_{PS\pm} \rangle}{\langle 0 | P^\pm | M_{PS\pm} \rangle}$ in the NG-phase.

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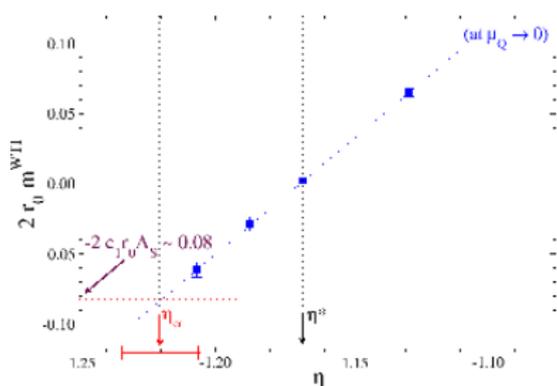
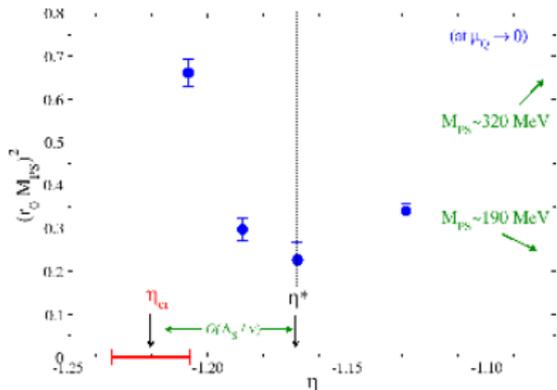
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At $\eta = \eta_{cr}$: $\begin{cases} M_{PS} \neq 0 \\ m^{WTI} \neq 0, \end{cases}$ Note $m^{WTI} = (\eta - \eta_{cr})v + c_1 \Lambda_s \xrightarrow{\eta = \eta_{cr}} m^{WTI} = c_1 \Lambda_s$

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$$\text{At } \eta = \eta_{cr} : \begin{cases} M_{PS} \neq 0 \\ m^{WTI} \neq 0, \end{cases} \quad \text{Note } m^{WTI} = (\eta - \eta_{cr})v + c_1 \Lambda_s \xrightarrow{\eta = \eta_{cr}} m^{WTI} = c_1 \Lambda_s$$

- m^{WTI} cancels at $\eta^* = \eta_{cr} - c_1 \Lambda_s / v \Rightarrow \eta_{cr} \neq \eta^* \leftrightarrow c_1 \Lambda_s \neq 0$
- At $\eta = \eta_{cr}$ for $\beta = 5.85$: $M_{PS} \sim 400 \text{ MeV}$ and $m_{bare}^{WTI} \sim 16 \text{ MeV}$.

(Preliminary results!)

Conclusions & Outlook

- We have presented a toy-model that exemplifies a *novel* NP mechanism for fermion mass generation.
R. Frezzotti and G.C. Rossi, PRD 2015, [arXiv:1402.0389 [hep-lat]]
- The **toy model** is a non-Abelian gauge model with an $SU(N_f = 2)$ -doublet of strongly interacting fermions coupled to scalars through Yukawa and Wilson-like terms: at the *critical point*, where (fermion) $\tilde{\chi}$ invariance is recovered in Wigner phase (up to UV-effects) the model is *conjectured* to give rise in NG phase to dynamical $\tilde{\chi}$ -SSB and hence to non-perturbative fermion mass generation.
- The main physical implications of the conjecture above can be *verified/falsified* by numerical simulations of the toy-model (rather cheap in the quenched approximation).

Conclusions & Outlook

- A *preliminary* study at one value of the lattice spacing (~ 0.12 fm) in the quenched approximation has been presented.
- We have discussed one possible method for the accurate determination of the critical value of the Yukawa coupling in the Wigner phase at which $\tilde{\chi}$ is restored. Then we explored the effects of dynamical SSB of the (restored) $\tilde{\chi}$ -symmetry in the NG phase which (barring cutoff effects) **look very well compatible with the generation of a non-zero (effective) fermion mass and $M_{PS} \sim O(\Lambda_s)$.**
- These findings have to be checked and verified at different values of the lattice spacing (*on-going work*) in order to confirm the **persistence of the dynamical mass generation mechanism in the continuum limit.**

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Thank you for your attention!

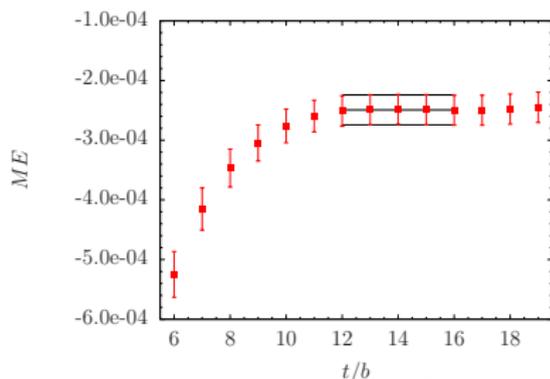
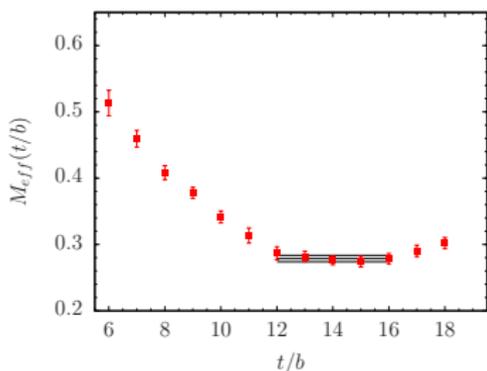
Extra slides

- Correlation function (example case $\eta = -1.020$ @ $a\mu_Q = 0.0224$ in

Wigner phase):

$$C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x) \tilde{D}^{S3}(y) \rangle$$

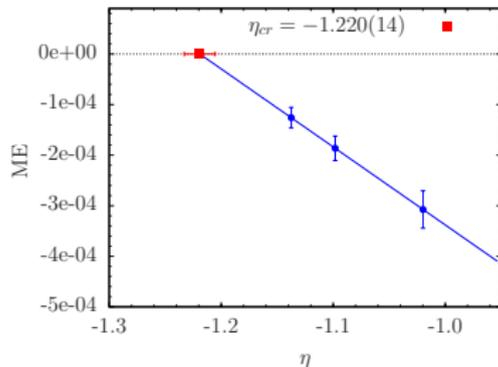
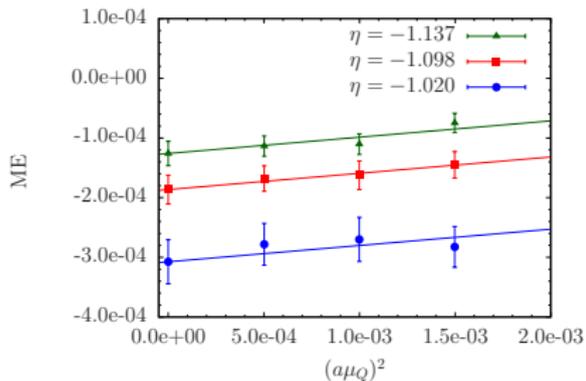
$$ME = \langle 0 | \tilde{J}_0^{V3} | M_S \rangle \langle M_S | \tilde{D}^{S3} | 0 \rangle$$



Extrapolation in $\mu_Q = 0$ and in η of

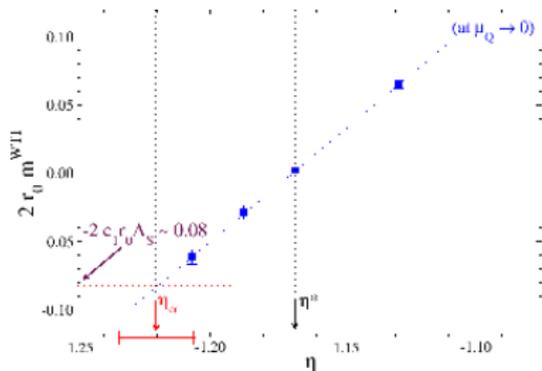
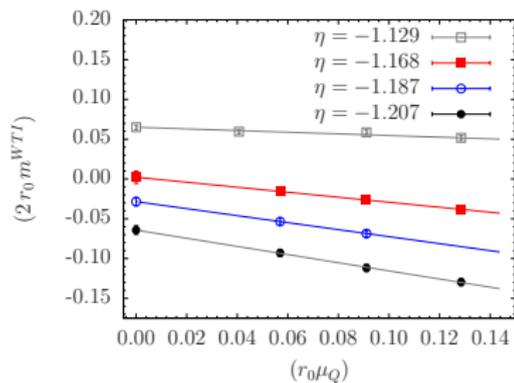
$$\text{ME} = \langle 0 | \tilde{J}_0^{V3} | M_S \rangle \langle M_S | \tilde{D}^{S3} | 0 \rangle$$

in order to determine η_{cr} where ME vanishes:



Extrapolation in $\mu_Q = 0$ (left panel) and η -dependence (right panel) for

$$2r_0 m^{WTI} = \frac{2r_0 \partial_0 \langle 0 | \tilde{J}_0^{A\pm} | M_{PS\pm} \rangle}{\langle 0 | P^\pm | M_{PS\pm} \rangle} \text{ in the NG-phase.}$$



Extrapolation in $\mu_Q = 0$ (left panel) and η -dependence (right panel) for $(r_0 M_{PS})^2$

