α_s from the HVP

Renwick Hudspith[1]

Randy Lewis [1], Kim Maltman [1], Eigo Shintani [2]

[1]York University

[2]Riken AICS

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renwick.james.hudspith@gmail.com

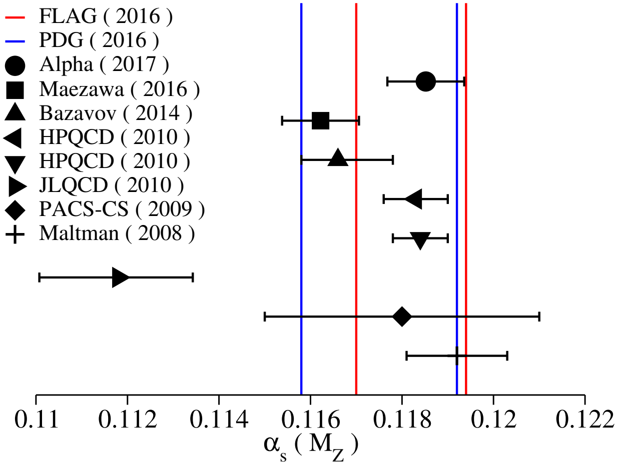

Figure: Results from [1, 2, 3, 4, 5, 6], PDG result is without the lattice data [7]. **Erratum value for JLQCD (2010)**.

WHY?

renwick.james.hudspith@gmail.com

- No $a^2 \to 0$ limit?

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- Other cut off effects?
- Possible μ-dependence?

Some PT results

$$D(Q^2,\mu^2) = D^{(0)}(Q^2,\mu^2) + \frac{m^2}{Q^2}D^{(2)}(Q^2,\mu^2) + \sum_{i=2}^{\infty}\frac{C_{2i}}{Q^{(2i)}},$$

$$D^{(0)}(Q^2,\mu^2) = \frac{1}{4\pi^2}\sum_{i=0}\left(\frac{\alpha(\mu)}{\pi}\right)^i\sum_{j=0}^{i-1}d_{ij}t^j \quad t = \ln\left(\frac{Q^2}{\mu^2}\right).$$

$$\Pi(Q^2,\mu^2) = C - \frac{1}{4\pi^2}\sum_{i=0}\left(\frac{\alpha(\mu)}{\pi}\right)^i\sum_{j=0}^{i-1}d_{ij}\frac{t^{j+1}}{j+1}.$$

$$\begin{aligned}\Delta(Q_1^2,Q^2,\mu^2) &= 4\pi^2\left(\Pi(Q_1^2,\mu^2)-\Pi(Q^2,\mu^2)\right)-1,\\ &= \sum_{i=1}\left(\frac{\alpha(\mu)}{\pi}\right)^i\sum_{j=1}^{i-1}d_{ij}\frac{t_1^{j+1}-t^{j+1}}{(j+1)(t_1-t)}.\end{aligned}$$

Lessons from FESRs

$$\int_0^{s_0} \mathrm{ds}\, w\left(\frac{s}{s_0}\right)\rho(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} \mathrm{ds}\, w\left(\frac{s}{s_0}\right)\Pi(s).$$

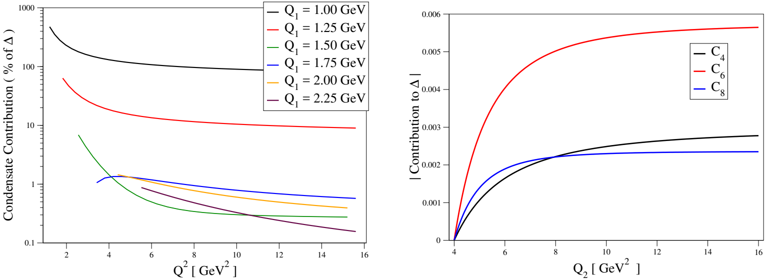
- FESR relation, powers of $\left(\frac{s}{s_0}\right)$ access condensates! $(s=-Q^2)$

$$-\frac{1}{2\pi i s_0}\oint_{|s|=s_0} \mathrm{ds}\left(\frac{s}{s_0}\right)^n \frac{C_{2i}}{(-s)^n} = (-1)^{n+1}\frac{C_{2n+2}}{s_0^{n+1}}\delta_{i,n+1}.$$

- Rough estimate for C_4, and C_6 and C_8 from [8]

$$C_4 = 0.0012\ \mathrm{GeV}^4, \quad C_6 = -0.0093\ \mathrm{GeV}^6, \quad C_8 = 0.0153\ \mathrm{GeV}^8.$$

FESR generalised condensates



(a) Contribution to Δ (b) Fixed $Q_1 = 2$ GeV magnitudes

Figure: Contribution to Δ of the effective condensates from the FESR relations.

Ensemble details

	Coarse			Fine			Superfine
Extent	$24^3 \times 64 \times 32$			$32^3 \times 64 \times 32$			$32^3 \times 64 \times 32$
a^{-1} (GeV)	1.7848(50)			2.3833(86)			3.148(17)
am_l	0.005	0.01	0.02	0.004	0.006	0.008	0.0047
m_π (GeV)	0.33	0.42	0.54	0.28	0.33	0.38	0.37
measurements	685	110	109	510	352	80	920
am_{res}	0.003076(58)			0.0006643(82)			0.0006296(58)
Z_V	0.71408(58)			0.74404(181)			0.77700(8)

Table: Simulation parameters for the ensembles used in this study. a^{-1}, Z_V and m_{res} have been measured in [9].

Lattice HVP

- Conserved-Local vector currents → Lattice Ward Identity

$$\sum_\mu \hat{Q}_\mu e^{iaQ_\mu/2}\Pi_{\mu\nu}(Q) = 0,\ \hat{Q}_\mu = 2\sin(aQ_\mu/2),\ Q_\mu = \frac{2\pi n_\mu}{L_\mu}.$$

- Reflection projection eliminates some H_4 invariants

$$\Pi_{\mu\nu}(\hat{Q}^2 : a^2) = \Pi_{\mu\nu}(Q^2) + \sum_{m,n;m+n\geq 2} C_{mn}a^{m+n}\hat{Q}_\mu^m\hat{Q}_\nu^n.$$

$$\Pi(\hat{Q}^2 : a^2) = \frac{1}{12}\sum_\mu\sum_{\nu\neq\mu}\frac{\Pi_{\mu\nu}(\hat{Q} : a^2) - \Pi_{\mu\nu}(r_\mu\hat{Q} : a^2)}{2\hat{Q}_\mu\hat{Q}_\nu}.$$

- Cylinder cut [10, 11]

$$|aQ_\mu - (aQ_\mu n_\mu)n_\mu| < aw\frac{2\pi}{L}.$$

Modelling of cut-off effects

- Expect dominant **rotation-preserving** higher-order terms

$$\Pi(\hat{Q}^2 : a^2) = \Pi(Q^2) \; + \; C_1(a^2\hat{Q}^2) \; + \; C_2(a^2\hat{Q}^2)^2.$$

- And a correction to the continuum coupling

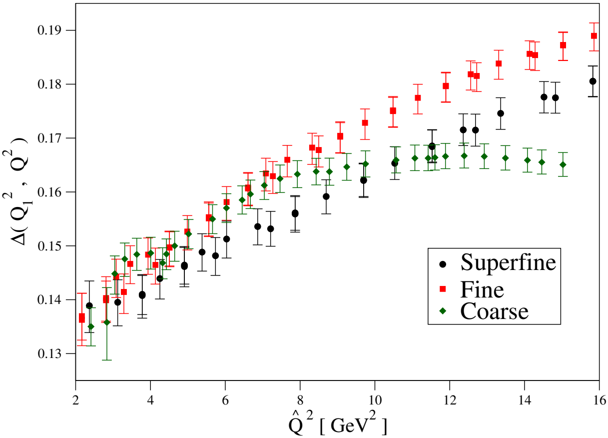
$$\alpha(\mu : a^2) = \alpha(\mu)\left(1 + C_\alpha a^2\right).$$

- Cannot fit $O(a^4)$ correction to coupling or $O((a^2\hat{Q}^2)^3)$ terms

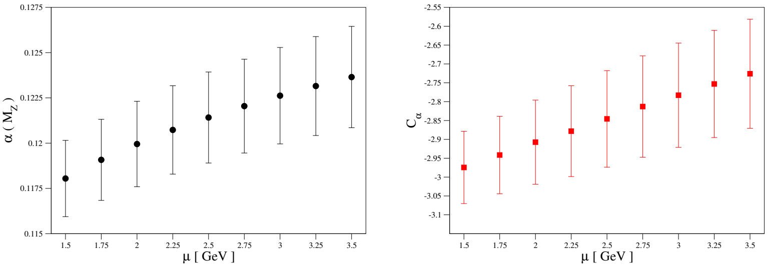
NB

All in all we will have **4 parameters** in our uncorrelated *simultaneous* fit.

Plot of our data



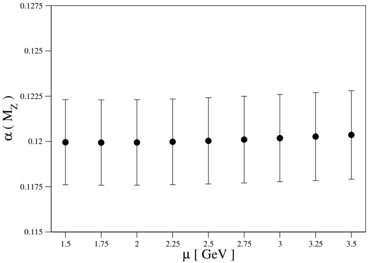
Single-scale analysis is disconcerting



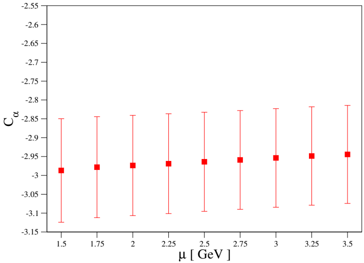
(a) $\alpha(M_Z)$ (b) C_α

Figure: With our data, single μ fit has **strong dependence on** μ.

Multiple scale fitting provides consistent results



(a) $\alpha(M_Z)$



(b) C_α

Figure: Multiple μ analysis with $m = 2, 2.25, 2.5, 2.75$ GeV **removes the dependence**.

Coarse ensemble is nothing but trouble

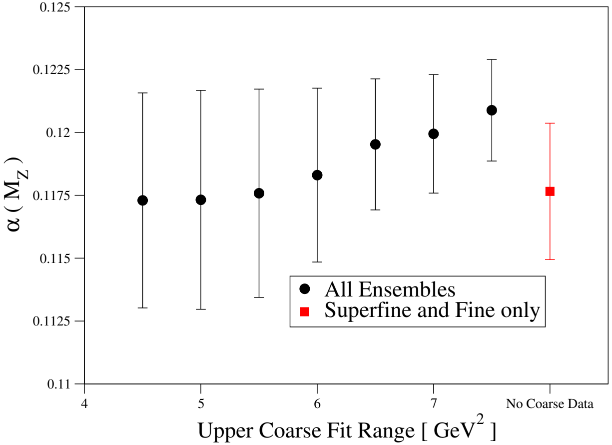


Figure: Where we trust the fit range for the Coarse ensemble all it does is add noise.

Still full of systematics

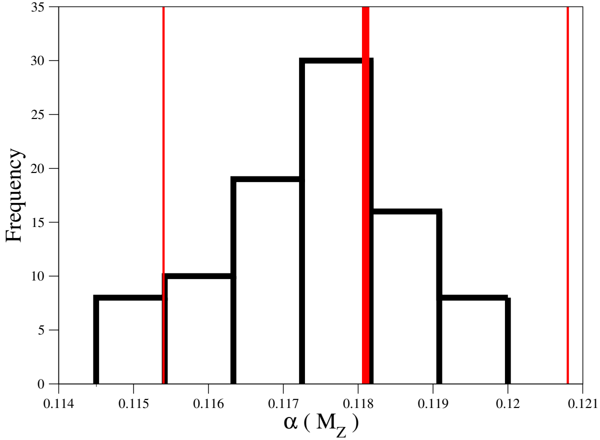


Figure: Histogram of central values for $\alpha(M_Z)$ when the fit range in $\hat{Q}^2$ is varied. Red is our central result with statistical errors.

99 systematics but loop order ain't one

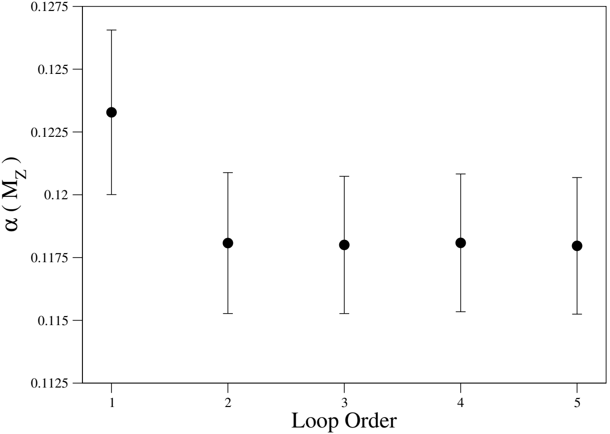


Figure: Result is not sensitive to loop order within error after 1 loop truncation.

Where does our result lie?

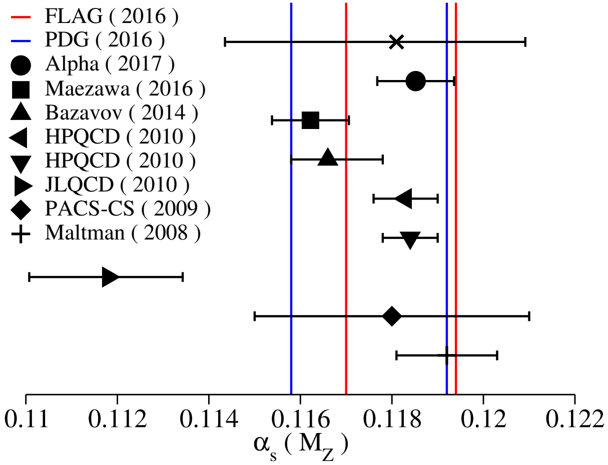


Figure: Results from [1, 2, 3, 4, 5, 6], PDG result is without the lattice data [7]. **Our preliminary result is the** $\times$.

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