Charge creation, finite size effects and infra-red photons in simulations of QCD plus QED

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Introduction

In lattice QED we do some unnatural things.
In QCD we normally create colour singlets (though there are exceptions.
In QED we frequently use source and sink operators which interpolate between the vacuum and a charged state. How will the electromagnetic fields behave when a charge suddenly appears or disappears?

Two model systems.
A finite Wilson line.
A charged fermion in perturbation theory.

Lattice QED in a Box

Now that we are simulating QED on the lattice we have some issues like this to think about. When we use an interpolating field to create a charged π^+ or proton from the vacuum, what is the EM field supposed to do? Maxwell's equations don't have a sub-paragraph saying what should happen when charge is not conserved.

Maxwell and Noether

In classical physics we have the Maxwell equations

$$\begin{aligned} \nabla \cdot E &= \rho \\ \nabla \times B - \frac{\partial E}{\partial t} &= J \end{aligned}$$

What is their status in QFT? Some might take them to be definitions, so that the charge density *is* the divergence of E, configuration by configuration. However, I would understand them as equations of motion. On the left-hand side we have quantities calculated from the photon fields, on the right hand side we have ρ or J calculated from the conserved (Noether) currents constructed from the charged particles, eg $\bar{\psi}\gamma_\mu\psi$.

Maxwell and Noether

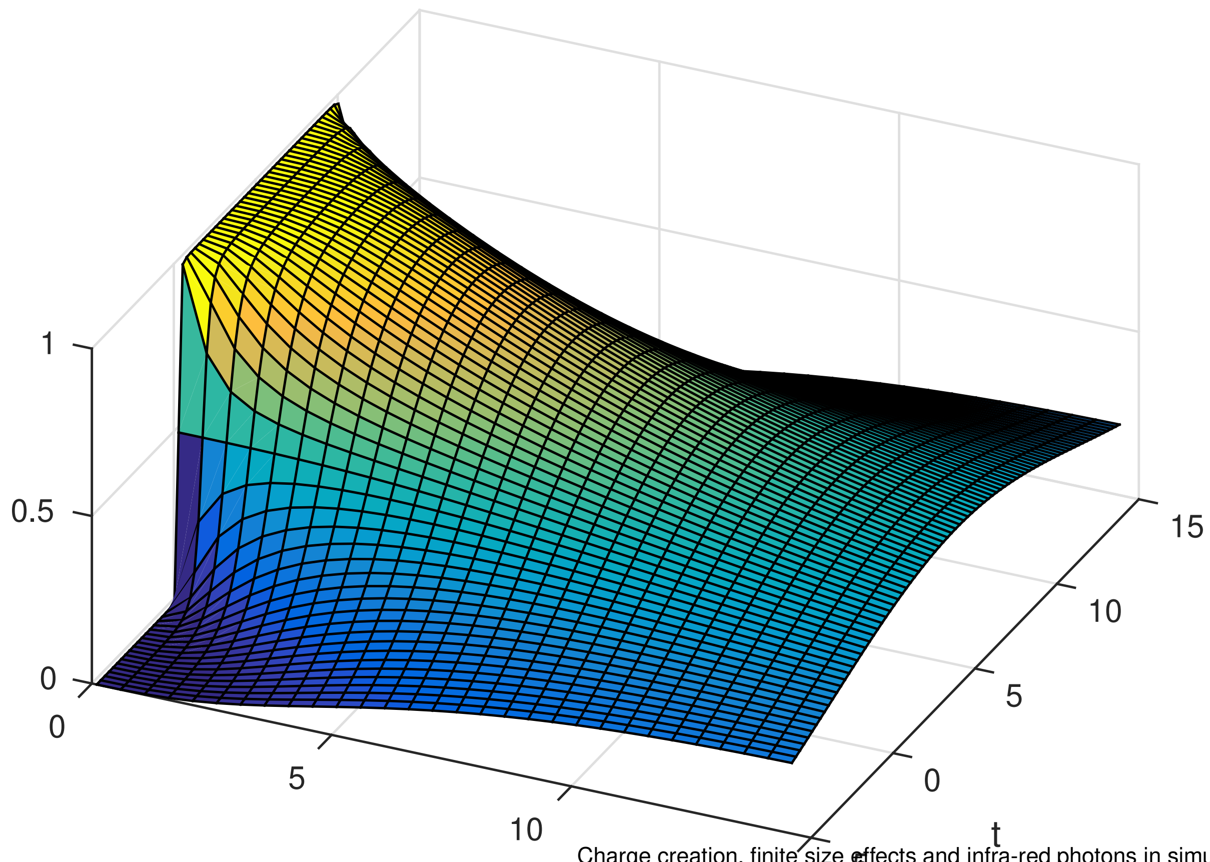
In this case the above equations are only true on average at best. If we use finite volume boundary conditions we may cause extra terms. If we create particles in a way that doesn't obey charge conservation, there is no way for the electric and magnetic fields to obey Maxwell's equations.

Maxwell and Noether

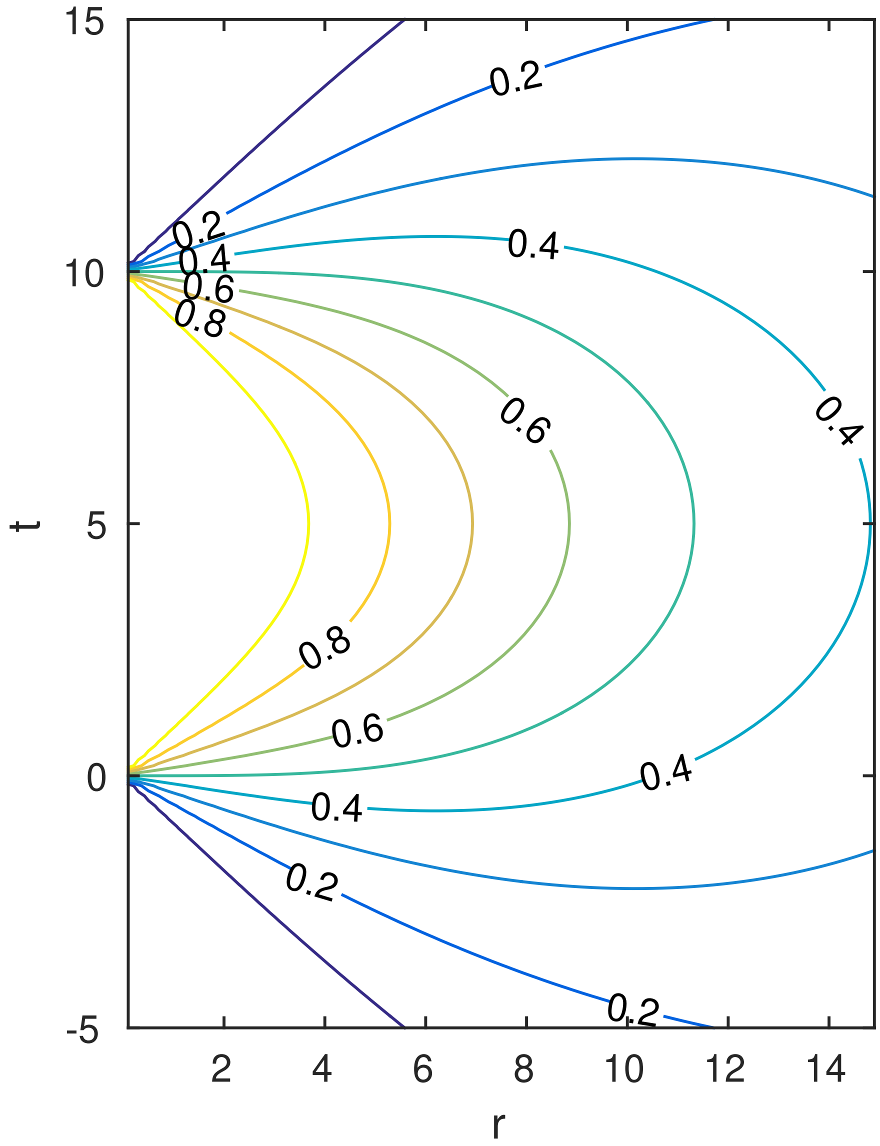
To get some idea of what might happen when we create charge ex nihilo, I have started looking at the following situation. We create a classical static charge at the origin at time 0, and annihilate it at time T. We then calculate the gauge field that minimises the action, and assume that this is the background about which the quantum fields will fluctuate.

Maxwell and Noether

We show the quantity $4\pi r^2 E_r$, which would be 1 if we had the expected radial field of a point charge. Close to the charge we do have the expected electric field, but farther out the field strength drops, as if there were some shielding taking place.



Maxwell and Noether



Maxwell and Noether

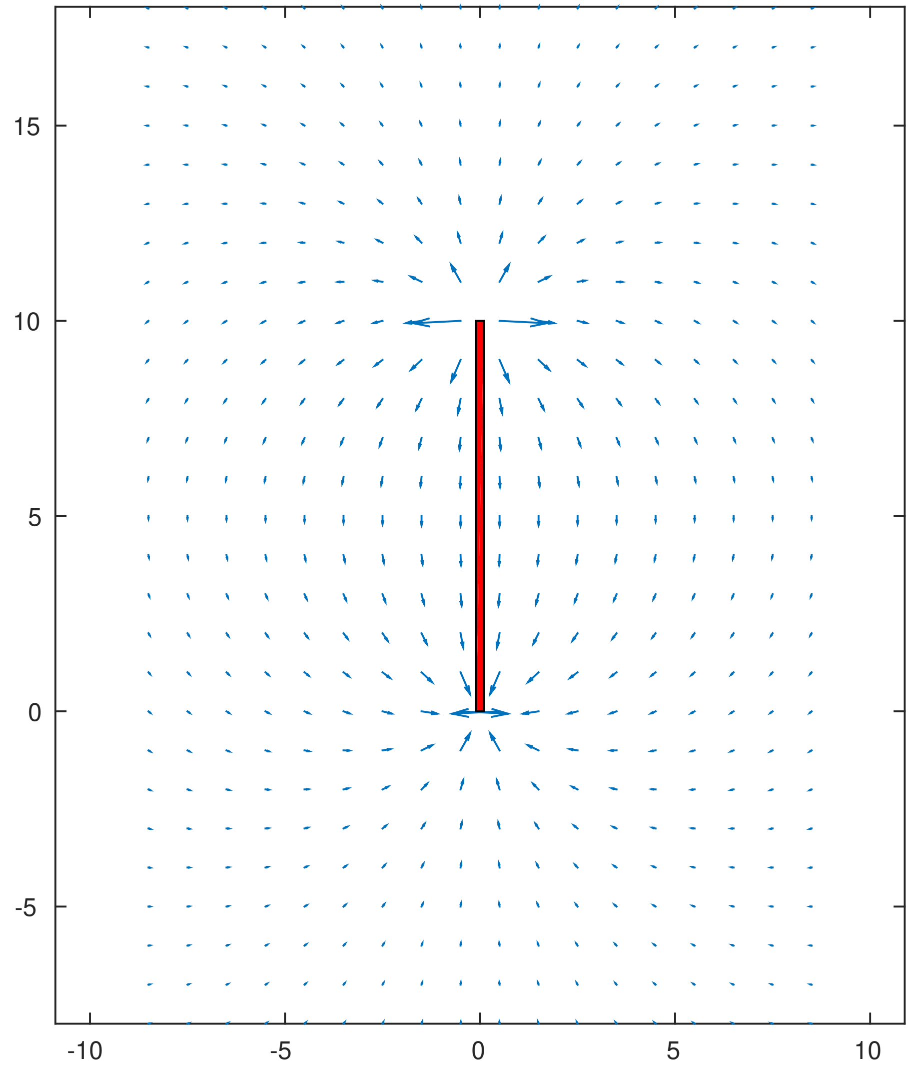
Gauss charge density $\nabla \cdot E$

Noether charge density $\bar{\psi}\gamma^4\psi$
What is happening to Gauss' law? If we look at $\nabla \cdot E$ and at $\partial E/\partial t$ they are not zero. The electromagnetic field acts as if there are additional charges and currents. We can calculate the current

$$P_\mu \equiv \partial_\nu F_{\mu\nu} \tag{1}$$

(the left hand side of Maxwell's equations, just involving the photon fields). This current is automatically conserved for any A_μ.

Maxwell and Noether



Maxwell and Noether

We see that this current has the expected δ function at the origin, where we put our classical charge (Polyakov line), but it also has a return flow of apparent charge, which is the cause (or maybe the effect) of the shielding.

Creating a charge in a box

Create a fixed unit charge $(+1)$ at the origin, at time zero.

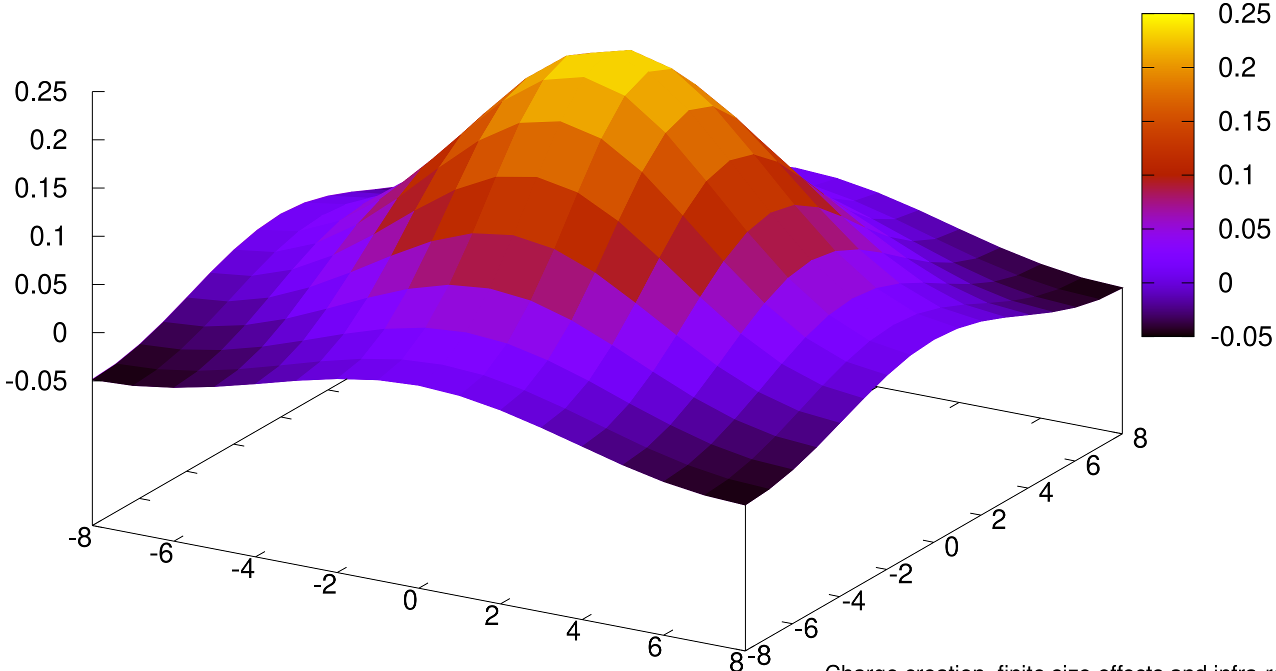
Noether charge: Zero before charge creation,
δ function afterwards.

Gauss charge $\nabla \cdot E$
More interesting.

Creating a charge in a box

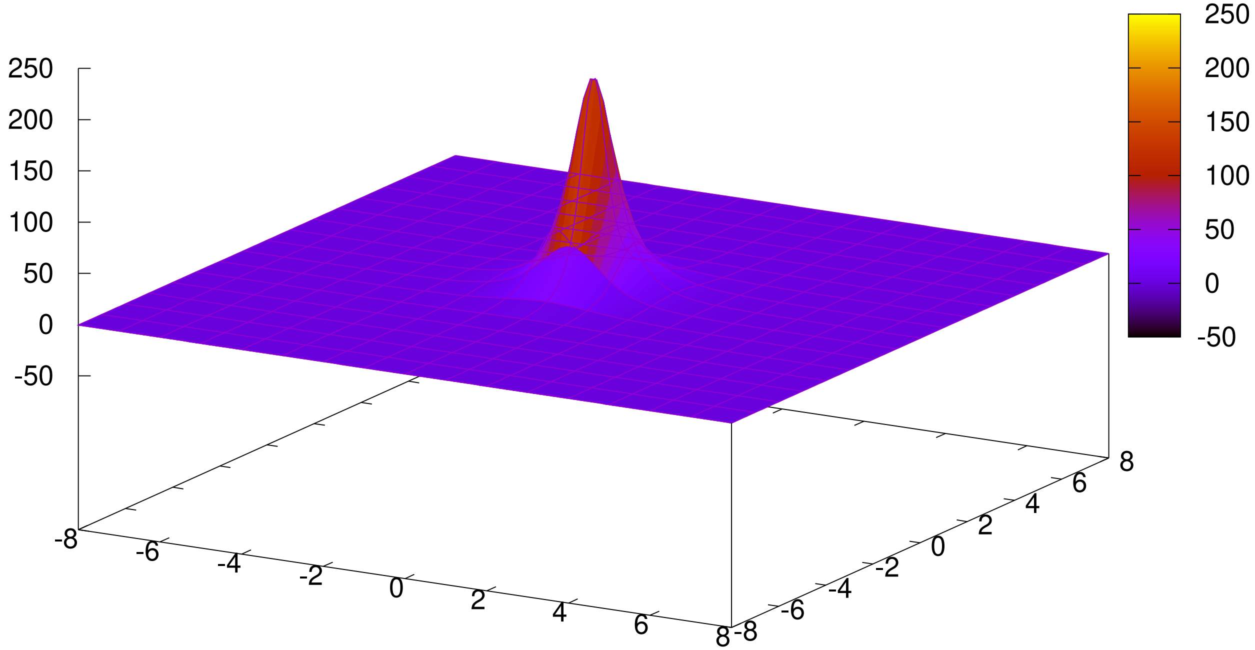
1) Before $t = 0$ the Gauss charge starts separating, positive charge gathers near the origin, negative charge outside, (so total Gauss charge always zero).

Remember, time is Euclidean, no causality.



Creating a charge in a box

2) Positive charge gathers at origin. At $t = 0_-$ we have a δ function with charge $+\frac{1}{2}$, balanced by diffuse background.



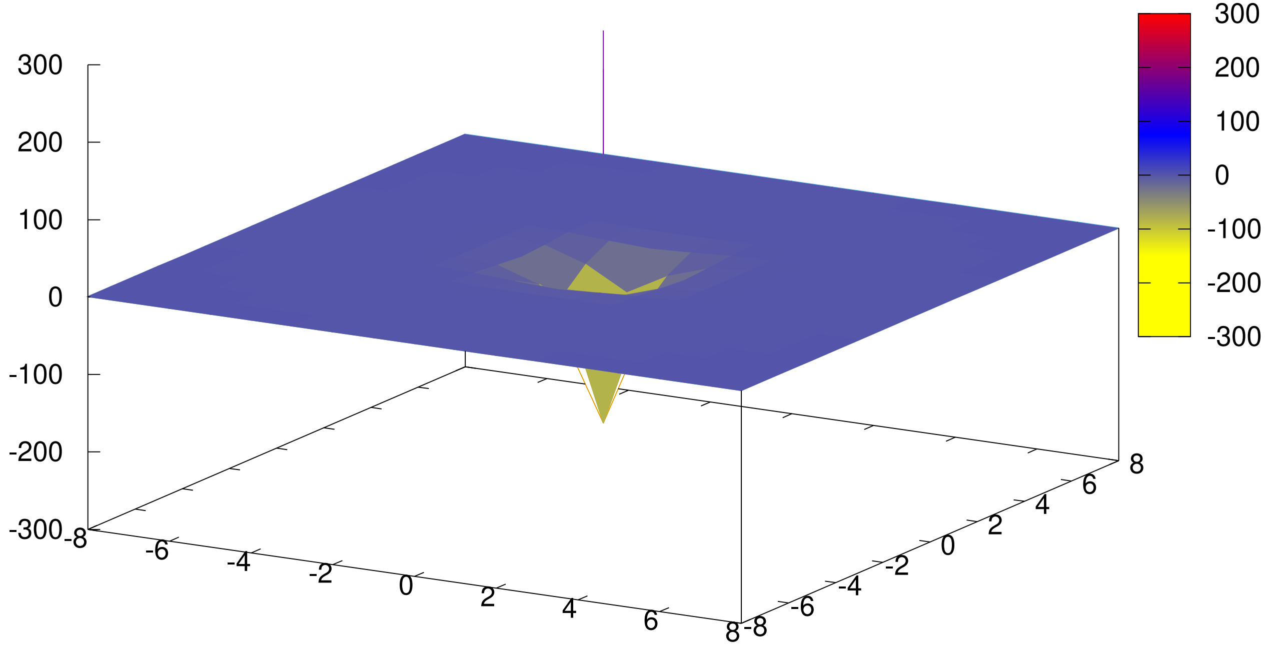
Creating a charge in a box

3) At $t = 0$ the classic charge (Noether charge) is created at the origin.

Gauss charge is still $+\frac{1}{2}$ at origin.

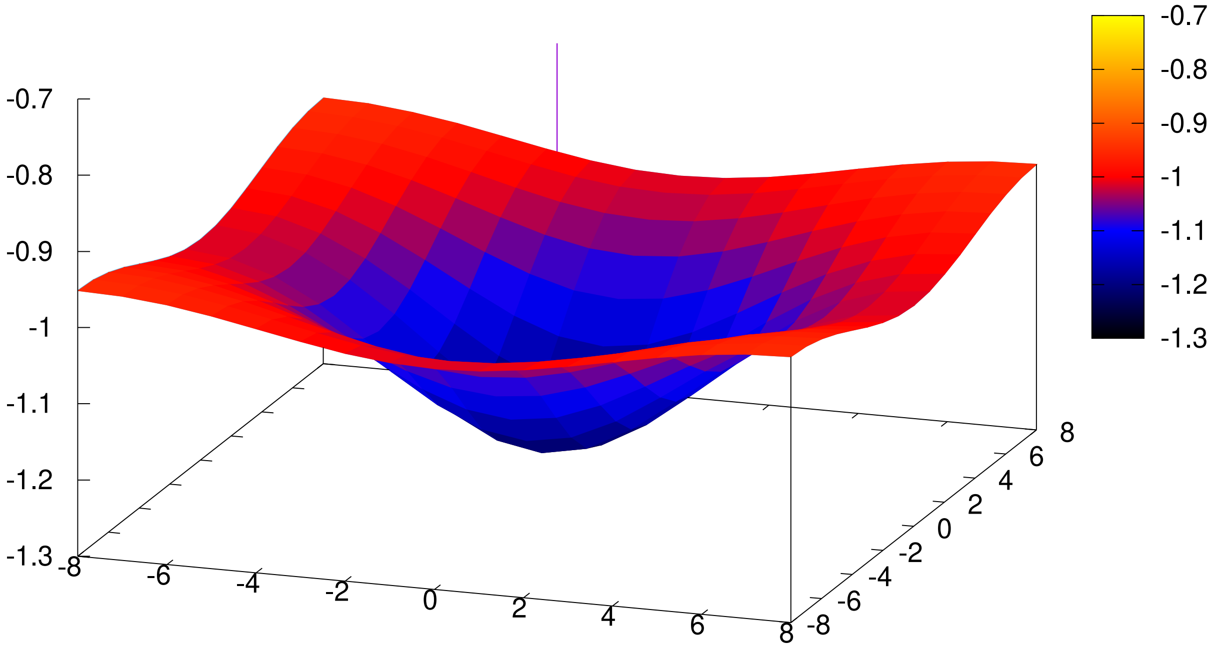
Creating a charge in a box

4) For small positive t the Gauss charge reveals itself as a δ function of charge $+1$, and a shielding cloud of charge $-\frac{1}{2}$.



Creating a charge in a box

5) The shielding cloud spreads through the lattice, leaving a unit δ at the origin, and a background charge of -1 evenly spread through the lattice.



Propagating a charged particle in a box

How does this spread of fields effect the propagator of a charged particle?
Test case, charged fermion, 1-loop calculation.
3-momentum $\vec{p} = 0$
Polarisation projector $\frac{1}{2}(1 + \gamma_0)$. Favours forward propagating state, suppresses backward propagator.

Charged particle not in a box

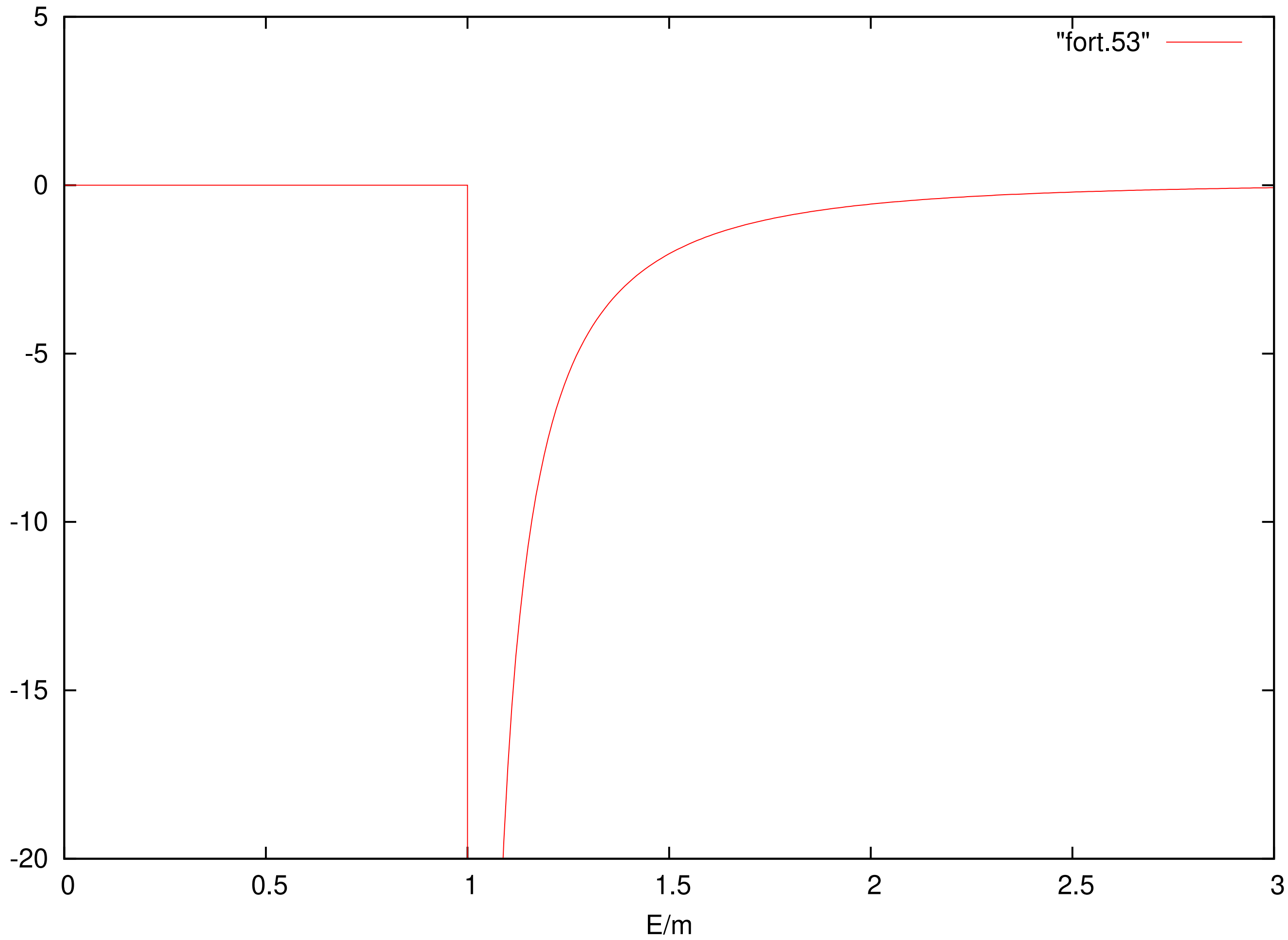
In infinite volume tree-level propagator has poles at $p^2 = m^2$.
Beyond tree-level also has a cut for $p^2 > m^2$.
Contribution of the cut to the (Euclidean) propagator at time t.

$$\frac{1}{2\pi} \int_m^\infty dE e^{-Et} \rho(E)$$

(Like a Laplace transform).
Discontinuity: $\rho(E)$ has a pole at $E = m$ (lower limit of integral).
Infra-red logarithms in the propagator.

Charged particle not in a box

Cut in QED



Charged particle not in a box

Although normalisation has IR logs, shape of propagator better defined.
Effective mass:

$$m + e^{mt}\frac{1}{2\pi}\int_m^\infty dE e^{-Et}(E-m)\rho(E)$$

The $(E-m)$ factor cancels the pole in ρ.
Integral can be done in closed form, answer in terms of exponential integral E_1.
At large time, cut contribution to effective mass

$$\sim \alpha_{EM}/t$$

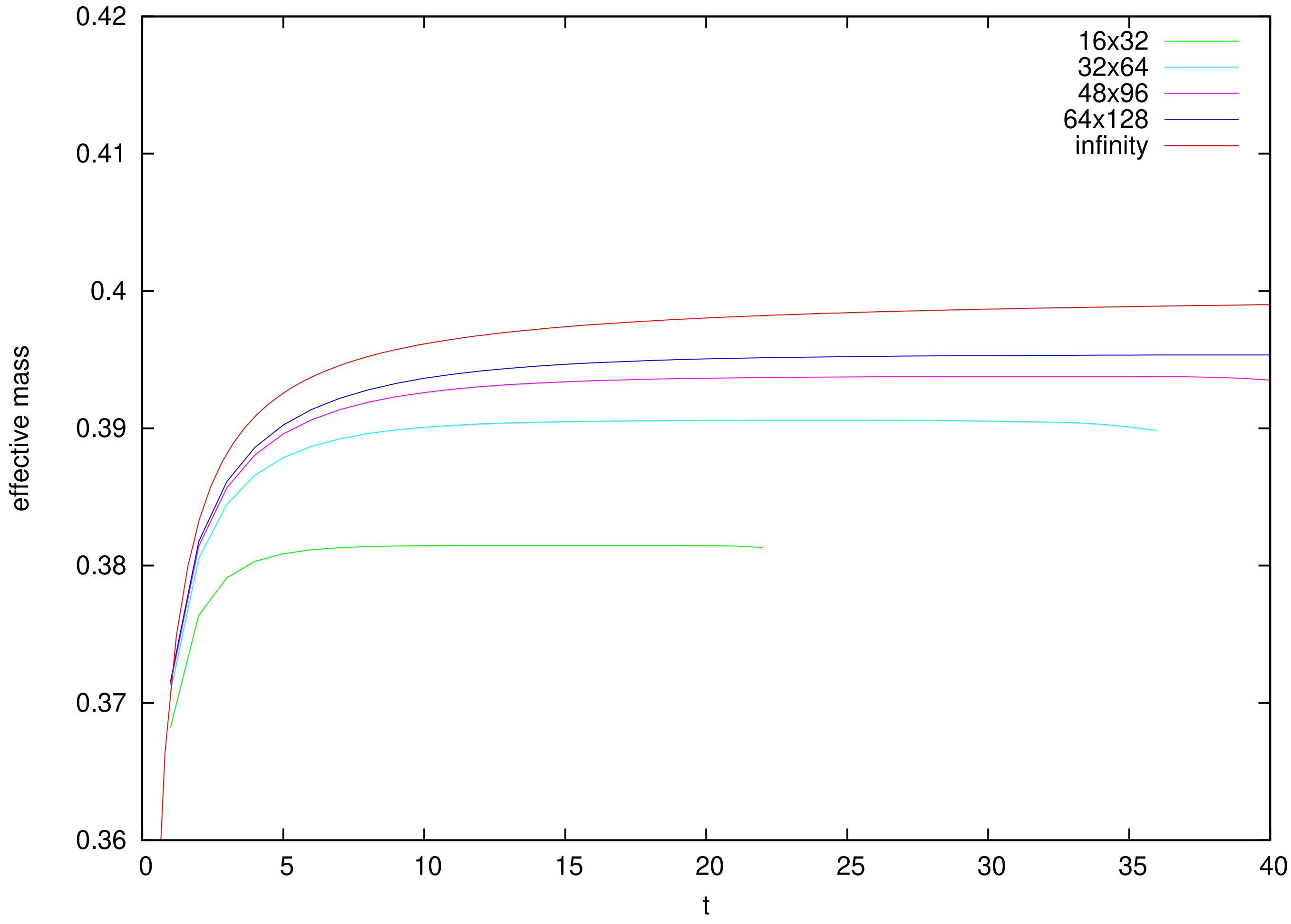
Propagating a charged particle in a box

In a box, the cut is replaced by a collection of discrete energy states.
At short times, many states contribute, effective mass still drops like $1/t$.
At larger time, $O(L_s)$, excited states drop out, just see the shifted fermion pole.

Davoudi and Savage

Propagating a charged particle in a box

Effective mass in QED



Conclusions

It takes time for the electric field around a newly created charged particle to grow. Initially finite size effects are small.

Complementary explanation: Telling the difference between a cut and a row of poles requires energy resolution comparable to the spacing between poles. Good energy resolution requires long times.

If lattices become really large in physical units, we will need to think about finite size effects on Greens functions, not on derived quantities (mass, energy).