Setting up a twisted mass QCD valence action for flavour physics

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In collaboration with:
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IFT UAM-CSIC

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Motivation: Flavour Physics

- Target: Determine CKM matrix elements through flavour-changing processes and check the consistency with SM.
- Sector: Charm physics, $|V_{cd}|$ and $|V_{cs}|$.
- Processes: Leptonic and semileptonic D decays.

	Exp [HFAG 2016, PDG 2017]		LQCD [FLAG-3]
$D_s^+ \to \tau^+ \nu_\tau$	$\delta\mathcal{B} \sim 2.3\text{-}3.6\%$,	$\delta\tau_{D_s} \sim 1.4\%$	$f_{D_s} \sim 0.5\%$
$D^+ \to \mu^+ \nu_\mu$	$\delta\mathcal{B} \sim 4.5\%$,	$\delta\tau_D \sim 0.7\%$	$f_D \sim 0.7\%$
$D \to \pi \ell \nu_\ell$	$f_+^\pi(q^2{=}0)\lvert V_{cd}\rvert \sim 1.3\%$		$f_+^\pi(q^2{=}0) \sim 4.4\%$
$D \to K \ell \nu_\ell$	$f_+^K(q^2{=}0)\lvert V_{cs}\rvert \sim 0.5\%$		$f_+^K(q^2{=}0) \sim 2.5\%$

- Preliminary results: light sector.

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Sea quarks: CLS. [1411.3982] [Talk by Mohler, Lattice 2017]

- Open boundary conditions in time.
  - Improve the scaling of the algorithm considerably: topological sectors become connected and topological charge can smoothly flow. [Lüscher: 1009.5877] [Lüscher, Schaefer: 1105.4749]
    - Autocorrelation times scale $\sim 1/a^2$.
    - Possibility to simulate at fine $a \quad \longrightarrow \quad$ heavy quark physics.
- Action:
  - Gauge action: Improved Lüscher-Weisz gauge action.
  - Fermion action: $N_f = 2+1$ Wilson fermions with a non-perturbative c_{sw}.
  - Renormalised chiral trajectory:

$$\phi_4 = 8t_0 \left(m_K^2 + \tfrac{1}{2} m_\pi^2\right) = \text{const.}$$

- Lattice spacing (fm):

$$a = 0.087,\ 0.077,\ 0.064,\ 0.050,\ \sim 0.039.$$

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Renormalised chiral trajectory [Bruno, Korzec, Schaefer: 1608.08900]

- Chiral trajectory:

$$\mathrm{tr} M_q = \text{const}, \quad M_q = \mathrm{diag}\left(m_{q_u}, m_{q_d}, m_{q_s}\right).$$

  - Chiral trajectory used in simulations.
  - Keep constant $\mathcal{O}\left(a \mathrm{tr} M_q\right)$ cutoff effects.
  - Slight deviation from renormalised chiral trajectory.
- Renormalised chiral trajectory:
  - $\phi_4 = 8t_0\left(m_K^2 + \frac{1}{2}m_\pi^2\right) = \text{const}$
    $\implies \quad \mathrm{tr} M_R = \text{const} + \mathcal{O}\left(m_q^2\right)$.
- Correct chiral trajectory on observable:
  - Method: Perform small correction in m through a Taylor exp.

$$f\left(m'\right) = f\left(m\right) + \left(m' - m\right)\frac{df}{dm},$$

$$\frac{df}{dm} = \sum_i \frac{\partial f}{\partial \overline{A}_i}\left[\left\langle \frac{\partial A_i}{\partial m}\right\rangle - \left\langle \left(A_i - \overline{A}_i\right)\left(\frac{\partial S}{\partial m} - \overline{\frac{\partial S}{\partial m}}\right)\right\rangle\right].$$

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Valence quarks: Wilson twisted mass

[Frezzotti et al.: hep-lat/0101001] [Frezzotti and Rossi: hep-lat/0306014]

[Pena et al.: hep-lat/0405028]

- Valence action in twisted basis:

$$\frac{1}{2}\gamma_\mu\left(\nabla_\mu^* + \nabla_\mu\right) - \frac{a}{2}\nabla_\mu^*\nabla_\mu + \frac{i}{4}ac_{sw}\sigma_{\mu\nu}\widehat{F}_{\mu\nu} + \mathbf{m}^0 + i\gamma_5\boldsymbol{\mu}^{\mathbf{0}}.$$

- At maximal twist $\omega = \pi/2$:

$$\mathbf{m}_0 = m_{cr}\mathbb{1}\ .$$

$$\boldsymbol{\mu}^{\mathbf{0}} = \mathrm{diag}\left(\mu_\ell, -\mu_\ell, \pm\mu_s, \mp\mu_c\right)\ .$$

- Properties:
  - Automatic $\mathcal{O}(a)$ improvement of physical observables at maximal twist save of: $\mathcal{O}\left(ag_0^4\mathrm{tr}M\right)$.
  - In particular, terms of order $\mathcal{O}(a\mu_c)$ are absent.
  - Simplifies renormalisation on some observables, e.g.: f_π.
  - SW term included in valence expected to reduce flavour breaking cutoff effects.

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Tuning to maximal twist

- Maximal twist:

$$\kappa_\ell \equiv \tilde{\kappa}_{cr} \quad \Longrightarrow \quad m_{12}^R\Big|_v \to 0.$$

- Matching:

1

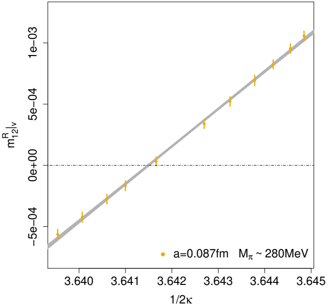
$$\mu_\ell^R\Big|_v \equiv m_{12}^R\Big|_s .$$

$$\frac{1}{\cancel{Z_P}}\,\mu_\ell = \frac{Z_A}{\cancel{Z_P}}\,m_{12}\left(1+\left(\tilde{b}_A-\tilde{b}_P\right)am_{12}+\left(\overline{b}_A-\overline{b}_P\right)a\mathrm{tr}M_q\right).$$

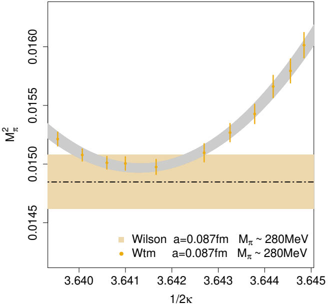
2 Alternatively:

$$M_\pi^2\Big|_v \equiv M_\pi^2\Big|_s .$$

$$M_\pi^2 \propto \sqrt{ m_{12}^R\big|_v^{\,2} + \mu_\ell^{R\,2} }$$



Preliminary

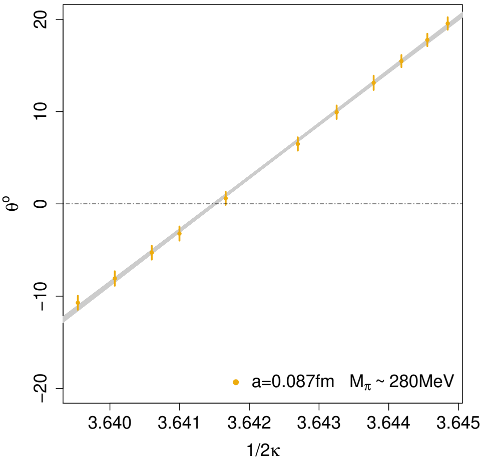


$$\mu_\ell^R\big|_v \equiv m_{12}^R\big|_s$$

$$\mathrm{Tr} M_q = \text{const}$$

Preliminary

$$\theta = \pi/2 - \omega = \pi/2 - \tan^{-1}\left(\frac{\mu_\ell^R}{m_{12}^R\big|_v}\right)$$

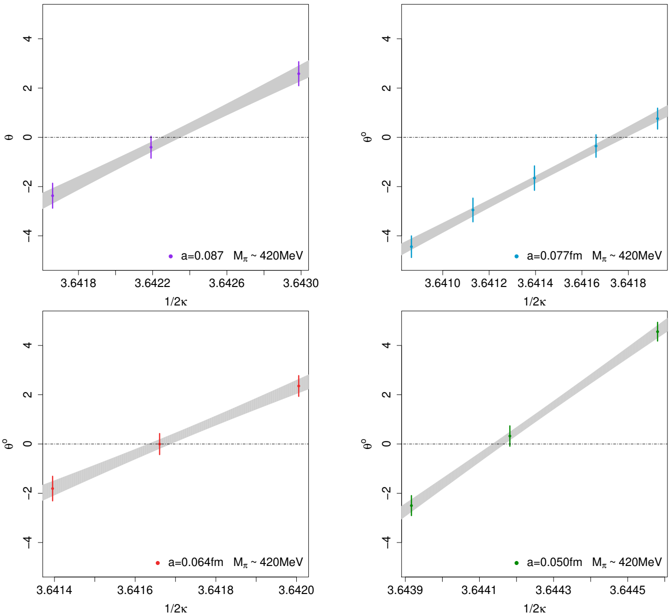


$$\mu_\ell^R\big|_v \equiv m_{12}^R\big|_s$$

$$\mathrm{Tr} M_q = \text{const}$$

Symmetric point: $m_\ell = m_s$, $M_\pi = 420\text{MeV}$, $\theta = \pi/2 - \omega$

Preliminary



$$\mu_\ell^R\Big|_v \equiv m_{12}^R\Big|_s$$

$$\phi_4 = \text{const}$$

M_π

Preliminary

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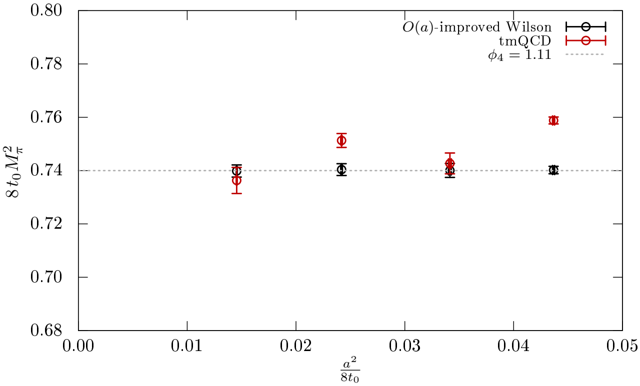

[Bruno et al. 1608.08900] [Javier Ugarrio et al., poster Lattice 2017.]

$\mu_\ell^R\big|_v \equiv m_{12}^R\big|_s$

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Continuum limit scaling of f_π

Preliminary

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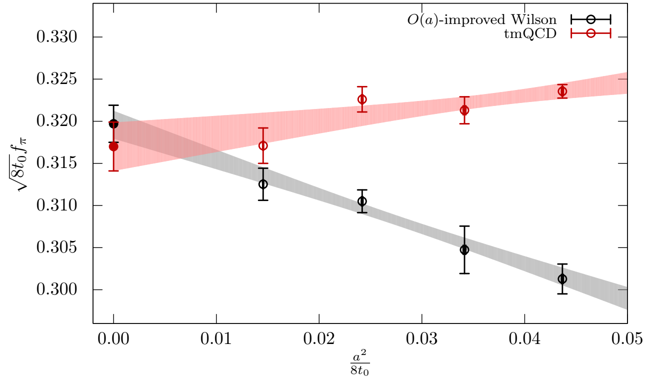

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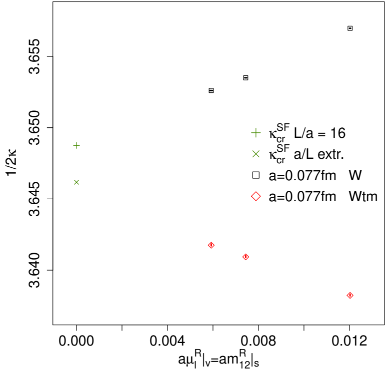
Check of κ_{cr}

Preliminary

Symmetric point: $m_\ell = m_s$, $a = 0.077$fm.



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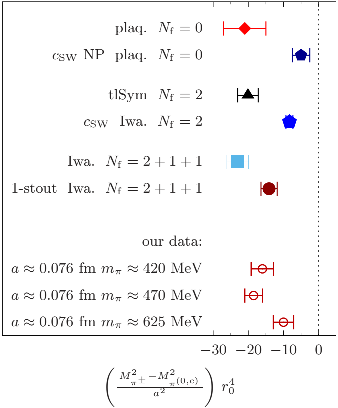
[ALPHA, Fritzch and Korzec 2017]

Pion mass splitting: charged vs neutral connected

Preliminary

Discretisation effects of $\mathcal{O}\left(a^2\right)$: $M^2_{\pi^\pm} - M^2_{\pi^{(0,c)}} = O(a^2)$



[χLF: hep-lat/0507032] [ALPHA: 0902.1074] [ETMC: 1303.3516,1507.05068]

- Conclusions:
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  - Matching of quark masses is simplified: same renormalisation factors in sea and valence sectors.
  - Universality check of f_π: important test of the mixed action.
  - The accuracy of Wtm results look promising.
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