$\pi\pi$ P – WAVE RESONANT SCATTERING FROM LATTICE QCD.

[arXiv:1704.05439]

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HPC-LEAP EUROPEAN JOINT DOCTORATES

In collaboration with C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, M. Petschlies, A. Pochinsky, G. Rendon, S. Syritsyn

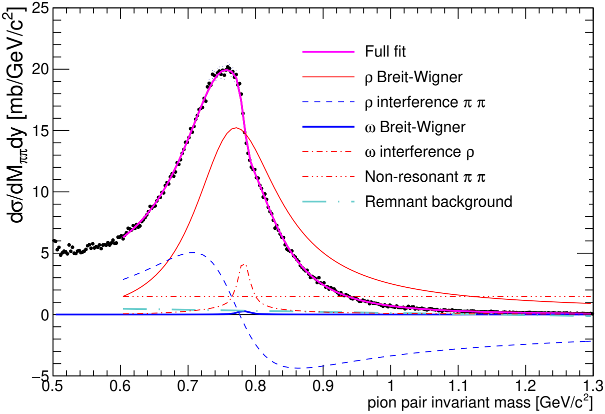
LATTICE, Granada, June 19, 2017

$\pi\pi$-scattering

- Importance of study

- In nature, ρ decays through the P-wave resonance with $\Gamma_{\rho\to\pi^+\pi^-}/\Gamma_{full} \sim 1 \implies$ lots of experimental data!

[STAR collab.(2017)]



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  [STAR collab.(2017)]
- Required to study $\pi\pi \to \pi\gamma$, wait till Friday at 18:30 by Dr. Luka Leskovec

The two most important parameters of ρ resonance are m_ρ and $g_{\rho-\pi\pi}$.(BENCHMARK)

We use the Lüscher formalism for scattering of two particles with equal masses.

Calculation Setup

Gauge Ensemble

- $N_f = 2+1$ Clover fermions.
- isotropic lattice.($32^3 \times 96$)
- $m_\pi L = 5.865(32)$
- m_π is low enough: ρ is unstable.

a(fm)	L(fm)	m_π(MeV)	m_K(MeV)	N_{config}
$0.11403(77)$	$3.649(25)$	317	530	1041

Lattice setup for scattering

$\vec{P}\ [\frac{2\pi}{L}]$	Little Group	Irrep Λ	J
$(0,0,0)$	O_h	T_1^-	$1^-, 3^-, \ldots$
$(0,0,1)$	D_{4h} (Dic_4)	A_2^- (A_1)	$1^-, 3^-, \ldots$
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Interpolating operators for scattering

Projection method

- Single hadron operators

[Feng, Jansen, Renner(2011)]

$$O_{\rho}^{\Lambda,\vec{P}}(t) = \frac{\dim(\Lambda)}{N_{LG(\vec{P})}} \sum_{\hat{R}\in LG(\vec{P})} \chi_{\Lambda}(\hat{R})\hat{R}\,\rho^{+}(t,\vec{P}),$$

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- Meson-Meson operators

$$O_{\pi\pi}^{\Lambda,\vec{P}}(t) = \frac{\dim(\Lambda)}{N_{LG(\vec{P})}} \sum_{\hat{R}\in LG(\vec{P})} \chi_{\Lambda}(\hat{R})\Big(\pi^{+}(t,\vec{P}/2+\hat{R}\vec{p})\,\pi^{0}(t,\vec{P}/2-\hat{R}\vec{p}) - \pi^{0}(t,\vec{P}/2+\hat{R}\vec{p})\,\pi^{+}(t,\vec{P}/2-\hat{R}\vec{p})\Big), \quad (1)$$

Construction of Correlation functions

Source smearing method

- enhance the overlap of pion $W\left[U_{\mathrm{APE}}\right] D^{-1} W\left[U_{\mathrm{APE}}\right]^{\dagger}$
  half-width full-maximum of the overlap profile $0.34 fm$.
- $(N, \alpha_{APE}) = (25, 2.5)$
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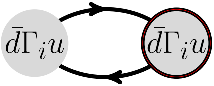
- Multigrid inverter for CPUs in QLUA(QOPQDP) has been used.

[USQCD software Qlua package]

Wick contractions

	$\bar{q}q$	$\pi\pi$
$\bar{q}q$		
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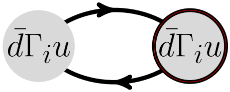
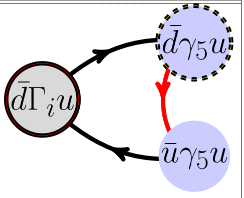
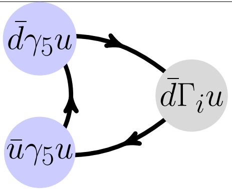
$\bar{q}q$		
$\pi\pi$		

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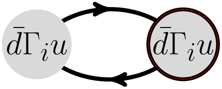
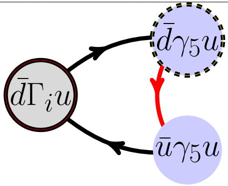
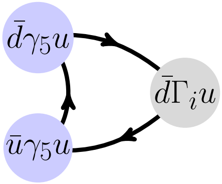
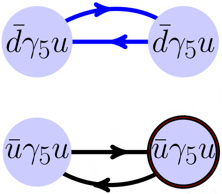
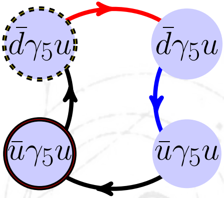
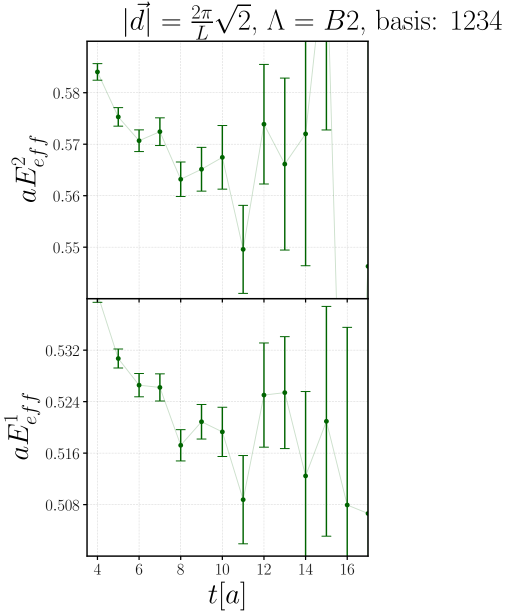
$\bar{q}q$		
$\pi\pi$		 

GEVP Analysis



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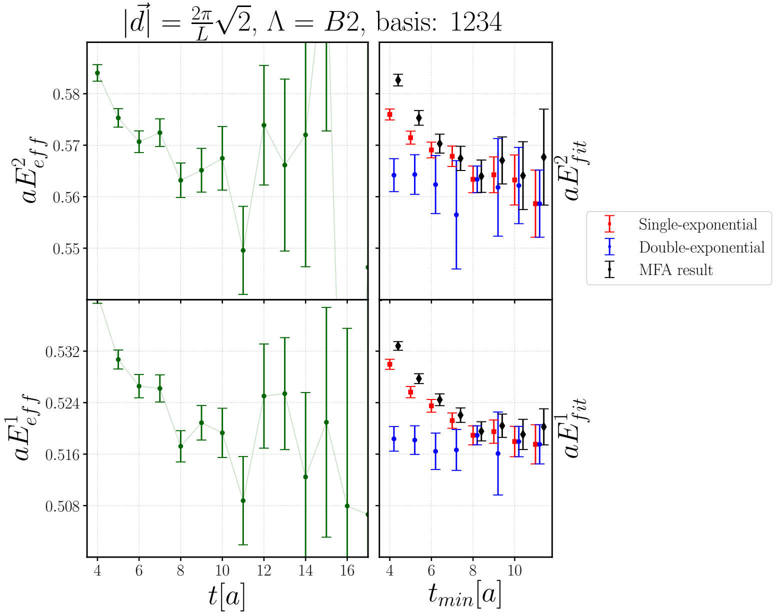

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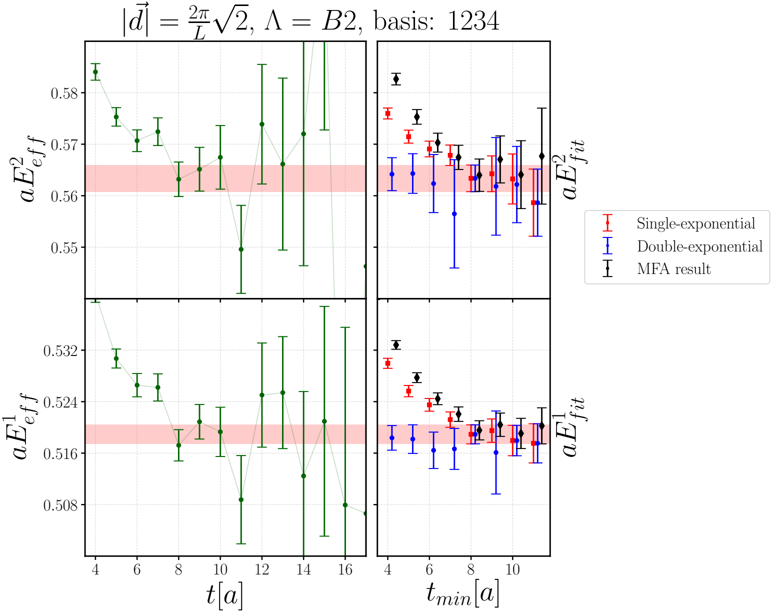

Lüscher Analysis

Quantization condition

$$\det\left(\mathbb{1} + i\, t_\ell(s)\,(\mathbb{1} + i\,\mathcal{M}^{\vec{\mathcal{P}}})\right) = 0,$$

where $t_\ell(s) = \frac{1}{\cot \delta_\ell(s) - i}$.

[Lüscher(1991)]

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[Lüscher(1991)]

$$\mathcal{M}^{\mathbf{d}}_{lm,l'm'} = \begin{array}{c} \\ 00 \\ 10 \\ 11 \\ 1-1 \end{array} \begin{array}{c} \begin{array}{cccc} 00 & 10 & 11 & 1-1 \end{array} \\ \begin{pmatrix} w_{00} & i\sqrt{3}w_{10} & i\sqrt{3}w_{11} & i\sqrt{3}w_{1-1} \\ -i\sqrt{3}w_{10} & w_{00} + 2w_{20} & \sqrt{3}w_{21} & \sqrt{3}w_{2-1} \\ i\sqrt{3}w_{1-1} & -\sqrt{3}w_{2-1} & w_{00} - w_{20} & -\sqrt{6}w_{2-2} \\ i\sqrt{3}w_{11} & -\sqrt{3}w_{21} & -\sqrt{6}w_{22} & w_{00} - w_{20} \end{pmatrix} \end{array},$$

$$w_{lm} = \frac{Z^{\vec{p}_{\pi\pi}}_{lm}(1; q^2)}{\pi^{3/2}\sqrt{2l+1}\,\gamma\, q^{l+1}} .$$

Phase Shift Parametrization

- BW I:

$$\Gamma_I(s) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{s},$$

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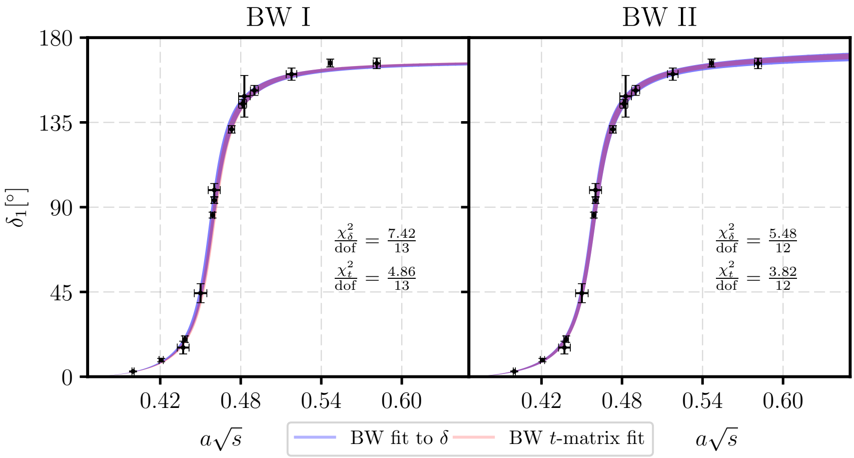
- BW II: Damping factor introduced.(Blatt-Weisskopf)

[Hippel, Quigg(1972)]

$$\Gamma_{II}(s) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{s}\frac{1+(k_R r_0)^2}{1+(kr_0)^2},$$

Phase shift results

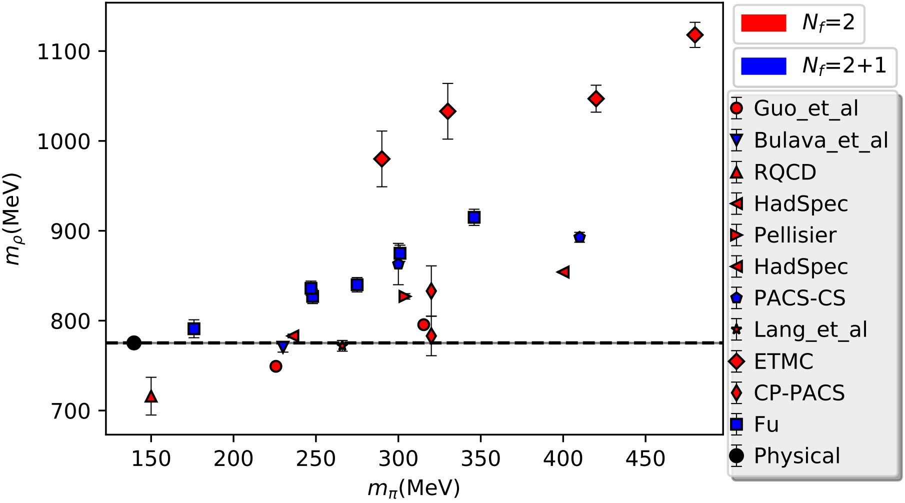


Results

Fit type	$\frac{\chi^2}{\text{dof}}$	am_ρ	$g_{\rho\pi\pi}$	$(ar_0)^2$
BW I Fit to δ_1	0.571	$0.4599(19)(13)$	$5.76(16)(12)$	
BW I t-matrix fit	0.374	$0.4609(16)(14)$	$5.69(13)(16)$	
BW II Fit to δ_1	0.457	$0.4600(18)(13)$	$5.79(16)(12)$	$8.6(8.0)(1.2)$
BW II t-matrix fit	0.318	$0.4603(16)(14)$	$5.77(13)(13)$	$9.6(5.9)(3.7)$

Our results in modern context

ρ meson mass comparison



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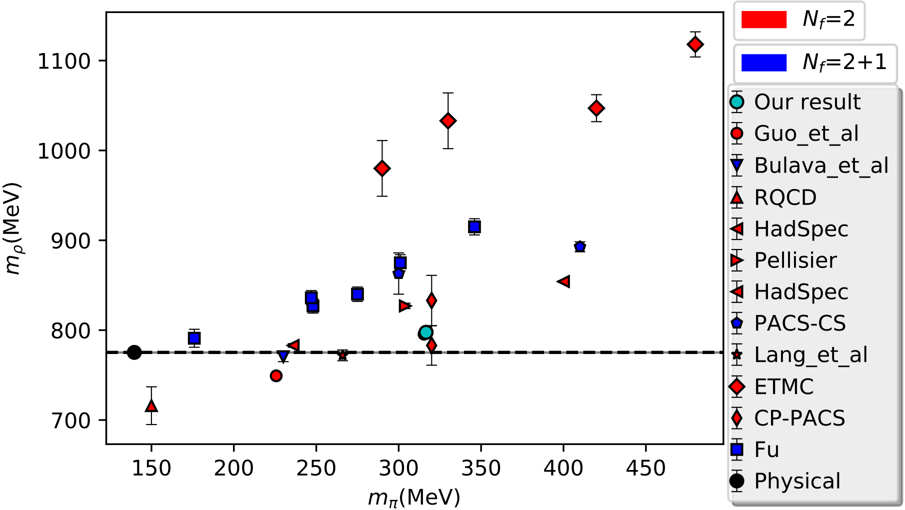

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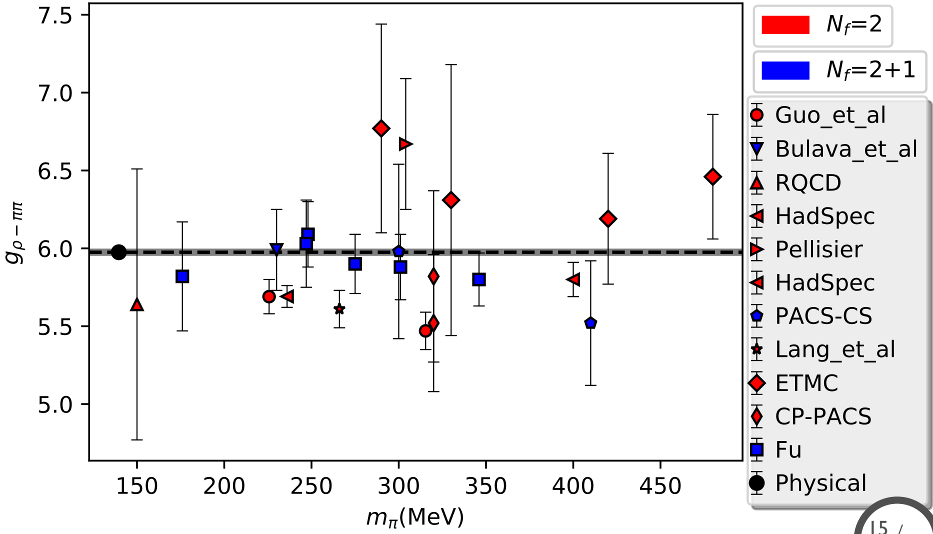

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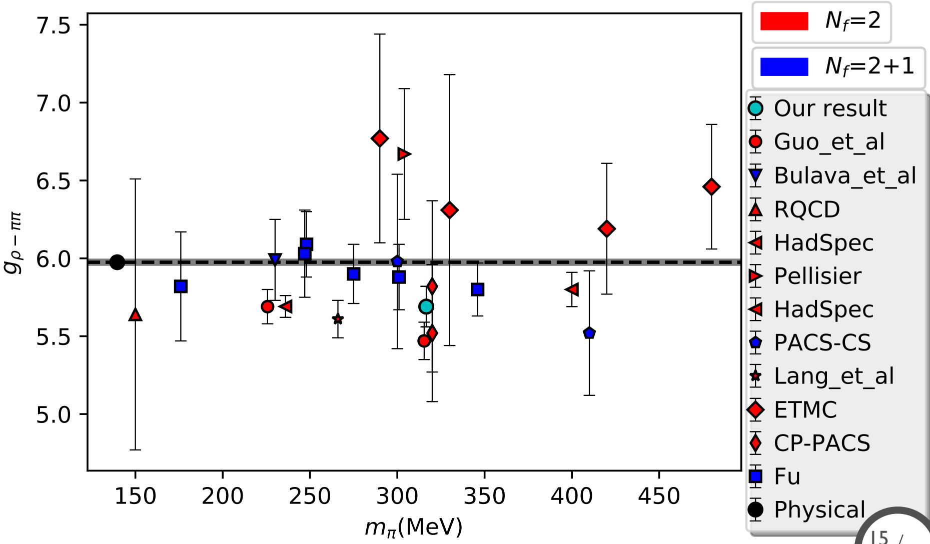

Conclusion

- We have achieved high statistical precision (0.35% for am_ρ and 2.3% for $g_{\rho-\pi\pi}$) by using source smearing method in the ρ resonance calculation.
- GEVP and MFA results are consistent.
- BW fit models with and without barriers, gave similar results, rendering BW_{II} redundant.
- Explicit phase shift calculation is equivalent to direct $t-$matrix fits.

Future work

- Higher volume($L \uparrow \implies \vec{p} \downarrow$) in the elastic region of $\pi\pi$ scattering, gives higher density of scattering states, implying more points for the phase shift plot.
- To do a similar calculation at a ”close-to” physical mass ensemble.[D6 and D7 ensemble, ongoing]
- To investigate the chiral extrapolation limit.
- To do a continuum limit extrapolation of our results, for comparison with nature.