The K_L - K_S Mass Difference

Chris Sachrajda

RBC-UKQCD Collaborations

(for Ziyuan Bai)

Department of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ
UK

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- $\Delta m_K = m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12}$ MeV is tiny on the scale of $\Lambda_{\rm QCD}$.
- This FCNC quantity is therefore an excellent one in which to search for new-physics effects.
- It is frequently said that Flavour Physics can probe scales which are unreachable in colliders.
  - Here, if we could reproduce the experimental Δm_K in the SM to 10% accuracy and if we imagine an effective new-physics $\Delta S = 2$ contribution $\frac{1}{\Lambda^2}(\bar{s}\cdots d)(\bar{s}\cdots d)$ then $\Delta \gtrsim (10^3 - 10^4)$ TeV.
- The calculation of Δm_K is one component of the RBC-UKQCD collaborations' programme of long-distance contributions in kaon physics, requiring the evaluation of matrix elements of bilocal operators of the form

$$\int d^4x \, \langle f \,|\, T\big[Q_1(x)\, Q_2(0)\big] \,|\, i \,\rangle \,.$$

  This includes rare-kaon decays and ϵ_K.

  X.Feng plenary, Tuesday 20/6 10.00,
  A.Lawson, following talk

- As well as computing the non-perturbative long-distance contributions from scales of $O(\Lambda_{\rm QCD})$, we aim to avoid the necessity of performing perturbation theory at the scale of m_c. For Δm_K this has proved particularly slowly convergent.

  J.Brod & M.Gorbahn, arXiv:1108.2036

The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

York University (Toronto)

Renwick Hudspith

- Physical value: $$\Delta m_K = 3.483(6) \times 10^{-12}\,\mathrm{MeV}$$

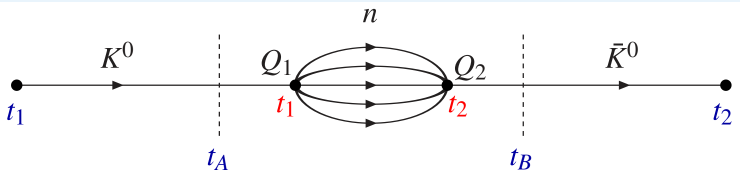
1. "Long-distance contribution to the $K_L - K_S$ mass difference,"
   N.H. Christ, T. Izubuchi, CTS, A. Soni and J. Yu,
   Phys.Rev. D88 (2013) 014508 (arXiv:1212.5931)
   Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses ($m_\pi = 421$ MeV) including only connected diagrams $\cdots$ but results were encouragingly in the right ball park.

2. "$K_L - K_S$ mass difference from Lattice QCD,"
   Z. Bai, N.H. Christ, T.Izubuchi, CTS, A.Soni and J. Yu,
   Phys.Rev.Lett. 113 (2014) 112003 (arXiv:1406.0916)
   All diagrams included on a $24^3 \times 64$ lattice with $a^{-1} = 1.729(28)$ GeV, $m_\pi = 330$ MeV, $m_K = 575$ MeV, $m_c^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 949$ MeV
   $$\Rightarrow \Delta m_K = 3.19(41)(96)\,\text{MeV}.$$

- Here I present an update of the computations and results at physical masses!
- Thanks to all my colleagues from RBC-UKQCD for helpful discussions and in particular to Ziyuan Bai for the analysis presented here.

Z.Bai, Ph.D. thesis (to be published)

From Correlation Functions to $\Delta m_K^{\rm FV}$



- Δm_K is given by

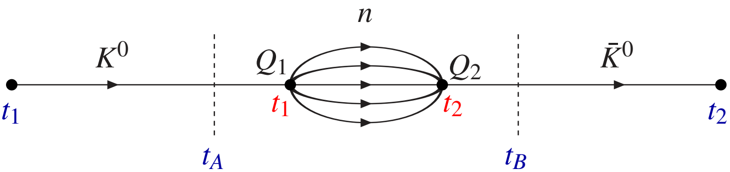
$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathcal{P}\sum_\alpha \frac{\langle \bar K^0\,|\,\mathcal{H}_W\,|\,\alpha\rangle\,\langle\alpha\,|\,\mathcal{H}_W\,|\,K^0\rangle}{m_K - E_\alpha} = 3.483(6)\times 10^{-12}\,\mathrm{MeV}.$$

- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A,t_B;t_i,t_f) \;=\; |Z_K|^2 e^{-m_K(t_f-t_i)}\sum_n \frac{\langle \bar K^0\,|\,\mathcal{H}_W\,|\,n\rangle\,\langle n\,|\,\mathcal{H}_W\,|\,K^0\rangle}{(m_K - E_n)^2}\times \left\{e^{(M_K-E_n)T} - (m_K-E_n)T - 1\right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\rm FV} \equiv 2\sum_n \frac{\langle \bar K^0\,|\,\mathcal{H}_W\,|\,n\rangle\,\langle n\,|\,\mathcal{H}_W\,|\,K^0\rangle}{(m_K - E_n)}\,.$$



$$C_4(t_A,t_B;t_i,t_f) = |Z_K|^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0\,|\,\mathcal{H}_W\,|\,n\rangle\,\langle n\,|\,\mathcal{H}_W\,|\,K^0\rangle}{(m_K-E_n)^2} \times \left\{ e^{(M_K-E_n)T} - (m_K-E_n)T - 1 \right\}.$$

- The presence of terms which grow exponentially in T is a generic feature of calculations of matrix elements of bilocal operators.
  - The freedom of adding $c_S\,\bar{s}d$ or $c_P\,\bar{s}\gamma^5 d$ can be used to remove two of the contributions.
- Finite-volume corrections: the extension of the Lüscher & Lellouch-Lüscher formalism to relate $\Delta m_K^{\rm FV}$ to Δm_K has been derived (see below).

N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170

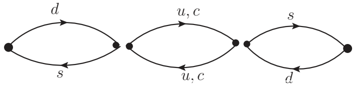
- The $\Delta S = 1$ effective Weak Hamiltonian takes the form:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

where the $\{Q_i^{qq'}\}_{i=1,2}$ are current-current operators, defined as:

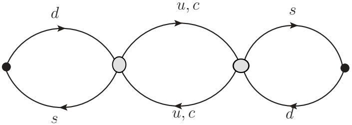
$$\begin{aligned} Q_1^{qq'} &= (\bar{s}_i \gamma^\mu (1-\gamma^5) d_i)\,(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_j) \\ Q_2^{qq'} &= (\bar{s}_i \gamma^\mu (1-\gamma^5) d_j)\,(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_i)\,. \end{aligned}$$

- As the two H_W approach each other, we have the potential of new ultraviolet divergences.
  - Taking the u-quark component of the operators $\Rightarrow$ a quadratic divergence.

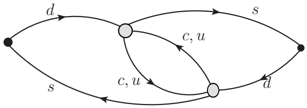


  - GIM mechanism & $V - A$ nature of the currents $\Rightarrow$ elimination of both quadratic and logarithmic divergences.
  - This is not the case for ϵ_K or for $K \to \pi\nu\bar{\nu}$ rare kaons decays.

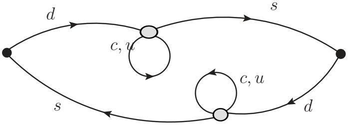
- There are four types of diagram to be evaluated:



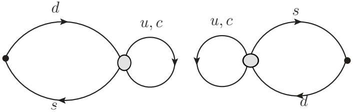
Type 1



Type 2



Type 3

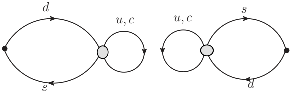


Type 4

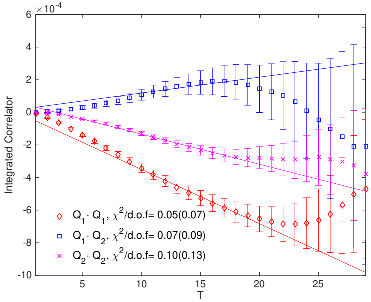
- The calculation was performed on a $64^3 \times 128 \times 12$ lattice with Möbius DWF and the Iwasaki gauge action. The inverse lattice spacing is 2.359(7) GeV, $m_\pi = 135.9(3)$ MeV and $m_K = 496.9(7)$ MeV.

  T.Blum et al., RBC-UKQCD Collabs., arXiv:1411.7017

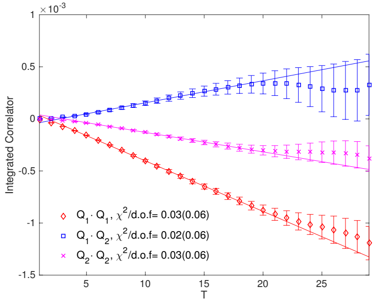
  Charm-physics studies with this action $\Rightarrow am_c \simeq 0.32 - 0.33$. We have used $am_c \simeq 0.31$ and studied the dependence on m_c (see below).
  - Preliminary results presented here obtained on a subset of 59 configurations for type 3 and type 4 diagrams, using AMA with “exact" propagators determined on 7 of the configurations and with “sloppy" stopping conditions for the remaining 52.
  - The more precise type 1 and type 2 diagrams were calculated on 11 configurations with exact stopping conditions for the propagators.
  - For the (disconnected) type-4 diagrams, which are the most noisy, we store the left- and right-hand sides separately in order to be able to vary the source-sink separations.



Correlation Functions - All results are preliminary



all diagrams



Type 1 & 2 diagrams only

- Lines here correspond to uncorrelated fits in the range $10 < T < 20$.

- Results given in units of 10^{-12} MeV.

Fit	ΔM_K	ΔM_K(tp 3&4 only)	ΔM_K(tp 1&2 only)
un-corr, 10:20	5.8(17)	-1.1(12)	7.0(13)
corr, 14:20	6.6(18)	-0.4(14)	8.5(9)
corr, 16:22	5.6(25)	-1.6(21)	7.5(15)

We follow the standard procedure:

1. NPR: $O^{\text{latt}} \to O^{\text{RI-SMOM}}$.
   This is performed on a smaller $32^3 \times 64$ with the same action and lattice spacing at $p_1^2 = p_2^2 \simeq 7\,\text{GeV}^2$.

$$Z^{\text{lat}\to\text{RI-SMOM}} = \begin{pmatrix} 0.6266 & -0.0437 \\ -0.0437 & 0.6266 \end{pmatrix}$$

2. Perturbation theory: $O^{\text{RI-SMOM}} \to O^{\overline{\text{MS}}}$ at one loop.

   Extension of C. Lehner and C. Sturm, arXiv:1104.4948

$$Q_i^{\overline{\text{MS}}} = (I + \Delta r)_{ij}\, Q_j^{\text{RI-SMOM}}$$

   with

$$\Delta r = \begin{pmatrix} -2.2817 \cdot 10^{-3} & 6.8452 \cdot 10^{-3} \\ 6.8452 \cdot 10^{-3} & -2.2817 \cdot 10^{-3} \end{pmatrix}$$

3. Use the perturbatively calculated Wilson coefficients in the $\overline{\text{MS}}$ scheme.

   G. Buchalla, A.J. Buras and M.E. Lautenbacher, hep-ph/9512380

- Results for ΔM_K calculated from different charm quark masses (uncorrelated fit in the range $10 \le T \le 20$) on 56 configurations:

m_c	0.25	0.28	0.31	0.34
ΔM_K	4.7(19)	5.1(18)	5.5(20)	5.9(21)

- Jackknife differences between ΔM_K calculated using different charm quark masses and ΔM_K from $m_c = 0.25$:

m_c	0.28	0.31	0.34
ΔM_K	0.38(66)	0.78(65)	1.23(80)

- Theoretically the FV correction is given by

N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170

$$\Delta m_K^{FV} = 2\mathcal{P}\int \mathrm{dE}\,\rho_V(E)\frac{f(E)}{m_K - E} - 2\sum_n \frac{f(E_n)}{m_K - E_n} = -2\left(f(m_K)\cot(h)\frac{\mathrm{dh}}{\mathrm{dE}}\right)_{E=m_K},$$

where

$$f(m_K) = {}_V\langle \bar{K^0}|H_W|(\pi\pi)_{E=m_K}\rangle_V\ {}_V\langle(\pi\pi)_{E=m_K}|H_W|K_0\rangle_V \quad \text{and} \quad h(k) = \delta(k) + \phi(k)\,.$$

- The phase-shift $\delta(k_{m_K})$ (and its derivative) is unknown from this calculation and so we can only estimate the FV correction.
- We do this very approximately by determining the scattering length $a_{\pi\pi}$ and using the linear approximation $\delta(k_{m_K}) = k_{m_K}\,a_{\pi\pi}$.
- With this approximation the FV correction is found to be $\ll$ statistical uncertainty $\Delta m_K = -0.27(18)\times 10^{-12}$ MeV.
- Further studies are needed to confirm that this is a general feature.

- We have performed the first calculation of the $K_L - K_S$ mass difference with physical quark masses.
- Our preliminary result based on an analysis of 59 configurations is

$$\Delta m_K = (5.5 \pm 1.7) \times 10^{-12}\,\text{MeV}\,,$$

to be compared to the physical value

$$(\Delta m_K)^{\text{phys}} = 3.483(6) \times 10^{-12}\,\text{MeV}\,.$$

- We plan to finish the present calculation by performing measurements on 160 configurations, aiming to reduce the uncertainty to about 1.0×10^{-12} MeV.
- Longer term, we plan to develop a strategy which will include an improved determination of Δm_K together with other elements of our kaon physics programme.