Lattice QCD studies on baryon interactions in the strangeness -2 sector

Kenji Sasaki (*YITP, Kyoto University*)

for HAL QCD Collaboration

HAL* (*H*adrons to *A*tomic nuclei from *L*attice) *QCD Collaboration

S. Aoki (*YITP*)	**T. Doi** (*RIKEN*)	**F. Etminan** (*Birjand U.*)	**S. Gongyo** (*RIKEN*)	**T. Hatsuda** (*RIKEN*)
Y. Ikeda (*RCNP*)	**T. Inoue** (*Nihon U.*)	**N. Ishii** (*RCNP*)	**T. Iritani** (*RIKEN*)	**D. Kawai** (*YITP*)
T. Miyamoto (*YITP*)	**K. Murano** (*RCNP*)	**H. Nemura** (*RCNP*)	**T. Aoyama** (*YITP*)	

Introduction

Aim : Nuclear structures and exotic states from QCD

Lattice QCD simulation

- **Advantageous for more strange quarks**
- **Signals getting worse as increasing the number of light quarks.**
- **Complementary role to experiment.**

S=-6

S=-5

S=-4

S=-3

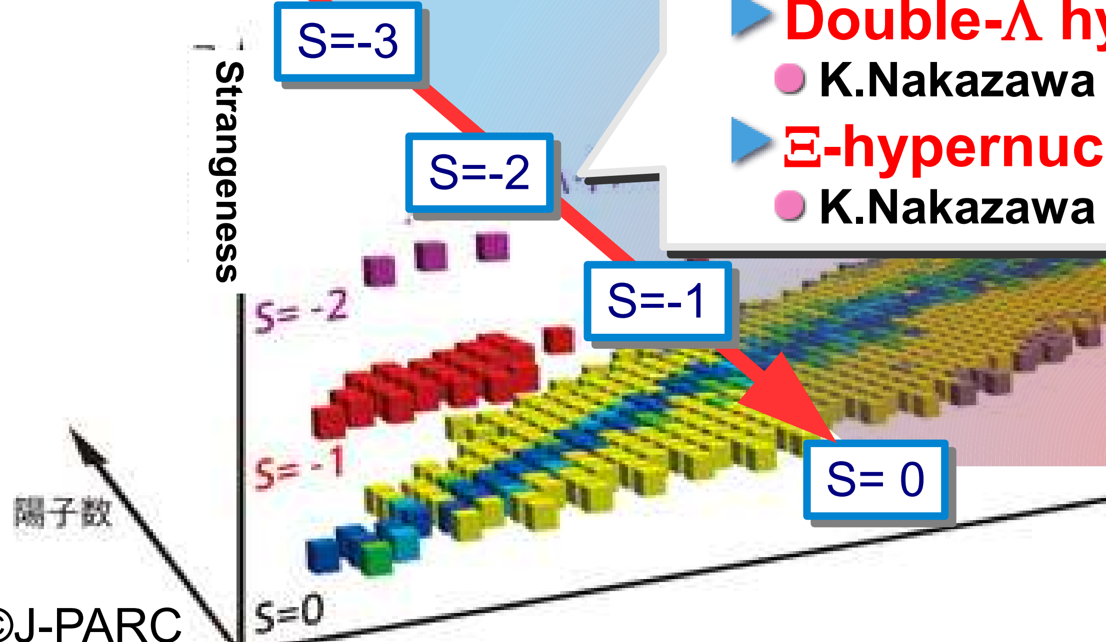
S=-2

S=-1

S= 0

Main topics of S=-2 multi-baryon system

- **H-dibaryon**
  - **R.L. Jaffe, PRL 38 (1977) 195**
- **Double-Λ hypernuclei**
  - **K.Nakazawa et al, KEK-E176 Collaboration**
- **Ξ-hypernuclei**
  - **K.Nakazawa et al, KEK-E373 Collaboration**



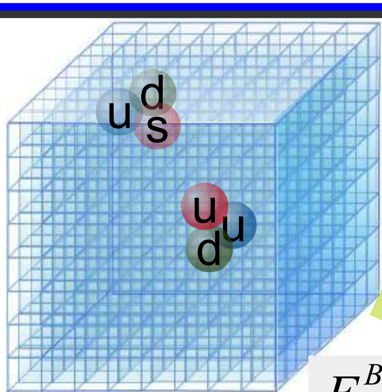
©J-PARC

Experiment

Derivation of hadronic interaction from QCD

Start with the fundamental theory, QCD

Lattice QCD simulation



Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

1. Measure the discrete energy spectrum, **E**
2. Put the E into the formula which connects E and **δ**

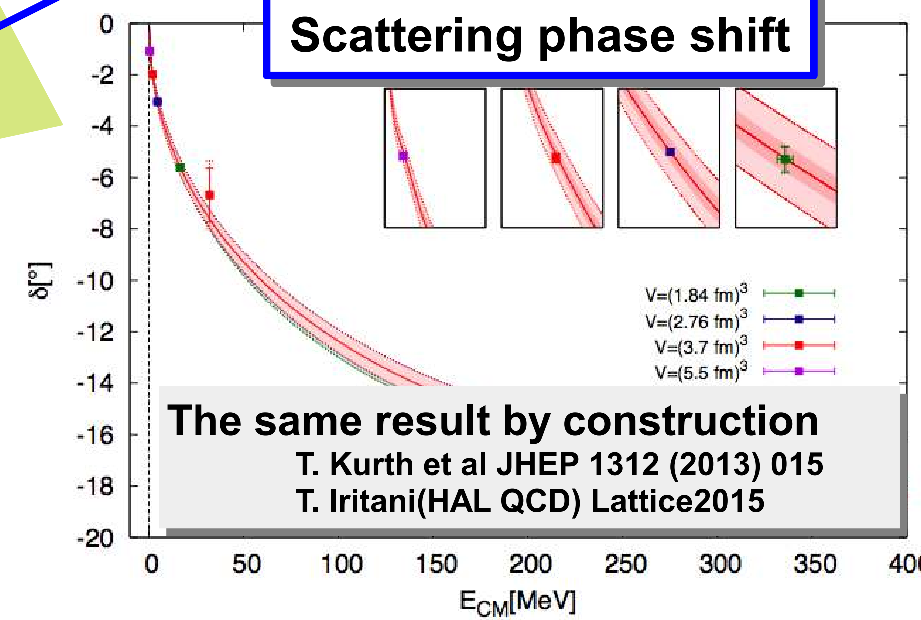
$$F^{B_1 B_2}{}_I(t,\vec{r}) = \langle 0 | B_1 B_2(t,\vec{r}) \overline{I}(t_0) | 0 \rangle = A_0 \Psi(\vec{r}, E_0) e^{-E_0(t-t_0)} + \cdots$$

HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

1. Measure the NBS wave function, **Ψ**
2. Calculate potential, **V**, through Schrödinger eq.
3. Calculate observables by scattering theory

Scattering phase shift



The same result by construction
T. Kurth et al JHEP 1312 (2013) 015
T. Iritani(HAL QCD) Lattice2015

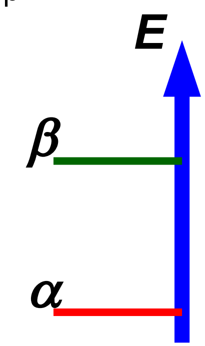
HAL QCD method (coupled-channel)

NBS wave function

$$\Psi^{\alpha}(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^{\alpha}(\vec{r})|E_i\rangle$$
$$\Psi^{\beta}(E_i,\vec{r})e^{-E_i t} = \langle 0|(B_1 B_2)^{\beta}(\vec{r})|E_i\rangle$$

$$\int dr\,\tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\delta_{\beta}^{\gamma}$$

$$R_E^{B_1 B_2}(t,\vec{r}) = \Psi_{B_1 B_2}(\vec{r},E)e^{(-E+m_1+m_2)t}$$



Leading order of velocity expansion and time-derivative method.

Modified coupled-channel Schrödinger equation

$$\begin{pmatrix} (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\alpha})R^{\alpha}_{E_0}(t,\vec{r}) \\ (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta})R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_0}(t,\vec{r}) \\ R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$\begin{pmatrix} \cdots \\ (-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta})R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} \cdots & \cdots(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ \cdots & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_1}(t,\vec{r}) \\ R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix}$$

$$\Delta^{\alpha}_{\beta} = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

S.Aoki et al [HAL QCD Collab.] Proc. Jpn. Acad., Ser. B, 87 509
K.Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

Considering two different energy eigen states

Potential

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^{\alpha}_{E0}(t,\vec{r}) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^{\alpha}_{E1}(t,\vec{r}) \\ (\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t})R^{\beta}_{E0}(t,\vec{r}) & (\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t})R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$

Baryon-baryon system with S=-2

Spin singlet states

Isospin	BB channels		
I=0	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
I=1	$N\Xi$	$\Lambda\Sigma$	——
I=2	$\Sigma\Sigma$	——	——

Spin triplet states

Isospin	BB channels		
I=0	$N\Xi$	——	——
I=1	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_S + 1 + \overline{10} + 10 + 8_A$$

$J^P=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

$J^P=1^+$, I=0

$$N\Xi \Leftrightarrow 8$$

$J^P=1^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ 10 \\ \overline{10} \end{pmatrix}$$

$J^P=0^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

$J^P=0^+$, I=2

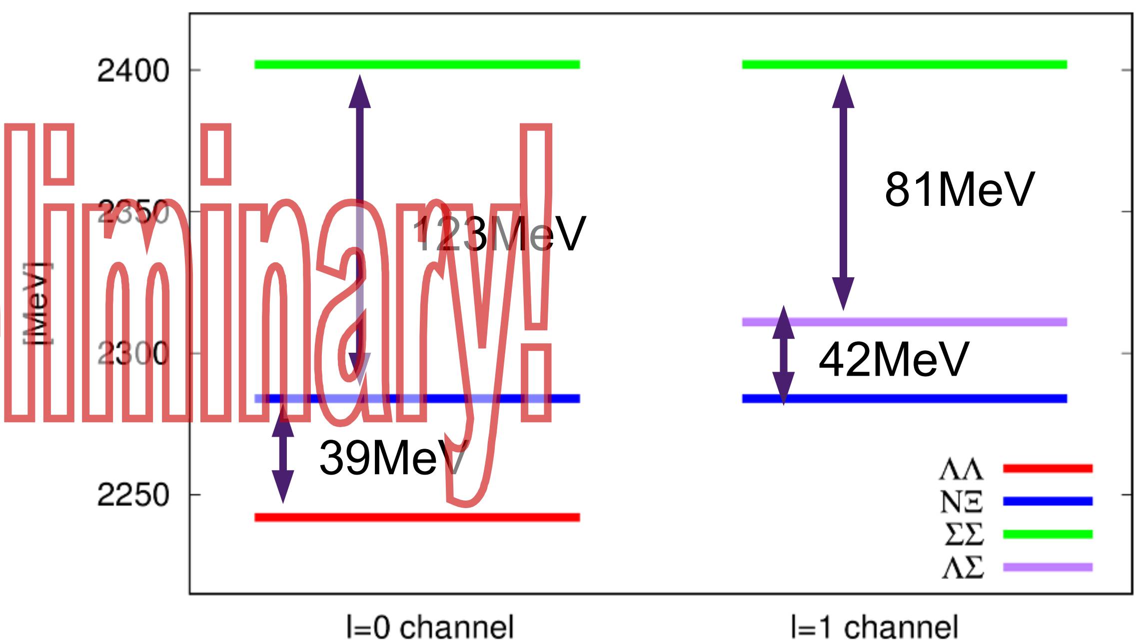
$$\Sigma\Sigma \Leftrightarrow 8$$

Features of flavor singlet interaction is embedded into the S=-2 $J^P=0^+$, I=0 system.

Numerical setup

- 2+1 flavor gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - a = 0.085 [*fm*], a^{-1} = 2.300 GeV.
  - 96^3x96 *lattice, L* = 8.21 [*fm*].
  - 414 confs x 84 sources x 4 rotations.
- Wall source is considered to produce S-wave B-B state.

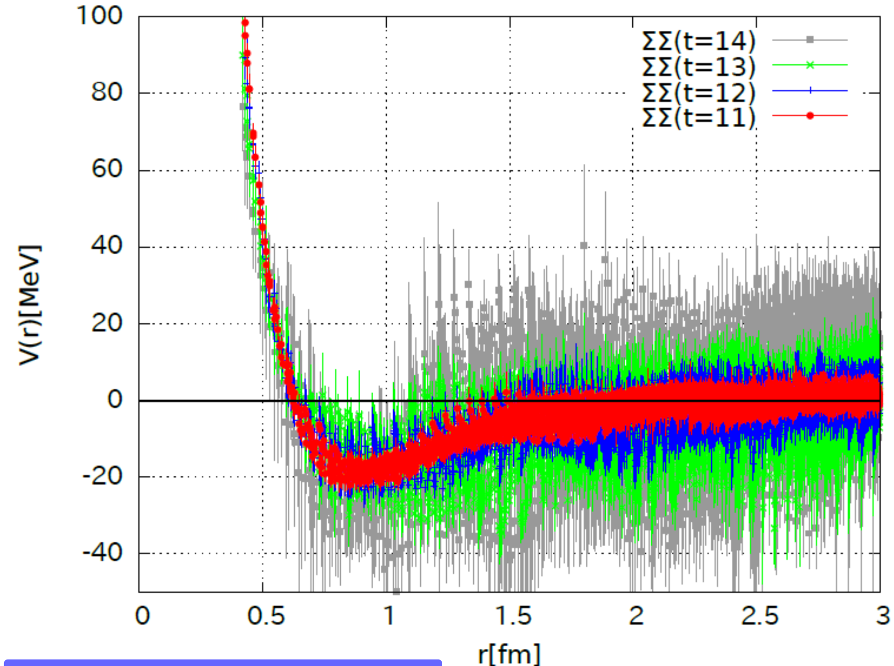
	Mass [MeV]
π	146
K	525
m_π/m_K	0.28
N	953±7
Λ	1123±3
Σ	1204±1
Ξ	1332±2



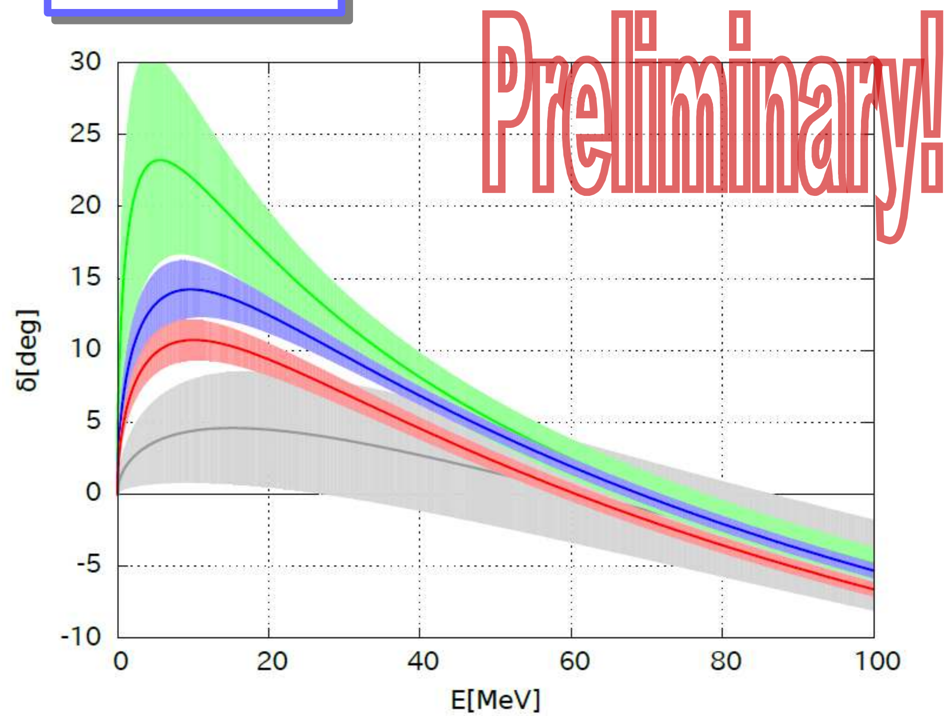
Preliminary!

ΣΣ (I=2) $^1S_0$ channel

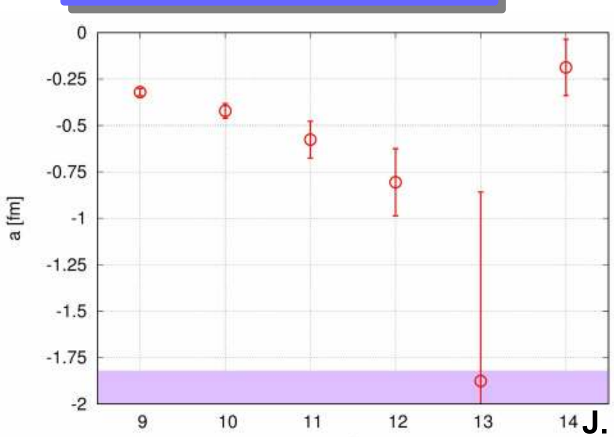
Potential



Phase shift



Scattering length

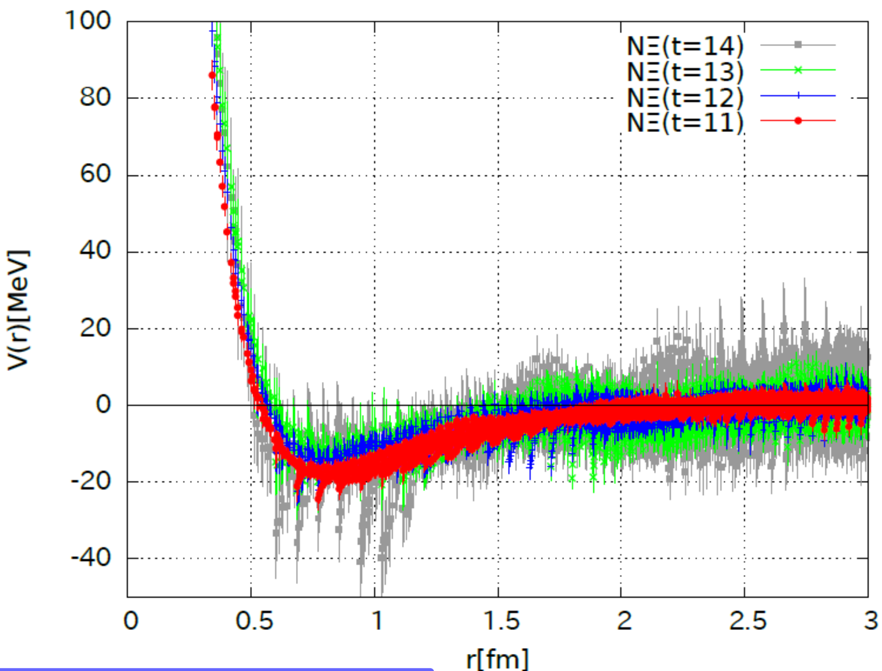


J. Haidenbauer et al, NPA954(2016)273

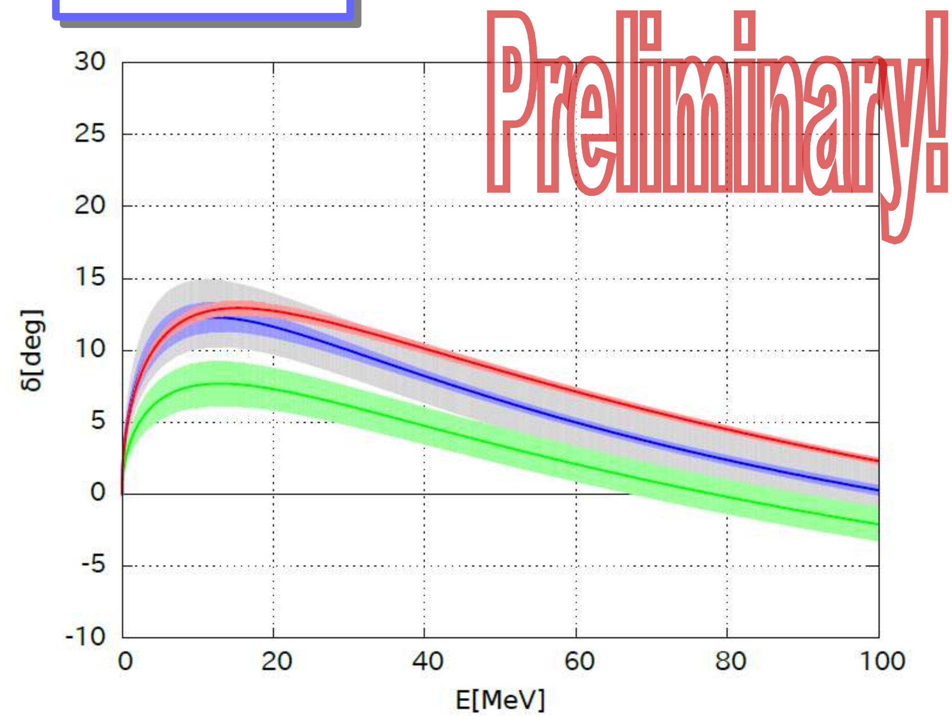
- ΣΣ (I=2) potential belongs to the 27plet in flavor SU(3) limit.
- The potential has an attractive pocket and repulsive core.
- The phase shift shows that potential is not saturated...

$N\Xi$ (I=0) $^3S_1$ channel (effective central potential)

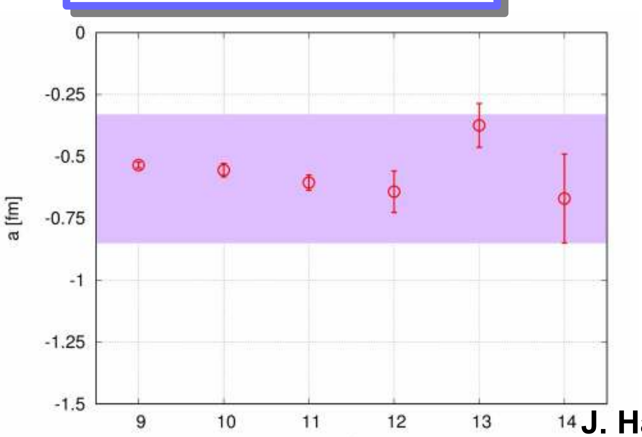
Potential



Phase shift



Scattering length

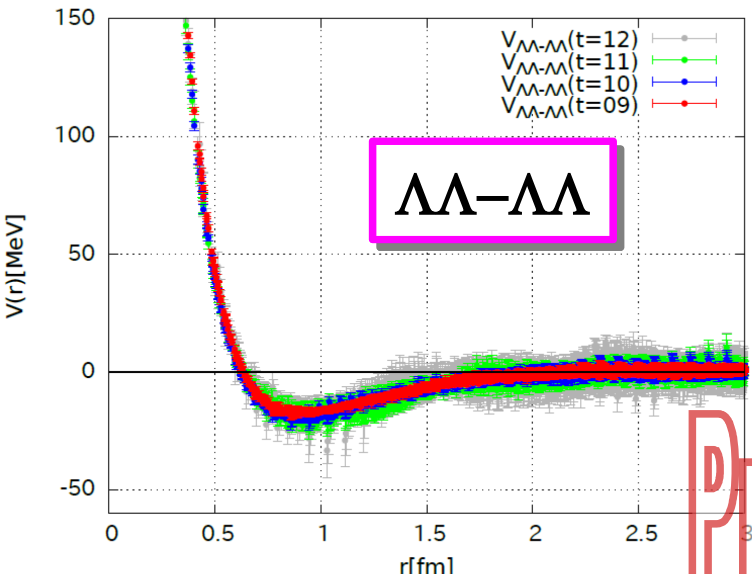


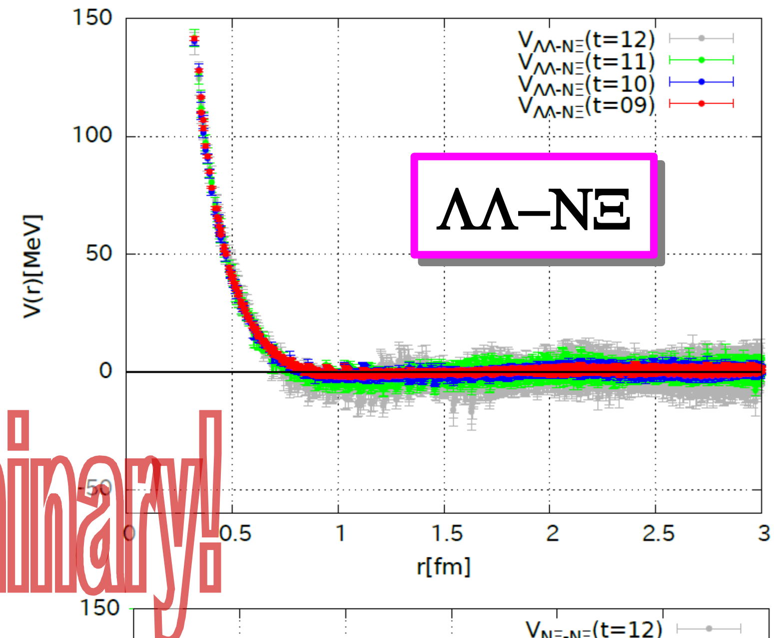
- Effective $N\Xi$ (I=0,spin triplet) central potential is plotted.
- It belongs to the 8a-plet in the SU(3) limit.
- Phase shifts at low energies are same within the error bars.

J. Haidenbauer et al, NPA954(2016)273

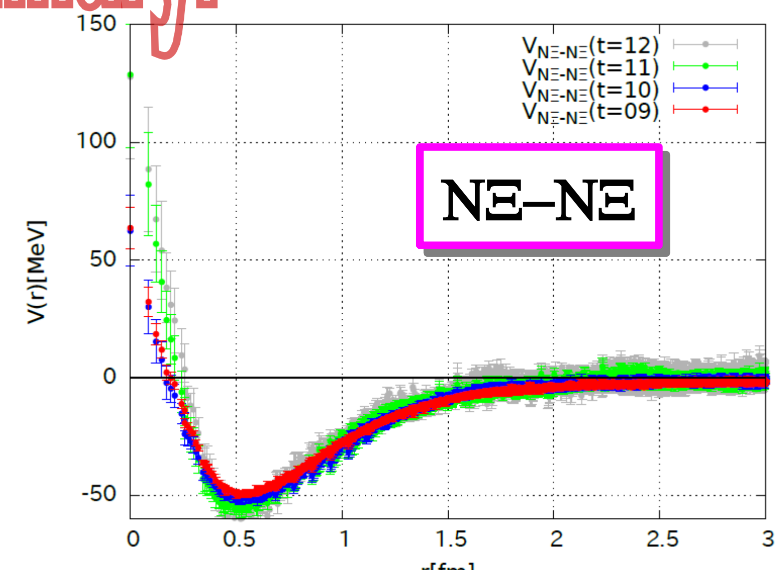
$\Lambda\Lambda$, $N\Xi$ (I=0) $^1S_0$ potential (2ch calc.)

- N_f = 2+1 full QCD with L = 8.1fm, m_π = 146 MeV





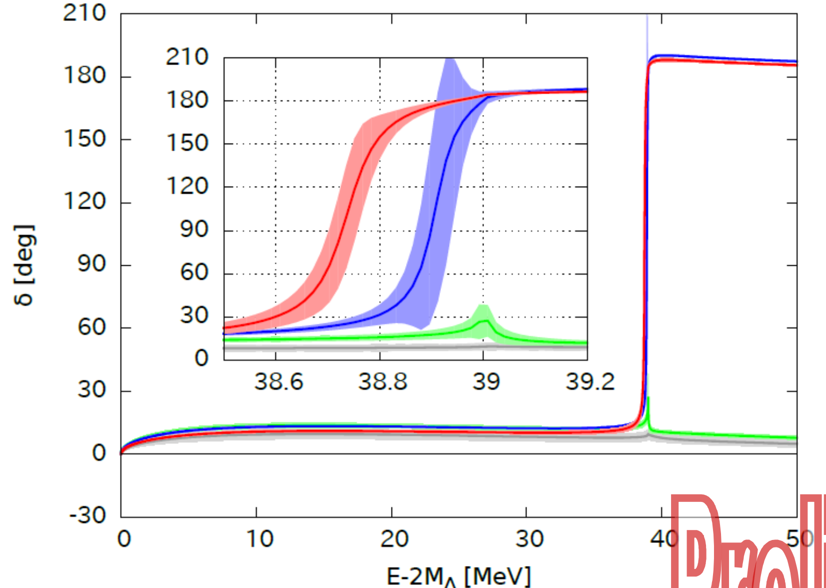
- Potential calculated by only using $\Lambda\Lambda$ and $N\Xi$ channels.
- Long range part of potential is almost stable against the time slice.
- Short range part of $N\Xi$ potential changes as time t goes.
- $\Lambda\Lambda$–$N\Xi$ transition potential is quite small in r > 0.7fm region



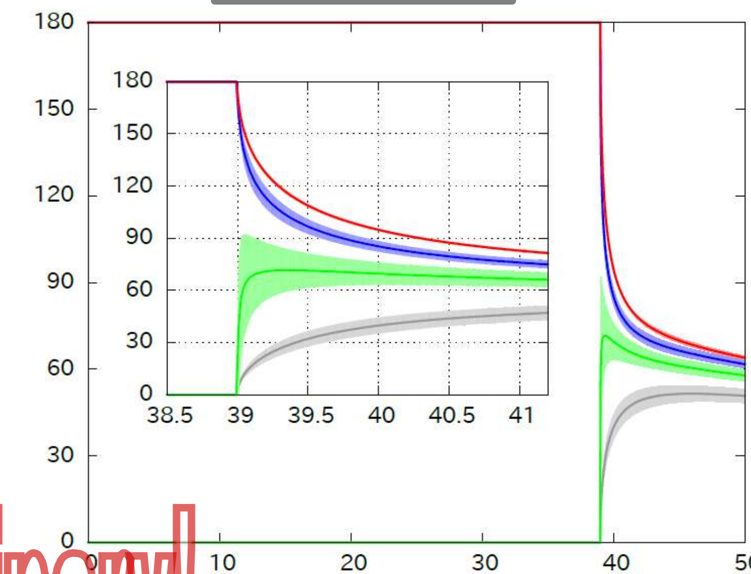
ΛΛ and NΞ phase shift and inelasticity

t=09
t=10
t=11
t=12

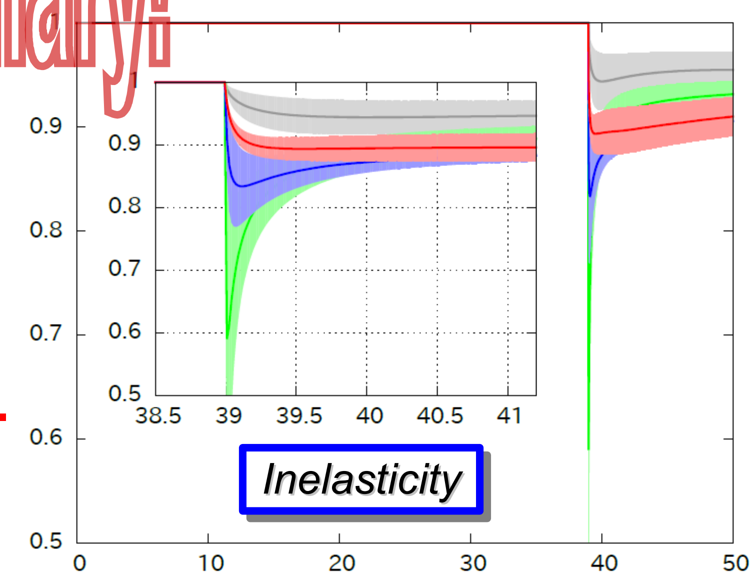
ΛΛ phase shift



NΞ phase shift



Preliminary!



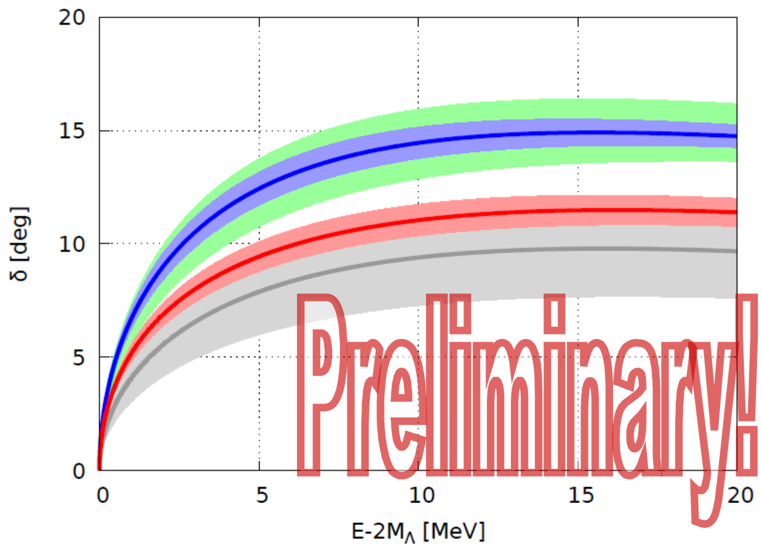
Inelasticity

- **ΛΛ and NΞ phase shift is calculated**
  by using 2ch effective potential.
- **A sharp resonance is found**
  below the NΞ threshold for t=9 and 10.

$\Lambda\Lambda$ scattering length

$t=09$
$t=10$
$t=11$
$t=12$

Phase shift



Λ	550	600	650	700
$a^{\Lambda\Lambda}_{1S0}$	-1.52	-1.52	-1.54	-1.67
$r^{\Lambda\Lambda}_{1S0}$	0.82	0.59	0.31	0.34

H. Polinder et al, PLB653(2007)29

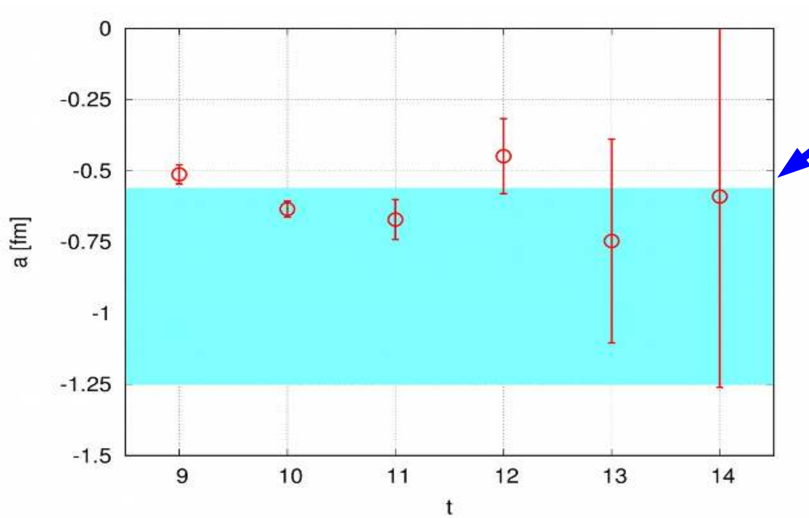
$a_{\Lambda\Lambda}=-0.821$fm

Y.Fujiwara et al, PPNP58(2007)439

$a_{\Lambda\Lambda}=-0.97$fm

Th.A.Rijken et al, Few-Body Syst 54(2013)801

Scattering length



$-1.25 < a_{\Lambda\Lambda} < -0.56$fm

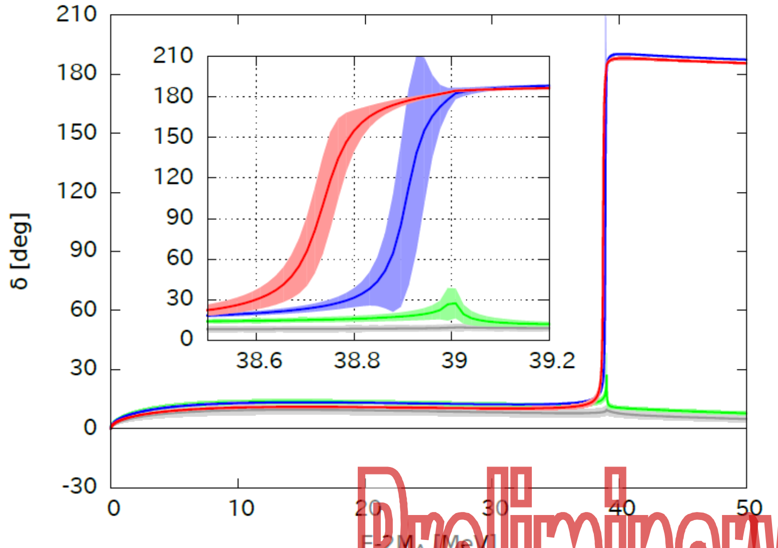
K.Morita et al, PRC91 (2015)024916

- **Scattering length in $\Lambda\Lambda$ (I=0) is almost saturated.**
- **Attraction is weaker than the phenomenological one.**

$\Lambda\Lambda$ and $N\Xi$ phase shift –comparison--

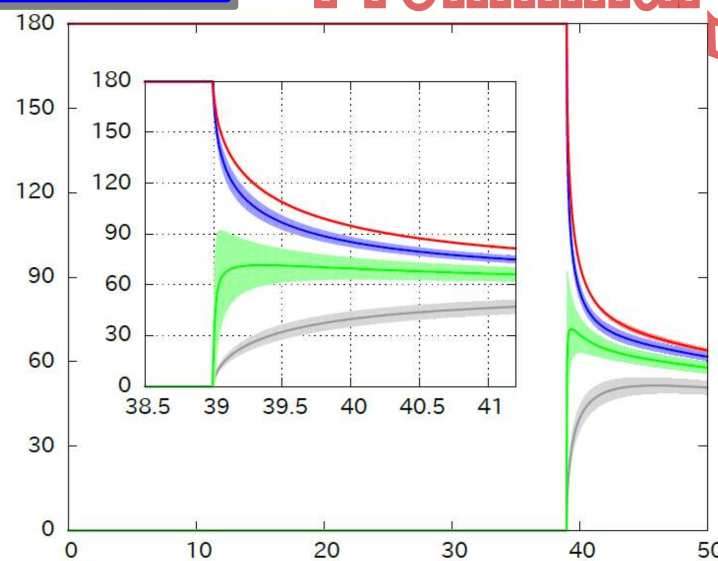
t=09
t=10
t=11
t=12

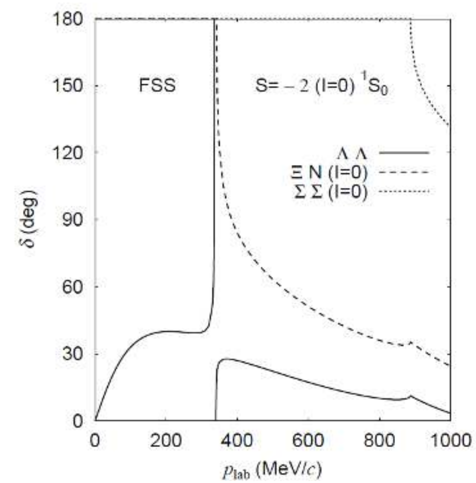
$\Lambda\Lambda$ phase shift

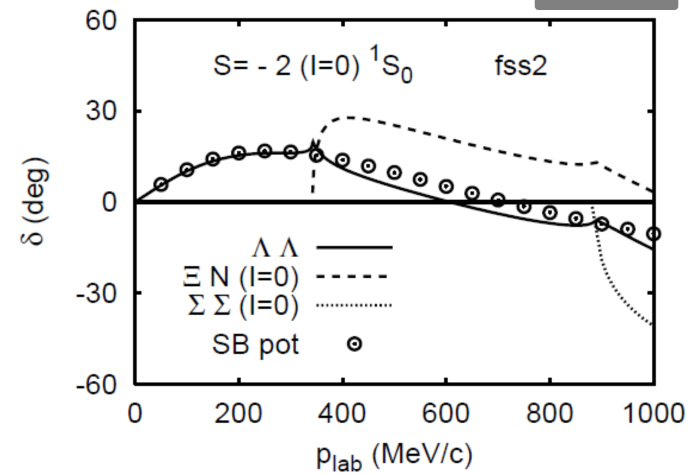


Preliminary!

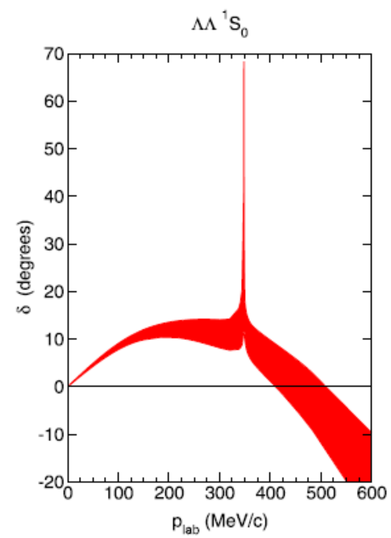
$N\Xi$ phase shift

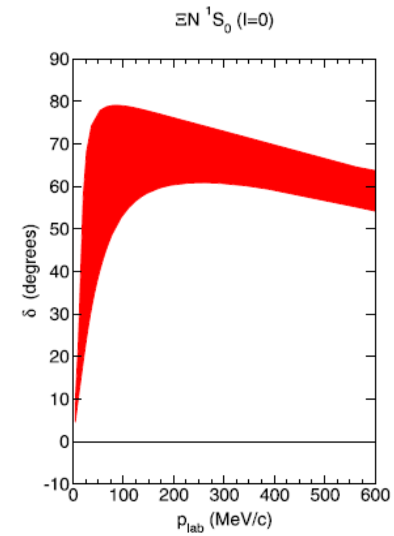






Y.Fujiwara et al, PPNP58(2007)439





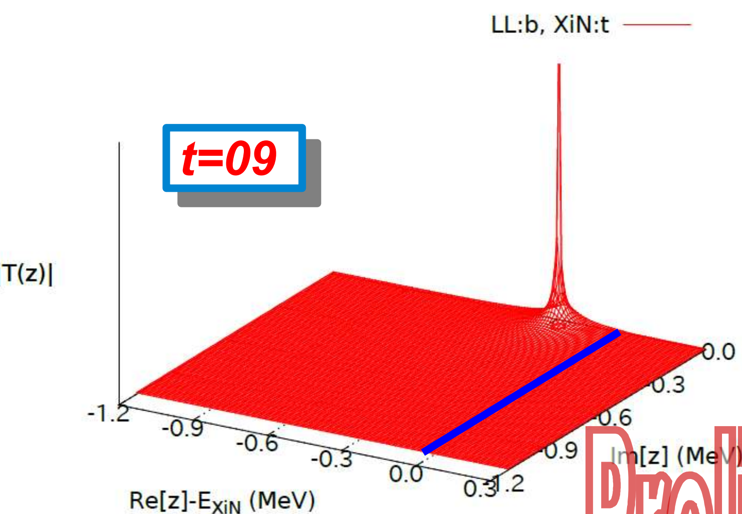
J. Haidenbauer et al, NPA954(2016)273

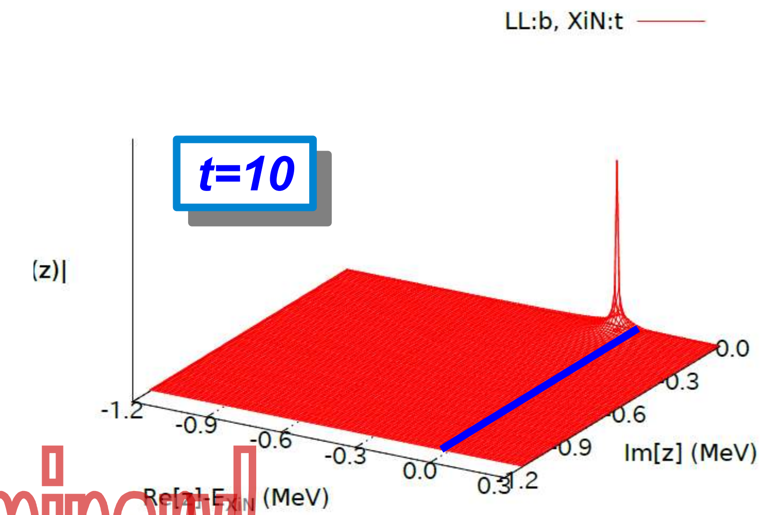
- **Our results are compatible with the phenomenological ones.**

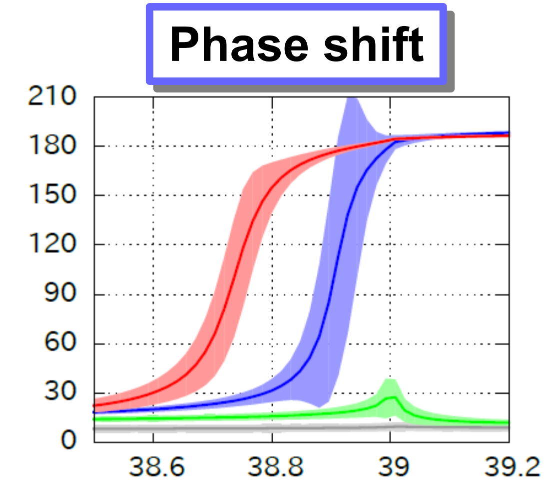
Pole search

- **N_f = 2+1 full QCD with L = 8.1fm, *m_π = 146 MeV***

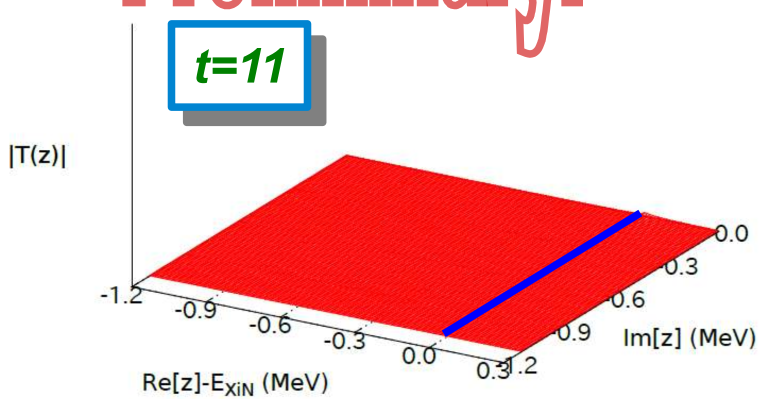
T-matrix ($\Lambda\Lambda$: unphysical, $N\Xi$: physical)

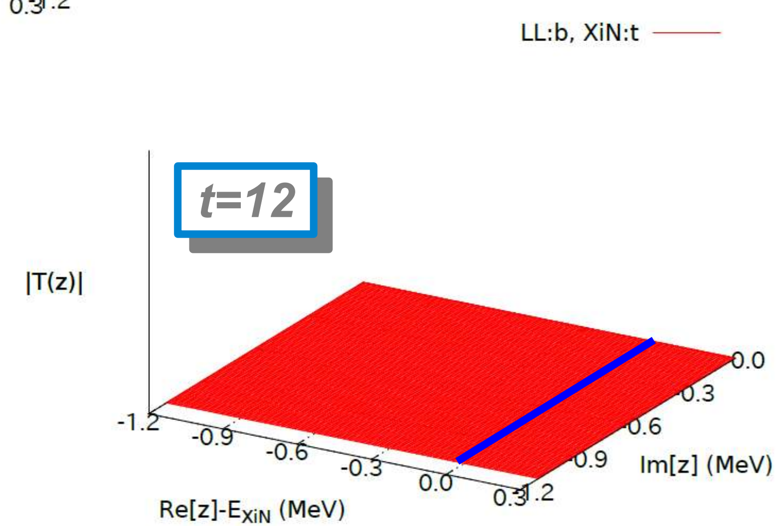






Preliminary!





Summary

- We have investigated coupled channel S=-2 baryonic interactions from lattice QCD.

- We find that
  - Potential in $\Lambda\Lambda$ $^1S_0$ channel is weakly attractive.
  - $N\Xi$ potential in $^1S_0$ channel is strongly attractive.

- We have studied dibaryon candidate
  - H-dibaryon channel
    - We perform $\Lambda\Lambda$–$N\Xi$ coupled channel calculation.
    - Sharp resonance is found just below the $N\Xi$ threshold in relatively small "t" region. (Time slice saturation is not achieved yet.)

→ Fate of H-dibaryon...

- We continue to study it by using higher statistical data.

Backup

Interests of S=-2 multi-baryon system

H-dibaryon

- The flavor singlet state with J=0 predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

Double-Λ hypernucleus

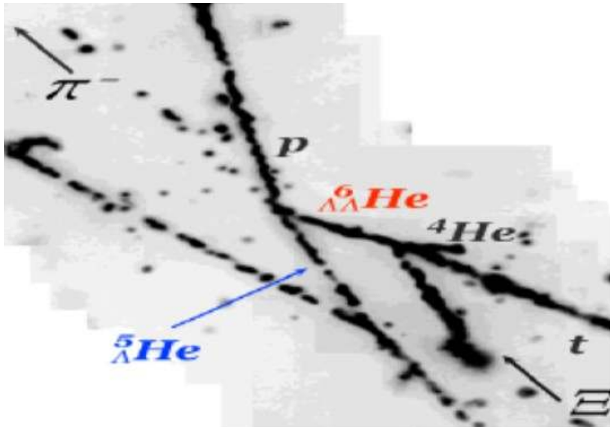
- Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 Collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV

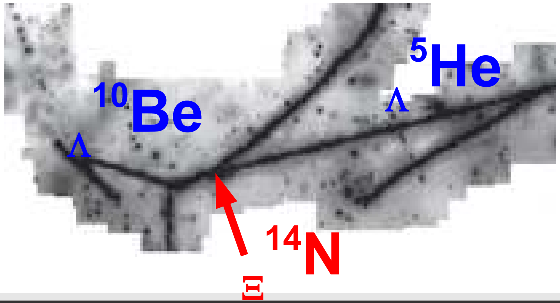


Ξ hypernucleus

- Conclusions of the "KISO Event"

K.Nakazawa and KEK-E373 Collaborators

Ξ–N attraction

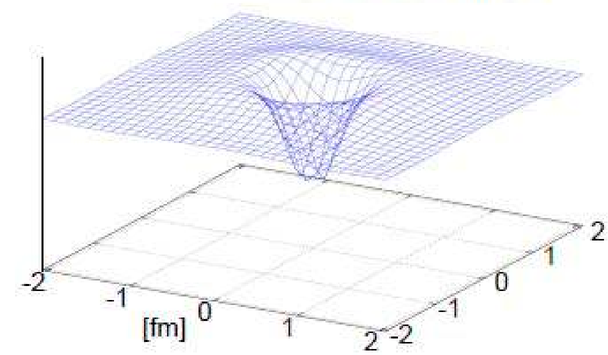


Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0|H_1^{\alpha}(t,\vec{x}+\vec{r})H_2^{\alpha}(t,\vec{x})|E\rangle$$

E : Total energy of the system



Local composite interpolating operators

$$B_{\alpha}=\epsilon^{abc}(q_a^T C\gamma_5 q_b)q_{c\alpha} \qquad D_{\mu\alpha}=\epsilon^{abc}(q_a^T C\gamma_{\mu} q_b)q_{c\alpha}$$

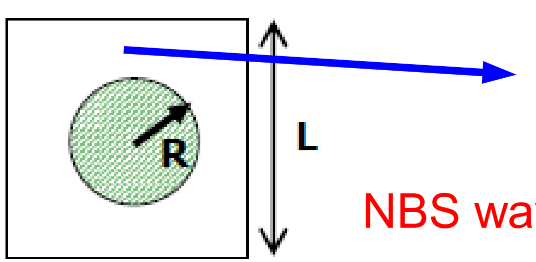
$$M=(\bar{q}_a\gamma_5 q_a)$$

Etc.....

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- Using the reduction formula,

C.-J.D.Lin et al.,NPB619 (2001) 467.

$$\Psi^{\alpha}(E,\vec{r}) = \sqrt{Z_{H_1}}\sqrt{Z_{H_2}}\left(e^{i\vec{p}\cdot\vec{r}}+\int\frac{d^3q}{2E_q}\frac{T(q,p)}{4E_p(E_q-E_p-i\epsilon)}e^{i\vec{q}\cdot\vec{r}}\right)$$



$$\Psi(E,\vec{r})\simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Phase shift is defined as

$$S\equiv e^{i\delta}$$

NBS wave function has a same asymptotic form with quantum mechanics.

(NBS wave function is characterized from phase shift)

N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent Schrödinger like equation

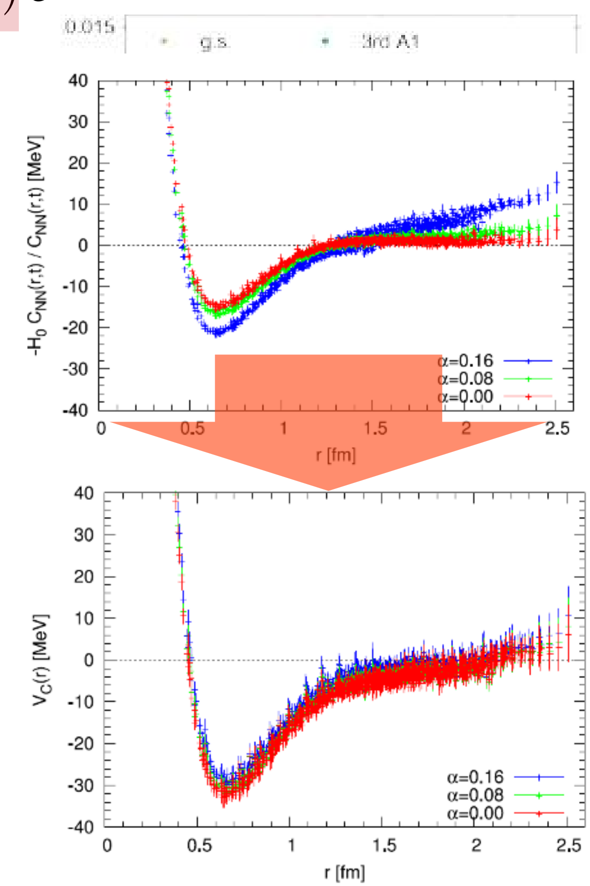
Start with the normalized four-point correlator.

$$R_I^{B_1B_2}(t,\vec{r}) = F^{B_1B_2}{}_I(t,\vec{r})\,e^{(m_1+m_2)t}$$
$$= A_0\Psi(\vec{r},E_0)e^{-(E_0-m_1-m_2)t} + A_1\Psi(\vec{r},E_1)e^{-(E_1-m_1-m_2)t} + \cdots$$

$$\left(\frac{p_0^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_0) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_0)d^3r'$$

$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(\frac{p_1^2}{2\mu}+\frac{\nabla^2}{2\mu}\right)\Psi(\vec{r},E_1) = \int U(\vec{r},\vec{r}\,')\Psi(\vec{r}\,',E_1)d^3r'$$



A single state saturation is not required!!

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

Derivative (velocity) expansion of U

$$U(\vec{r},\vec{r}\,') = \left[V_C(r)+S_{12}V_T(r)\right]+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

HAL QCD method

NBS wave function

$$\Psi(E,\vec{r})e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})|E,t_0\rangle$$

E : Total energy of system

- In asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec{r})=K(E,\vec{r})$

$$\Psi(E,\vec{r}) \simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Modified Schrödinger equation

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

N. Ishii et al Phys. Lett. B712(2012)437

Derivative expansion

$$U(\vec{r},\vec{r}\,') = V_C(r)+S_{12}V_T(r)+\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)+O(\nabla^2)$$

K. Murano et al Phys.Lett. B735 (2014) 19

Potential

$$V(\vec{r}) = \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r})/R_I^{B_1B_2}(t,\vec{r})$$

H-dibaryon channel

Keys to understand H-dibaryon state

A strongly bound state predicted by Jaffe in 1977 using MIT bag model.

H-dibaryon state is

- **SU(3) flavor singlet [uuddss], strangeness S=-2.**
- **spin and isospin equals to zero, and $J^P = 0^+$**

▶ **Strongly attractive interaction is expected in flavor singlet channel.**

- **Short range one-gluon exchange contributions**

  Strongly attractive Color Magnetic Interaction

- **Symmetry of two-baryon system (Pauli principle)**

  Flavor singlet channel is free from Pauli blocking effect

	27	8	1	$\overline{10}$	10	8
Pauli		forbidden	allowed		forbidden	
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

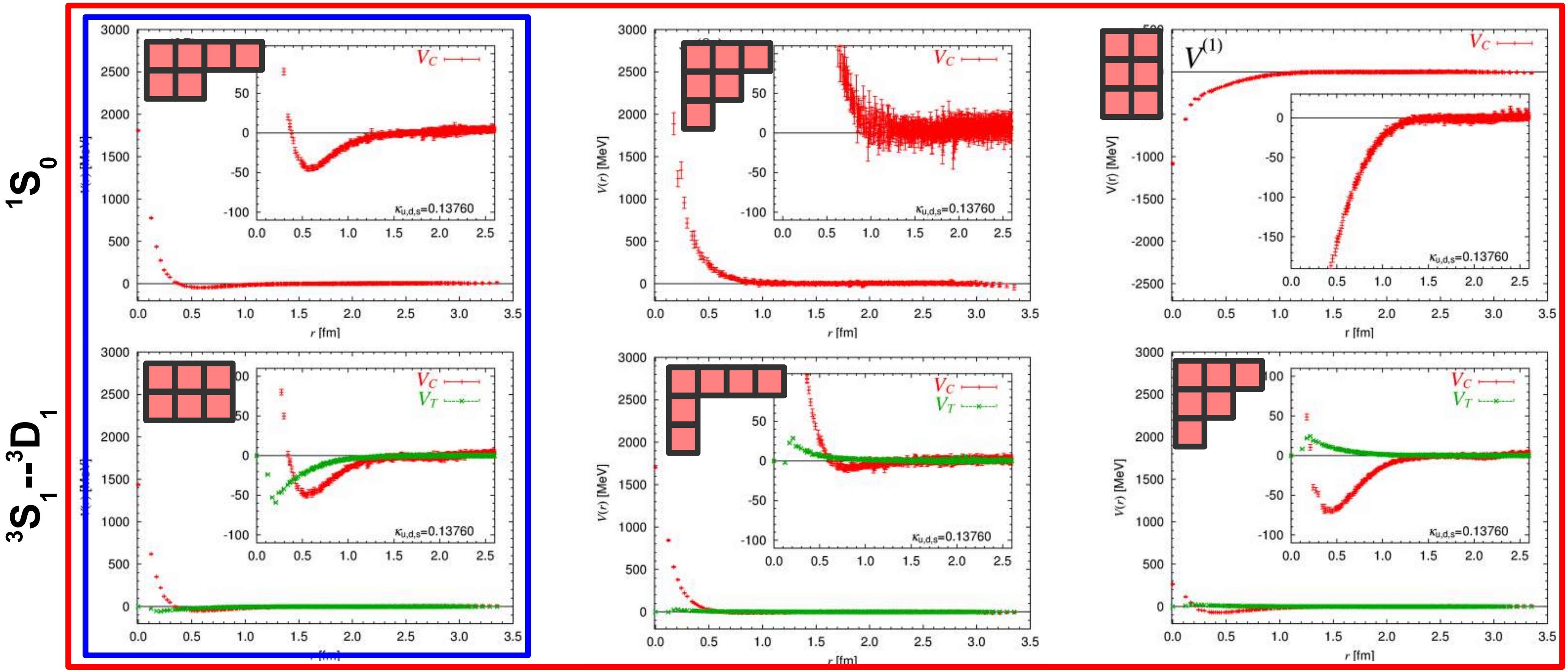
Oka, Shimizu and Yazaki NPA464 (1987)

$J^p=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

B-B potentials in SU(3) limit

m_π= 469MeV



Two-flavors

Three-flavors

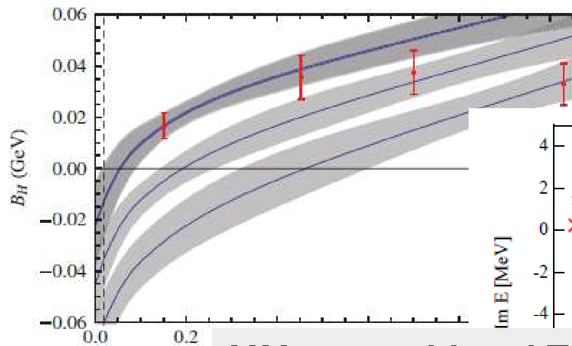
- Quark Pauli principle can be seen at around short distances
  - No repulsive core in flavor singlet state
  - Strongest repulsion in flavor 8s state
- **Possibility of bound H-dibaryon in flavor singlet channel.**

Works on H-dibaryon state

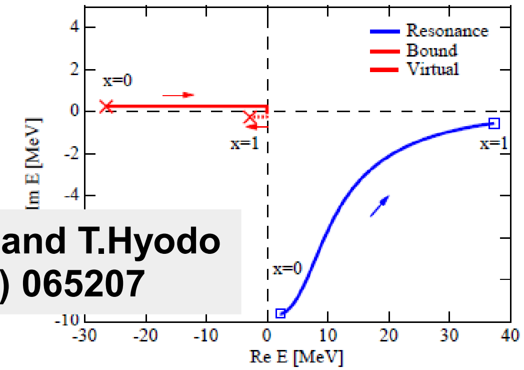
Theoretical status

Several sort of calculations and results
(bag models, NRQM, Quenched LQCD….)

There were no conclusive result.



P. E. Shanahan et al
PRL 107(2011) 092004



Y.Yamaguchi and T.Hyodo
PRC 94 (2016) 065207

Chiral extrapolations of recent LQCD data

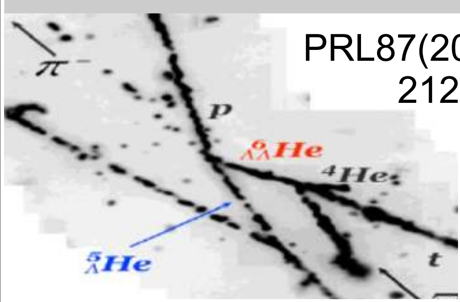
Unbound or resonance

Experimental status

"NAGARA Event"

K.Nakazawa et al
KEK-E176 & E373 Coll.

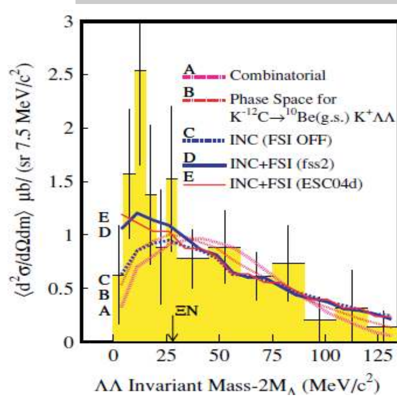
PRL87(2001) 212502



- Deeply bound dibaryon state is ruled out

"$^{12}C(K^-,K^+\Lambda\Lambda)$ reaction"

C.J.Yoon et al KEK-PS E522 Coll.



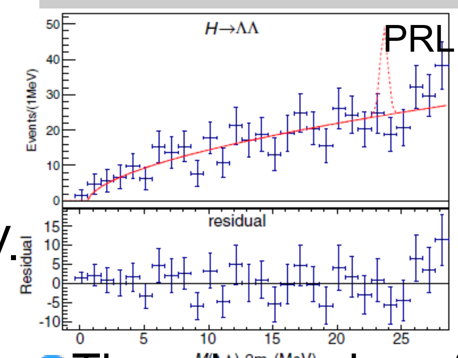
PRC75(2007) 022201(R)

- Significance of enhancements below 30 MeV.

Larger statistics
J-PARC E42

"Y(1S) and Y(2S) decays"

B.H. Kim et al Belle Coll.

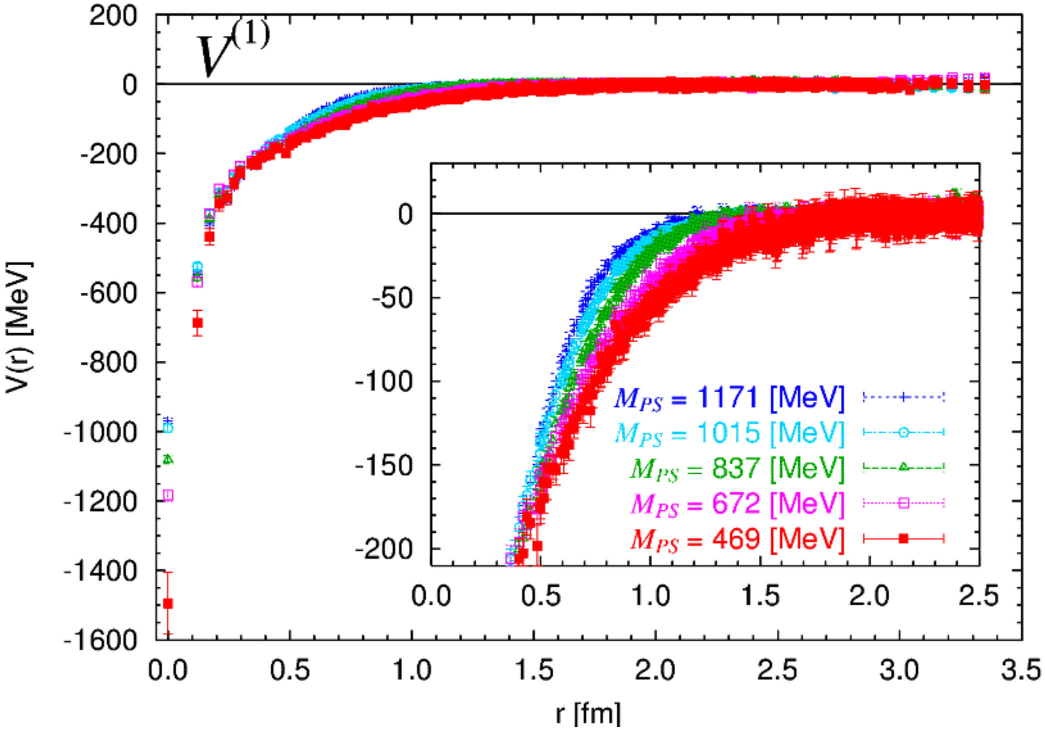


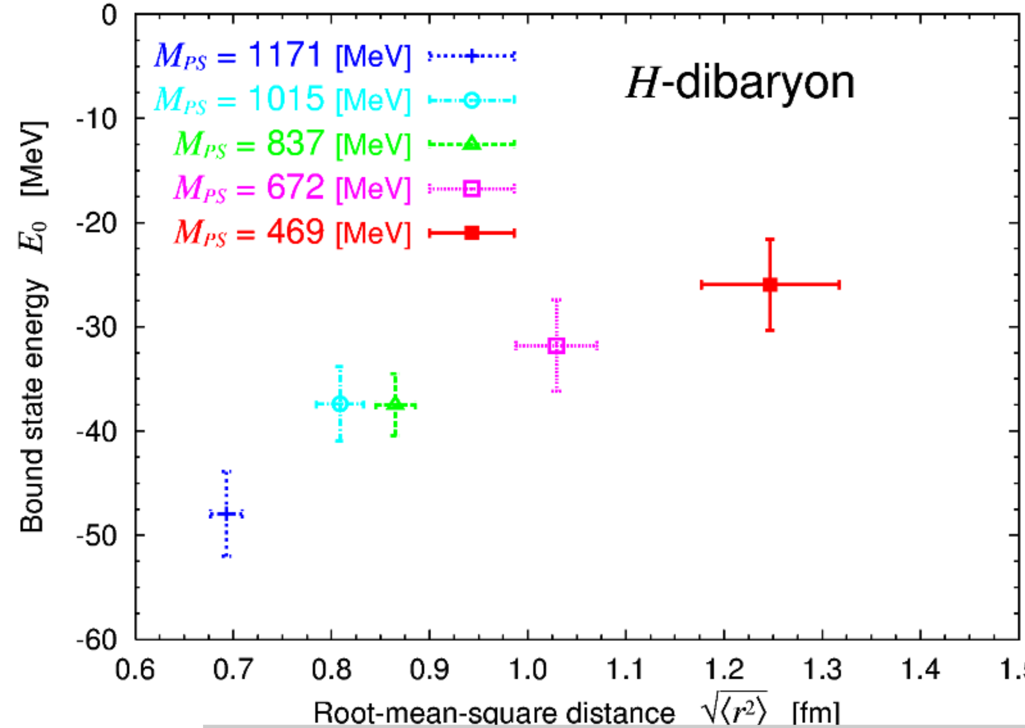
PRL110(2013) 222002

- There is no sign of near threshold enhancement.

Hunting for H-dibaryon in SU(3) limit

Strongly attractive interaction is expected in flavor singlet channel.





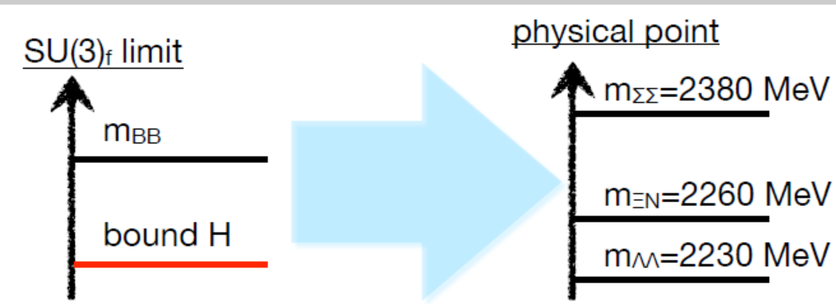
T.Inoue et al[HAL QCD Coll.] NPA881(2012) 28

- **Strongly attractive potential was found in the flavor singlet channel.**
- **Bound state was found in this mass range with SU(3) symmetry.**

What happens at the physical point?

▶ **SU(3) breaking effects**

- **Threshold separation**
- **Changes of interactions**



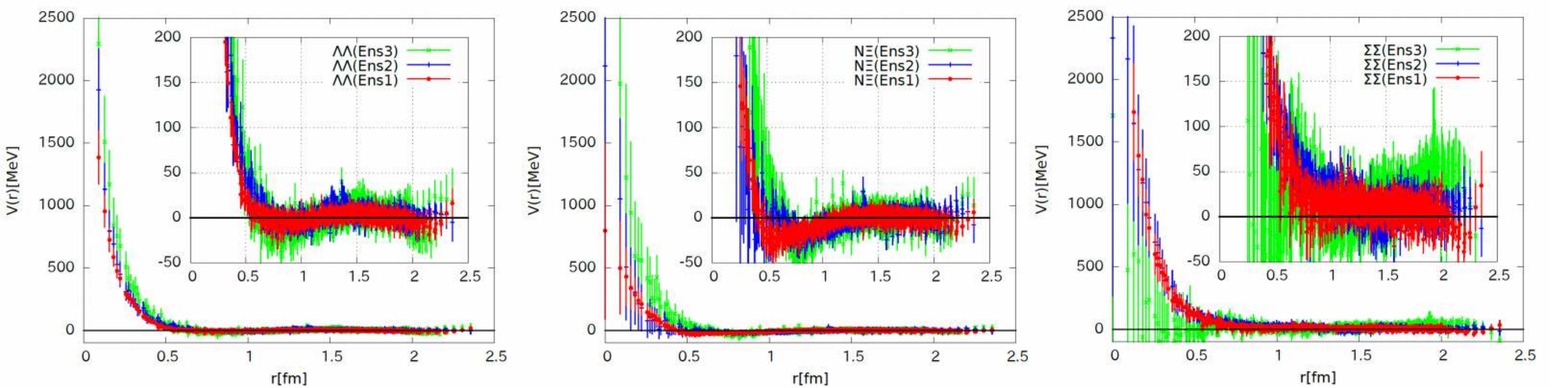
Non-trivial contributions

ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ channel

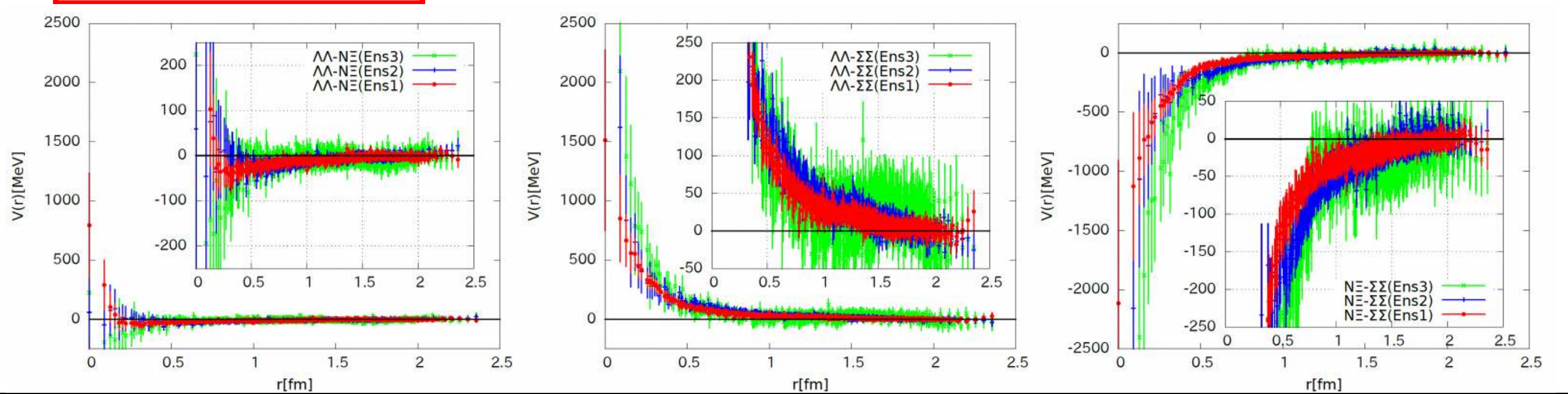
Esb1 : mπ= 701 MeV
Esb2 : mπ= 570 MeV
Esb3 : mπ= 411 MeV

- **N_f = 2+1 full QCD with L = 2.9fm**

Diagonal elements



Off-diagonal elements

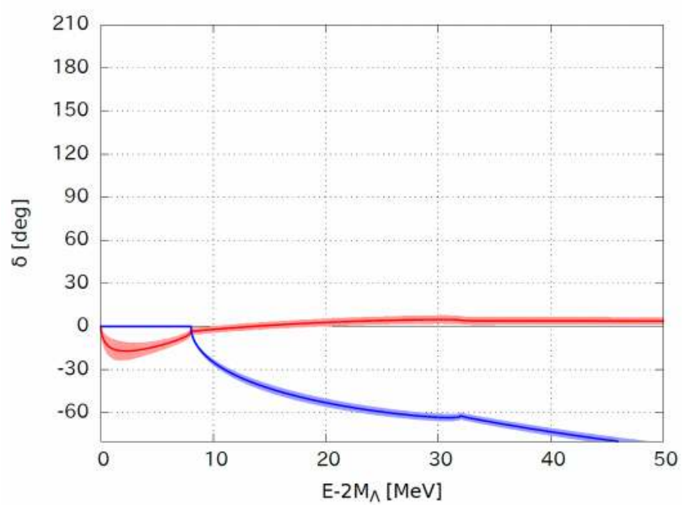


$\Lambda\Lambda$ and $N\Xi$ phase shifts

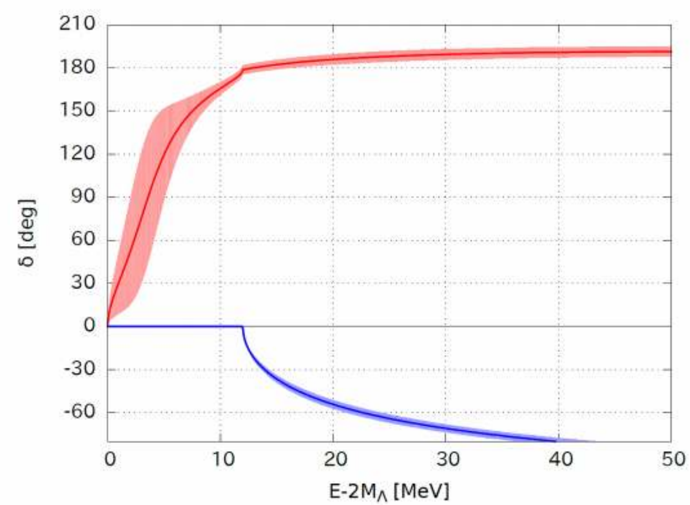
- N_f = 2+1 full QCD with L = 2.9fm

Preliminary!

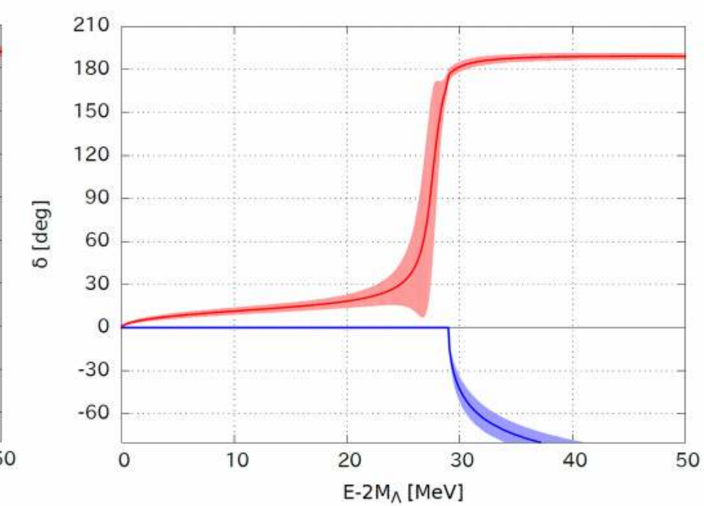
m_π = 700 MeV



m_π = 570 MeV

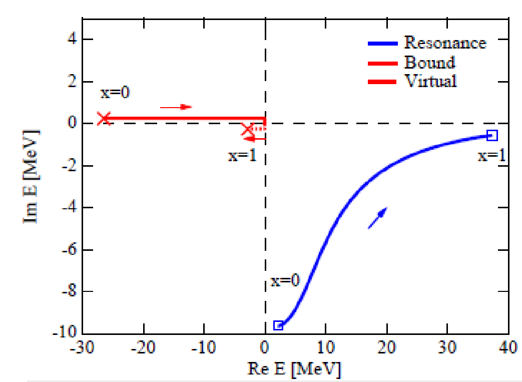


m_π = 410 MeV



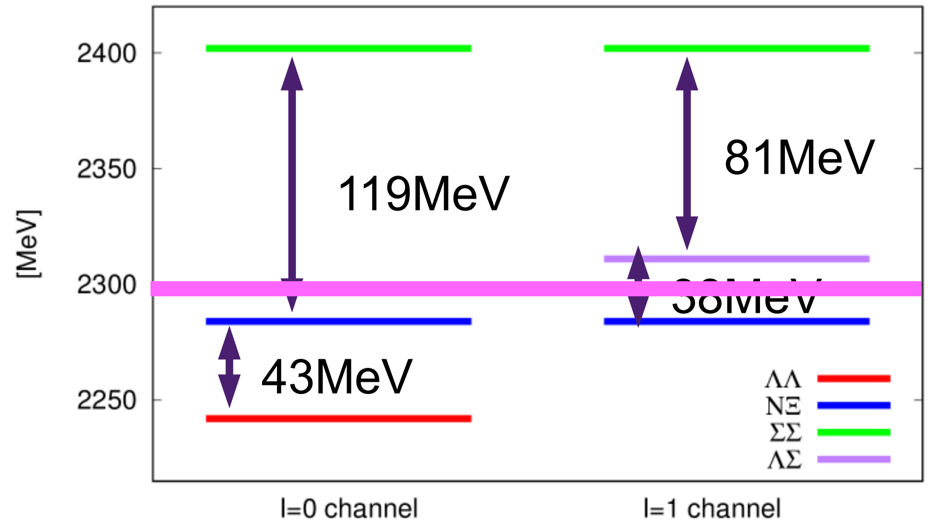
- m_π = 700 MeV : bound state
- m_π = 570 MeV : resonance near $\Lambda\Lambda$ threshold
- m_π = 410 MeV : resonance near $N\Xi$ threshold..

Go to the physical point simulation!



Y.Yamaguchi and T.Hyodo PRC 94 (2016) 065207

Effective ΛΛ and NΞ potential



Effective two channel potential

Original coupled channel equation

$$\begin{pmatrix} (E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda})R^{\Lambda\Lambda}(\vec{r},t) \\ (E^{\Xi N}-H_0^{\Xi N})R^{\Xi N}(\vec{r},t) \\ (E^{\Sigma\Sigma}-H_0^{\Sigma\Sigma})R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} V^{\Lambda\Lambda}_{\Lambda\Lambda}(\vec{r}) & V^{\Lambda\Lambda}_{\Xi N}(\vec{r}) & V^{\Lambda\Lambda}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Xi N}_{\Lambda\Lambda}(\vec{r}) & V^{\Xi N}_{\Xi N}(\vec{r}) & V^{\Xi N}_{\Sigma\Sigma}(\vec{r}) \\ V^{\Sigma\Sigma}_{\Lambda\Lambda}(\vec{r}) & V^{\Sigma\Sigma}_{\Xi N}(\vec{r}) & V^{\Sigma\Sigma}_{\Sigma\Sigma}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \\ R^{\Sigma\Sigma}(\vec{r},t) \end{pmatrix}$$

Truncation of ΣΣ channel

Reduced coupled channel equation

$$\begin{pmatrix} (E^{\Lambda\Lambda}-H_0^{\Lambda\Lambda})R^{\Lambda\Lambda}(\vec{r},t) \\ (E^{\Xi N}-H_0^{\Xi N})R^{\Xi N}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \overline{V^{\Lambda\Lambda}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Lambda\Lambda}_{\Xi N}}(\vec{r}) \\ \overline{V^{\Xi N}_{\Lambda\Lambda}}(\vec{r}) & \overline{V^{\Xi N}_{\Xi N}}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\Lambda\Lambda}(\vec{r},t) \\ R^{\Xi N}(\vec{r},t) \end{pmatrix}$$

Effective ΛΛ–NΞ potential

- The same scattering phase shift would be expected in a low energy region.
- Non-locality (energy dependence, higher derivative contribution) of potential matrix could be enhanced.

SU(3) feature of BB interaction

In view of quark degrees of freedom

Oka, Shimizu and Yazaki NPA464 (1987)

- Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.
- Strengths of repulsive core in YN and YY interaction are largely depend on their flavor structures.
- For the s-wave BB system, no repulsive core is predicted in flavor singlet state which is known as H-dibaryon channel.

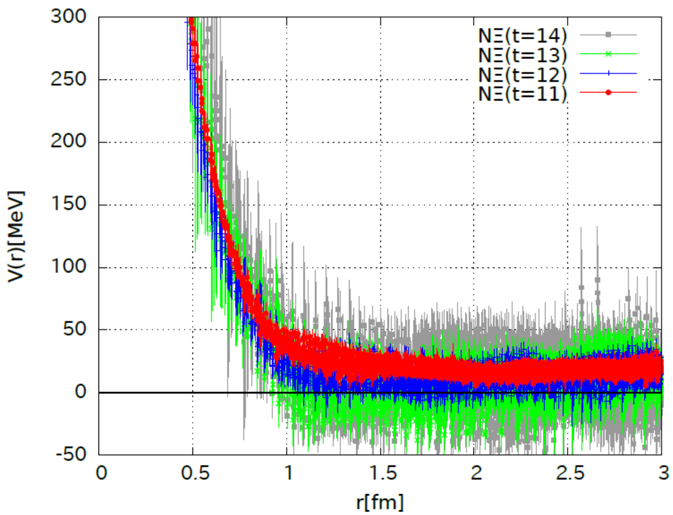
- **Flavor symmetry aspect**

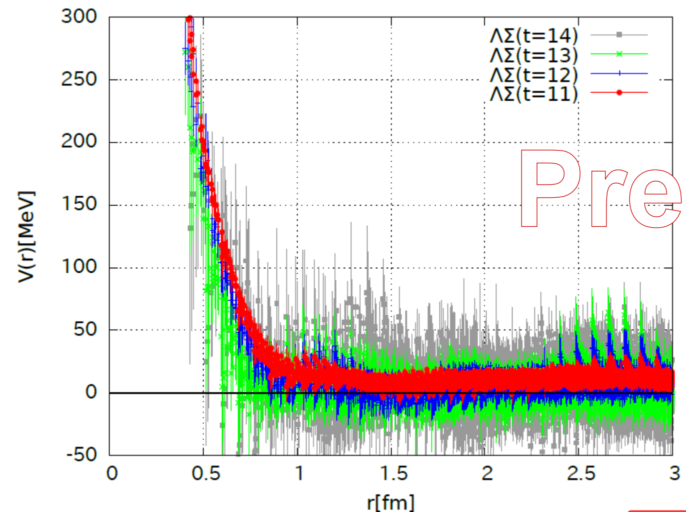
	1	8s	27	8a	10	10*
Pauli	allowed	forbidden	---	allowed	forbidden	---
CMI	attractive	repulsive	repulsive	repulsive	repulsive	repulsive
Short range int.	**attractive**	**repulsive**	**repulsive**	**repulsive**	**repulsive**	**repulsive**

$N\Xi$, $\Lambda\Sigma$ (I=1) $^1S_0$ channel

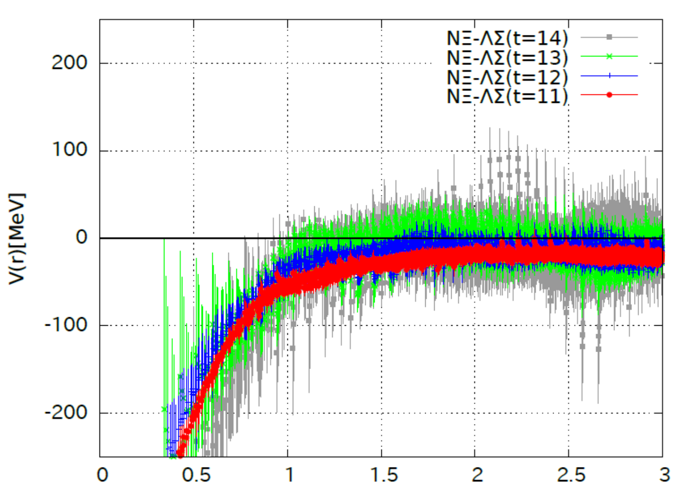
▶ N_f = 2+1 full QCD m_π=146MeV with L = 8.1fm

Diagonal elements





Off-diagonal elements



$$\begin{pmatrix} N\,\Xi \\ \Sigma\,\Lambda \end{pmatrix} = \frac{1}{5}\begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix}\begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

- Diagonal elements are repulsive in whole range.
- Diagonal $N\Xi$ potential is strongly repulsive.
- Potentials are not saturated in this time range.

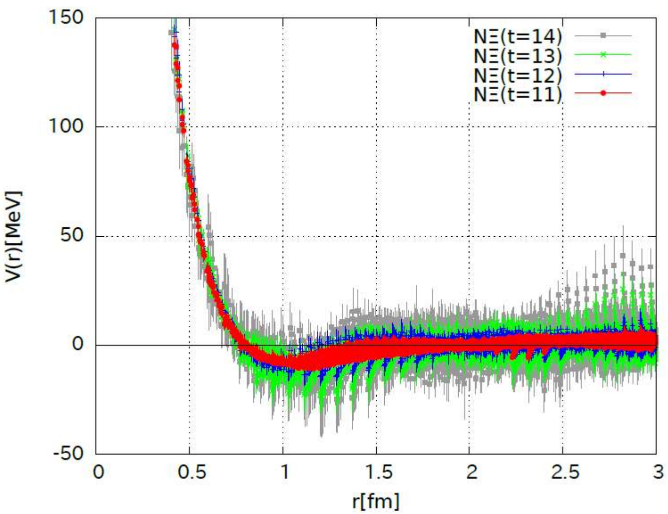
NΞ, ΛΣ, ΣΣ (I=1) $^3S_1$ *channel (eff. pot.)*

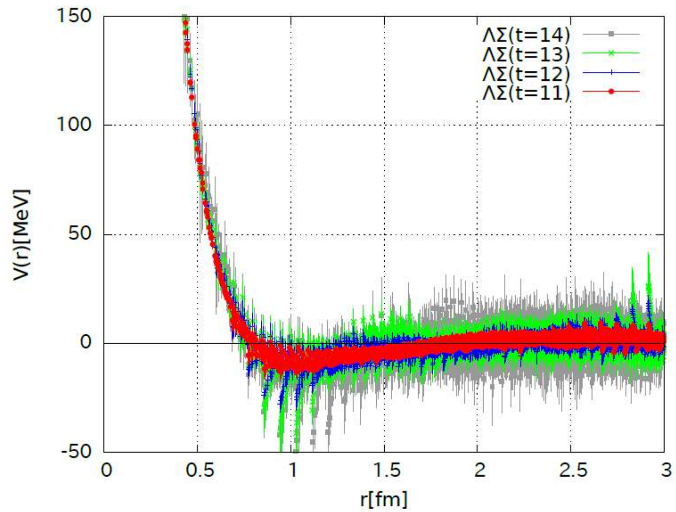
t=09
t=10
t=11
t=12

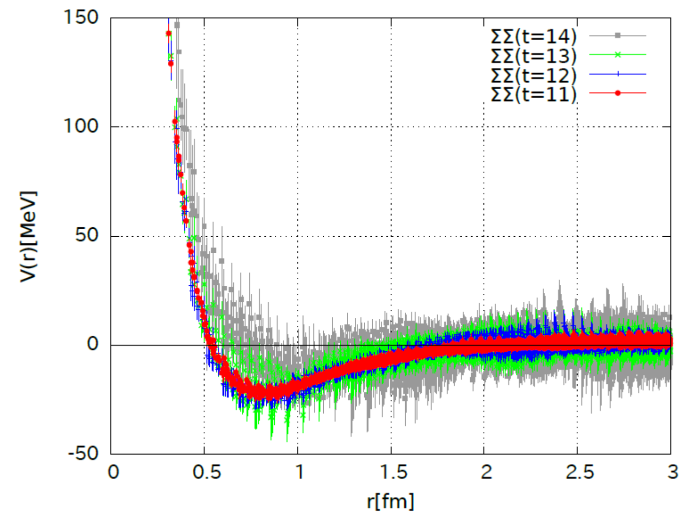
- N_f = 2+1 full QCD mπ=146MeV with L = 8.1fm

Preliminary!

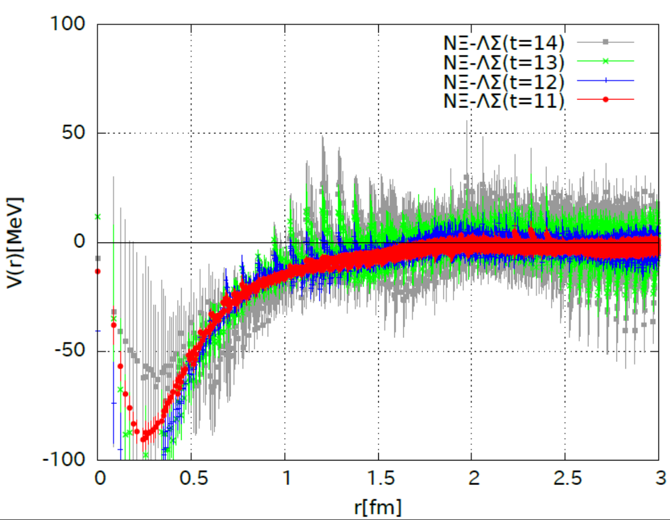
Diagonal elements

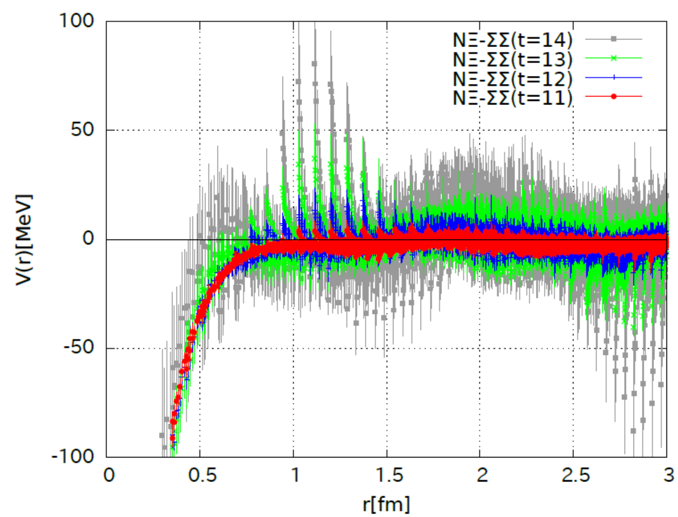


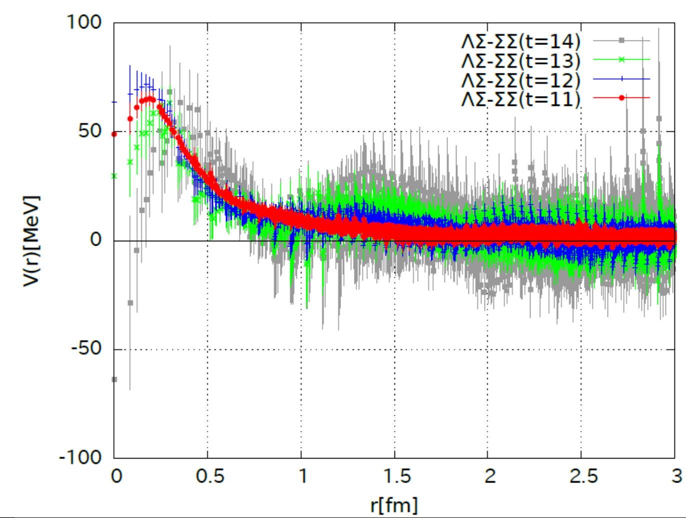




Off-diagonal elements







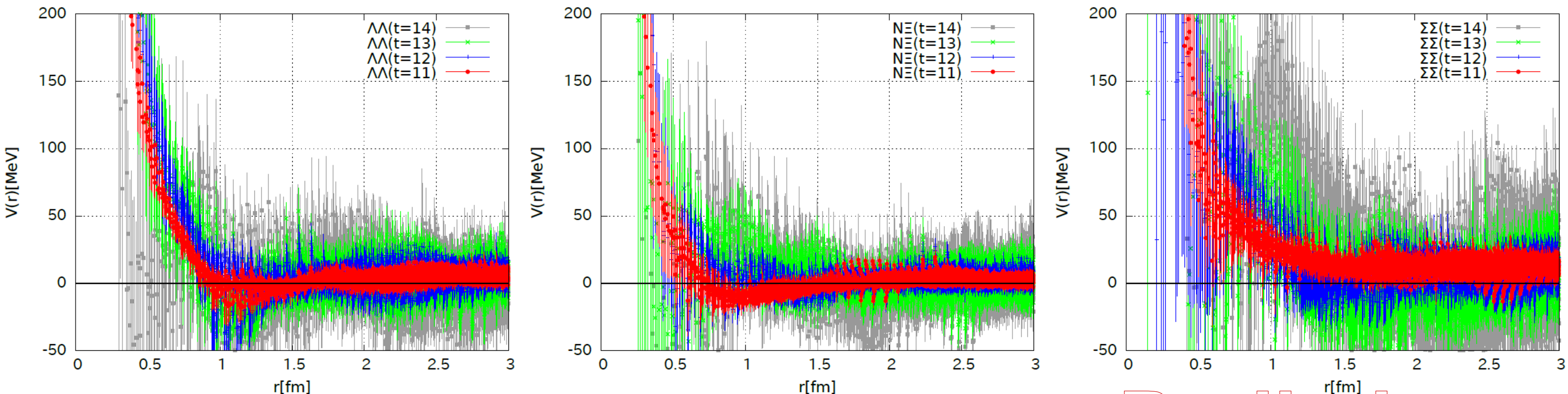
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

t=09
t=10
t=11
t=12

- N_f = 2+1 full QCD m_π=146MeV with L = 8.1fm

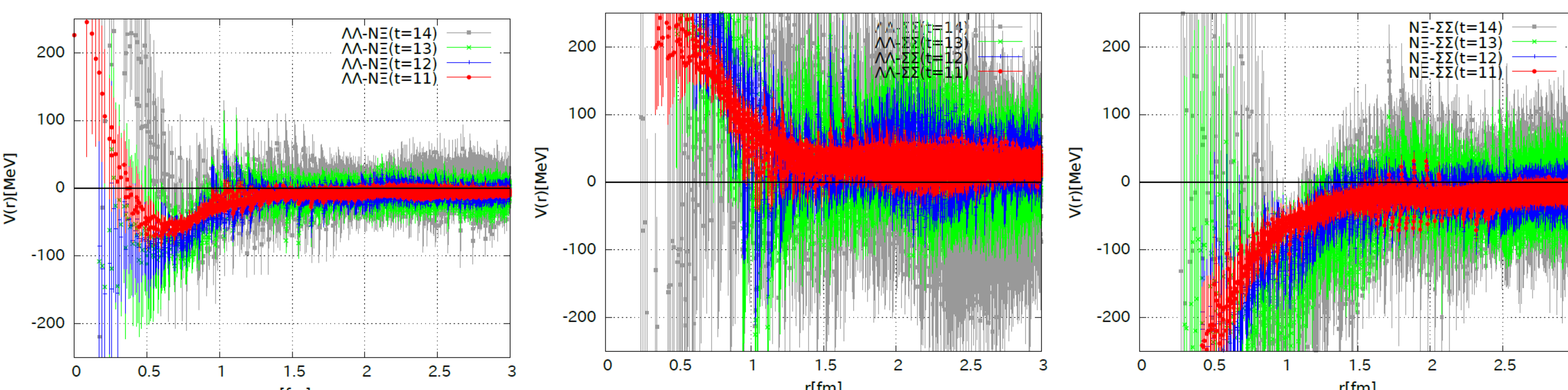
$$\begin{pmatrix}\Lambda\Lambda\\ N\Xi\\ \Sigma\Sigma\end{pmatrix} = \frac{1}{\sqrt{40}}\begin{pmatrix}-\sqrt{5} & -\sqrt{8} & \sqrt{27}\\ \sqrt{20} & \sqrt{8} & \sqrt{12}\\ \sqrt{15} & -\sqrt{24} & -1\end{pmatrix}\begin{pmatrix}1\\ 8\\ 27\end{pmatrix}$$

Diagonal elements



Preliminary!

Off-diagonal elements



Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

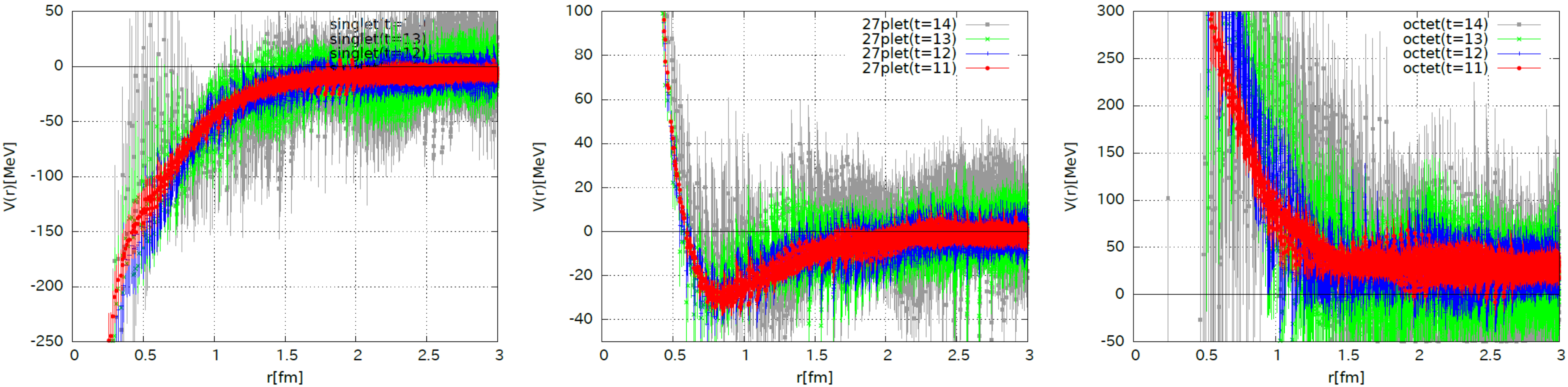
Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effective measure of the SU(3) breaking effect.

1, 8, 27plet (I=0) $^1S_0$ channel

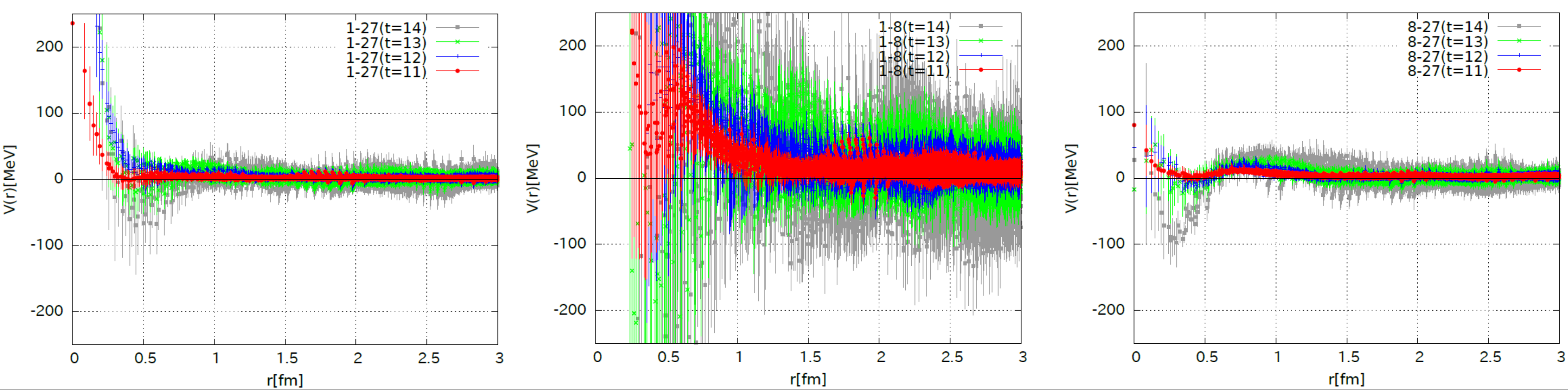
t=09
t=10
t=11
t=12

- N_f = 2+1 full QCD mπ=146MeV with L = 8.1fm

Diagonal elements

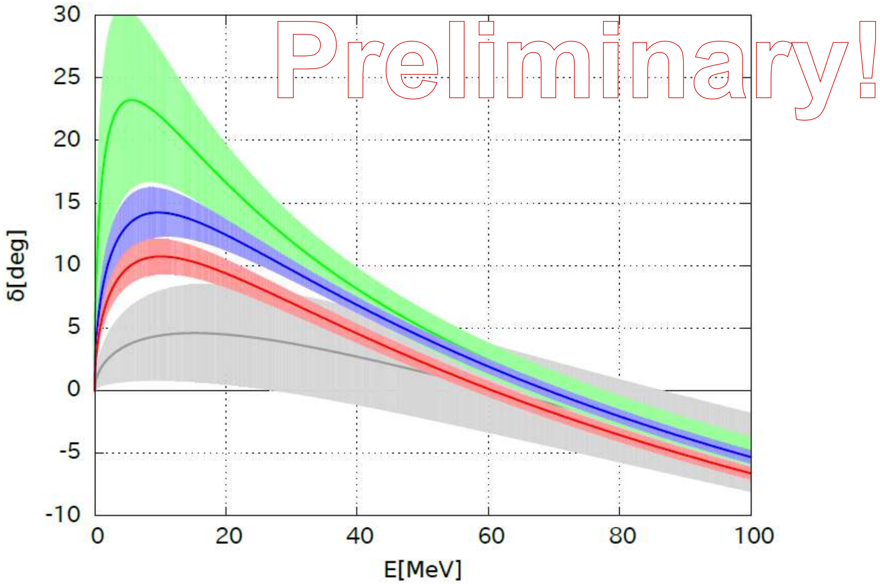


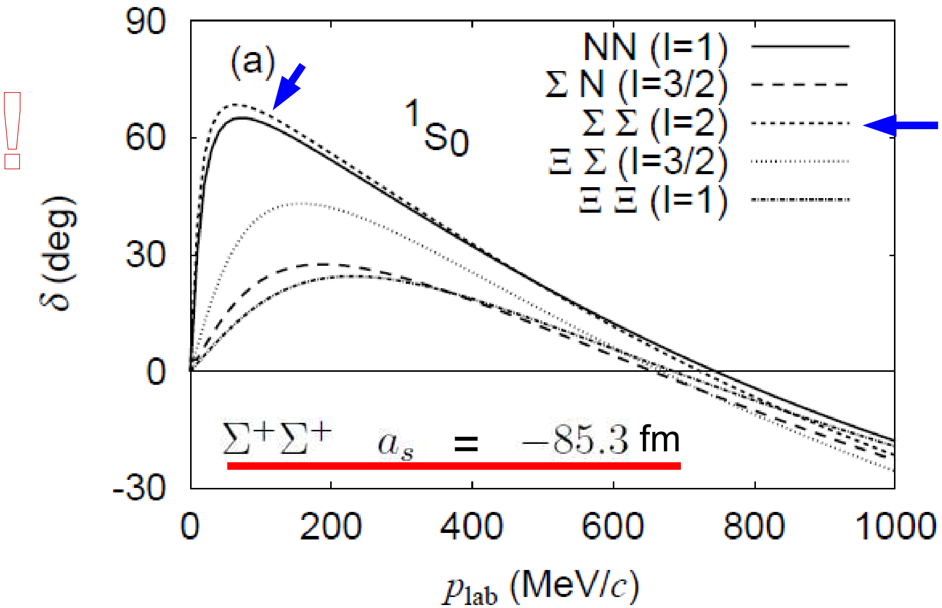
Off-diagonal elements



ΣΣ (I=2) $^1S_0$ channel –comparison--

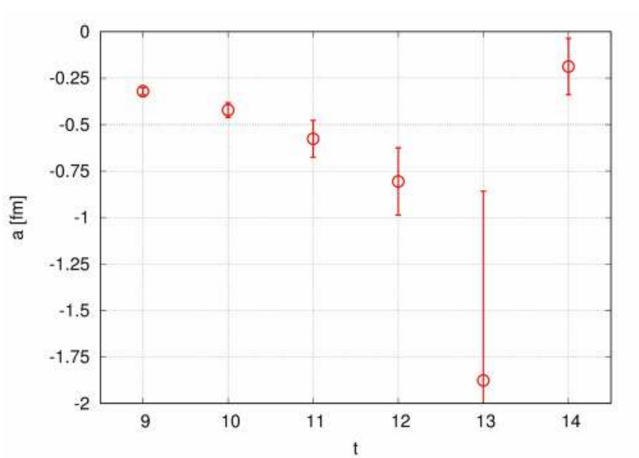
Phase shift





Y.Fujiwara et al, PPNP58(2007)439

Scattering length



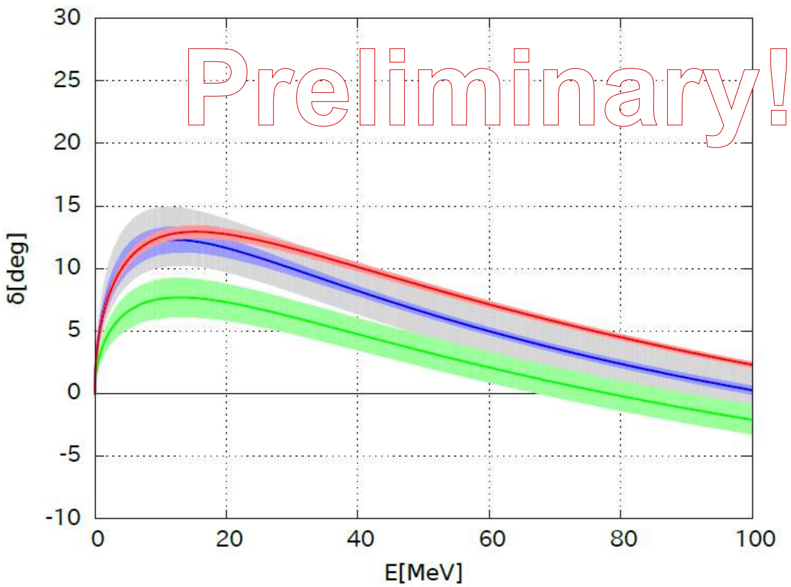
Λ		NLO			
		500	550	600	650
$\Sigma^+\Sigma^+$	a_{1S0}	-2.19	-1.94	-1.83	-1.82

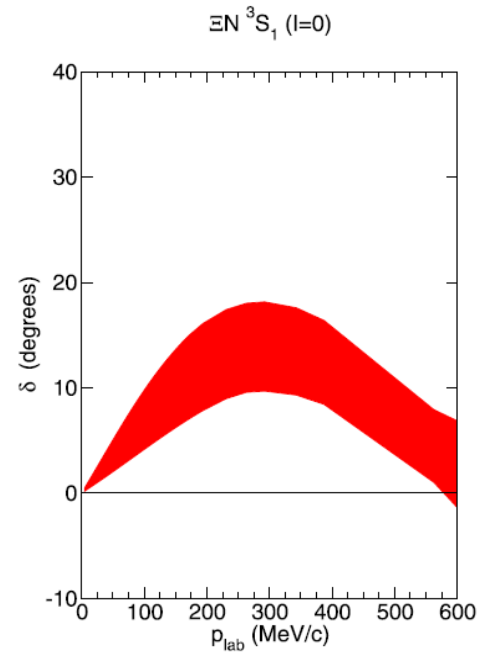
J. Haidenbauer et al, NPA954(2016)273

- **Scattering length in ΣΣ (I=2) channel is not saturated.**
- **Attraction is weaker than the phenomenological one.**

NΞ (I=0) $^3S_1$ channel –comparison--

Phase shift

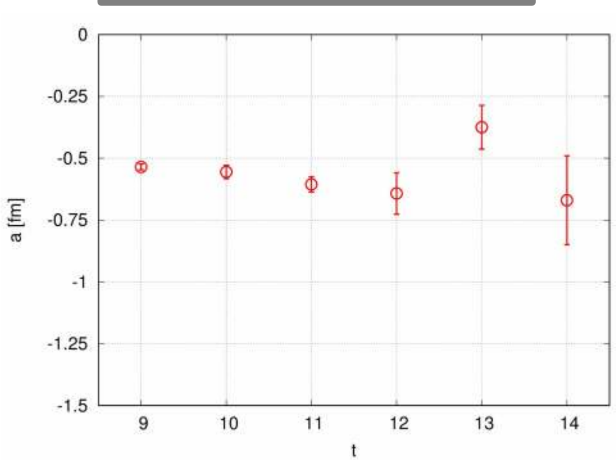




J. Haidenbauer et al, NPA954(2016)273

Λ		NLO			
		500	550	600	650
$I = 0$	$a_{^3S1}$	−0.33	−0.39	−0.62	−0.85
	$r_{^3S1}$	−6.87	−1.77	1.00	1.42

Scattering length



- Phase shift and scattering length are saturated (?).
- They are comparable with the results of NLO EFT calculation.