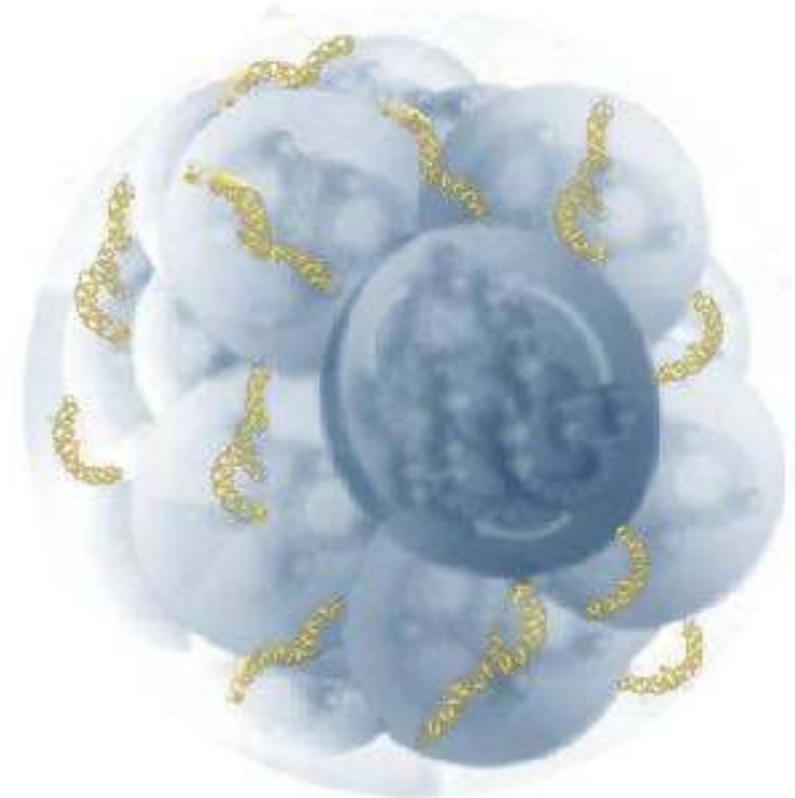
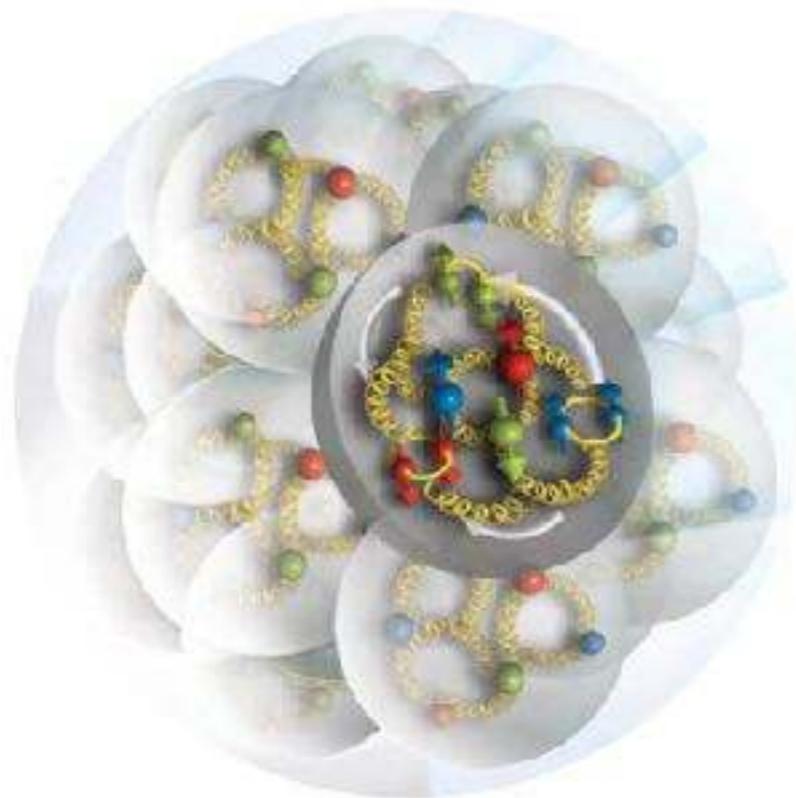


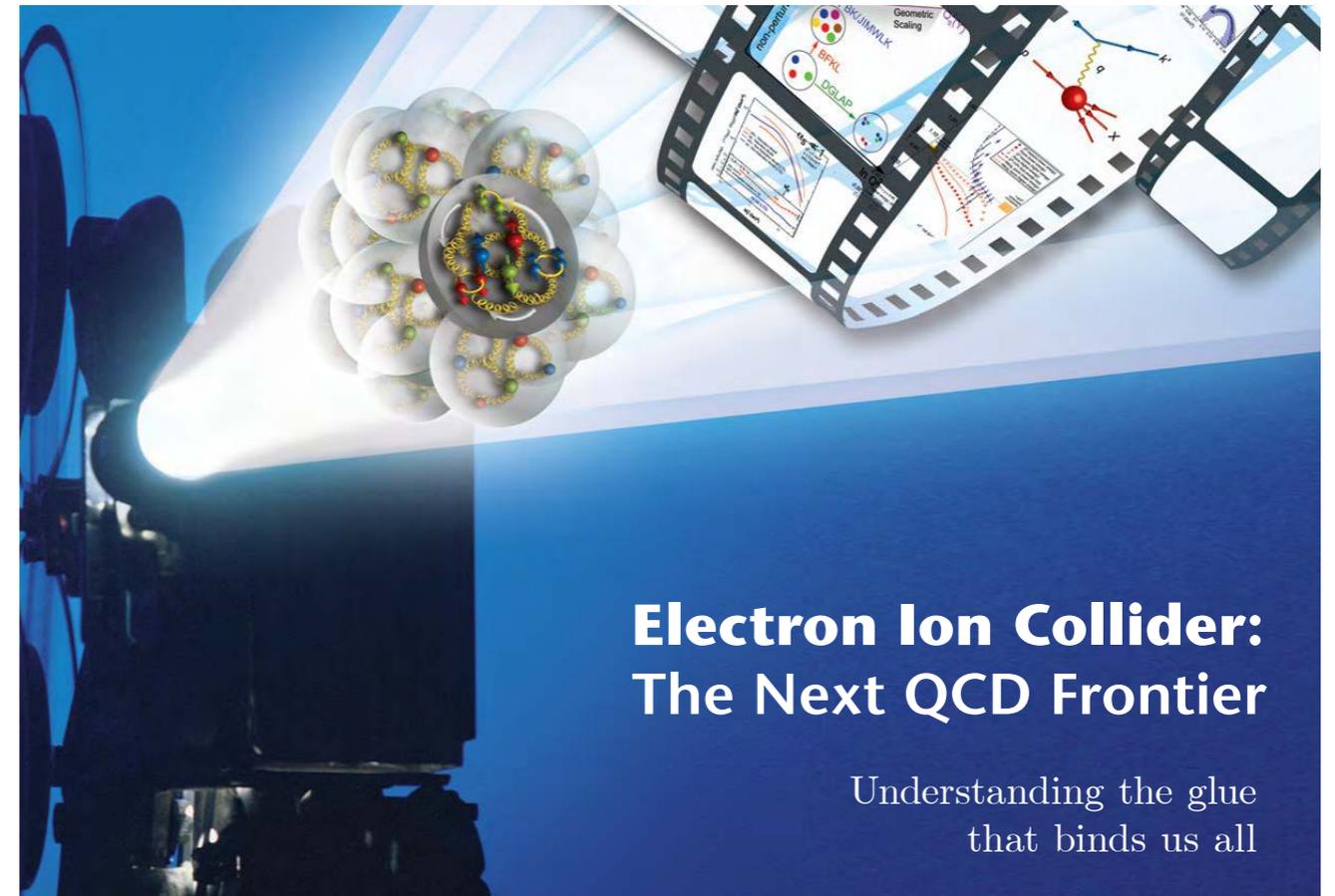
# Gluon Structure of Hadrons and Nuclei



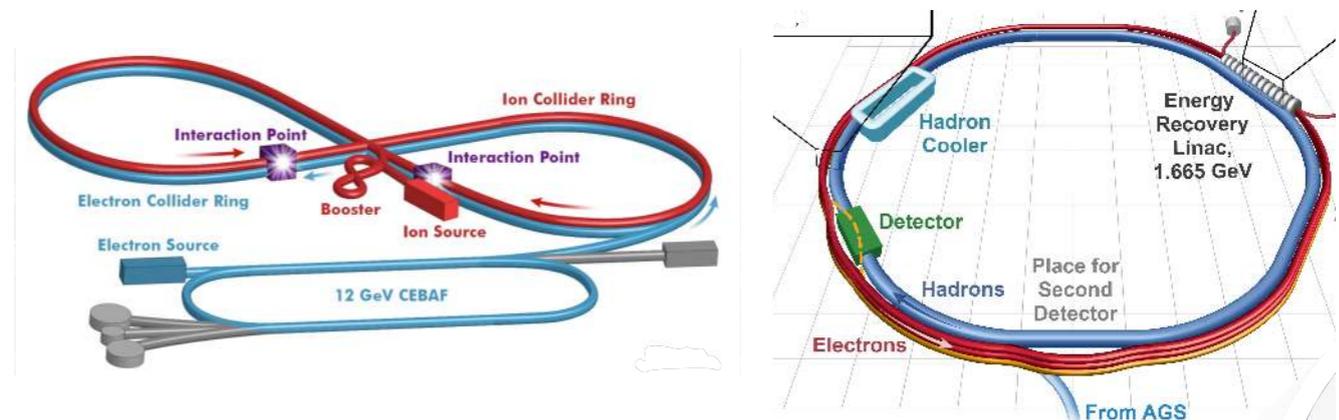
Phiala Shanahan  
MIT

# Gluon Structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluonic structure (beyond gluon density) relatively unexplored
- Electron-Ion Collider
  - Priority in 2015 nuclear physics long range plan
  - “Understanding the glue that binds us all”
- Insights from Lattice QCD?



Cover image from EIC whitepaper arXiv:1212.1701



# Gluon Structure from LQCD

1

How much do gluons contribute to the proton's

- Momentum
- Spin
- Mass

2

What is the 3D gluon distribution of a proton

- PDFs
- GPDs
- TMDs
- 'Gluon radius'

3

How is the gluon structure of a proton modified in a nucleus

- Gluonic 'EMC' effect

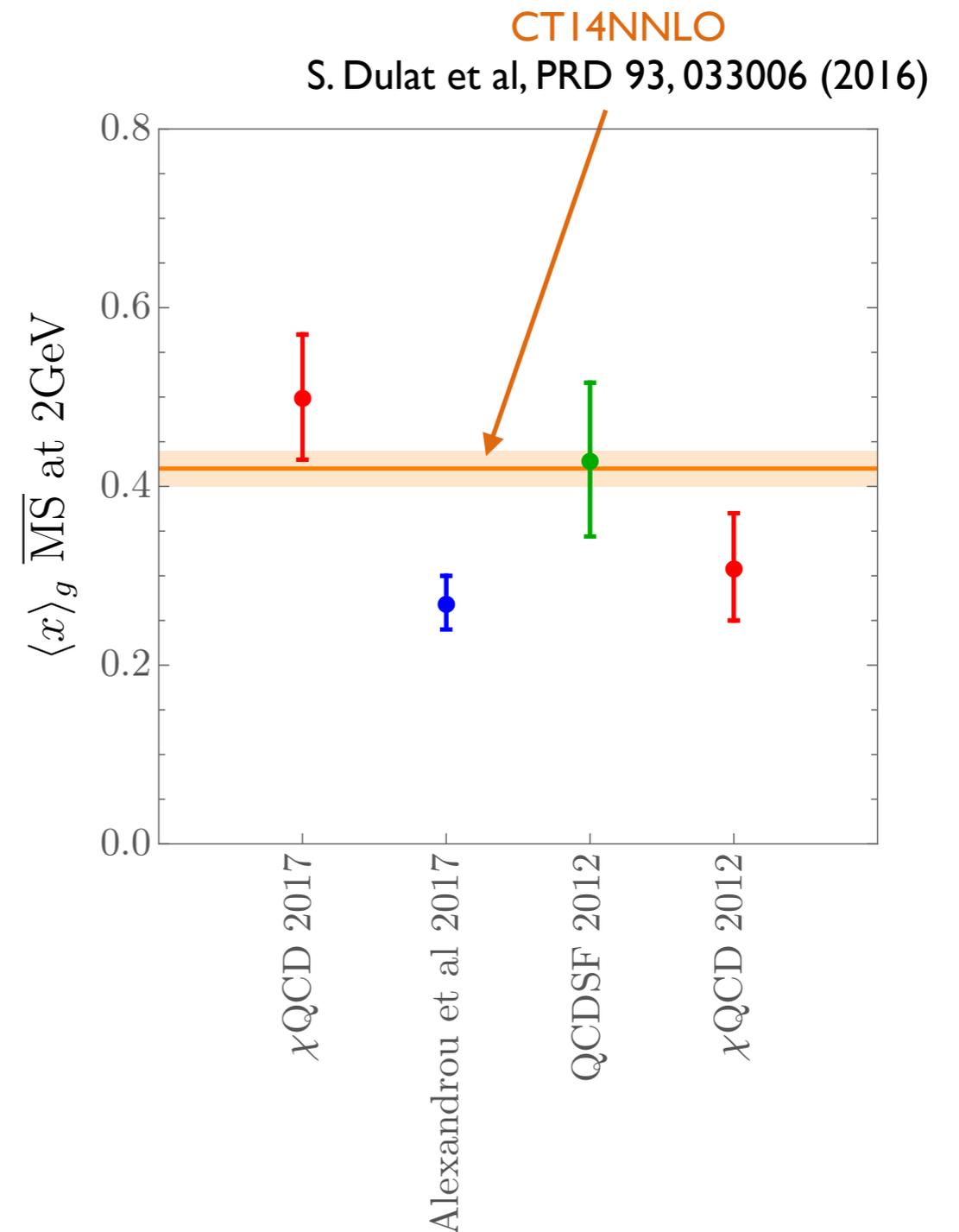
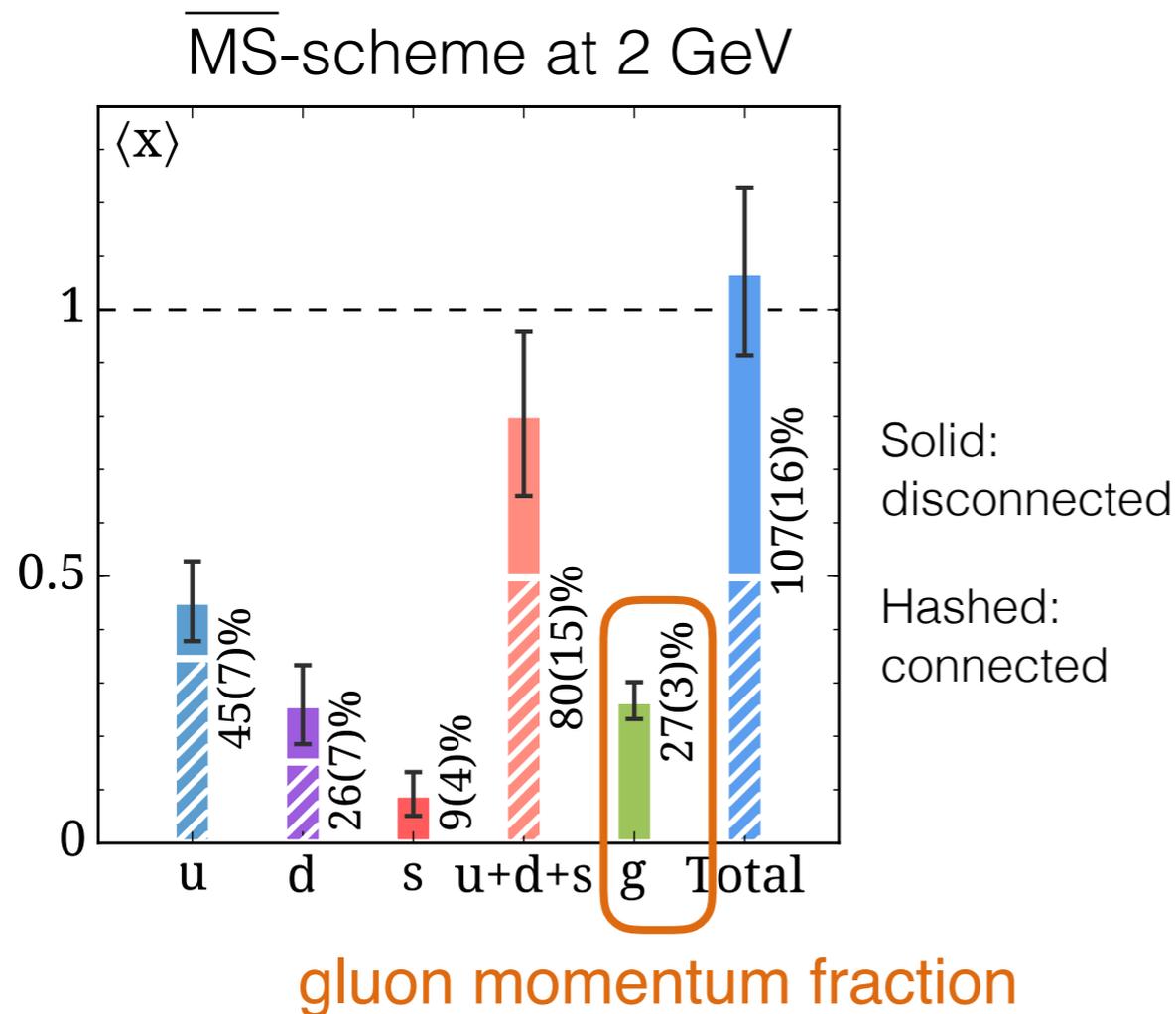
# Nucleon momentum decomposition

## Glun Momentum fraction

- Two direct calculations at the physical point since last year

C.Alexandrou et al., arXiv:1706.02973

Y.-B.Yang et al.,  $\chi$ QCD, in preparation



# Nucleon spin decomposition

Two decompositions of the proton spin:

- Ji (1996)

$$J_N = \sum_{q=u,d,s,c\dots} \left( \frac{1}{2} \Delta\Sigma_q + L_q \right) + J_g$$

quark orbital angular momentum

quark helicity

gluon spin

- Jaffe-Manohar (1990)

$$J_N = \sum_{q=u,d,s,c\dots} \left( \frac{1}{2} \Delta\Sigma_q + \mathcal{L}_q \right) + \Delta g + \mathcal{L}_g$$

gluon helicity

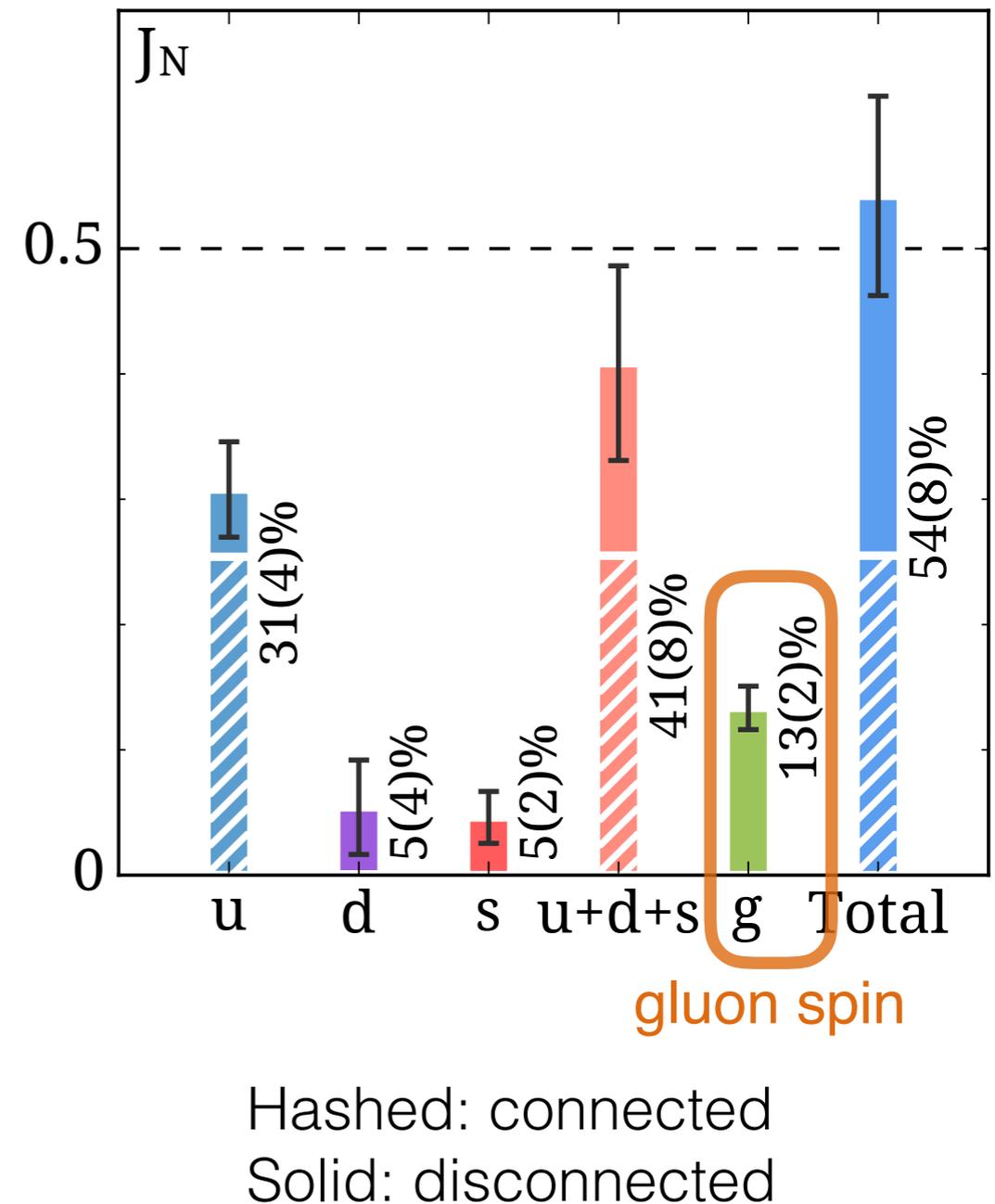
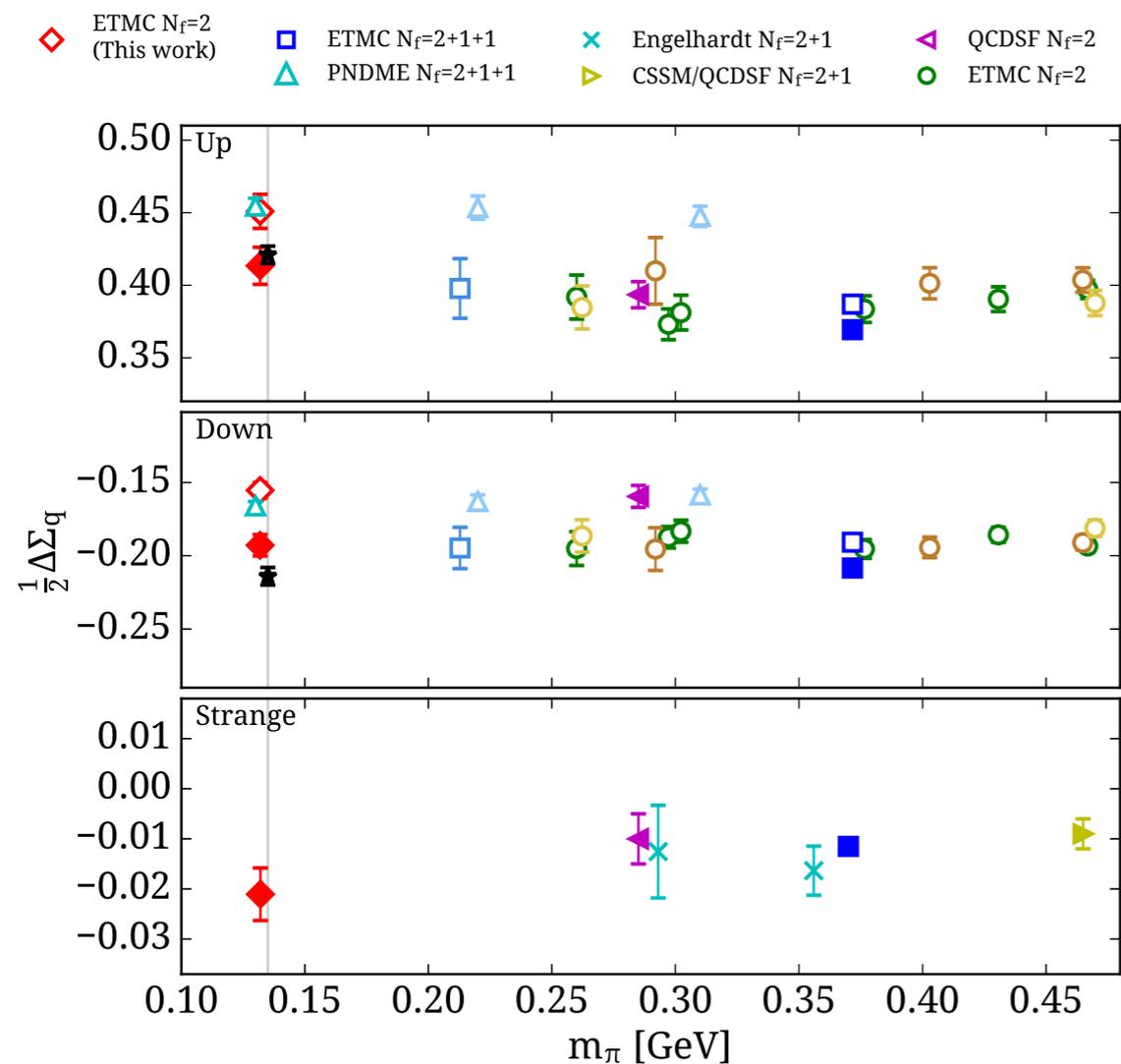
Interpolation between decompositions: [M. Engelhardt, PRD 95 094505 \(2017\)](#)

# Ji spin decomposition

C. Alexandrou et al., arXiv:1706.02973

- Physical pion mass
- All terms calculated directly

$\overline{\text{MS}}$ -scheme at 2 GeV



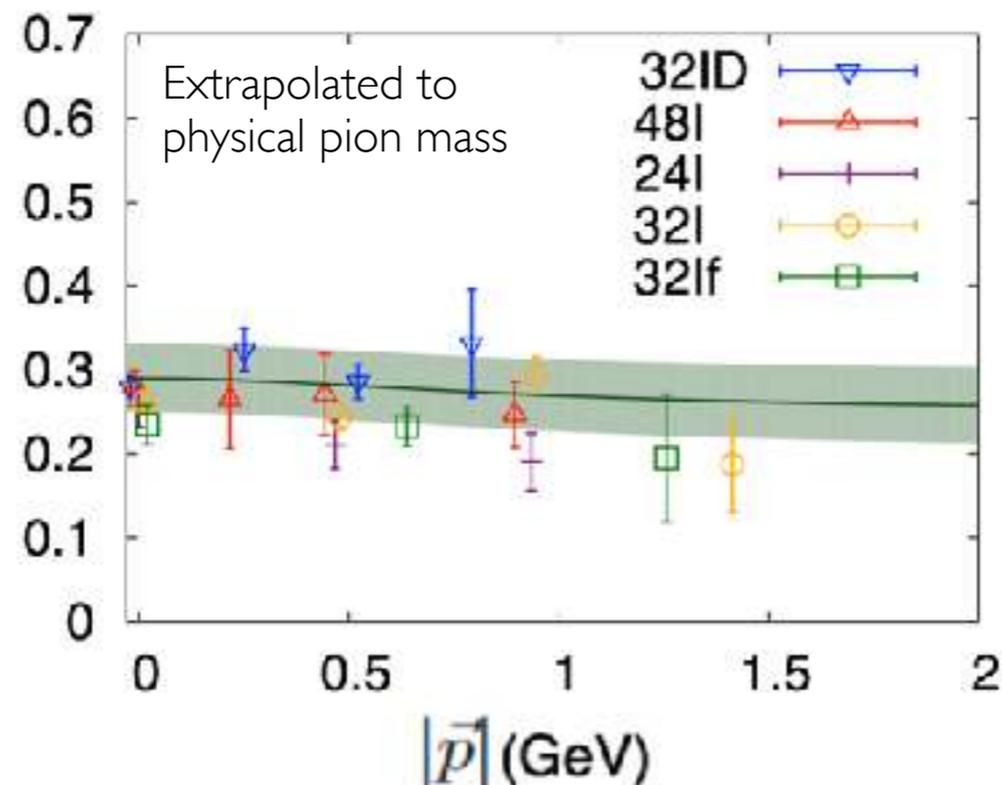
# J-M spin decomposition

Y.-B. Yang et al., PRL 118, 102001 (2017)

## Gluon Helicity

- Can't be calculated directly
- Match to calculable ME in infinite momentum frame limit using large momentum effective theory

LaMET: X. Ji et al., PRL 111, 112002 (2013)



gluon helicity

de Florian et. al, Phys.Rev.Lett. 113, 012001 (2014)

# Gluonic Transversity

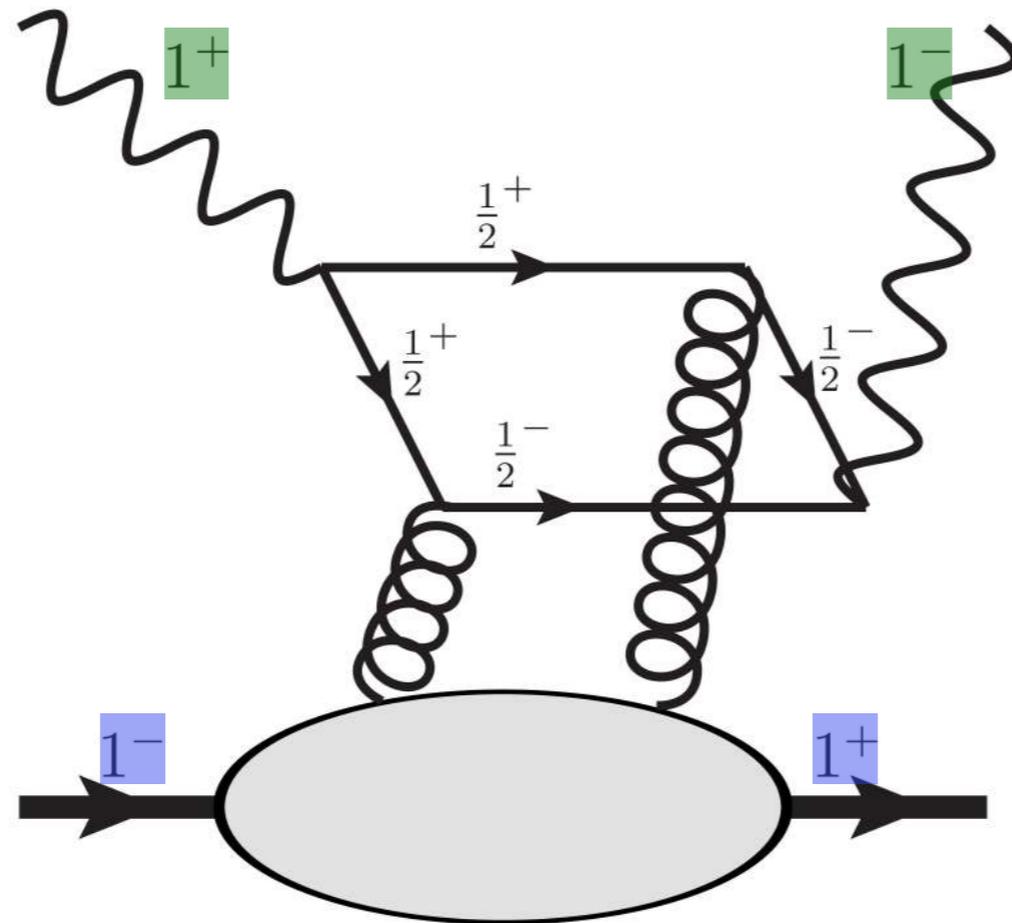
Targets with  $J \geq 1$  have leading twist gluon parton distribution  $\Delta(x, Q^2)$ : double helicity flip [Jaffe & Manohar 1989]

- **Unambiguously gluonic**: no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin  $\geq 1$
- **Experimentally measurable** in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab Lol 2015
  - Polarised nuclei at EIC
- **Moments calculable in LQCD**



# Gluonic Transversity

Double helicity flip structure function  $\Delta(x, Q^2)$



Changes both photon and target helicity by 2 units

# Gluonic Transversity

## Double helicity flip structure function $\Delta(x, Q^2)$

- **Hadrons:** Gluonic Transversity (parton model interpretation)

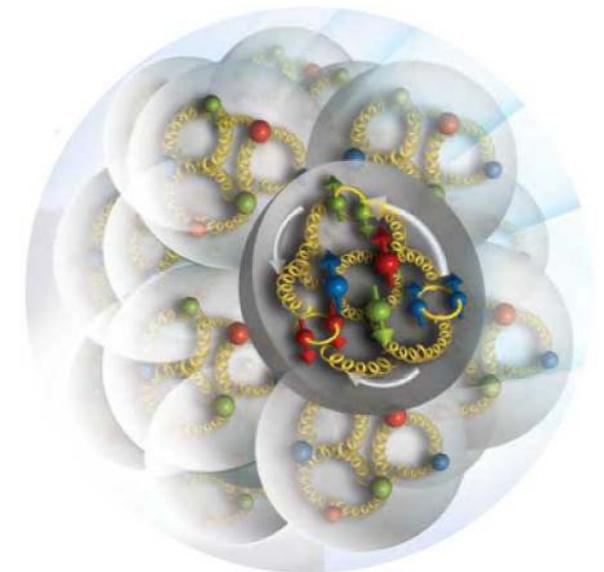
$$\Delta(x, Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 x^2 \int_x^1 \frac{dy}{y^3} [g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(x, Q^2)]$$

$g_{\hat{x}, \hat{y}}(y, Q^2)$ : probability of finding a gluon with momentum fraction  $y$  linearly polarised in  $\hat{x}$ ,  $\hat{y}$  direction

- **Nuclei:** Exotic Glue

gluons not associated  
with individual nucleons  
in nucleus

$$\begin{aligned} \langle p | \mathcal{O} | p \rangle &= 0 \\ \langle N, Z | \mathcal{O} | N, Z \rangle &\neq 0 \end{aligned}$$



# Gluonic Transversity

## Double helicity flip structure function $\Delta(x, Q^2)$

- **Hadrons:** Gluonic Transversity (parton model interpretation)

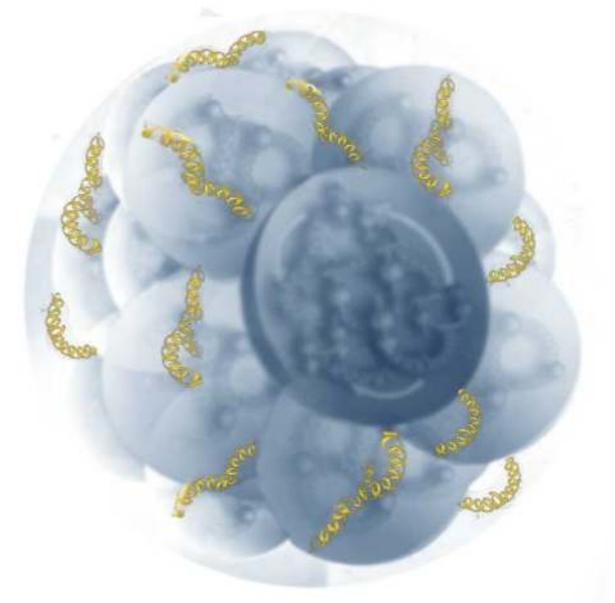
$$\Delta(x, Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 x^2 \int_x^1 \frac{dy}{y^3} [g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(x, Q^2)]$$

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# Gluonic Transversity

Moments of  $\Delta(x, Q^2)$  are calculable in LQCD

$$\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n+2}, \quad n = 2, 4, 6 \dots,$$

Moment of Structure Function Reduced Matrix Element

Determined by matrix elements of local gluonic operators

$$\langle pE' | \underline{S} [G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \dots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}] | pE \rangle$$

Symmetrise in  $\mu_1, \dots, \mu_n$ , trace subtract in all free indices

$$= (-2i)^{n-2} \underline{S} [(p_\mu E'_{\mu_1} - p_{\mu_1} E'_\mu) (p_\nu E_{\mu_2} - p_{\mu_2} E_\nu) + (\mu \leftrightarrow \nu)] p_{\mu_3} \dots p_{\mu_n} A_n(Q^2) \dots,$$

Reduced Matrix Element

# First LQCD Calculation

## Glue transversity of the $\phi$ meson

- First moment in  $\phi$  meson (simplest spin-1 system  $\rightarrow$  nuclei)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

$L/a$	$T/a$	$\beta$	$am_l$	$am_s$
24	64	6.1	-0.2800	-0.2450
$a$ (fm)	$L$ (fm)	$T$ (fm)	$m_\pi$ (MeV)	$m_K$ (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
$m_\phi$ (MeV)	$m_\pi L$	$m_\pi T$	$N_{\text{cfg}}$	$N_{\text{src}}$
1040(3)	6.390	17.04	1042	$10^5$

- Many systematics not addressed (yet)

- Quark mass effects
- Volume effects

- Discretisation
- Renormalisation

Alexandrou et al. arXiv:1611.06901

# LQCD Calculation

- Discrete lattice: rotational symmetry  $\rightarrow$  hypercubic symmetry
- Take linear combinations of operators that transform irreducibly under hypercubic group: safe from mixing

e.g., for  $\mathcal{O}_{\mu\nu\mu_1\mu_2}^{(E)} = G_{\mu\mu_1}^{(E)} G_{\nu\mu_2}^{(E)}$  use  $\mathcal{O}_{1,1}^{(E)} = \frac{1}{8\sqrt{3}} \left( -2\mathcal{O}_{1122}^{(E)} + \mathcal{O}_{1133}^{(E)} + \mathcal{O}_{1144}^{(E)} + \mathcal{O}_{2233}^{(E)} + \mathcal{O}_{2244}^{(E)} - 2\mathcal{O}_{3344}^{(E)} \right)$

$$C_{jk}^{2\text{pt}}(t, \vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle \eta_j(t, \vec{x}) \eta_k^\dagger(0, \vec{0}) \rangle$$

$$= Z_\phi \left( e^{-Et} + e^{-E(T-t)} \right) \sum_{\lambda} \epsilon_j^{(E)}(\vec{p}, \lambda) \epsilon_k^{(E)*}(\vec{p}, \lambda)$$

$$C_{jk}^{3\text{pt}}(t, \tau, \vec{p}) = \sum_{\vec{x}} \sum_{\vec{y}} e^{i\vec{p}\cdot\vec{x}} \langle \eta_j(t, \vec{p}) \mathcal{O}(\tau, \vec{y}) \eta_k^\dagger(0, \vec{0}) \rangle$$

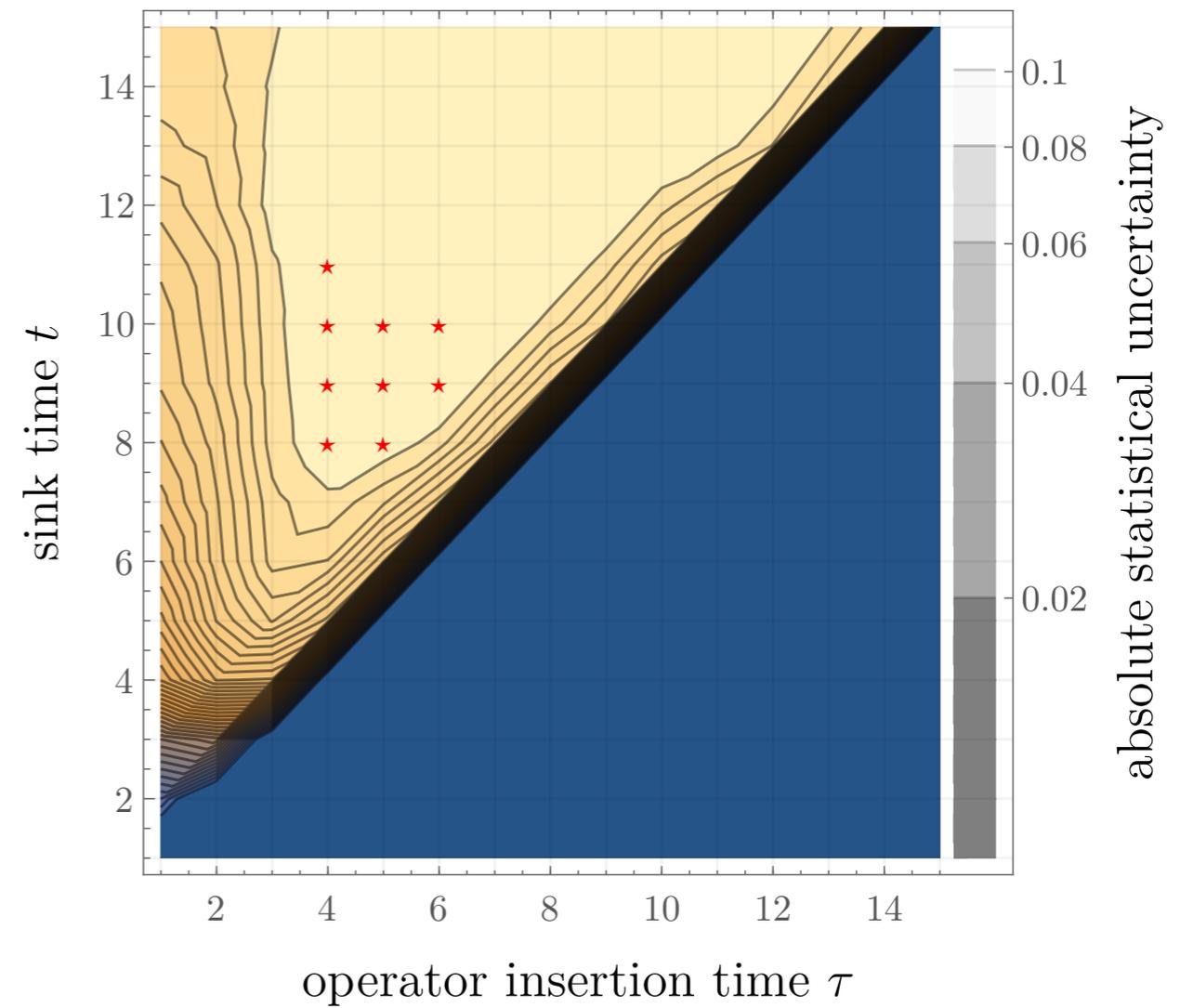
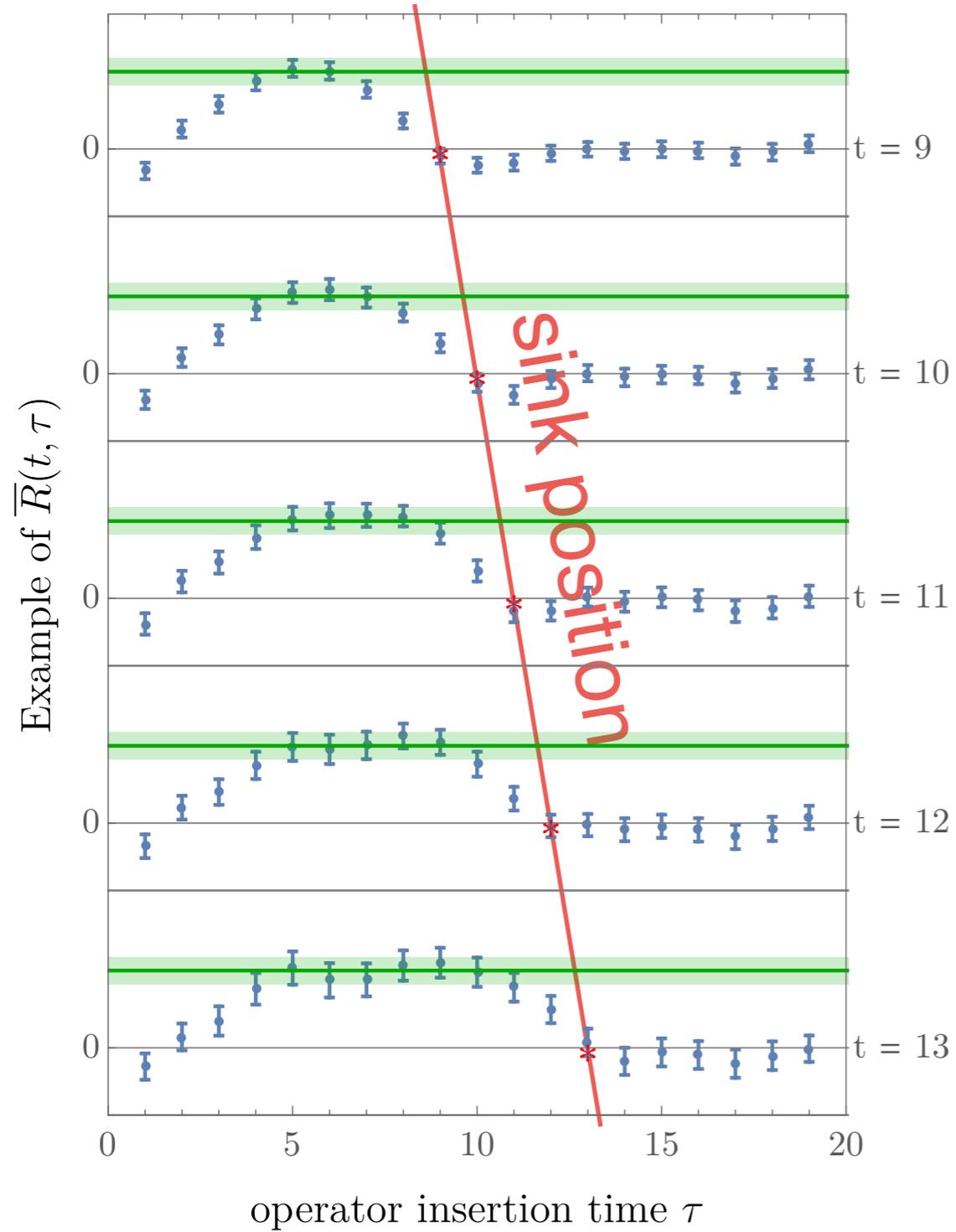
$$= Z_\phi e^{-Et} \sum_{\lambda\lambda'} \epsilon_j^{(E)}(\vec{p}, \lambda) \epsilon_k^{(E)*}(\vec{p}, \lambda') \langle \vec{p}, \lambda | \mathcal{O} | \vec{p}, \lambda' \rangle$$

$$R_{jk}(t, \tau, \vec{p}) = \frac{C_{jk}^{3\text{pt}}(t, \tau, \vec{p}) + C_{jk}^{3\text{pt}}(T-t, T-\tau, \vec{p})}{C_{jk}^{2\text{pt}}(t, \vec{p})}$$

- All polarisation combinations (j,k)
- Boost momenta up to (1,1,1)
- Examine all elements of each hypercubic irrep.

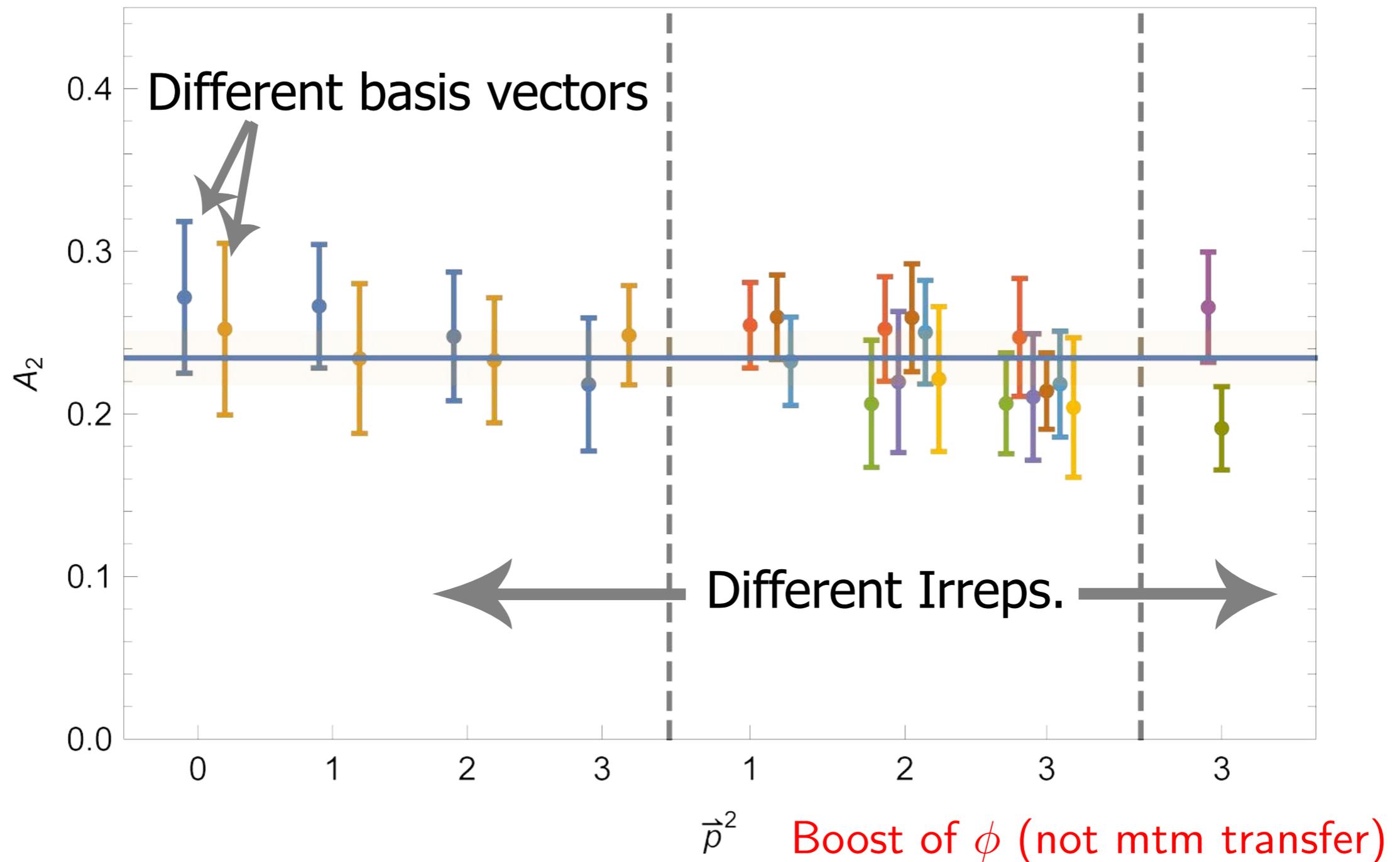
ratio depends on  
polarisations,  
momentum,  
operator

# LQCD Calculation



# LQCD Calculation

W. Detmold, PES, PRD 94 (2016), 014507



# Soffer-type Bounds

Constraint relating **transversity**, **spin-indep.** and **spin-dep.** distributions

For quark distributions in spin 1/2 state:

$$|\overset{\text{Transversity}}{\delta q(x)}| \leq \frac{1}{2} \left( \overset{\text{Spin-independent}}{q(x)} + \underset{\text{Spin-dependent}}{\Delta q(x)} \right)$$

**Analogue for first moments of gluon distributions?**

- Need to calculate moments of spin independent gluon distribution (first moment of spin-dependent gluon distribution vanishes by operator symmetries)

# Spin-indep. gluon structure

W. Detmold, PES, PRD 94 (2016), 014507

Spin-independent gluon operator:

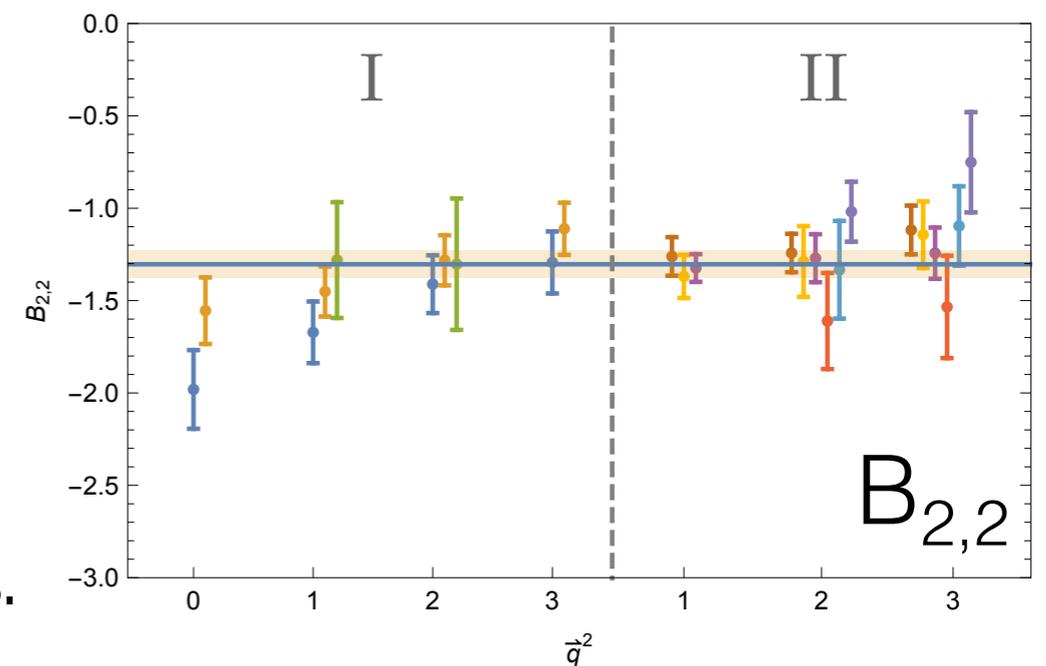
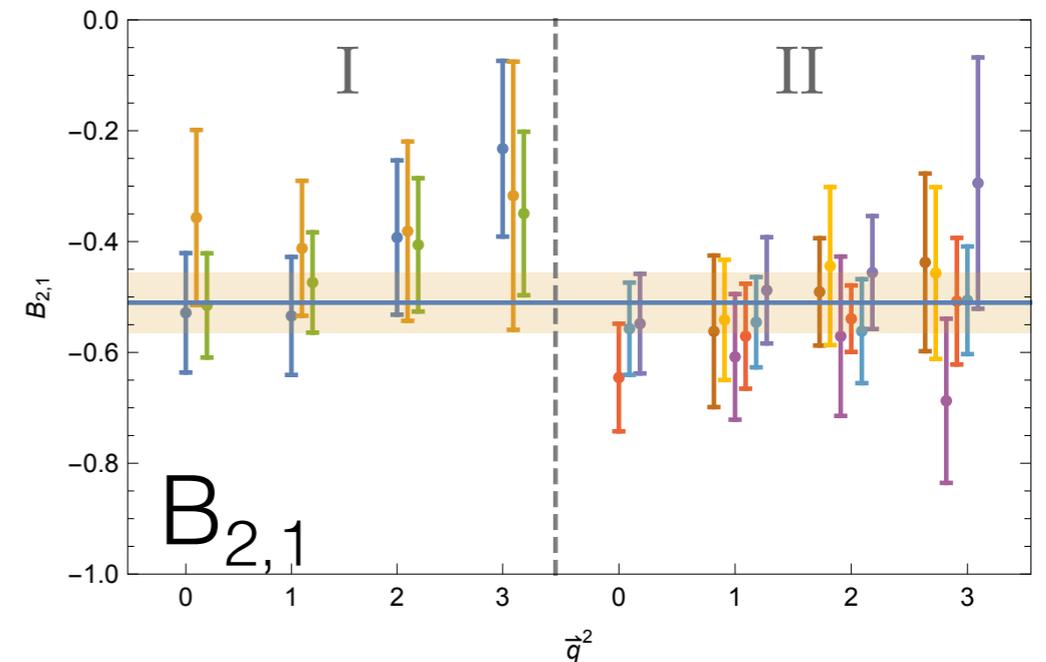
$$\bar{\mathcal{O}}_{\mu_1 \dots \mu_n} = S \left[ G_{\mu_1 \alpha} \overleftrightarrow{D}_{\mu_3} \dots \overleftrightarrow{D}_{\mu_n} G_{\mu_2}{}^\alpha \right]$$

Matrix elements at n=2 define lowest moment of structure functions

$$\begin{aligned} \langle pE' | \bar{\mathcal{O}}_{\mu_1 \mu_2} | pE \rangle \\ = S \left[ M^2 E_{\mu_1}^* E_{\mu_2} \right] B_{2,1}(\mu^2) \\ + S \left[ (E \cdot E'^*) p_{\mu_1} p_{\mu_2} \right] B_{2,2}(\mu^2) \end{aligned}$$

Two reduced matrix elements

- Analysis as in transversity case
- Mixing with quark ops. neglected, pQCD calcs. shown that it is small: Alexandrou 1611.06901



# Soffer-type Bounds

Soffer-type bound for leading moments of gluon distributions (spin-1 state):

$$|A_2| \leq \frac{1}{24} (5B_{2,1} - 6B_{2,2})$$

Annotations:   
 - A pink arrow labeled "Transversity" points to  $|A_2|$ .   
 - A green arrow labeled "Spin-independent" points to  $5B_{2,1}$ .   
 - A green arrow labeled "Spin-independent" points to  $6B_{2,2}$ .   
 - A blue arrow labeled "Spin-dependent" points to the constant  $0$  on the right side of the inequality.

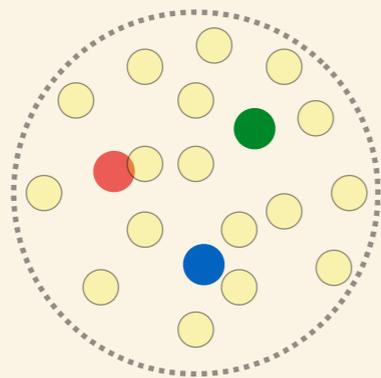
$$|0.24| \leq \frac{1}{24} [5(-0.5) - 6(-1.4)] = 0.24$$

Soffer-like bound approximately saturated

# Gluon Radii

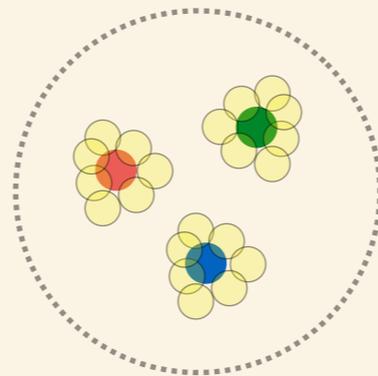
How does the gluon radius of a proton compare to the quark/charge radius?

Bag Model



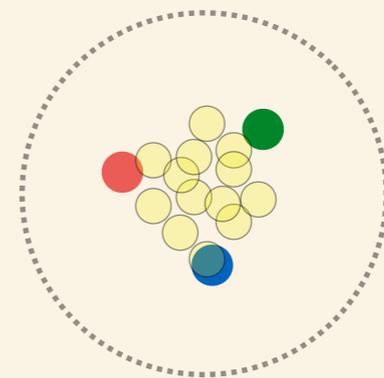
gluon radius  $>$  charge radius

Constituent Quark Model



gluon radius  $\sim$  charge radius

LQCD with heavy quarks



gluon radius  $<$  charge radius

Or is the picture more complicated?

# Gluon Generalised FFs

## Matrix elements of the spin-independent gluon structure function

- Off-forward matrix elements are complicated:

$$\begin{aligned}
 & \langle p' E' | S [G_{\mu\alpha} i \overleftrightarrow{D}_{\mu_1} \dots i \overleftrightarrow{D}_{\mu_n} G_{\nu}^{\alpha}] | p E \rangle \\
 &= \sum_{\substack{m \text{ even} \\ m=0}}^n \left\{ \begin{aligned}
 & B_{1,m}^{(n+2)}(\Delta^2) M^2 S [E_{\mu} E'_{\nu}^* \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \\
 & + B_{2,m}^{(n+2)}(\Delta^2) S [(E \cdot E'^*) P_{\mu} P_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \\
 & + B_{3,m}^{(n+2)}(\Delta^2) S [(E \cdot E'^*) \Delta_{\mu} \Delta_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \\
 & + B_{4,m}^{(n+2)}(\Delta^2) S [((E'^* \cdot P) E_{\mu} P_{\nu} + (E \cdot P) E'_{\mu}^* P_{\nu}) \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \\
 & + B_{5,m}^{(n+2)}(\Delta^2) S [((E'^* \cdot P) E_{\mu} \Delta_{\nu} - (E \cdot P) E'_{\mu}^* \Delta_{\nu}) \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \\
 & + \frac{B_{6,m}^{(n+2)}(\Delta^2)}{M^2} S [(E \cdot P)(E'^* \cdot P) P_{\mu} P_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \\
 & + \frac{B_{7,m}^{(n+2)}(\Delta^2)}{M^2} S [(E \cdot P)(E'^* \cdot P) \Delta_{\mu} \Delta_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n}] \end{aligned} \right\}.
 \end{aligned}$$

# Gluon Generalised FFs

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 & + B_{2,m}^{(n+2)}(\Delta^2) S [ (E \cdot E') P_{\mu} P_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n} ] \\
 & + B_{3,m}^{(n+2)}(\Delta^2) S [ (E \cdot P) P_{\mu} P_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n} ] \\
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 & + \frac{B_{7,m}^{(n+2)}(\Delta^2)}{M^2} S [ (E \cdot P)(E' \cdot P) \Delta_{\mu} \Delta_{\nu} \Delta_{\mu_1} \dots \Delta_{\mu_m} P_{\mu_{m+1}} \dots P_{\mu_n} ] \end{aligned} \right\}.
 \end{aligned}$$

Many gluonic radii:  
Defined by slope of each  
form factor at  $Q^2=t=0$

# Gluon Generalised FFs

- Complicated over and under-determined systems of equations (different choices of polarisation and boost at same momentum transfer)
- Some GFFs suppressed by orders of magnitude
- Some GFFs related by symmetries at some momenta

**Simplest example:**  
 Transversity GFFs  
 One basis (2 vectors)  
 Mtm 1 (lattice units)

$$\begin{pmatrix}
 0.604 & 0.0424 & 0 & 0 & 0 & 0 & 0.0588 & 0 \\
 0.592 & -2.45 \times 10^{-3} & 0.0785 & -0.0785 & 6.58 \times 10^{-3} & -0.0992 & -0.103 & -4.15 \times 10^{-3} \\
 0.485 & 0.0429 & 0 & 0 & 0 & 0 & 0.0379 & 0 \\
 0.481 & 0.0431 & -3.02 \times 10^{-5} & 3.02 \times 10^{-5} & -2.53 \times 10^{-6} & -4.03 \times 10^{-7} & 0.0374 & -1.69 \times 10^{-8} \\
 0.475 & -3.29 \times 10^{-3} & 0.0791 & -0.0791 & 6.59 \times 10^{-3} & -0.0791 & -0.0824 & -3.29 \times 10^{-3} \\
 0.353 & -7.97 \times 10^{-4} & 0.0385 & -0.0385 & 3.28 \times 10^{-3} & -0.0598 & -0.0631 & -2.54 \times 10^{-3} \\
 0.347 & -0.0382 & 0 & 0 & 0 & 0 & 0.0962 & 0 \\
 0.258 & 0.0806 & 0 & 0 & 0 & 0 & -0.0374 & 0 \\
 0.258 & 0.0808 & 0 & 0 & 0 & 0 & -0.0379 & 0 \\
 0.253 & 0.101 & -8.60 \times 10^{-4} & 8.60 \times 10^{-4} & -7.20 \times 10^{-5} & 6.32 \times 10^{-7} & -0.0588 & 2.65 \times 10^{-8} \\
 0.239 & -1.66 \times 10^{-3} & 0.0401 & -0.0401 & 3.29 \times 10^{-3} & -0.0393 & -0.0402 & -1.61 \times 10^{-3} \\
 0.238 & -1.65 \times 10^{-3} & 0.0396 & -0.0396 & 3.29 \times 10^{-3} & -0.0396 & -0.0412 & -1.65 \times 10^{-3} \\
 0.228 & -0.0581 & 8.30 \times 10^{-4} & -8.30 \times 10^{-4} & 6.94 \times 10^{-5} & -1.04 \times 10^{-6} & 0.0962 & -4.33 \times 10^{-8} \\
 0.228 & -0.0379 & 0 & 0 & 0 & 0 & 0.0758 & 0 \\
 0.0590 & -0.0109 & 0.139 & -0.139 & 0.0112 & -4.97 \times 10^{-3} & -3.94 \times 10^{-4} & -8.24 \times 10^{-6} \\
 0.0578 & -2.56 \times 10^{-4} & 9.42 \times 10^{-3} & -9.42 \times 10^{-3} & 3.89 \times 10^{-4} & -4.65 \times 10^{-3} & 2.51 \times 10^{-4} & 5.25 \times 10^{-6} \\
 0.0338 & 1.59 \times 10^{-3} & -0.128 & 0.128 & -0.0107 & 3.18 \times 10^{-4} & 0.0154 & 1.33 \times 10^{-5} \\
 0.0183 & 6.36 \times 10^{-3} & -1.29 \times 10^{-4} & 1.29 \times 10^{-4} & 3.84 \times 10^{-4} & 4.84 \times 10^{-3} & 5.99 \times 10^{-3} & 5.18 \times 10^{-6} \\
 0.0155 & -4.78 \times 10^{-3} & -0.128 & 0.128 & -0.0111 & -4.52 \times 10^{-3} & 9.41 \times 10^{-3} & 8.14 \times 10^{-6} \\
 1.19 \times 10^{-3} & -0.0106 & 0.129 & -0.129 & 0.0108 & -3.22 \times 10^{-4} & -6.45 \times 10^{-4} & -1.35 \times 10^{-5} \\
 0.549 & 2.44 \times 10^{-3} & 0 & 0 & 0 & 0 & 0.0895 & 0 \\
 0.546 & -1.88 \times 10^{-3} & 0.0676 & -0.0676 & 5.69 \times 10^{-3} & -0.0918 & -0.0960 & -3.86 \times 10^{-3} \\
 0.498 & 0.0710 & 0 & 0 & 0 & 0 & 0.0123 & 0 \\
 0.480 & -2.37 \times 10^{-3} & 0.0685 & -0.0685 & 5.70 \times 10^{-3} & -0.0799 & -0.0828 & -3.33 \times 10^{-3} \\
 0.429 & 0.0714 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0.424 & 0.0834 & -5.14 \times 10^{-4} & 5.14 \times 10^{-4} & -4.30 \times 10^{-5} & 1.33 \times 10^{-7} & -0.0123 & 5.55 \times 10^{-9} \\
 0.412 & 2.85 \times 10^{-3} & 0 & 0 & 0 & 0 & 0.0657 & 0 \\
 0.412 & -2.85 \times 10^{-3} & 0.0685 & -0.0685 & 5.70 \times 10^{-3} & -0.0685 & -0.0714 & -2.85 \times 10^{-3} \\
 0.409 & -8.65 \times 10^{-3} & 4.61 \times 10^{-4} & -4.61 \times 10^{-4} & 3.86 \times 10^{-5} & -8.30 \times 10^{-7} & 0.0771 & -3.47 \times 10^{-8} \\
 0.0674 & -6.43 \times 10^{-3} & 0.0856 & -0.0856 & 6.70 \times 10^{-3} & -5.55 \times 10^{-3} & -8.26 \times 10^{-5} & -1.73 \times 10^{-6} \\
 0.0656 & 4.96 \times 10^{-4} & -9.21 \times 10^{-4} & 9.21 \times 10^{-4} & -6.37 \times 10^{-6} & -0.0119 & -0.0132 & -5.32 \times 10^{-4} \\
 0.0514 & -0.0685 & 0 & 0 & 0 & 0 & 0.0771 & 0 \\
 0.0347 & -0.0124 & 0.155 & -0.155 & 0.0127 & -3.05 \times 10^{-3} & -6.00 \times 10^{-4} & -1.26 \times 10^{-5} \\
 0.0327 & 5.99 \times 10^{-3} & -0.0692 & 0.0692 & -6.03 \times 10^{-3} & -2.50 \times 10^{-3} & 5.17 \times 10^{-4} & 1.08 \times 10^{-5} \\
 0.0301 & 4.59 \times 10^{-3} & -0.0738 & 0.0738 & -5.95 \times 10^{-3} & 2.98 \times 10^{-3} & 0.0123 & 1.07 \times 10^{-5} \\
 0.0285 & -1.84 \times 10^{-3} & -0.147 & 0.147 & -0.0126 & -2.43 \times 10^{-3} & 0.0143 & 1.24 \times 10^{-5} \\
 0.0171 & 0.0685 & 0 & 0 & 0 & 0 & -0.0657 & 0 \\
 0.0146 & 0.0920 & -9.75 \times 10^{-4} & 9.75 \times 10^{-4} & -8.17 \times 10^{-5} & 9.63 \times 10^{-7} & -0.0895 & 4.03 \times 10^{-8} \\
 1.59 \times 10^{-3} & 6.43 \times 10^{-3} & 0.0736 & -0.0736 & 6.61 \times 10^{-3} & 5.40 \times 10^{-3} & -1.97 \times 10^{-3} & -1.71 \times 10^{-6}
 \end{pmatrix}
 \begin{pmatrix}
 A_{1,0}^{(2)}(1) \\
 A_{2,0}^{(2)}(1) \\
 A_{3,0}^{(2)}(1) \\
 A_{4,0}^{(2)}(1) \\
 A_{5,0}^{(2)}(1) \\
 A_{6,0}^{(2)}(1) \\
 A_{7,0}^{(2)}(1) \\
 A_{8,0}^{(2)}(1)
 \end{pmatrix}
 =
 \begin{pmatrix}
 0.179(36) \\
 0.150(38) \\
 0.152(30) \\
 0.154(37) \\
 0.129(32) \\
 0.056(31) \\
 0.067(41) \\
 0.056(35) \\
 0.069(21) \\
 0.093(36) \\
 0.028(32) \\
 0.041(27) \\
 0.012(33) \\
 0.029(30) \\
 0.024(11) \\
 -0.005(21) \\
 -0.0056(96) \\
 -0.002(11) \\
 0.009(16) \\
 0.0162(91) \\
 0.086(26) \\
 0.131(31) \\
 0.155(33) \\
 0.086(33) \\
 0.098(16) \\
 0.094(17) \\
 0.088(27) \\
 0.114(25) \\
 0.075(27) \\
 0.034(25) \\
 -0.006(22) \\
 -0.001(31) \\
 0.022(11) \\
 0.014(16) \\
 0.0010(16) \\
 0.0008(85) \\
 0.018(23) \\
 0.001(29) \\
 0.005(18)
 \end{pmatrix}$$

# Gluon Generalised FFs

- Complicated over and under-determined systems of equations (different choices of polarisation and boost at same momentum transfer)
- Some GFFs suppressed by orders of magnitude
- Some GFFs related by symmetries at some momenta

**Simplest example:**  
 Transversity GFFs  
 One basis (2 vectors)  
 Mtm 1 (lattice units)

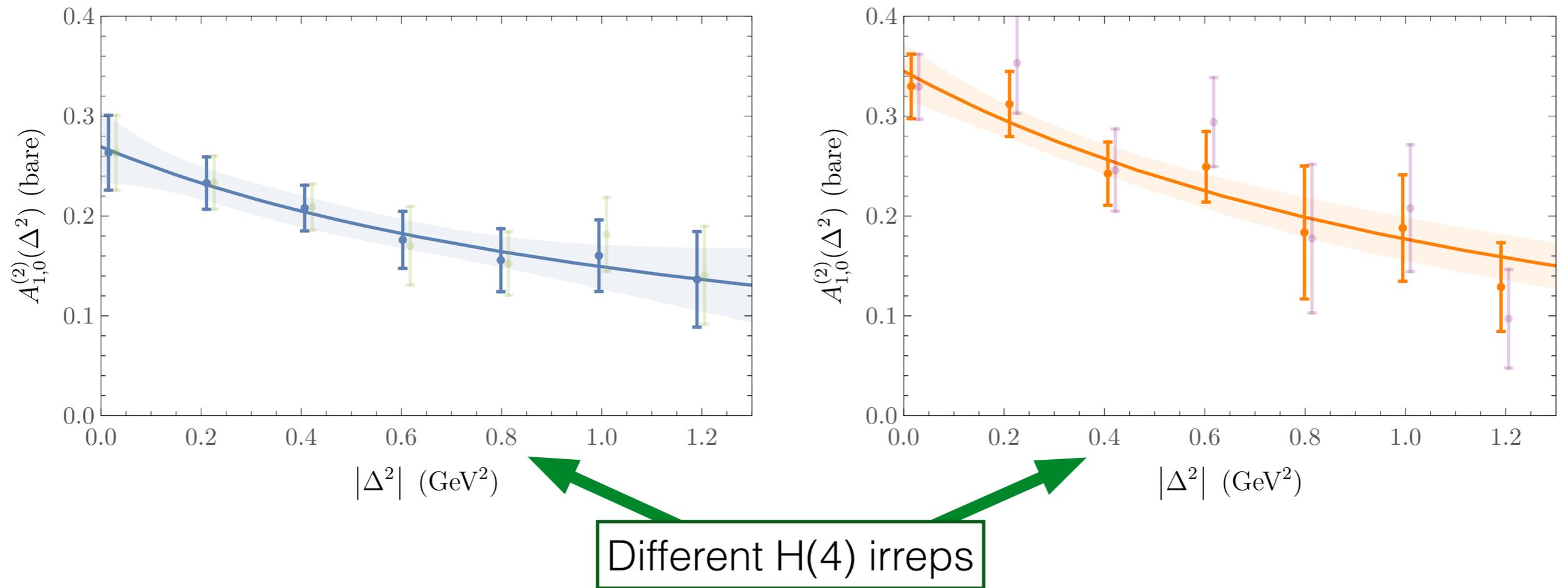
0.604	0.0424	0	0	0	0	0.0588	0		
0.592	$-2.45 \times 10^{-3}$	0.0785	-0.0785	$6.58 \times 10^{-3}$	-0.0992	-0.103	$-4.15 \times 10^{-3}$		
0.485	0.0429	0	0	0	0	0.0379	0		
0.481	0.0431	$-3.02 \times 10^{-5}$	$3.02 \times 10^{-5}$	$-2.53 \times 10^{-6}$	$-4.03 \times 10^{-7}$	0.0374	$-1.69 \times 10^{-8}$		0.179(36)
0.475	$-3.29 \times 10^{-3}$	0.0791	-0.0791	$6.59 \times 10^{-3}$	-0.0791	-0.0824	$-3.29 \times 10^{-3}$		0.150(38)
0.353	$-7.97 \times 10^{-4}$	0.0385	-0.0385	$3.28 \times 10^{-3}$	-0.0598	-0.0631	$-2.54 \times 10^{-3}$		0.152(30)
0.347	-0.0382	0	0	0	0	0.0962	0		0.154(37)
0.258	0.0806	0	0	0	0	-0.0374	0		0.129(32)
0.258	0.0808	0	0	0	0	-0.0379	0		0.056(31)
0.253	0.101	$-8.60 \times 10^{-4}$	$8.60 \times 10^{-4}$	$-7.20 \times 10^{-5}$	$6.32 \times 10^{-7}$	-0.0588	$2.65 \times 10^{-8}$		0.067(41)
0.239	$-1.66 \times 10^{-3}$	0.0401	-0.0401	$3.29 \times 10^{-3}$	-0.0393	-0.0402	$-1.61 \times 10^{-3}$		0.056(35)
0.238	$-1.65 \times 10^{-3}$	0.0396	-0.0396	$3.29 \times 10^{-3}$	-0.0396	-0.0412	$-1.65 \times 10^{-3}$		0.069(21)
0.228	-0.0581	$8.30 \times 10^{-4}$	$-8.30 \times 10^{-4}$	$6.94 \times 10^{-5}$	$-1.04 \times 10^{-6}$	0.0962	$-4.33 \times 10^{-8}$		0.093(36)
0.228	-0.0379	0	0	0	0	0.0758	0		0.028(32)
0.0590	-0.0109	0.139	0.139	0.0112	$4.07 \times 10^{-3}$	$2.04 \times 10^{-4}$	$8.24 \times 10^{-6}$	$(1^{(2)}(1))$	0.041(27)
0.0578	$-2.56 \times 10^{-4}$	$4.2 \times 10^{-3}$							0.012(33)
0.0338	$1.59 \times 10^{-3}$								0.088(27)
0.0183	6.36								0.114(25)
0.0155	$-4.78 \times 10^{-3}$	0.128							0.075(27)
$1.19 \times 10^{-3}$	-0.0106	0.129							0.034(25)
0.549	$2.44 \times 10^{-3}$	0	0	0	0	0.0895	0	$A_{5,0}^{(2)}(1)$	0.086(26)
0.546	$-1.88 \times 10^{-3}$	0.0676	-0.0676	$5.69 \times 10^{-3}$	-0.0918	-0.0960	$-3.86 \times 10^{-3}$	$A_{6,0}^{(2)}(1)$	0.131(31)
0.498	0.0710	0	0	0	0	0.0123	0	$A_{7,0}^{(2)}(1)$	0.155(33)
0.480	$-2.37 \times 10^{-3}$	0.0685	-0.0685	$5.70 \times 10^{-3}$	-0.0799	-0.0828	$-3.33 \times 10^{-3}$	$A_{8,0}^{(2)}(1)$	0.086(33)
0.429	0.0714	0	0	0	0	0	0		0.098(16)
0.424	0.0834	$-5.14 \times 10^{-4}$	$5.14 \times 10^{-4}$	$-4.30 \times 10^{-5}$	$1.33 \times 10^{-7}$	-0.0123	$5.55 \times 10^{-9}$		0.094(17)
0.412	$2.85 \times 10^{-3}$	0	0	0	0	0.0657	0		0.088(27)
0.412	$-2.85 \times 10^{-3}$	0.0685	-0.0685	$5.70 \times 10^{-3}$	-0.0685	-0.0714	$-2.85 \times 10^{-3}$		0.114(25)
0.409	$-8.65 \times 10^{-3}$	$4.61 \times 10^{-4}$	$-4.61 \times 10^{-4}$	$3.86 \times 10^{-5}$	$-8.30 \times 10^{-7}$	0.0771	$-3.47 \times 10^{-8}$		0.075(27)
0.0674	$-6.43 \times 10^{-3}$	0.0856	-0.0856	$6.70 \times 10^{-3}$	$-5.55 \times 10^{-3}$	$-8.26 \times 10^{-5}$	$-1.73 \times 10^{-6}$		0.034(25)
0.0656	$4.96 \times 10^{-4}$	$-9.21 \times 10^{-4}$	$9.21 \times 10^{-4}$	$-6.37 \times 10^{-6}$	-0.0119	-0.0132	$-5.32 \times 10^{-4}$		-0.006(22)
0.0514	-0.0685	0	0	0	0	0.0771	0		-0.001(31)
0.0347	-0.0124	0.155	-0.155	0.0127	$-3.05 \times 10^{-3}$	$-6.00 \times 10^{-4}$	$-1.26 \times 10^{-5}$		0.022(11)
0.0327	$5.99 \times 10^{-3}$	-0.0692	0.0692	$-6.03 \times 10^{-3}$	$-2.50 \times 10^{-3}$	$5.17 \times 10^{-4}$	$1.08 \times 10^{-5}$		0.014(16)
0.0301	$4.59 \times 10^{-3}$	-0.0738	0.0738	$-5.95 \times 10^{-3}$	$2.98 \times 10^{-3}$	0.0123	$1.07 \times 10^{-5}$		0.0010(16)
0.0285	$-1.84 \times 10^{-3}$	-0.147	0.147	-0.0126	$-2.43 \times 10^{-3}$	0.0143	$1.24 \times 10^{-5}$		0.0008(85)
0.0171	0.0685	0	0	0	0	-0.0657	0		0.018(23)
0.0146	0.0920	$-9.75 \times 10^{-4}$	$9.75 \times 10^{-4}$	$-8.17 \times 10^{-5}$	$9.63 \times 10^{-7}$	-0.0895	$4.03 \times 10^{-8}$		0.001(29)
$1.59 \times 10^{-3}$	$6.43 \times 10^{-3}$	0.0736	-0.0736	$6.61 \times 10^{-3}$	$5.40 \times 10^{-3}$	$-1.97 \times 10^{-3}$	$-1.71 \times 10^{-6}$		0.005(18)

Target a subset of "dominant GFFs"

# Gluon Transversity GFFs

W. Detmold, PES, PRD 94 (2016), 014507 + W. Detmold, D. Pefkou, PES arXiv:1703.08220

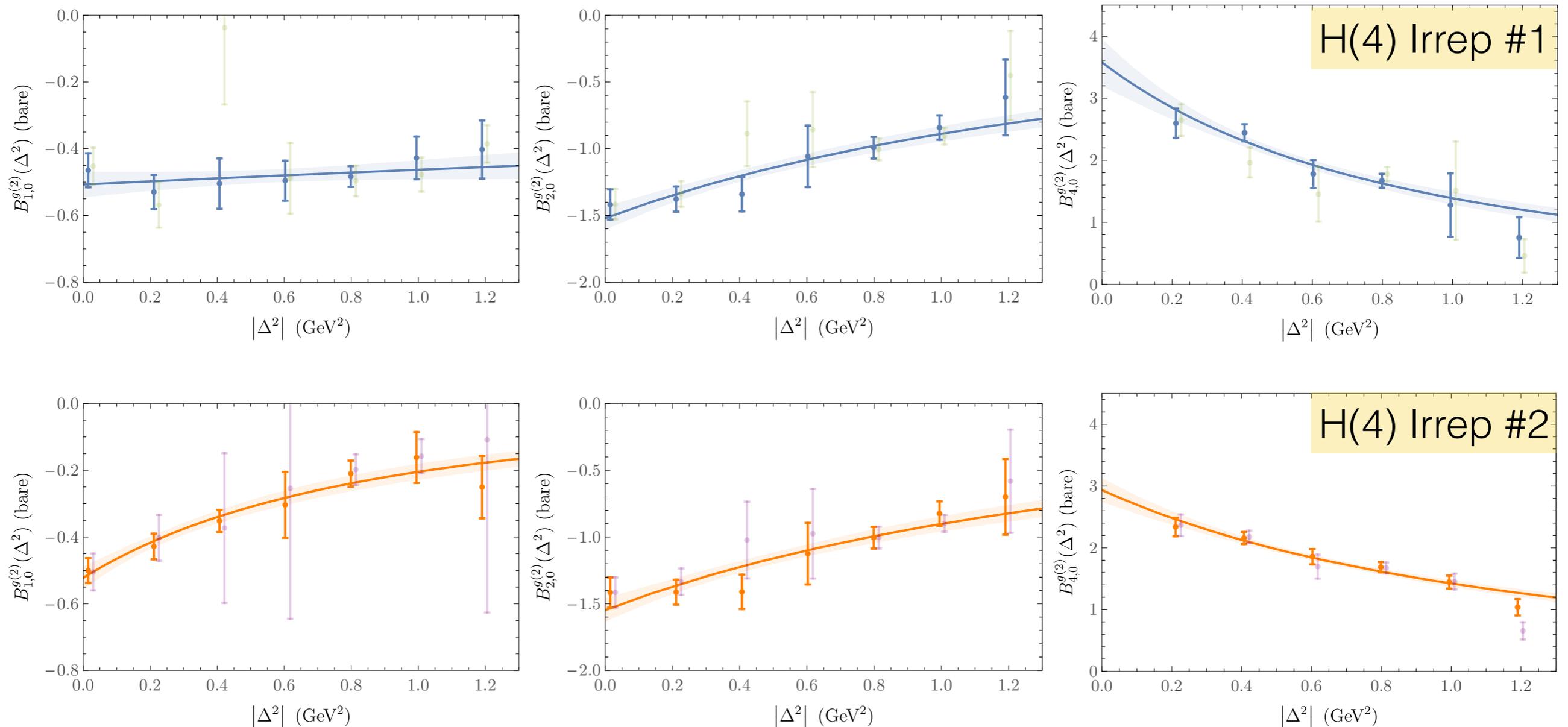
One GFF can be resolved for all momenta



# Spin-Indep. Gluon GFFs

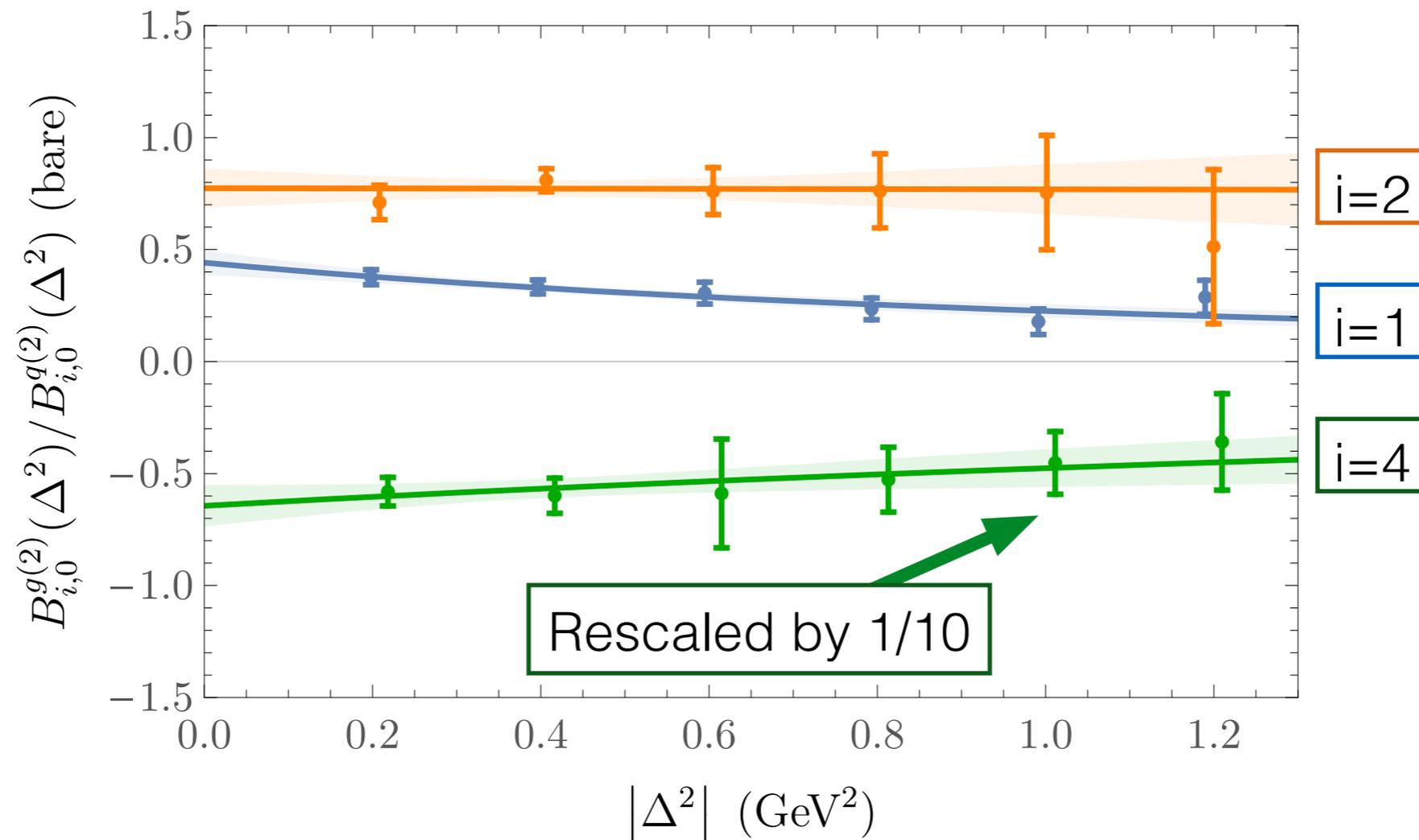
W. Detmold, PES, PRD 94 (2016), 014507 + W. Detmold, D. Pefkou, PES arXiv:1703.08220

Three GFFs can be resolved for all momenta



# Quark and Gluon GFFs

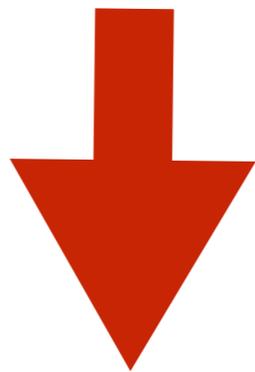
Ratio of gluon to quark unpolarised GFFs



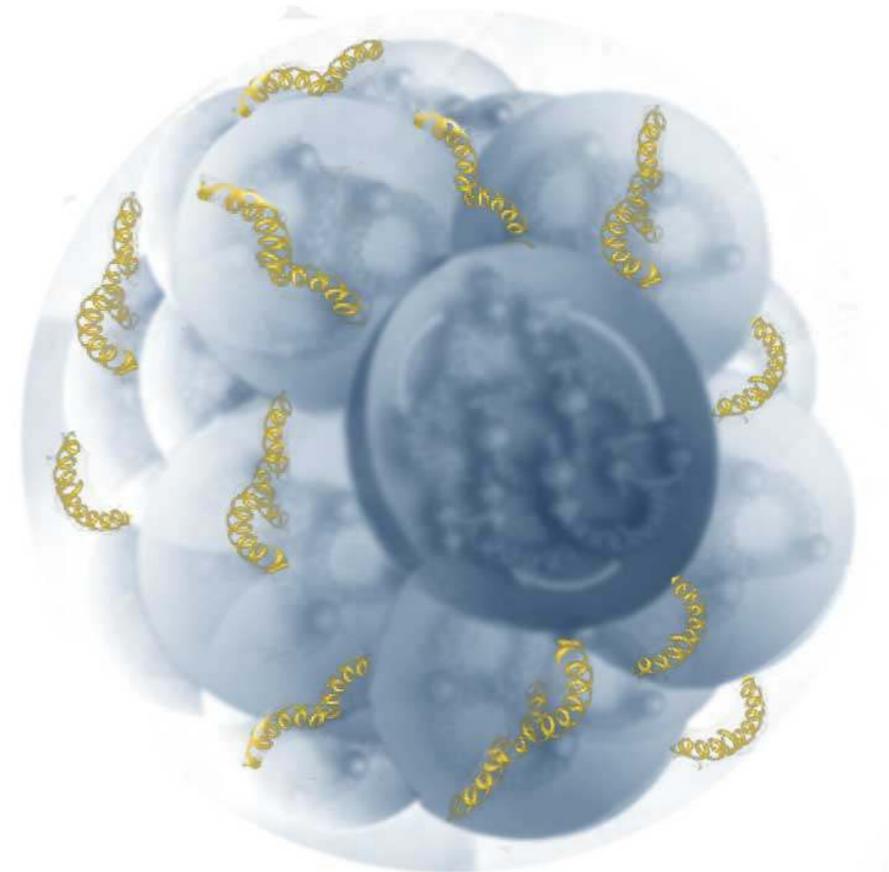
Gluon vs quark radius is a non-trivial question  
Much more complicated than intuitive pictures

# Glue structure of nuclei

- **First investigations:**  
 $\phi$  meson  
simplest spin-1 system (has fwd limit gluon transversity)



- **Phenomenologically relevant:**  
nucleon, nuclei



# Gluon structure - nuclei

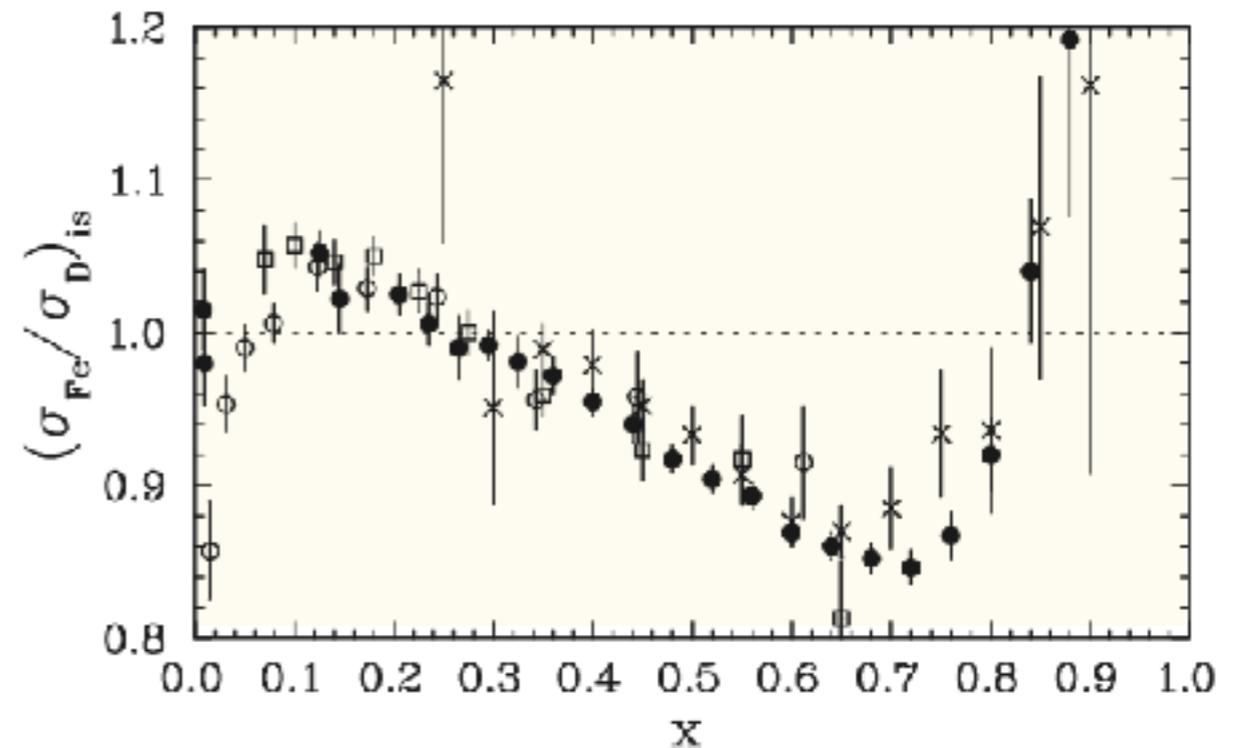
## European Muon Collaboration (1983):

Modification of per-nucleon cross section of nucleons bound in nuclei

Precise understanding of nuclear targets essential for DUNE expt: extraction of neutrino mass hierarchy, mixing parameters

Ratio of structure function  $F_2$  per nucleon for iron and deuterium

$$F_2(x, Q^2) = \sum_{q=u,d,s..} xz_q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]$$

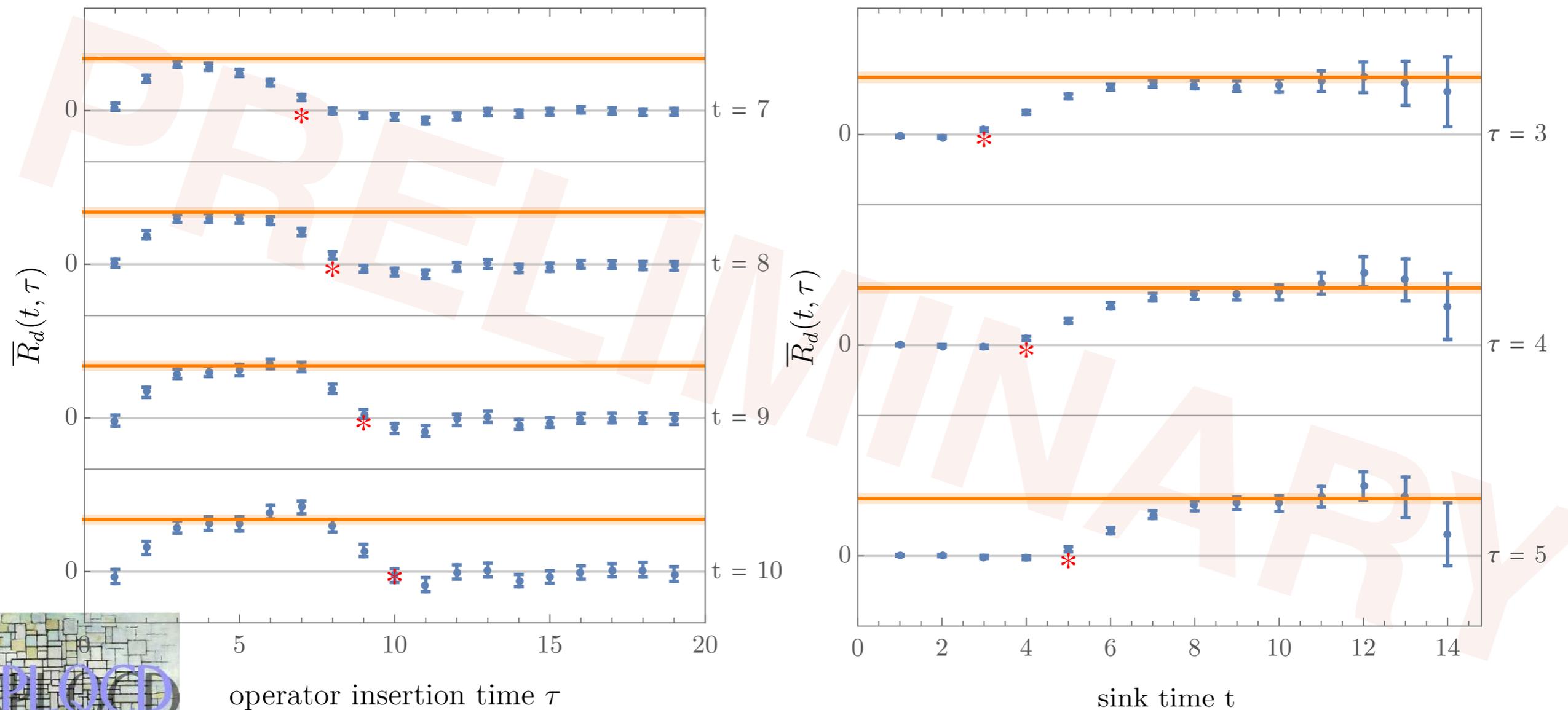


What is the gluonic analogue of the EMC effect?

# Nuclear glue, $m_\pi \sim 450$ MeV

NPLQCD Collaboration, in preparation

Signals for spin-independent gluon operator in deuteron



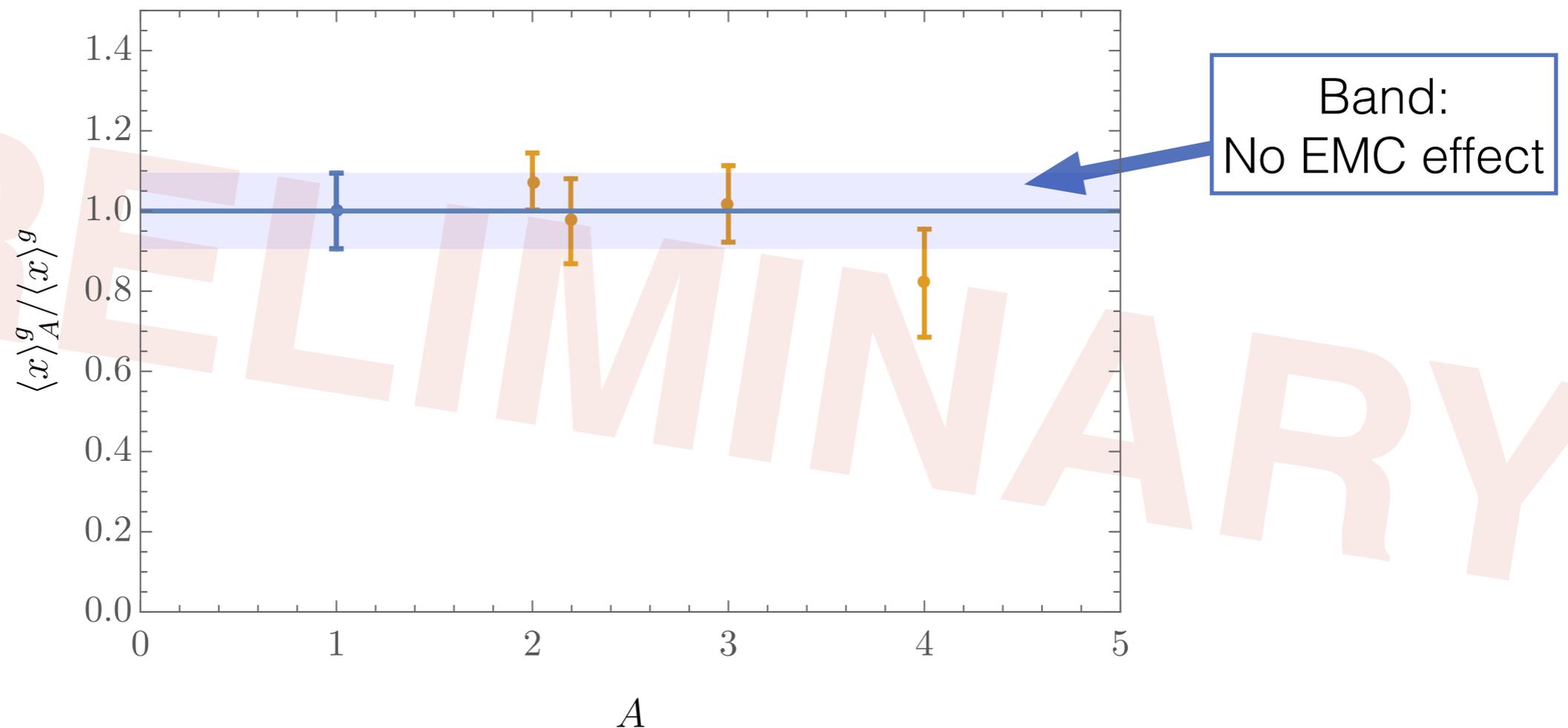
operator insertion time  $\tau$

sink time  $t$

# Gluon EMC Effect at $m_\pi \sim 450$ MeV

NPLQCD Collaboration, in preparation

- Spin-independent structure function in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators



# Gluon structure circa 2025

- Electron-Ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
  - Work towards a complete 3D picture of parton structure (moments,  $x$ -dependence of PDFs, GPDs, TMDs)
  - $\Delta(x, Q^2)$  has an interesting role
    - Purely gluonic
    - Non-nucleonic: directly probe nuclear effects
  - Compare quark and gluon distributions in hadrons and nuclei
- Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature

