Restoring canonical partition functions from imaginary chemical potential

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Our collaboration.

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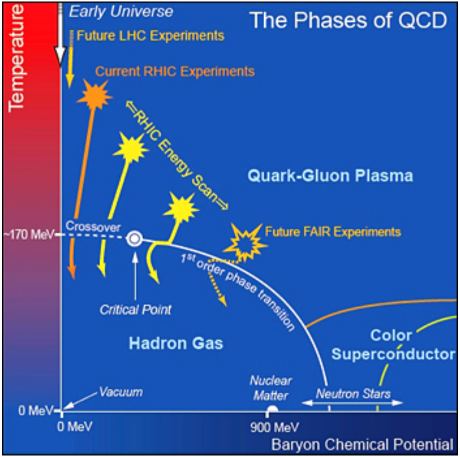
The phases of QCD.



Experiments:

- RHIC
- LHC
- J-PARC
- FAIR
- NICA

Ab initio theoretical calculations are required.

$\Rightarrow$ *LATTICE*

LQCD with non-zero chemical potential

In LQCD we have the following relation for the Dirac operator:

$$\det [M(\mu_B)]^* = \det [M(-\mu_B^*)]$$

μ_B is real $\rightarrow$ $\det [M(\mu_B)]$ is complex $\rightarrow$ no importance sampling

Techniques, which are now being used in LQCD at $\mu_B \neq 0$:

- Taylor expansion;
- analytic continuation;
- reweighting;
- complex Langevin;
- density of states;
- ...

Canonical approach

Suppose that we have conserved charge: $\left[\hat{H},\ \hat{Q}\right] = 0$. In this case

$$\mathbb{Z}_{GC}\left(T,\ \mu_q\right) = \mathrm{Tr}\, e^{-\left(\hat{H}-\mu_q \hat{N}\right)/T} = \sum_{n=-\infty}^{\infty} \langle n|\, e^{-\hat{H}/T}\, |n\rangle\ e^{n\mu_q/T}$$

$$\equiv \sum_{n=-\infty}^{\infty} \mathbb{Z}_C\left(n,\ T\right) e^{n\mu_q/T}, \quad \text{where} \quad e^{\mu_q/T} - \text{is fugacity.}$$

- $\mathbb{Z}_C\left(n,\ T\right)$ (or Z_n) are calculable in LQCD up to some norm. constant
- once Z_{GC} is known, baryon density, susceptibilities may be studied
- another conserved charges may be added in the same way

How we can calculate Z_n?

For pure imaginary $\mu_q = \imath\mu_I$:

$$\mathbb{Z}_{GC}\left(T, \imath\mu_I\right) = \sum_{n=-\infty}^{\infty} Z_n(T)\, e^{\imath n\mu_I/T}$$

Z_n must be the same for any μ_q

Inverse transformation:

$$Z_n(T) = \frac{1}{2\pi}\int_0^{2\pi} d\left(\frac{\mu_I}{T}\right) \mathbb{Z}_{GC}\left(T, \imath\mu_I\right)\, e^{-\imath n\mu_I/T}$$

$\leftarrow$ We need to calculate $\mathbb{Z}_{GC}$

Possible ways:

- calculation of the $\mathbb{Z}_{GC}$ with reweighting;
- integration of the n_B (new method, Denis talk).

Firstly reweighting method $\rightarrow$

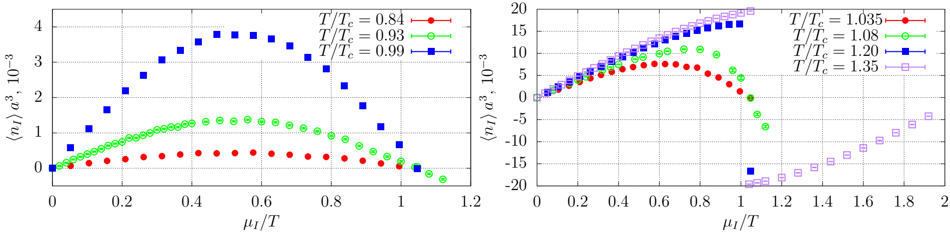
$$\mathbb{Z}_{GC}\left(\imath\mu_I\right) = \left\langle \left[\frac{\det\Delta\left(\imath\mu_I\right)}{\det\Delta\left(\mu_0\right)}\right]^{N_f} \right\rangle_{\mu_0} \mathbb{Z}_{GC}\left(\mu_0\right)$$

Integration method.

Definition of baryon density is $\frac{n_q}{T^3} = \frac{1}{VT^2}\frac{\partial}{\partial \mu_q} \ln \mathbb{Z}_{GC}$

Baryon density for imaginary μ can be computed on the lattice

$$\frac{n_q}{T^3} = \frac{N_f N_t^3}{N_s^3 \mathbb{Z}_{GC}} \int \mathcal{D}U e^{-S_G} (\det \Delta(\mu_q))^{N_f} \mathrm{Tr} \left[\Delta^{-1} \frac{\partial \Delta}{\partial \mu_q / T}\right]$$

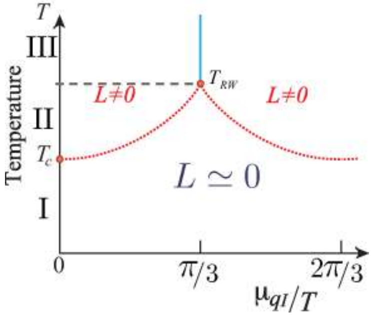


$\int n_I(\theta) d\theta$

$(\theta = \mu_I/T)$

- Numerical integration (Trapezoidal, Simpson's rules),
- Analytical integration of splines (line, cubic),
- Using anzats (sum of sin's, polynom, ...).

[A. Roberge and N. Weiss,
Nucl. Phys. B275, 734 (1986)]



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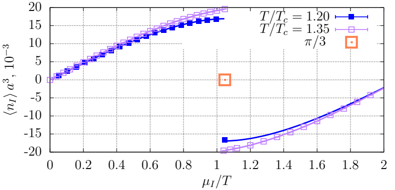

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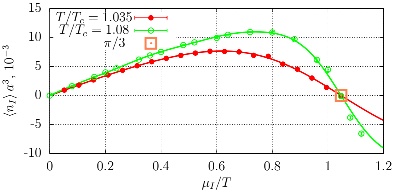
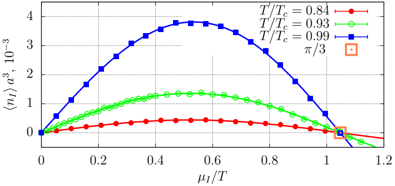


I:

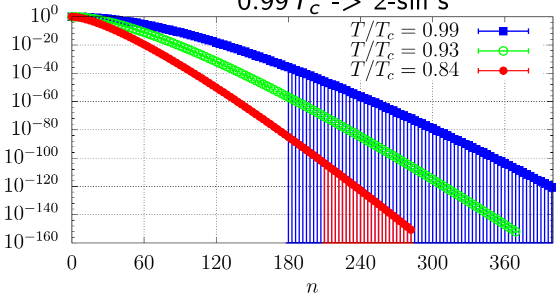


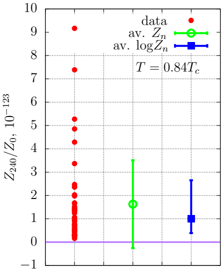
III. Polynomial dependence,
II. Sum of sin's,
I. Sine dependence.

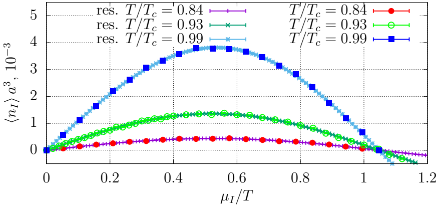
First regime: $T < T_c$.

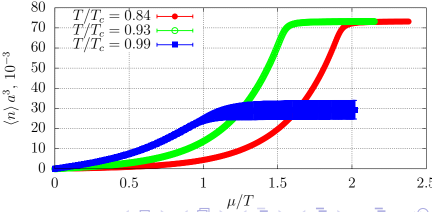
We use sin's anzats:
$0.84\,T_c$ -> 1-sin
$0.93\,T_c$ -> 1-sin
$0.99\,T_c$ -> 2-sin's

HRG



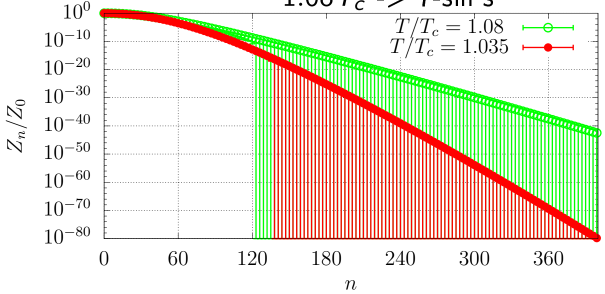


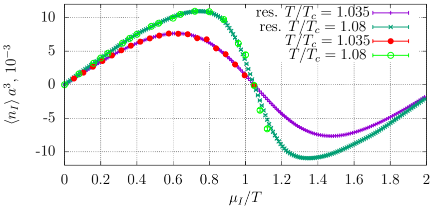


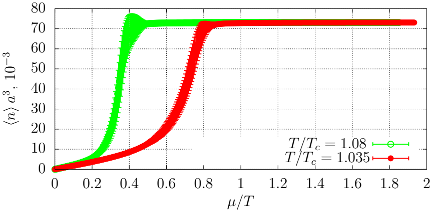


Second regime: $T_c < T < T_{RW}$.

We use sin's anzats: $1.035\,T_c$ -> 3-sin's
$1.08\,T_c$ -> 7-sin's

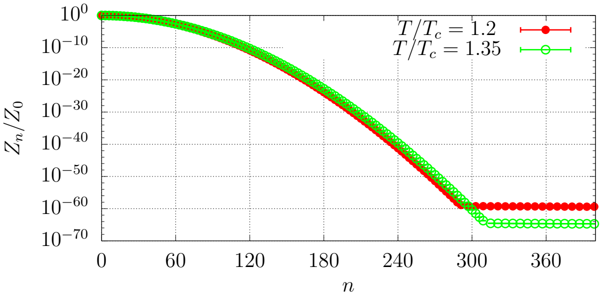




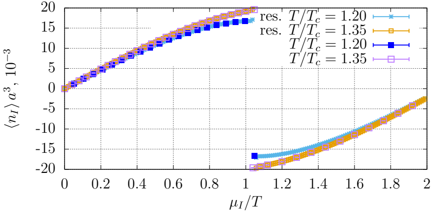


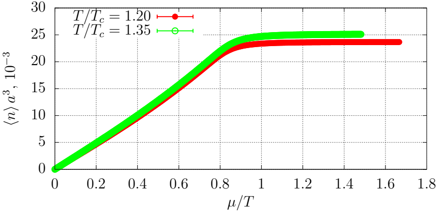
Third regime: $T > T_{RW}$.

We use polynomial anzats: $1.2T_c$, $1.35T_c$ -> $f(x) = ax + bx^3$

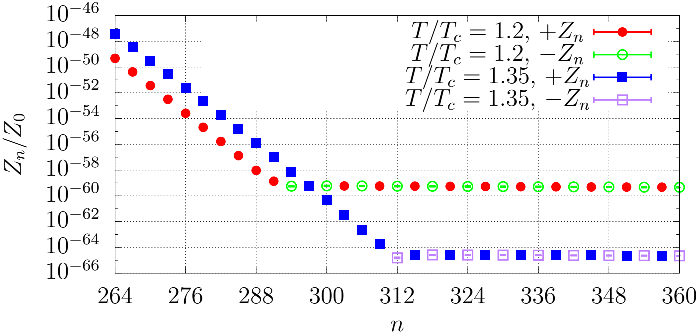


Please ZOOM it!





Zoom of Z_n.



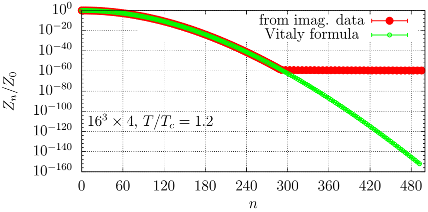
Negative Z_n??? What happened?

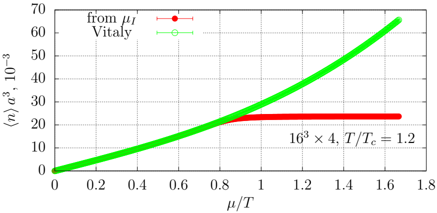
$$-\log Z_n/Z_0 = -\frac{16a^2}{b}\left[\left(-x+\frac{1}{3}\sqrt{1+x^2}\right)\left(\sqrt{1+x^2}+x\right)^{1/3}+\left(x+\frac{1}{3}\sqrt{1+x^2}\right)\left(\sqrt{1+x^2}-x\right)^{1/3}-\frac{2}{3}\right]$$

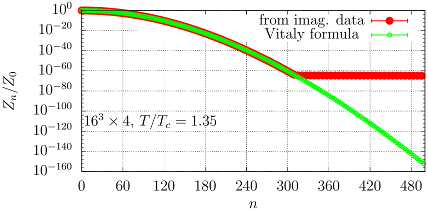
$$x = \frac{\sqrt{27b}}{128a^{3/2}}n$$

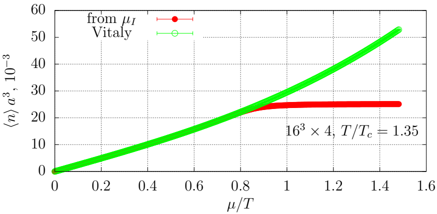
[V. Bornyakov]

Comparison with Vitaly formula.







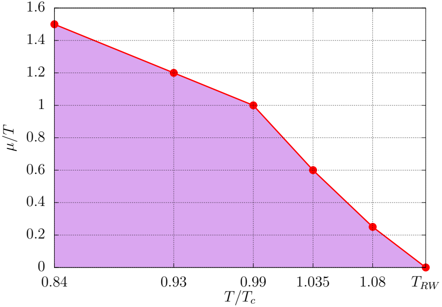


BUT – without negative Z_n we can not restore RW phase transition,

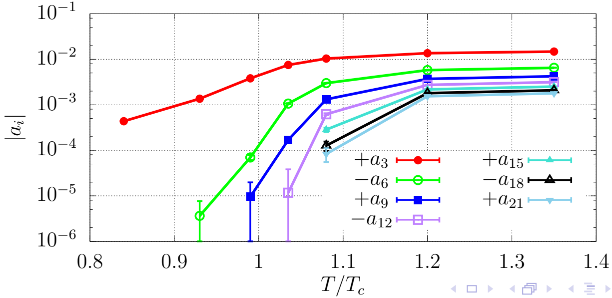
GOOD – positive Z_n is enough for restore $\langle n \rangle$ until $\mu_B/T < 2.5$.

Limitation of sin's anzats.

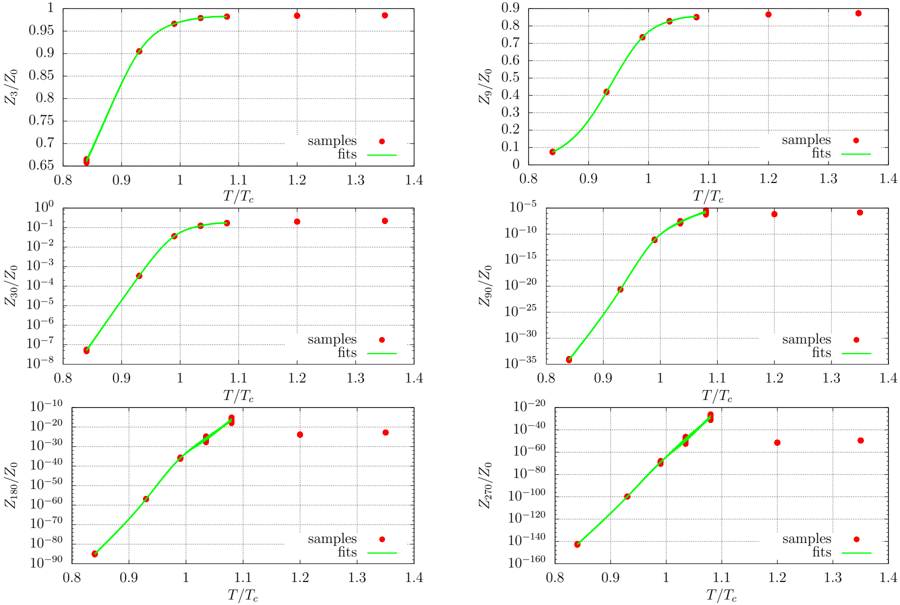
1. We must use only positive Z_n.
2. Using sin's anzats produce addition limitation in real μ.
$\sum_n a_n \sin(3n\mu_I/T) \to \sum_n a_n \sinh(3n\mu/T) \xrightarrow[3n\mu/T \gg 1]{} a_{n_{max}} e^{3n_{max}\mu/T}$



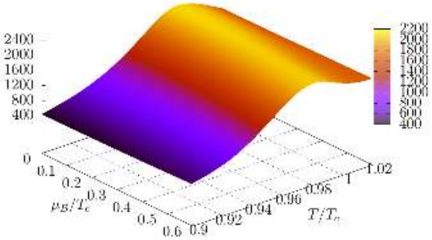
[qualitative fig.]



T dependence in Z_n.



Search crossover line.

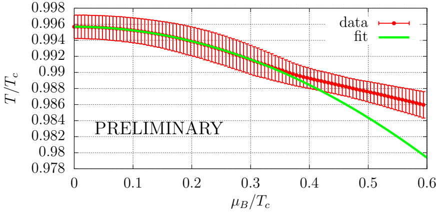


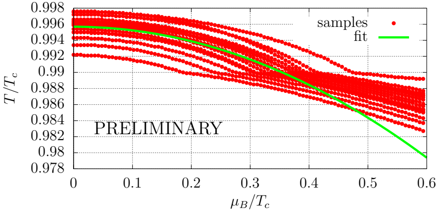
- $\frac{\chi}{T^2} = \frac{\partial^2}{\partial \mu^2} \log \mathbb{Z}_{GC}$ from Z_n,
- $\frac{\partial \chi}{\partial T}$ – numerically,
- Find position of maximal of $\frac{\partial \chi}{\partial T}$.

C. line: $T/T_c = a + b(\mu_B/T_c)^2$.

$$a = 0.9956 \pm 0.0015,$$
$$b = -0.0453 \pm 0.0099.$$

[Only 16 samples]





$\Longrightarrow$ We need to use more natural $Z_n(T/T_c)$ dependence.

Thank you for attention!