Friedrich-Schiller-Universität Jena

Spectroscopy of four-dimensional $\mathcal{N}=1$ supersymmetric $SU(3)$ Yang-Mills theory

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Research Training Group
Quantum and Gravitational Fields

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Motivation

- Extended standard model with supersymmetry
- So far no experimental verification
- Important building block of Super-QCD:
  $\mathcal{N}=1$ Super-Yang-Mills theory (SYM) with gauge group $SU(3)$ in 4 spacetime dimensions
- Investigations of bound states and confinement need non-perturbative methods $\rightarrow$ lattice simulations

Table of content

$\mathcal{N}=1$ Super-Yang-Mills theory

Fields

- Gauge boson (gluon) $A_\mu(x)$ in the adjoint representation
- Super partner (gluino) $\lambda(x)$ is Majorana fermion in the adjoint representation

On-shell Lagrange density

$$\mathcal{L}_{\mathsf{SYM}} = \mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\not{D}\lambda \qquad\qquad \right)$$

Supersymmetry

Relation between fermionic matter particles and bosonic force particles

$$\delta_\epsilon A_\mu = \mathrm{i}\bar{\epsilon}\gamma_\mu\lambda, \quad \delta_\epsilon\lambda = \mathrm{i}\sigma_{\mu\nu}F^{\mu\nu}\epsilon$$

$\mathcal{N}=1$ Super-Yang-Mills theory

Fields

- Gauge boson (gluon) $A_\mu(x)$ in the adjoint representation
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On-shell Lagrange density

$$\mathcal{L}_{\mathsf{SYM}} = \mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{2}\bar{\lambda}\not{D}\lambda - \frac{m_{\mathrm{g}}}{2}\bar{\lambda}\lambda\right)$$

Supersymmetry

Relation between fermionic matter particles and bosonic force particles

$$\delta_\epsilon A_\mu = \mathrm{i}\bar{\epsilon}\gamma_\mu\lambda, \quad \delta_\epsilon\lambda = \mathrm{i}\sigma_{\mu\nu}F^{\mu\nu}\epsilon$$

Softly broken by gluino mass term

Symmetries

Classical symmetries of the SYM action

- LORENTZ transformations
- Local gauge transformations
- Scale transformations

Symmetries

Classical symmetries of the SYM action with $m_{\mathrm{g}} = 0$

- Lorentz transformations
- Local gauge transformations
- Scale transformations
- Global chiral $U(1)_{\mathrm{A}}$ symmetry
- Supersymmetry

Symmetries

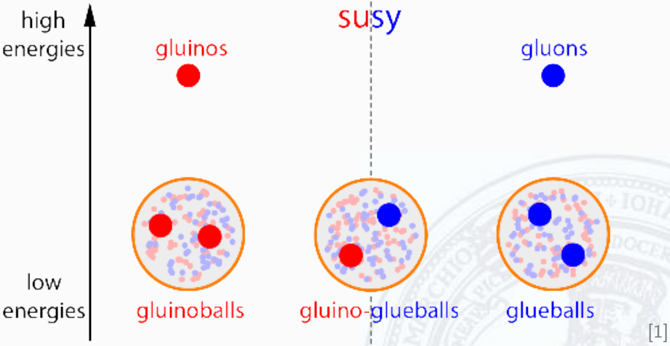
Chiral symmetry breaking in $SU(3)$ SYM theory

- Global chiral $U(1)_\mathsf{A}$ symmetry: $\lambda \mapsto \mathrm{e}^{\mathrm{i}\alpha\gamma_5}\lambda$
- Due to anomaly only $\mathbb{Z}_6$ remnant symmetry

$$\lambda \mapsto \mathrm{e}^{\mathrm{i}\frac{2\pi n}{6}\gamma_5}\lambda \quad \text{with} \quad n \in \{1, \ldots, 6\}$$

- Spontaneously broken to $\mathbb{Z}_2$ symmetry in consequence of gluino condensate $\langle\bar{\lambda}\lambda\rangle \neq 0 \rightarrow 3$ different vacua

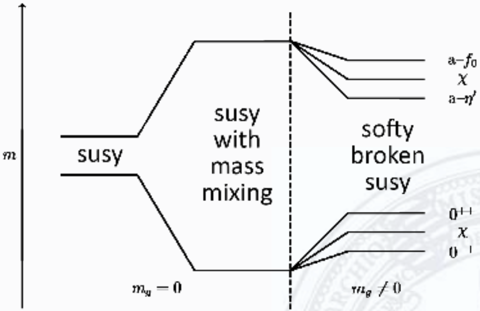
Particle spectrum of $\mathcal{N}=1$ SYM theory



- At high energies: gluons and gluinos like a free gas
- At low energies: confinement
  $\rightarrow$ no free colour charges, only colour-neutral states

[1] Piemonte: „N=1 supersymmetric Yang-Mills theory on the lattice" (dissertation)

Particle spectrum of $\mathcal{N}=1$ SYM theory



[2]

- Glueballs 0^{++} & 0^{-+}
- Gluino-glueballs χ
- Gluinoballs a-η' & a-f_0

[2]Sandbrink: „Numerische Bestimmung von Quarkpotential, Glueball-Massen und Phasenstruktur in der N=1 supersymmetrischen Yang-Mills-Theorie“ (dissertation)

The critical gluino mass m_g

- Lattice breaks chiral symmetry & supersymmetry
  $\rightarrow$ counter term $\sim$ gluino mass
- Fine tuning so that gluino becomes massless in the continuum limit
- Gluino can't be measured directly
  $\rightarrow$ (unphysical) adjoint pion $m^2_{a-\pi} \propto m_g$ can be measured easily
- Fine tuning assures restoration of the chiral symmetry and the supersymmetry in the continuum limit

Lattice Action

Gauge Action

- Symanzik improved Lüscher-Weisz gauge action

$$S_{\mathrm{g}}[\mathcal{U}] = \frac{\beta}{N_{\mathrm{c}}}\left(\frac{5}{3}\sum_{\square}\mathrm{tr}(\mathbb{1} - \mathrm{Re}\,\mathcal{U}_{\square}) - \frac{1}{12}\sum_{\square\square}\mathrm{tr}(\mathbb{1} - \mathrm{Re}\,\mathcal{U}_{\square\square})\right)$$

- $\mathcal{O}(a^2)$ lattice artefacts

Fermion Action

- adjoint representation $[\mathcal{V}_\mu(x)]_{ab} = 2\,\mathrm{tr}\left[\mathcal{U}_\mu^\dagger(x)\,T_a^{\mathrm{F}}\mathcal{U}_\mu(x)\,T_b^{\mathrm{F}}\right]$
- Wilson-Dirac operator

$$D_{\mathrm{W}}(x,y) = \delta_{x,y} - \kappa\sum_{\mu=\pm1}^{\pm4}(\mathbb{1} - \gamma_\mu)\,\mathcal{V}_\mu(x)\,\delta_{x+\mu,y}$$

- $\mathcal{O}(a\ \ )$ lattice artefacts

Lattice Action

Gauge Action

- Symanzik improved Lüscher-Weisz gauge action

$$S_{\mathrm{g}}[\mathcal{U}] = \frac{\beta}{N_{\mathrm{c}}}\left(\frac{5}{3}\sum_{\square}\mathrm{tr}(\mathbb{1}-\mathrm{Re}\,\mathcal{U}_{\square}) - \frac{1}{12}\sum_{\square\square}\mathrm{tr}(\mathbb{1}-\mathrm{Re}\,\mathcal{U}_{\square\square})\right)$$

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Fermion Action

- adjoint representation $[\mathcal{V}_\mu(x)]_{ab} = 2\,\mathrm{tr}\left[\mathcal{U}_\mu^\dagger(x)\,T_a^{\mathrm{F}}\,\mathcal{U}_\mu(x)\,T_b^{\mathrm{F}}\right]$
- Wilson-Dirac operator with irrelevant clover term

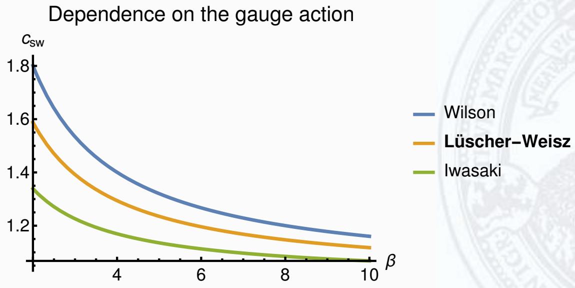
$$D_{\mathrm{W}}(x,y) = \delta_{x,y} - \kappa\sum_{\mu=\pm1}^{\pm4}(\mathbb{1}-\gamma_\mu)\,\mathcal{V}_\mu(x)\,\delta_{x+\mu,y} - c_{\mathrm{SW}}\frac{\kappa}{4}\sigma_{\mu\nu}F^{\mu\nu}\delta_{x,y}$$

- $\mathcal{O}(a^2)$ lattice artefacts with proper Sheikholeslami-Wohlert coefficient c_{SW}

Determination of c_{SW}

Possibilities

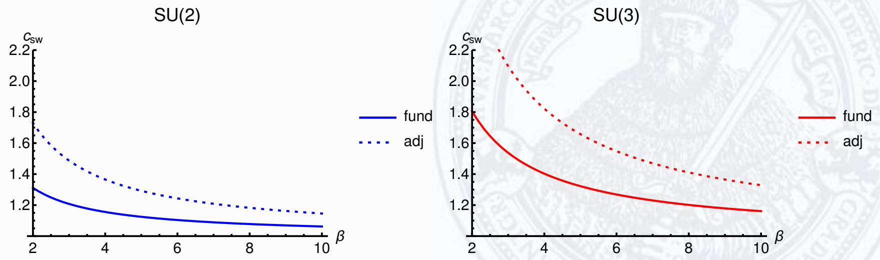
- Perturbation theory
  - Calculating quark-gluon vertices for **massless** fermions



Determination of c_{SW}

Possibilities

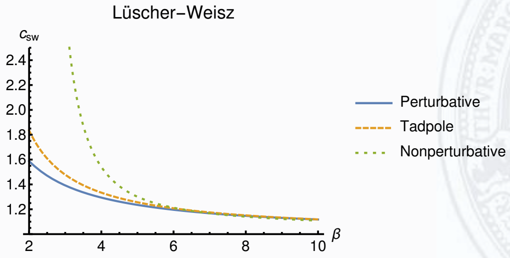
- Perturbation theory for Wilson gauge action [JHEP 05 (2013)]
  - Calculating quark-gluon vertices for **massless** fermions
  - One-loop: $c_{\mathrm{SW}} = 1 + \left(0.16764(3)\, C_{\mathrm{rep}} + 0.01503(3)\, N_{\mathrm{c}}\right) g^2$
    with quadratic Casimir invariant $C_{\mathrm{fund}} = \frac{N_c^2-1}{2N_{\mathrm{c}}}$, $C_{\mathrm{adj}} = N_{\mathrm{c}}$



Determination of c_{SW}

Possibilities

- Perturbation theory
- Tadpole improvement [Phys.Rev. D48 (1993)]
  - Mean link u_0, depends upon theory
  - Gauge-invariant definition $u_0 \equiv \langle \frac{1}{N_c} \mathrm{tr}\,(U_{\text{plaq}}) \rangle^{1/4}$
  - Tree-level: $c_{SW} = \frac{1}{u_0^3}$



Determination of c_{SW}

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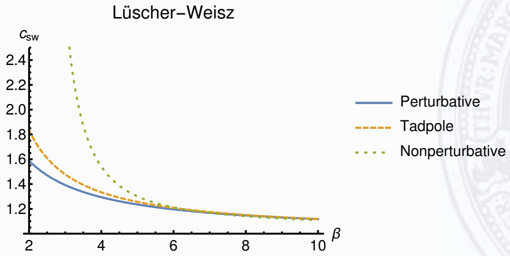
Possibilities

- Perturbation theory
- Tadpole improvement
- Nonperturbative with the Schrödinger functional [Nucl.Phys. B491 (1997)]



Determination of c_{SW}

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Possibilities

- Perturbation theory
- Tadpole improvement
- Nonperturbative with the Schrödinger functional
- Nonperturbative with **physical fine tuning**?

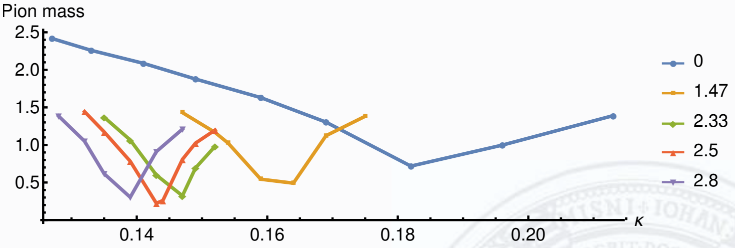
Physical fine tuning of c_{SW}

Distinctive feature of Super-Yang-Mills theory

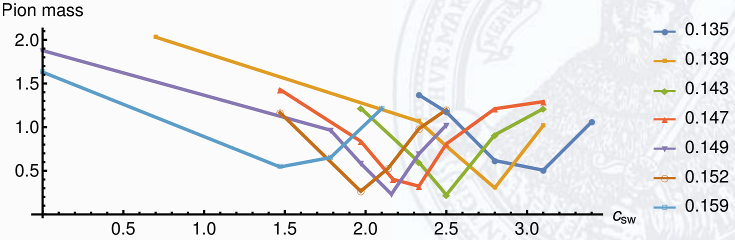
- Pion is not a physical particle
- Pion mass vanishes in the continuum theory
- Its lattice mass is purely an effect of broken supersymmetry & chiral symmetry
- Empirical tuning of the pion mass to find the point which is nearest to the continuum with respect to
  - Discretization errors
  - Symmetries

Parameter Scan

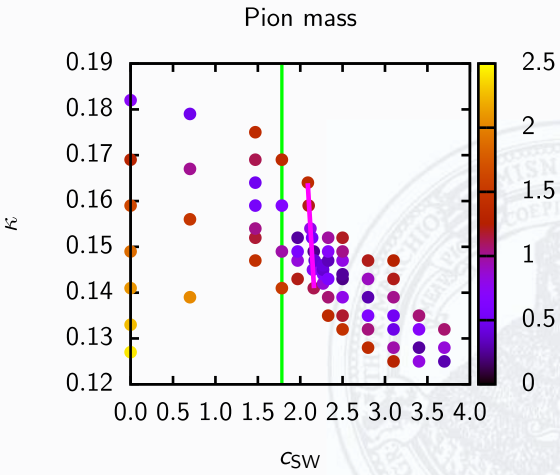
$c_{SW} = \text{const.}$



$\kappa = \text{const.}$



Parameter Scan



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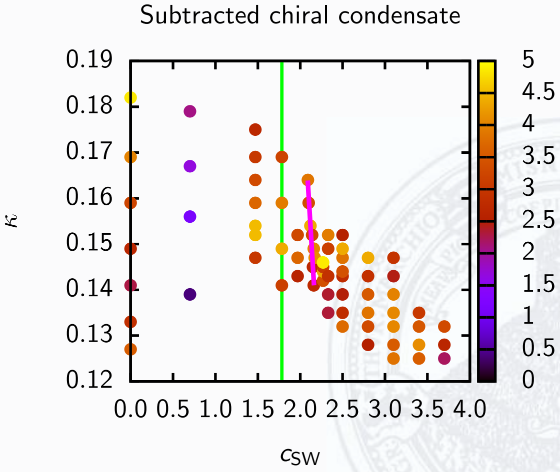
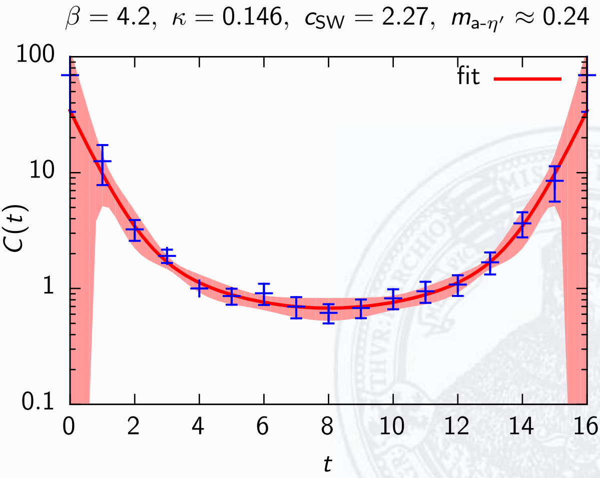
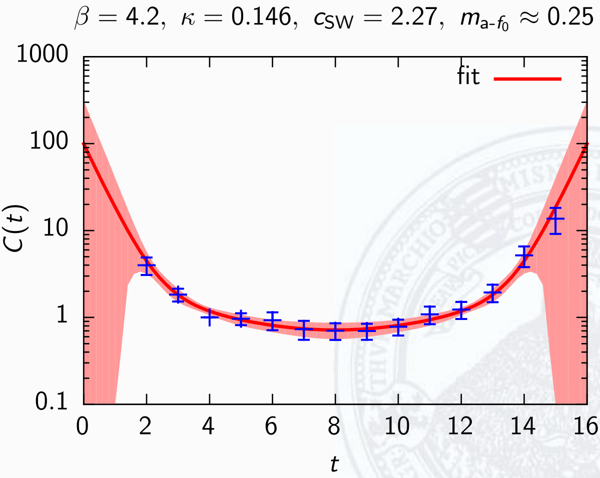


Particle masses



Particle masses



Summary

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- Lattice breaks supersymmetry
- Gluino condensate breaks remnant chiral symmetry
- Fine tuning of bare gluino mass m_g (or c_{SW}?) necessary
- Reduce lattice artefacts with improved lattice action
- Nonperturbative physical tuning of the c_{SW} coefficient?
- First results for masses of adjoint mesons

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Outlook

- Further investigations of c_{SW} dependence on β, κ
- Determination of c_{SW} with the Schrödinger functional
- Provide a basis for Super-QCD simulations