

Generating a Non-Perturbative Mass Gap using Feynman Diagrams in Asymptotically Free Theories

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Motivation

Asymptotically free theories are defined at a “free” fixed point with a marginally relevant coupling.

So it seems plausible that a “**non-perturbative**” approach based on an expansion in powers of “some” bare coupling U , should be able to describe the physics.

Recently, Feynman diagrammatic Monte Carlo methods have been developed to solve many problems non-perturbatively.

Van Houcke, Kozik, Prokof'ev and Svistunov, 2008

Can such an expansion generate the non-perturbative mass-gap in an asymptotically free theory?

How can it even work?

Non-perturbative mass gap $M = \Lambda \exp(-\beta/U)$

There is an essential singularity at $U=0$!

- So how can an expansion in U even make sense?

Here we address this question in a two dimensional model:
(gauge theories bring other subtleties)

Other recent work: $O(N)$, $U(N) \times U(N)$ models, (large N , $V = \infty$)

Buividovich, Davoudi, 1705.03368

This work: A Gross Neveu type Model (N, V finite)

Basic Idea

Define the theory with both a UV and IR regulator

- A finite lattice model.

The partition function has a completely convergent expansion in U .

- In our Gross-Neveu model it is a polynomial of order U^V
- But the series is actually convergent more rapidly!

This convergent expansion must become singular in the continuum and thermodynamic limit!

But perhaps at some high order “before” it becomes singular we can see the origin of the mass gap?

2d GN Models

Gross, Neveu (1974)

Continuum action:

$$S = \int d^2x \left\{ \sum_{a=1}^N \bar{\psi}_a \gamma^\mu \partial_\mu \psi_a - \frac{U}{2} \left(\sum_{a=1}^N \bar{\psi}_a \psi_a \right)^2 \right\}$$

Dimensions:

$$[\psi] = [\bar{\psi}] = \frac{1}{2} \quad [U] = 0$$

1-loop Beta function: (asymptotic freedom)

$$\mu \frac{\partial U}{\partial \mu} = -\frac{N}{\pi} U^2 \quad M_{\text{phys}} \approx \Lambda \exp\left(-\frac{\pi}{NU}\right)$$

↑
non-perturbative mass gap

Lattice Model (reduced staggered fermions):

$$S = \frac{1}{2} \sum_{x,y} \psi_{x,i} M_{x,y} \psi_{y,i} - U \sum_x \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}$$

Partition function:

$$\begin{aligned} Z &= Z_0 \int [d\psi] e^{-\frac{1}{2} \psi^T M \psi} \prod_x e^{U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}} \\ &= Z_0 \int [d\psi] e^{-\frac{1}{2} \psi^T M \psi} \prod_x (1 + U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}) \\ &= Z_0 \sum_k U^k \sum_{[x,k]} \left(\int [d\psi_1] e^{-\frac{1}{2} \psi_1^T M \psi_1} \psi_{x_1,1} \psi_{x_2,1} \psi_{x_3,1} \cdots \psi_{x_k,1} \right)^4 \\ Z &= \sum_{k=0}^V U^k z_k \quad \text{A polynomial!} \end{aligned}$$

The k-th order term is the sum over Feynman diagrams:

Van Houscke, Kozik, Prokof'ev and Svistunov, 2008

$$z_k = Z_0 \sum_{[x,k]} \left(\int [d\psi_1] e^{-\frac{1}{2} \psi_1^T M \psi_1} \psi_{x_1,1} \psi_{x_2,1} \psi_{x_3,1} \cdots \psi_{x_k,1} \right)^4$$

$$\int [d\psi_1] e^{-\frac{1}{2} \psi_1^T M \psi_1} \psi_{x_1,1} \psi_{x_2,1} \psi_{x_3,1} \cdots \psi_{x_k,1}$$

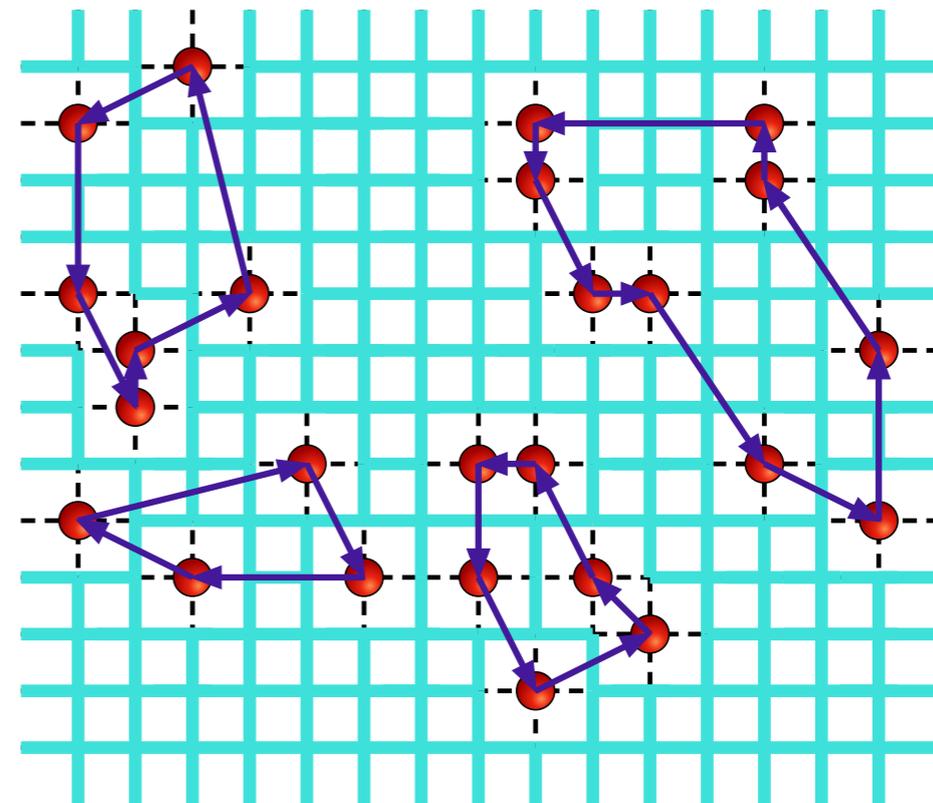
$$= \text{Pf}(M) \text{Pf}(W([x, k]))$$

We choose Z_0 so that

$$z_k = \sum_{[x,k]} \left(\text{Pf}(W[x, k]) \right)^4$$

contains disconnected diagrams

k x k matrix

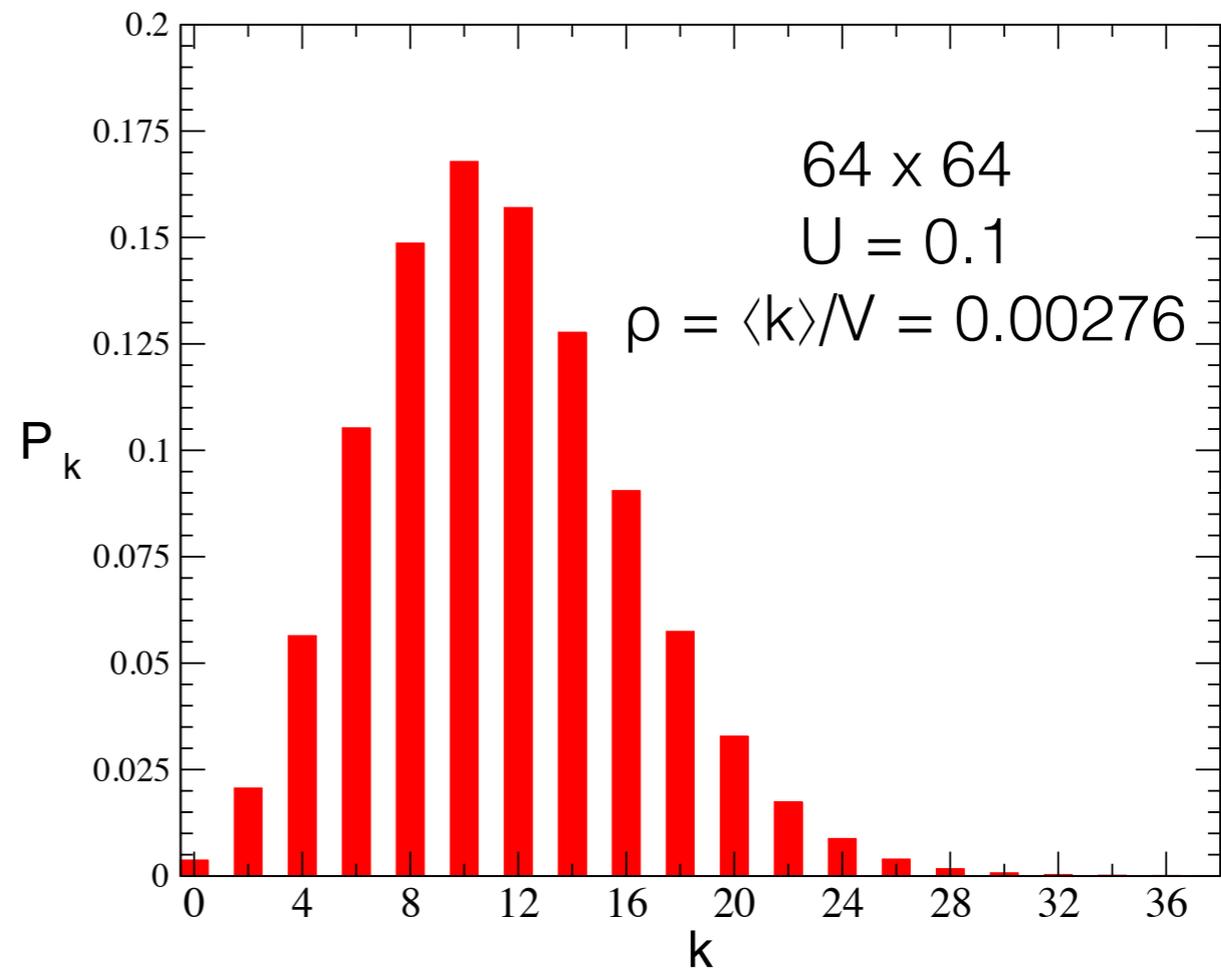


[x,k] configuration

Statistics of “k” in Z

We can define the probability distribution $P_k(U)$ of number of vertices

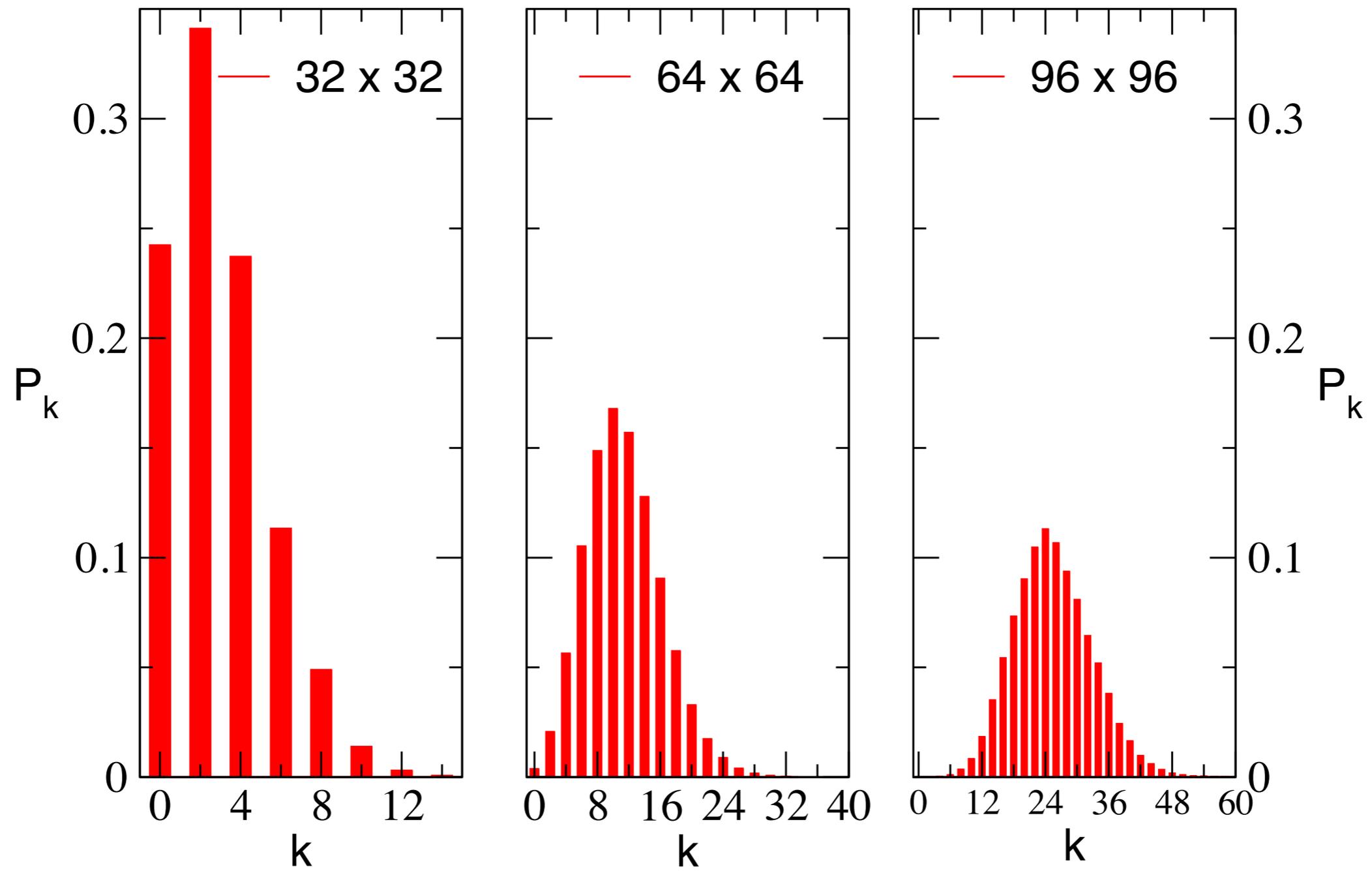
$$P_k(U) = \frac{U^k z_k}{Z}$$



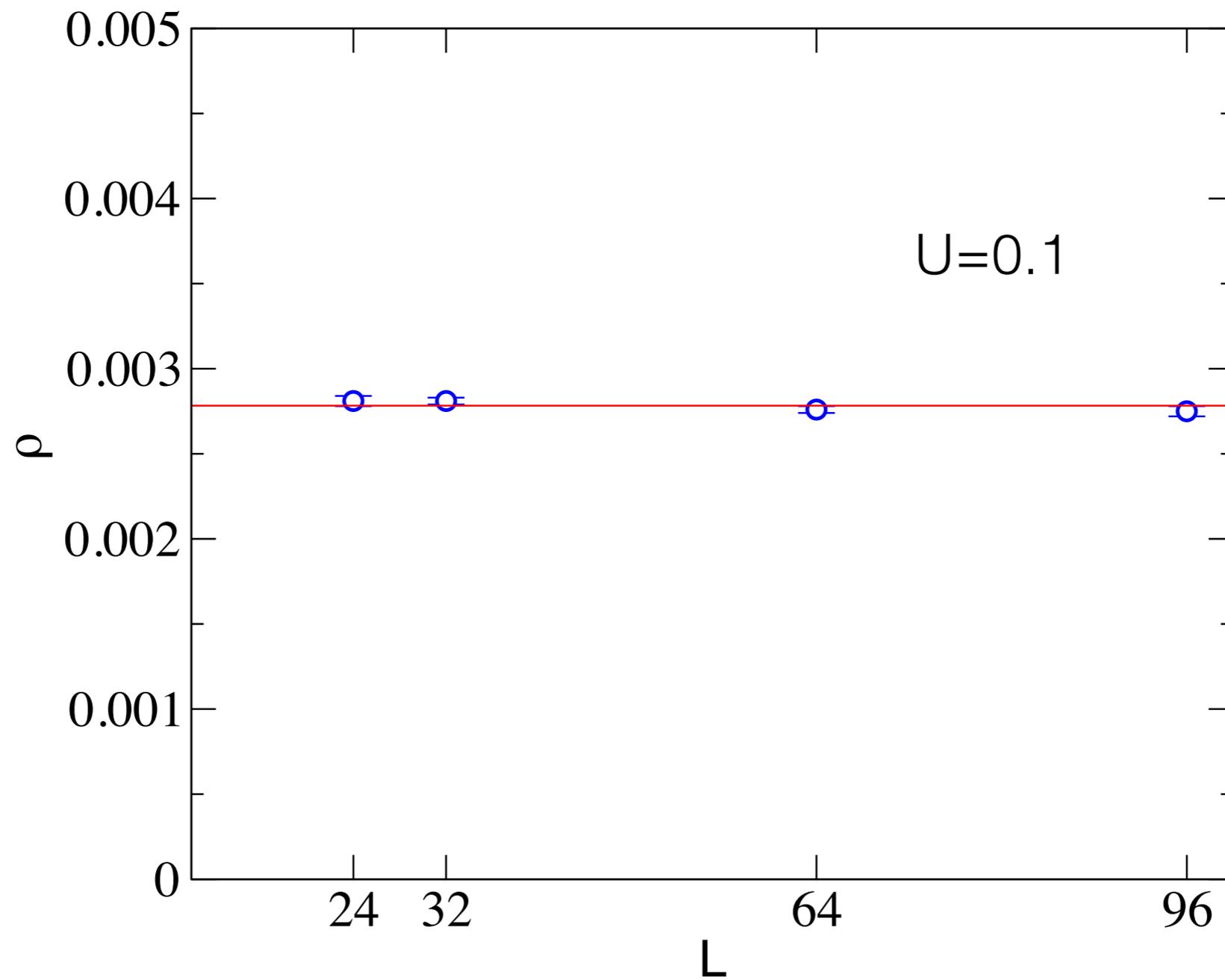
Since $V = 4096$, we learn that $Z(U)$ is a “convergent” series in U !

Change in the distribution as a function of L

U=0.1



Average density of vertices remains the same



Why is the density of vertices a constant?

Because $Z(U) = \exp(f(U)V)$

We begin with

$$Z(U) = \sum_k z_k U^k$$

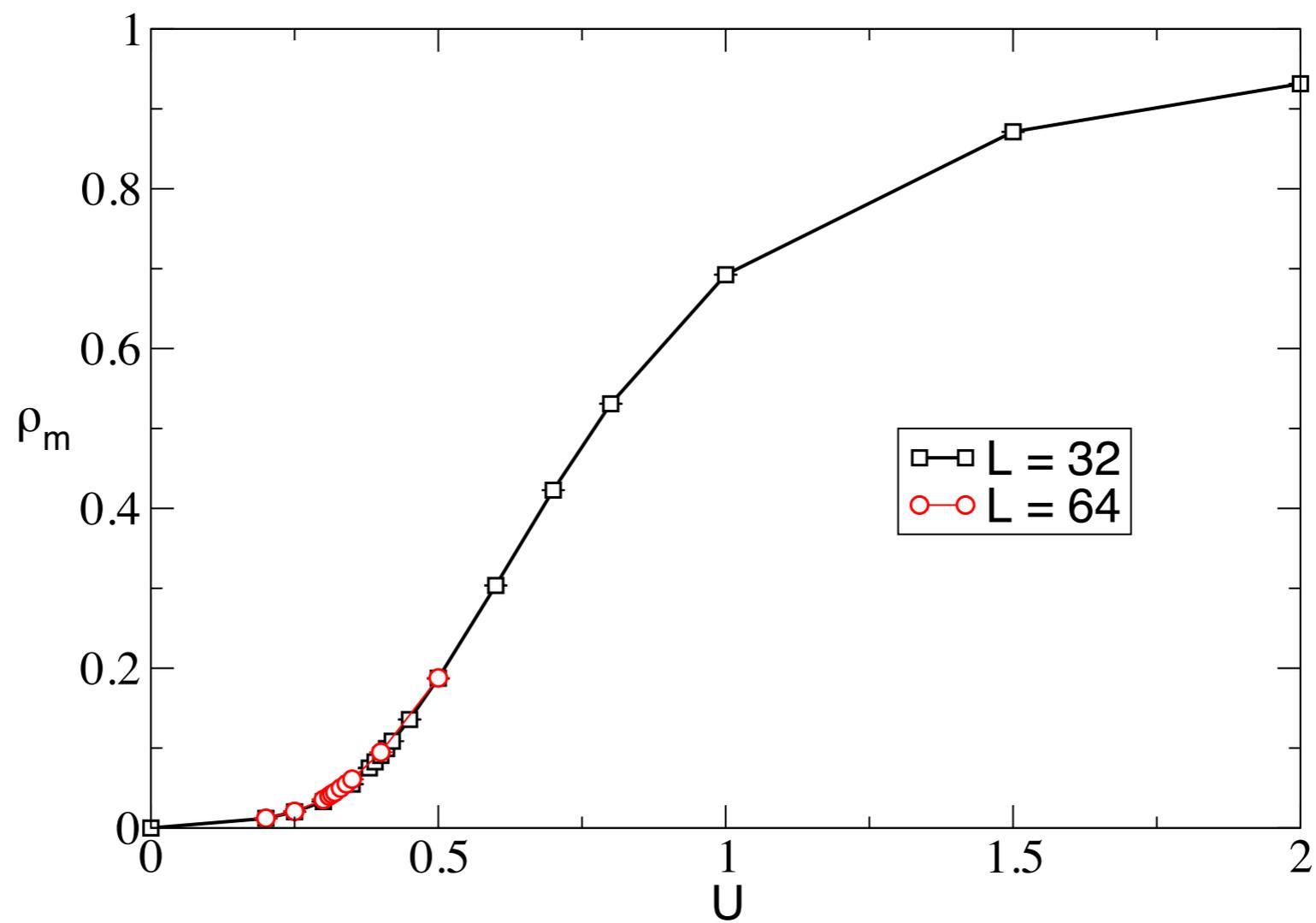
$$\langle k \rangle = \frac{1}{Z(U)} \sum_k k z_k U^k$$

$$\langle k \rangle = U \frac{\partial}{\partial U} \ln(Z(U))$$

If we assume $Z(U) = \exp(f(U)V)$

$$\rho(U) = \frac{\langle k \rangle}{V} = U \frac{\partial}{\partial U} f(U)$$

$\rho(U)$ as a function of U



In perturbation theory we usually compute $f(U)$

$$Z(U) = \exp(f(U)V)$$

disconnected diagrams

connected diagrams

Here $Z(U)$ is convergent. But it is possible that $f(U)$ is not!

$$f(U) = \frac{1}{V} \log(Z(U))$$

But we can extract $f(U)$ from $Z(U)$ through the relation

$$f(U) = \int_0^U \rho(U) dU$$

No singularities!

Non-perturbative Mass Gap

Susceptibility $\chi_1(L) = \frac{1}{Z} \int [d\psi] e^{-S} \psi_{0,1} \psi_{0,2} \left(\sum_y \psi_{y,2} \psi_{y,1} \right)$

As L becomes large

$$\chi_1(L) = \text{logarithmic divergence} \quad (\text{no mass gap})$$

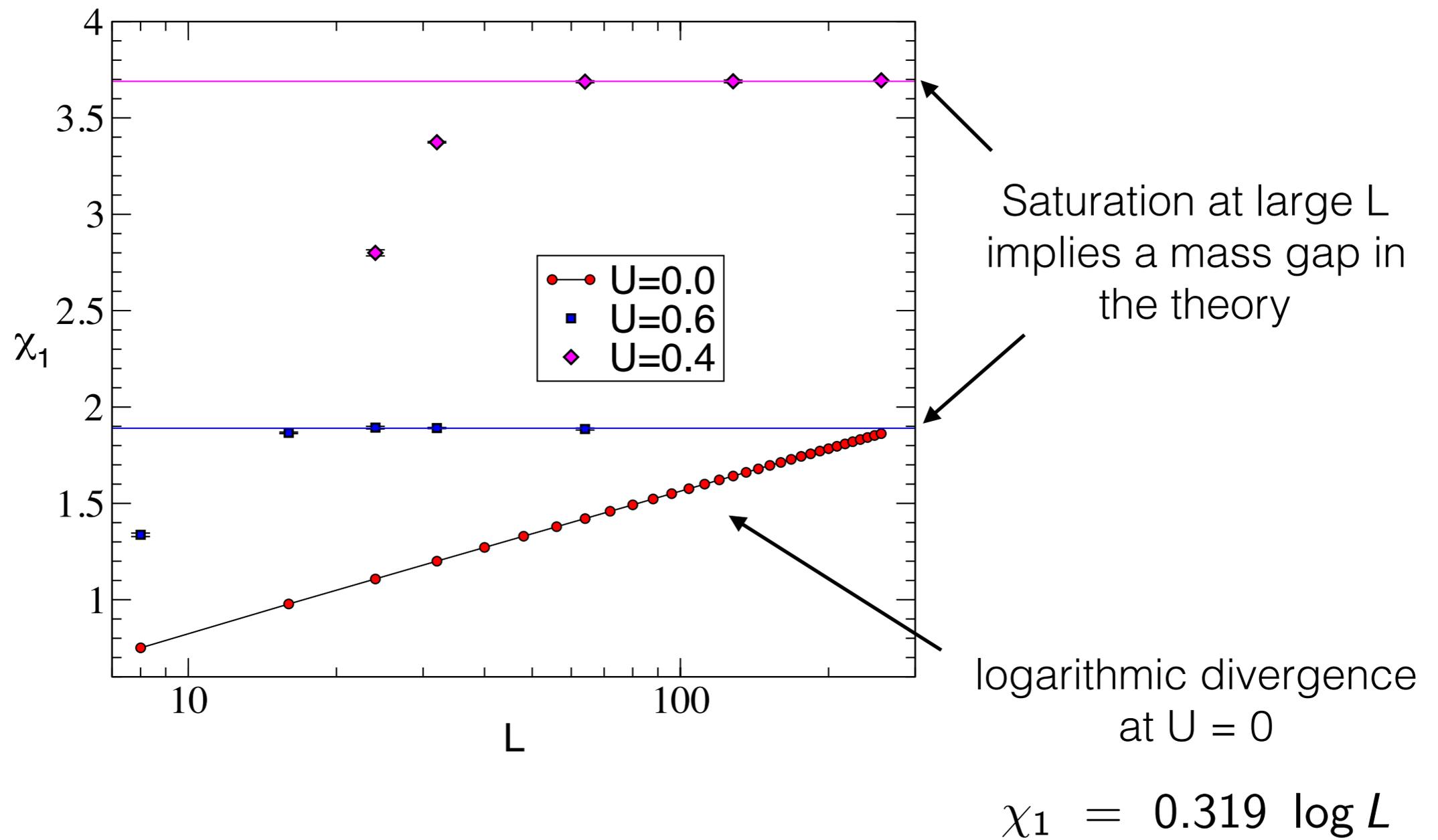
$$\chi_1(L) = C \quad (\text{constant}) \quad (\text{non-perturbative mass gap develops})$$

Diagrammatic Expansion

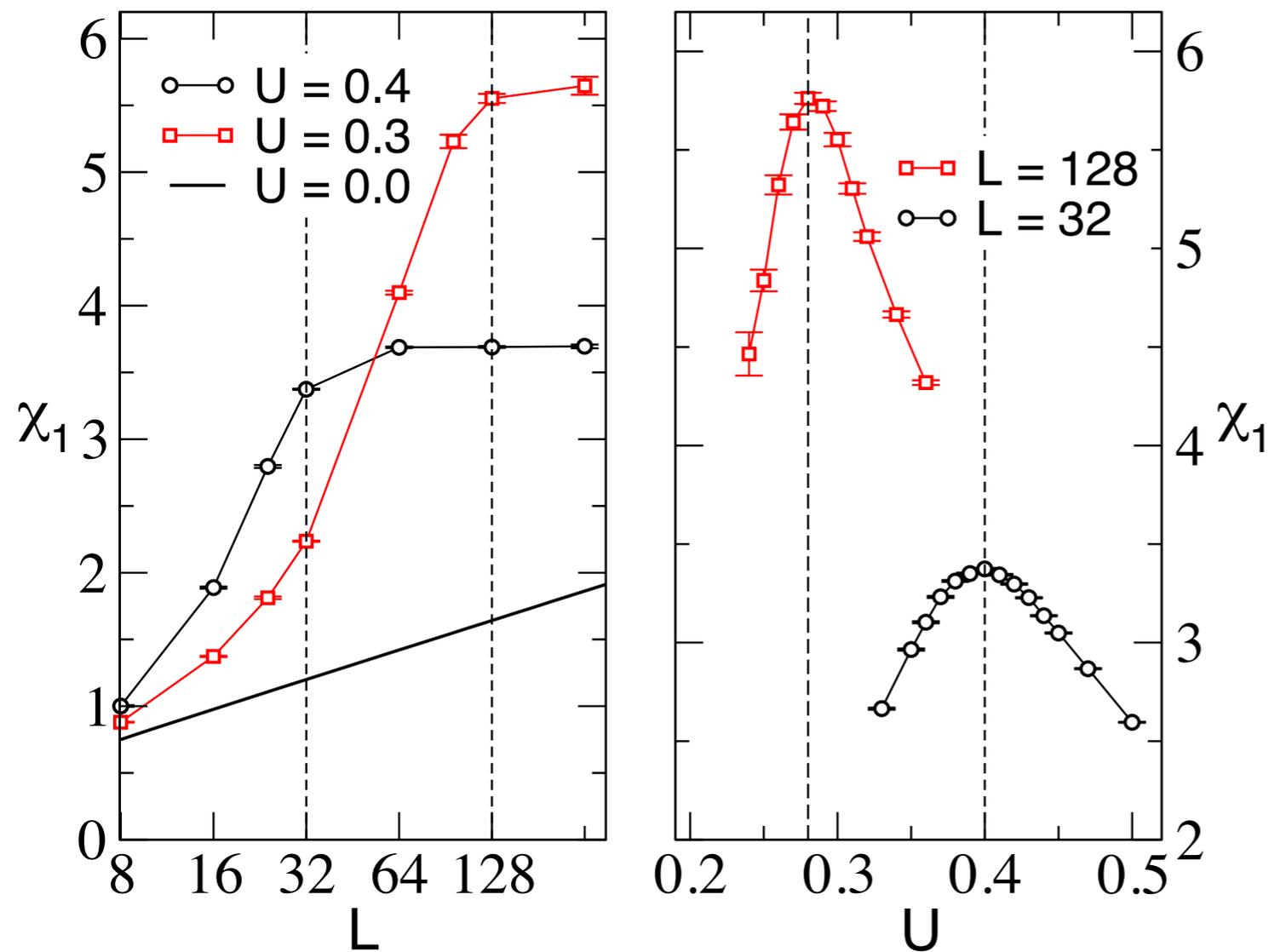
$$\chi_1(L) = \sum_k C_k P_k(U)$$

↑
only connected diagrams

Behavior as a function of L



Mass scale visible through the “Pseudo-Critical Coupling”



$U_{\text{peak}}(L)$ = pseudo-critical coupling

Define $M(U)$ such that

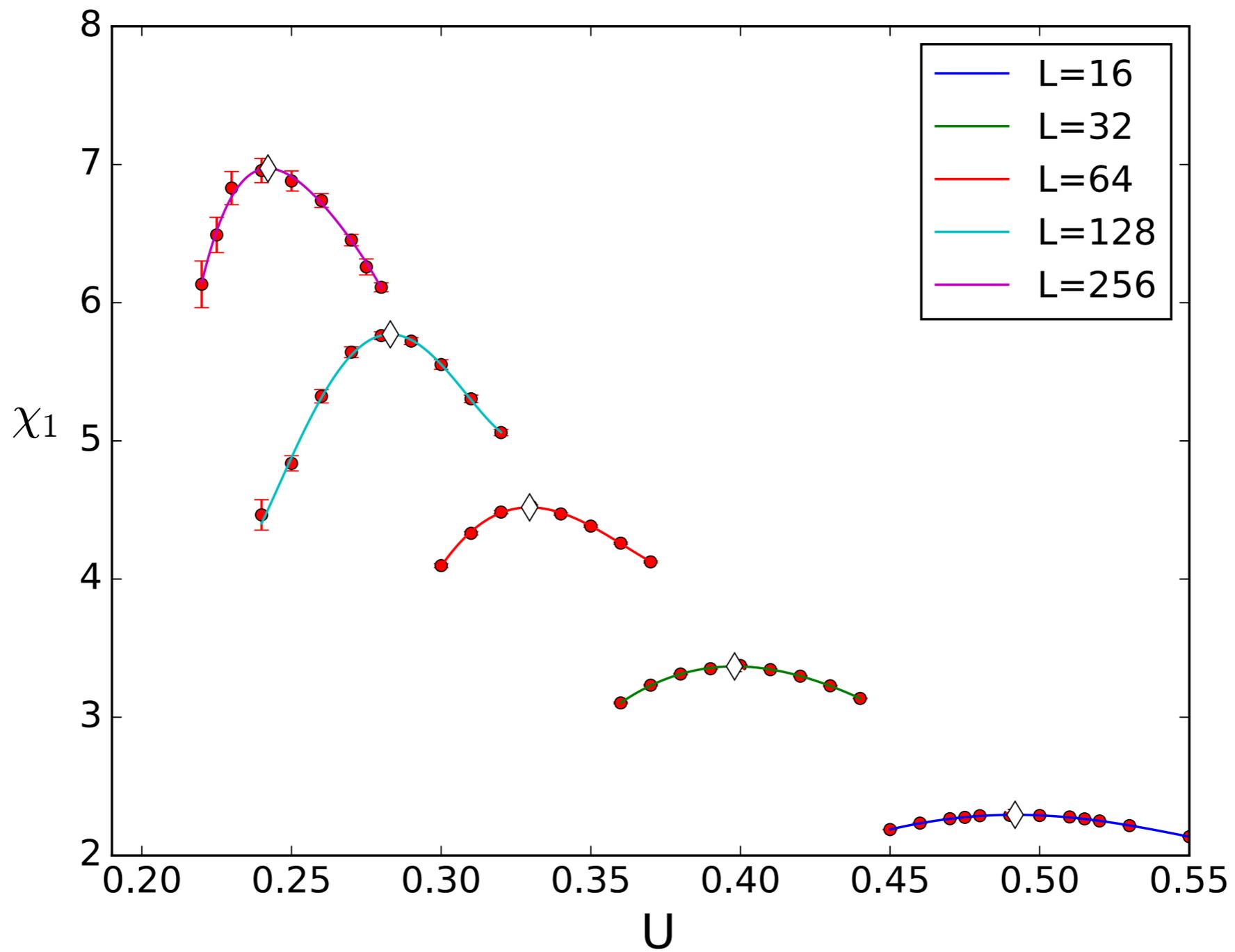
$M(U) < 1/L$ when $U < U_{\text{peak}}$

$M(U) > 1/L$ when $U > U_{\text{peak}}$

$$M(U_{\text{peak}}) = 1/L$$

This definition of Mass gap roughly agrees with the values of L where the susceptibility saturates

Pseudo-critical couplings change with L

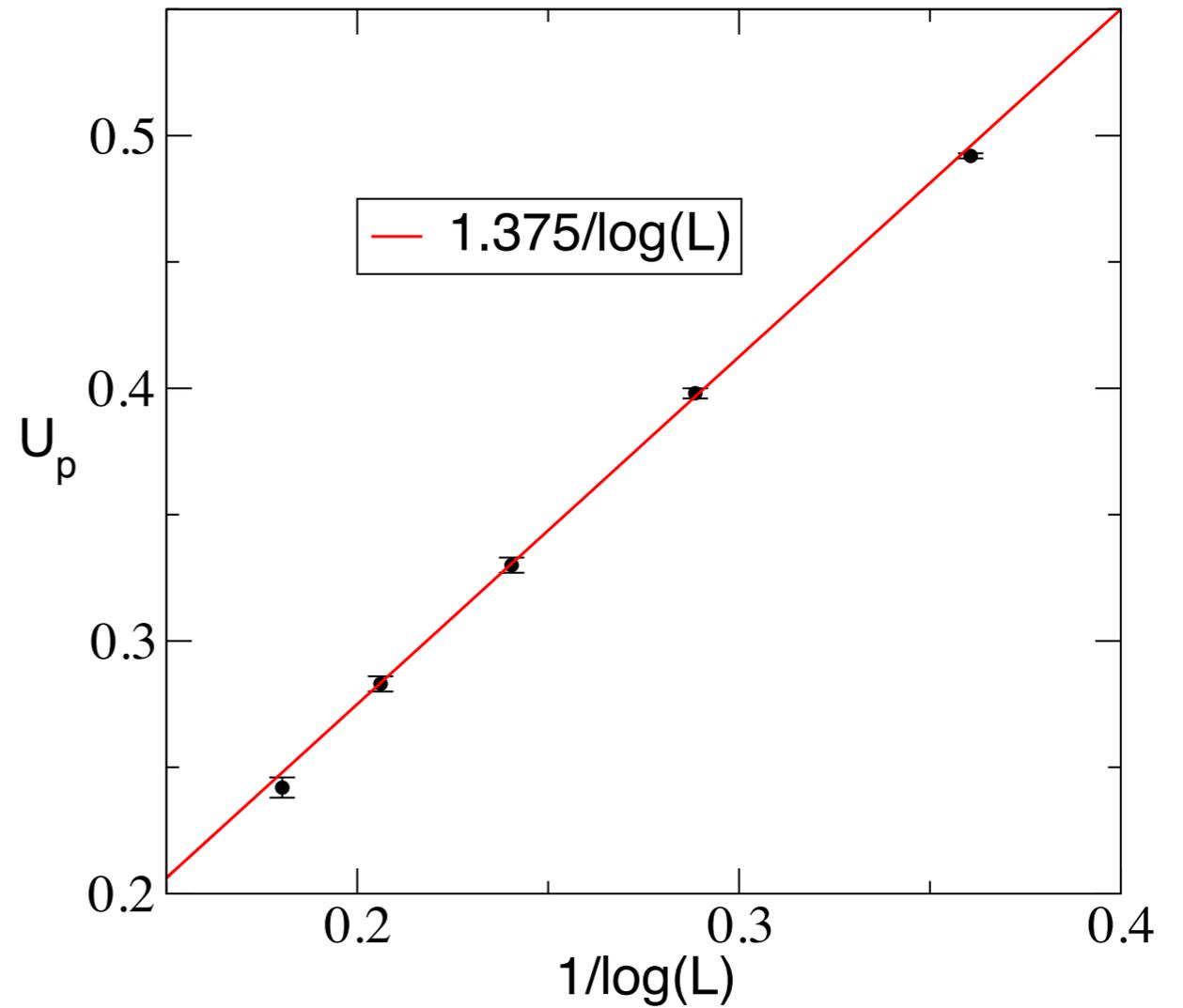


Mass gap scales exponentially with U as expected!

$$U_p = \frac{\beta}{\log(\Lambda L)}$$

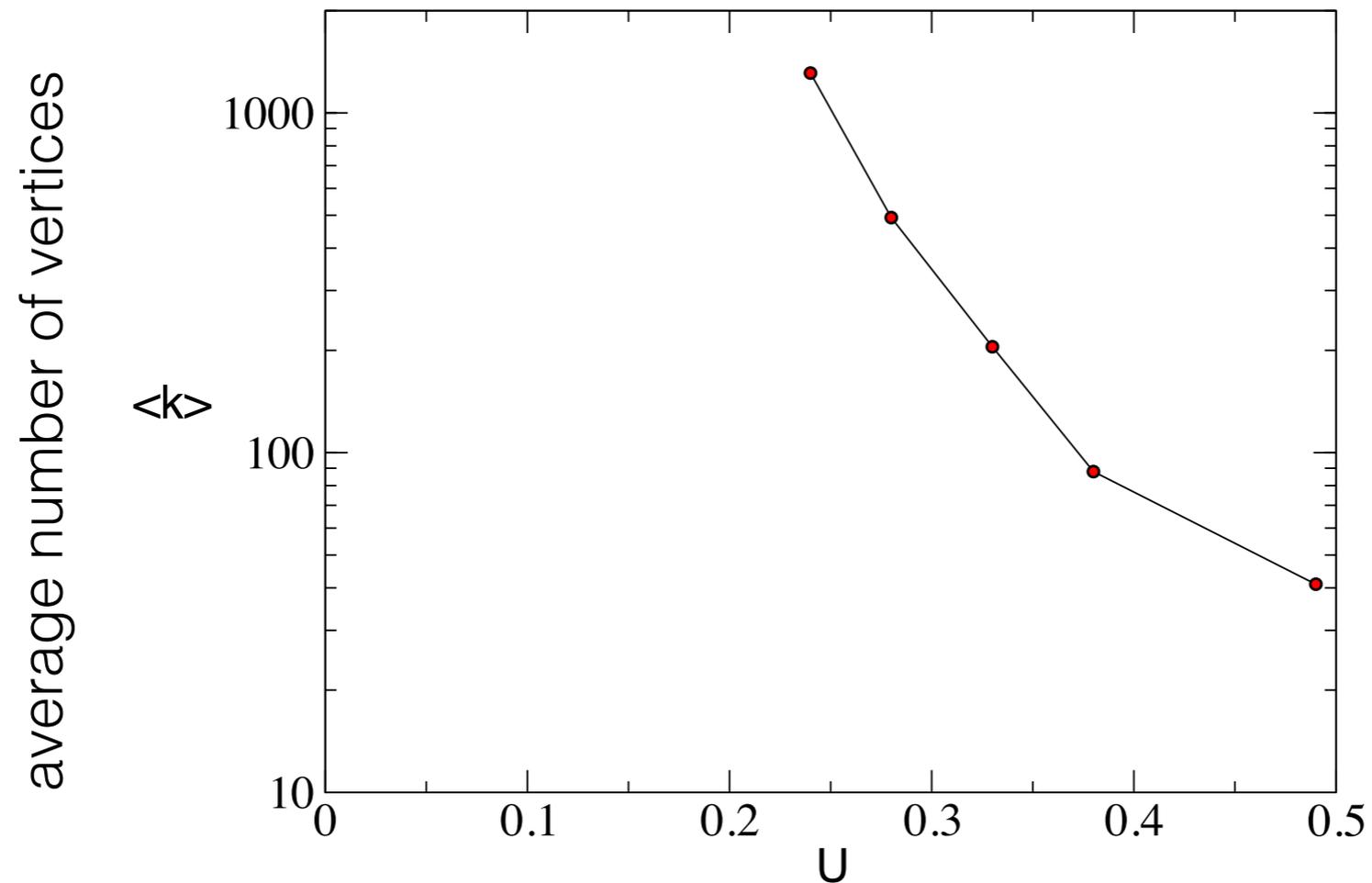
↑
mass gap

$$M \sim \frac{1}{L} = \Lambda \exp\left(-\frac{\beta}{U_p}\right)$$



RG on the lattice is complicated, difficult to predict β

At what order of the perturbative expansion does the mass gap appear?



Feynman diagrams can generate the mass gap at very high orders!

Conclusions

Using a finite UV and IR cutoff we can use “weak coupling diagrammatic approaches” to study **some** asymptotically free theories.

The series for the partition function $Z(U)$ is convergent.

Mass gap appears at very high orders in the expansion.

Questions:

Can we explore the series expansion directly (instead of a MC method)?

Can we extend these type of ideas to Yang-Mills theory and QCD?