Applying the density of states method to QCD at finite chemical potential

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Emberi Erőforrások
Minisztériuma

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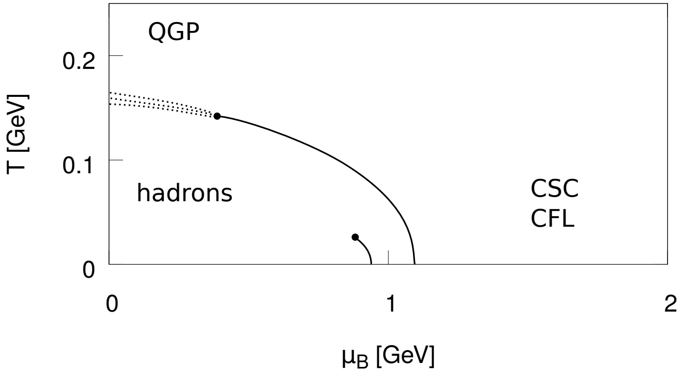
The $\mu_B - T$ phase diagram of QCD



Refs: [Aoki et al, 0611014; Aarts, 1512.05145 (rev.)]

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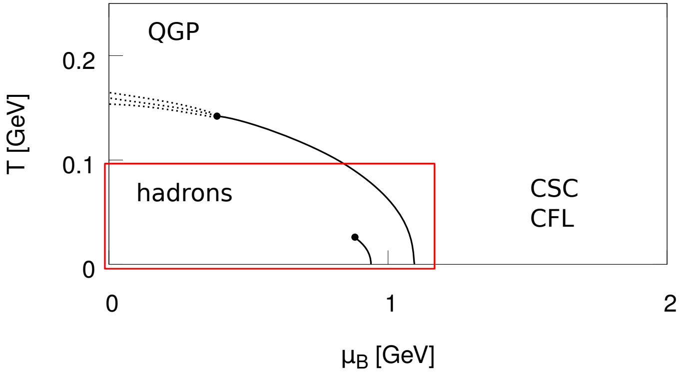

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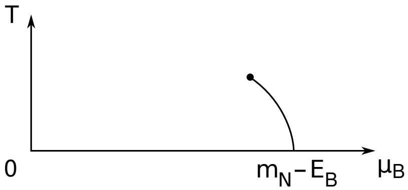
The low temperature region

Ref.: [T. Cohen, 0307089]



Silver Blaze property
until $\mu_B < m_N - E_B$

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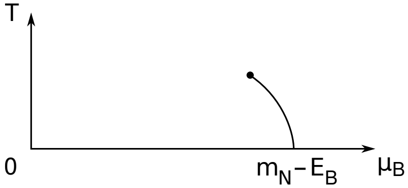

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Silver Blaze property
until $\mu_I < m_\pi$

QCD_I and QCD_B

N_f flavors of staggered quarks ($\mu = \mu_I$ in QCD_I, $\mu = \mu_B/3$ in QCD_B):

$$\mathcal{Z}_I(\lambda) = \int \mathcal{D}U \ \det(M(\mu)^\dagger M(\mu) + \lambda^2)^{N_f/8} \, \mathrm{e}^{-S_g}$$

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How to approach $\lambda = 0$?

"Careful way" : $\quad \langle \mathrm{n} \rangle_{\mathrm{B},\lambda=0} = \lim_{\lambda, 1/\mathrm{V} \to 0} \langle \mathrm{n} \rangle_{\mathrm{B},\lambda}$

QCD_I and QCD_B

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How to approach $\lambda = 0$?

"Less rigorously"[1] : $\quad \langle \mathrm{n} \rangle_{\mathrm{B},\lambda=0} = \frac{\mathrm{T}}{\mathrm{V}} \frac{\partial \ln \mathcal{Z}_{\mathrm{B}}(\lambda)}{\partial \mu}\Big|_{\lambda=0}$

[1] It's wrong for the pion condensate!

Reweighting to finite μ_B

N_f flavors of staggered quarks ($\mu = \mu_B/3$):

$$\mathcal{Z}_B(\lambda = 0) = \int \mathcal{D}U \det M(\mu)^{N_f/4} \mathrm{e}^{-S_g}$$

$$\langle O \rangle_B = \frac{1}{\mathcal{Z}_B(0)} \int \mathcal{D}U \mathrm{e}^{-S_g} \det M(\mu)^{N_f/4} O$$

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$$\det M(\mu)^{N_f/4} = |\det M(\mu)|^{N_f/4} \underbrace{\frac{\det M(\mu)^{N_f/4}}{|\det M(\mu)|^{N_f/4}}}_{w(\mu)}$$

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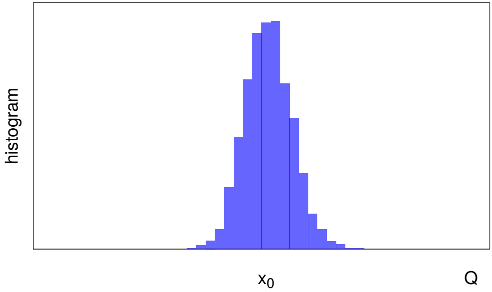

Reweighting to finite μ_B and the overlap problem

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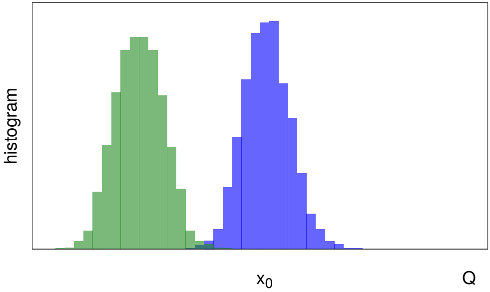
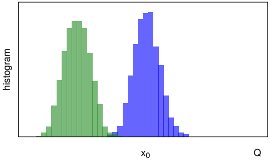


The density of states method

$$\langle O \rangle = \frac{\int \mathcal{D}U \, O \, f[U;p)}{\int \mathcal{D}U \, f[U;p)}$$



Suppose one can not simulate using f, but with g.
In order to reduce the overlap problem:

- do constrained simulations $\left(\delta(Q[U]-x) \approx N \exp\left\{-\frac{PV_4}{2}(Q-x)^2\right\}\right)$
- measure the histogram $\rho(x) = \langle \delta(Q-x) \rangle_g$ and expectation values in the constrained ensemble $(Z_{constr,x} = \rho(x))$

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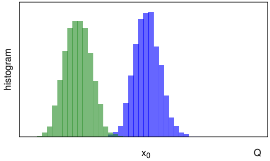

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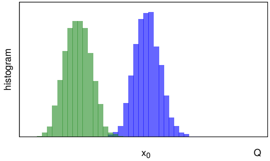

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Simulations: stagg. quarks, using $g = \det(M(\mu)^\dagger M(\mu) + \lambda^2)^{N_f/8} \, \mathrm{e}^{-S_g}$.

Constrained quantities:

- gauge action
- pion condensate

Refs: [K. Langfeld, 1610.09856,
J. Ambjorn et al., 0208025,
Fodor et al. 0701022,
Aoki et al. 1410.7421, ...]

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Constraining the pion condensate:

$$S' = S + S_c, \qquad S_c = \frac{PV_4}{2}(\pi - x)^2$$

$$\langle \pi \rangle_I = \frac{T}{V} \frac{\partial \ln \mathcal{Z}_I(\lambda)}{\partial \lambda} = \frac{T}{V} \frac{N_f}{8} 2\lambda \left\langle \mathrm{Tr} \left(M(\mu)^\dagger M(\mu) + \lambda^2 \right)^{-1} \right\rangle_I$$

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Using noise vectors (stochastic measurement):

$$\pi = \frac{N_f}{8} 2\lambda \frac{1}{N_v} \sum_{i=1}^{N_v} \eta_i^\dagger \left(M(\mu)^\dagger M(\mu) + \lambda^2 \right)^{-1} \eta_i, \qquad \langle \eta_i^\dagger \eta_j \rangle = \delta_{i,j}$$

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Using dynamical pseudofermion fields

The density of states method

$$\mathcal{Z}_I(\lambda) = \int \mathcal{D}U \; \det(M(\mu)^\dagger M(\mu) + \lambda^2)^{N_f/8} \, \mathrm{e}^{-S_g}$$

$$= \int \mathcal{D}U \prod_{j=1}^{N_{pf}} \mathcal{D}\phi_j^\dagger \mathcal{D}\phi_j \exp\left\{ -\sum_{j=1}^{N_{pf}} \phi_j^\dagger (M^\dagger M + \lambda^2)^{-\frac{N_f}{8N_{pf}}} \phi_j - S_g \right\}$$

The ϕ_js are not integrated during the HMC, but generated according to the above distribution at the beginning of each trajectory.

The density of states method

$$\mathcal{Z}_I(\lambda) = \int \mathcal{D}U \; \det(M(\mu)^\dagger M(\mu) + \lambda^2)^{N_f/8} \, \mathrm{e}^{-S_g}$$

$$\mathcal{Z}_{\phi,I}(\lambda) = \int \mathcal{D}U \prod_{j=1}^{N_{pf}} \mathcal{D}\phi_j^\dagger \mathcal{D}\phi_j \exp\left\{ -\sum_{j=1}^{N_{pf}} \phi_j^\dagger (M^\dagger M + \lambda^2)^{-\frac{N_f}{8N_{pf}}} \phi_j - S_g \right\}$$

Now let them be dynamical!

Then the expectation value of the pion condensate is

$$\langle \pi_\phi \rangle_I = \frac{T}{V} \frac{\partial \ln \mathcal{Z}_{\phi,I}(\lambda)}{\partial \lambda}$$

and the operator is

$$\pi_\phi = \frac{N_f}{8} \, 2\lambda \, \frac{1}{N_{pf}} \sum_{i=1}^{N_{pf}} \phi_i^\dagger \left(M(\mu)^\dagger M(\mu) + \lambda^2 \right)^{-\frac{N_f + 8N_{pf}}{8N_{pf}}} \phi_i .$$

Results

(Preliminary)

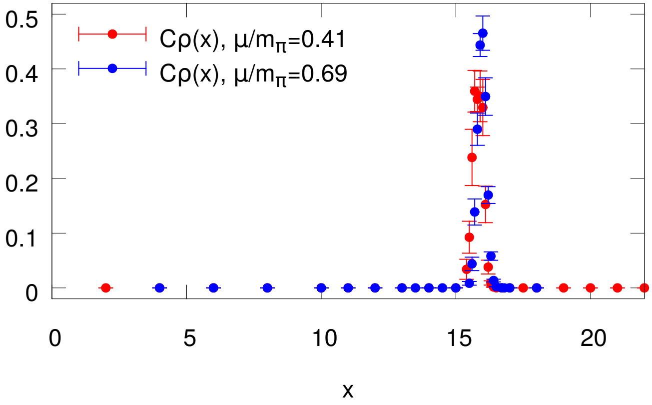
Simulation setup

- N_f flavors of staggered fermions (we used $N_f = 4$ and 2)
- tree-level Symanzik improved gauge action
- 4 stout smearings
- small, coarse lattices ($4^3 \times (8, 16), 6^3 \times (6, 8, 12)$, $a \sim 0.33$ fm)
- pion mass is 335 MeV at $N_f = 4$, 200 MeV at $N_f = 2$
- no. of trajectories: 20k. . .50k (at some region only 2k. . .8k) (every 10th were used for measurement)

Results – constraining the gauge action

$N_f = 4, \quad 4^3x8 \quad (\; ma = 0.05, \lambda a = 0.01, P = 8 \;)$
$a = 0.3288\,\mathrm{fm}, \; \mathrm{T} \approx 75\,\mathrm{MeV}$

Pion condensation starts at $\mu/m_\pi = 0.5$ in QCD_I.
Quark number density should be near zero below $\mu/m_N \approx 0.33$ at $T \sim 0$.



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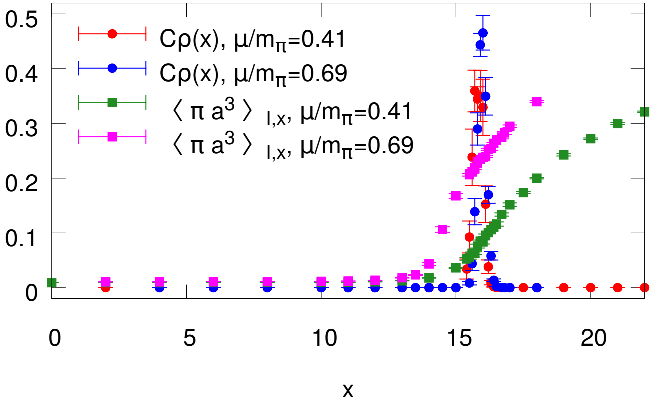

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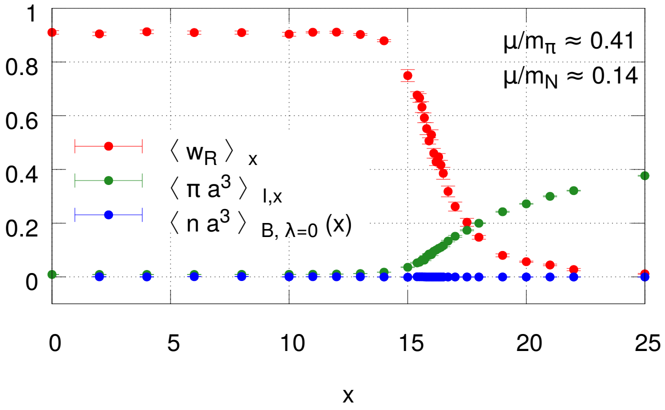

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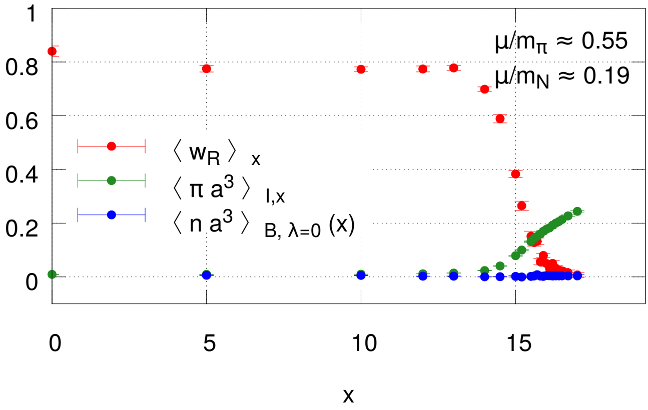

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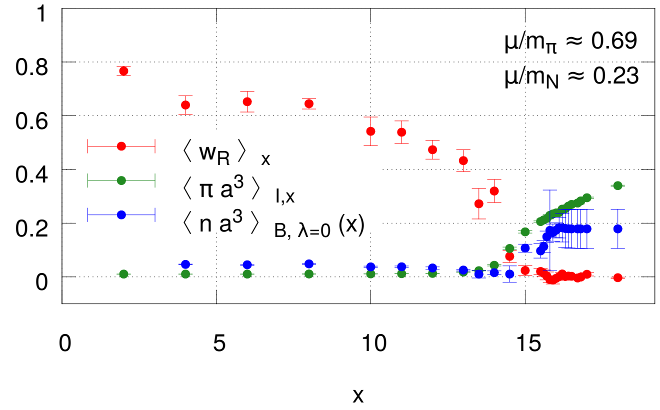

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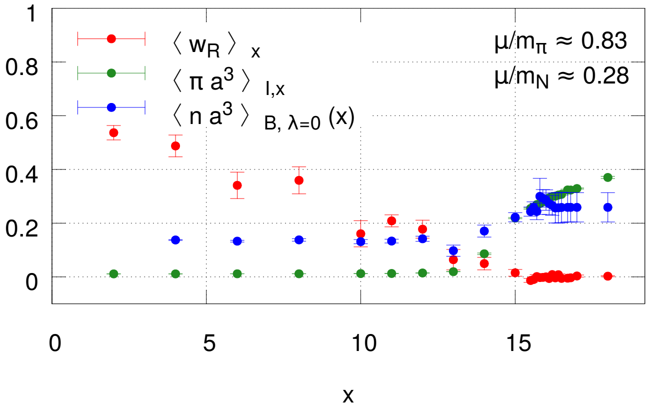

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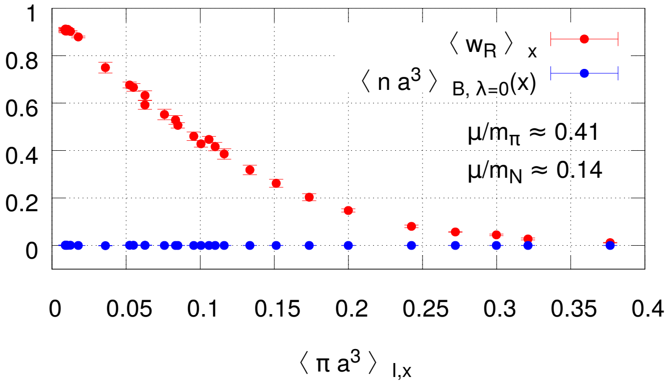

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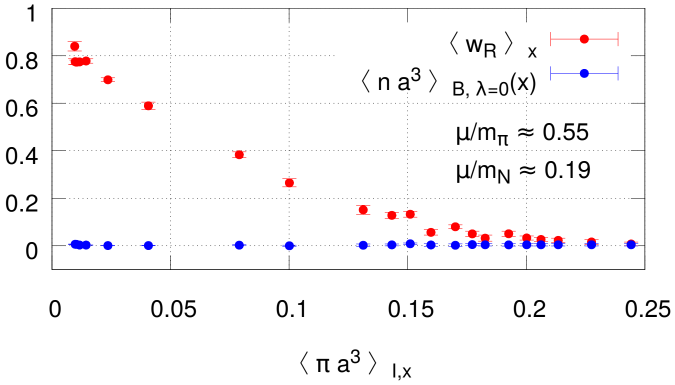

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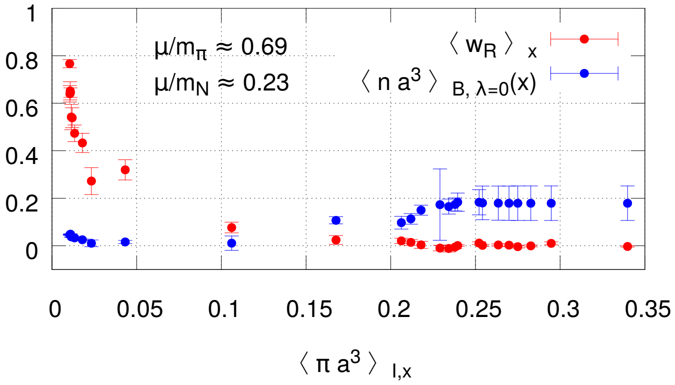

Results – constraining the pion condensate

$N_f = 2, \;\; 6^4$
$a = 0.3447\,\mathrm{fm}, \;\; \mathrm{T} \approx 95\,\mathrm{MeV}$

Pion condensation starts at $\mu/m_\pi = 0.5$ in QCD_I.
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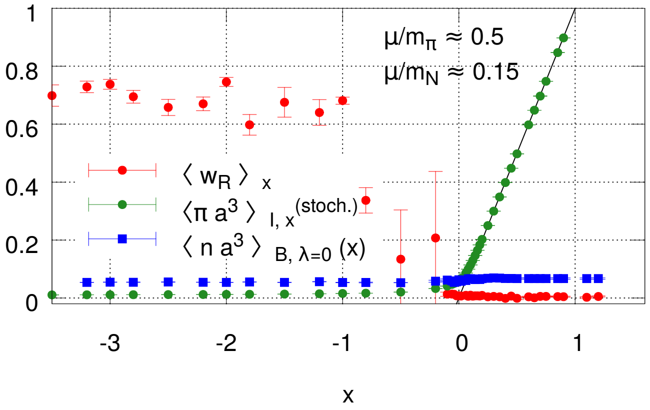
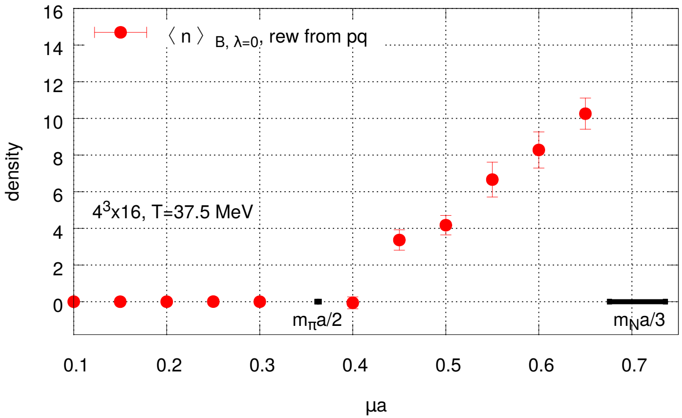


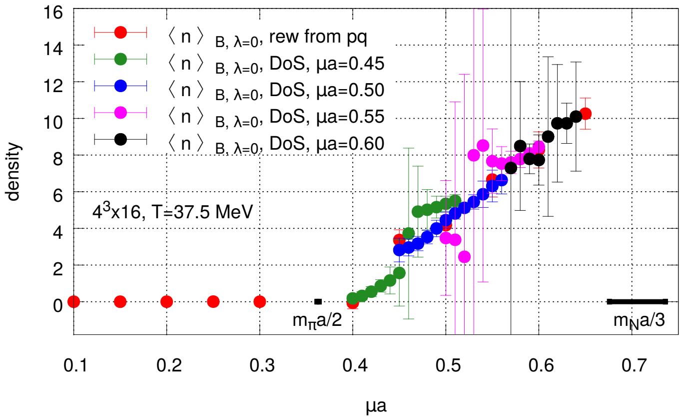
Results

$N_f = 4$, $4^3 x16$
$a = 0.3288$ fm, T ≈ 37.5 MeV



Results

$N_f = 4,\ 4^3x16$
$a = 0.3288\ \mathrm{fm},\ \mathrm{T} \approx 37.5\ \mathrm{MeV}$



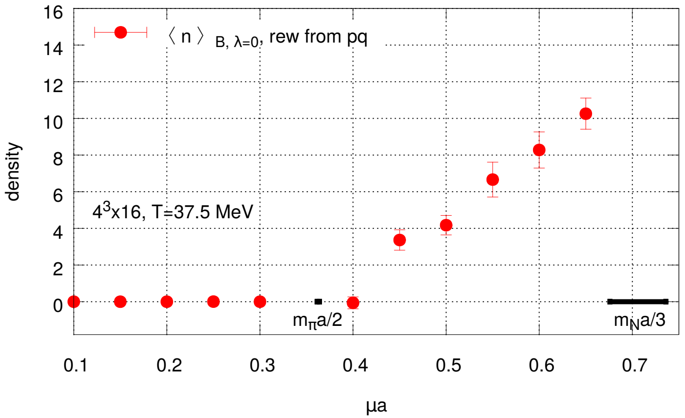
Results

$N_f = 4,\ 4^3x16$

$a = 0.3288\ \mathrm{fm},\ \mathrm{T} \approx 37.5\ \mathrm{MeV}$

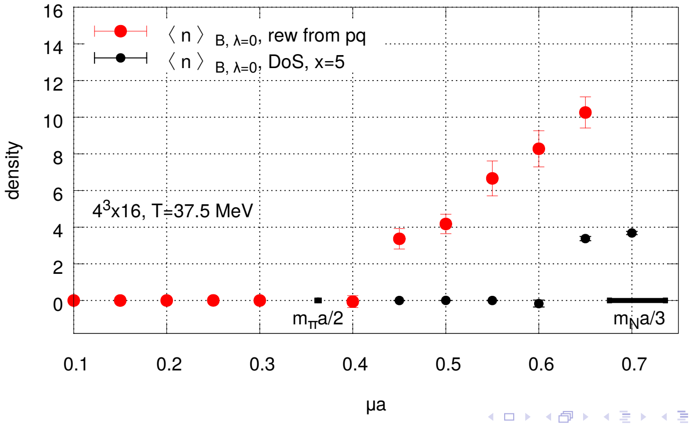
What happens when we change the upper limit of the integrals?

(Is it correct to do that?)



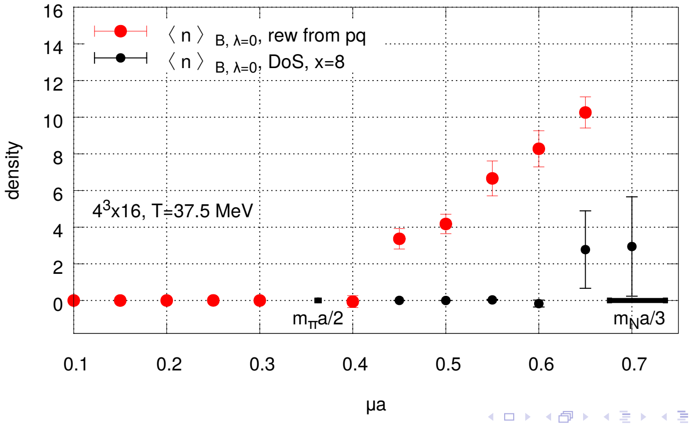
Results

$N_f = 4$, 4^3x16
$a = 0.3288\,\mathrm{fm}$, $\mathrm{T} \approx 37.5\,\mathrm{MeV}$
What happens when we change the upper limit of the integrals?
(Is it correct to do that?)
upper limit: x=5



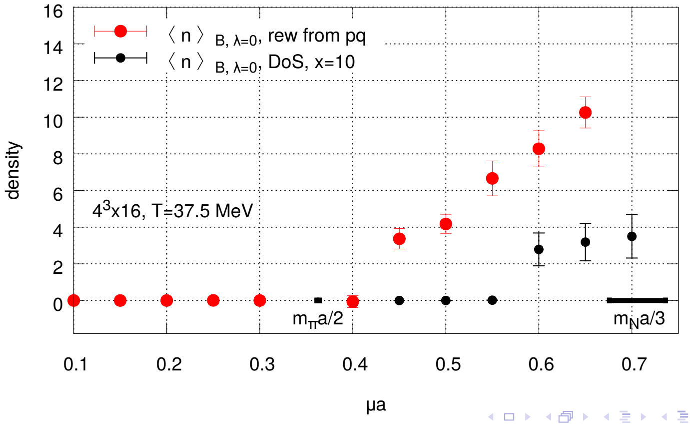
Results

$N_f = 4$, 4^3x16
$a = 0.3288\ \mathrm{fm}$, $\mathrm{T} \approx 37.5\ \mathrm{MeV}$
What happens when we change the upper limit of the integrals?
(Is it correct to do that?)
upper limit: x=8



Results

$N_f = 4$, 4^3x16
$a = 0.3288$ fm, $T \approx 37.5$ MeV
What happens when we change the upper limit of the integrals?
(Is it correct to do that?)
upper limit: x=10



Results

$N_f = 4$, 4^3x16

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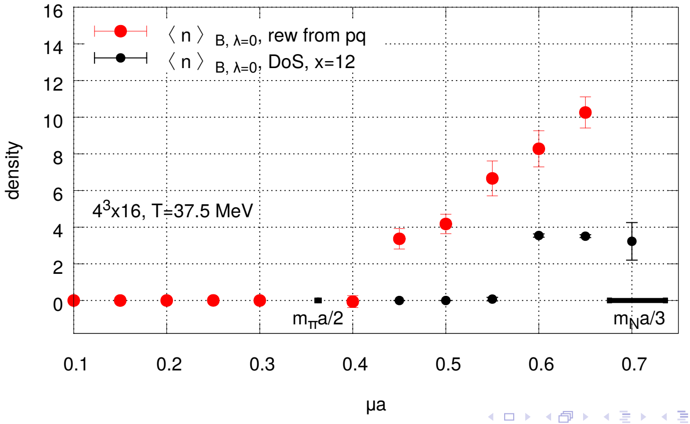
What happens when we change the upper limit of the integrals?

(Is it correct to do that?)

upper limit: x=12



Results

$N_f = 4$, $4^3 x16$
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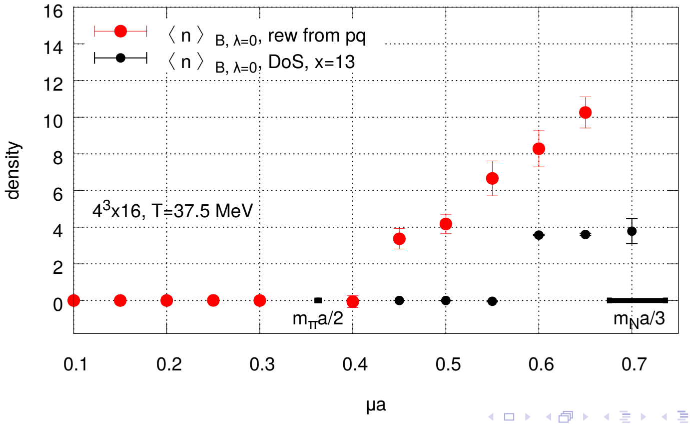
What happens when we change the upper limit of the integrals?
(Is it correct to do that?)
upper limit: x=13



Results

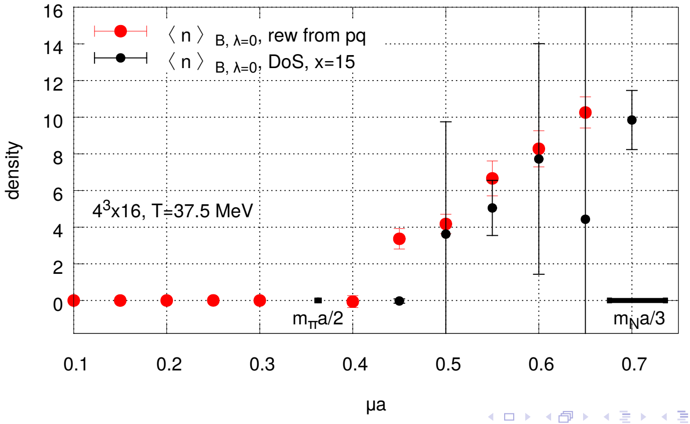
$N_f = 4$, 4^3x16

$a = 0.3288\,\text{fm}$, $\text{T} \approx 37.5\,\text{MeV}$

What happens when we change the upper limit of the integrals?

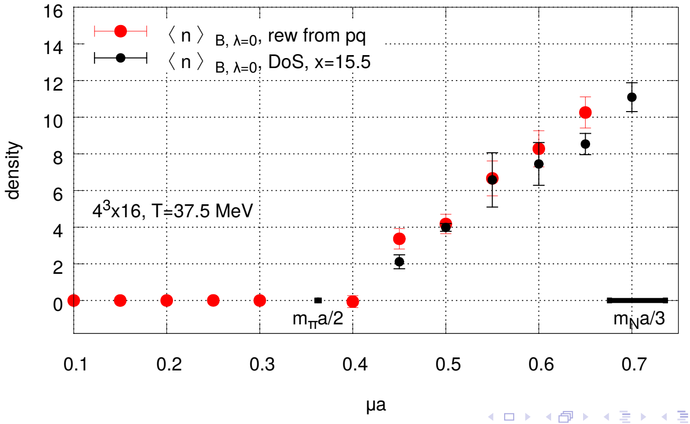
(Is it correct to do that?)

upper limit: x=15



Results

$N_f = 4$, 4^3x16
$a = 0.3288\,\mathrm{fm}$, $\mathrm{T} \approx 37.5\,\mathrm{MeV}$
What happens when we change the upper limit of the integrals?
(Is it correct to do that?)
upper limit: x=15.5



Results

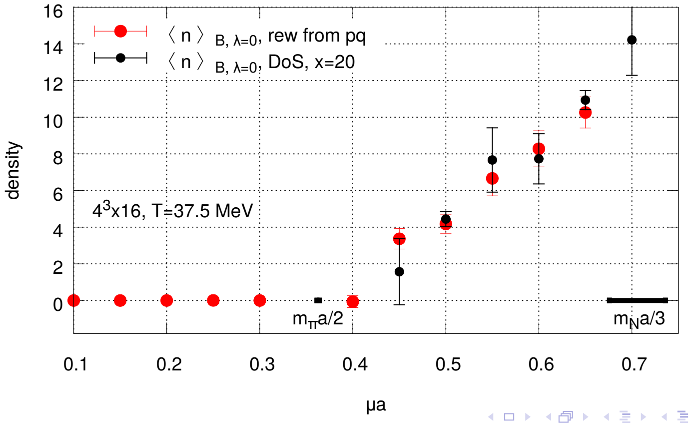
$N_f = 4$, 4^3x16

$a = 0.3288\,\text{fm}$, $\mathrm{T} \approx 37.5\,\mathrm{MeV}$

What happens when we change the upper limit of the integrals?

(Is it correct to do that?)

upper limit: x=20



Summary

Observations

- Full DoS integral seems to give the same results as reweighting from phasequenched theory
- By suppressing the condensation of pions in QCD_I we can get closer to QCD_B

Future directions

- study the effect of λ and P (rew. to finite λ, $\lambda \to 0$, $P \to \infty$)
- larger lattices and/or finer lattices
- compare the results to those of free hadrons on the lattice
- ...

Summary

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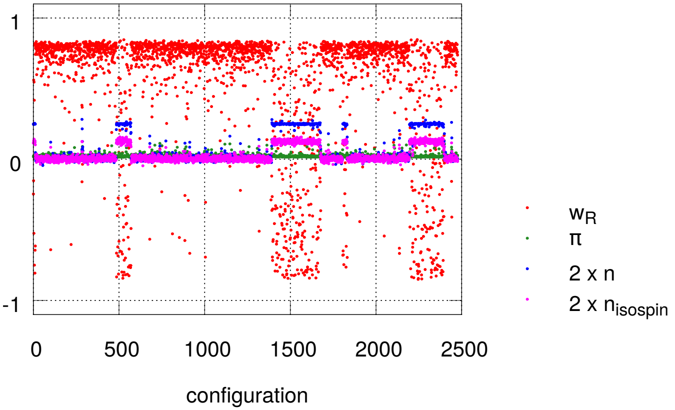
Thank you for your attention!

Backups

Results – constraining the gauge action

As μa approaches $m_N a/3$ from below: errors are sometimes unreliable. (Simulations can "freeze" to a wrong region, where the weights are smaller.)

On $4^3 \times 16$ lattices ($\mu/m_\pi \approx 0.76$, $\mu/m_N \approx 0.25$):



Results – constraining the gauge action

Above the onset of QCD_B, the problem seems to be absent (or milder).

$N_f = 4, \quad 6^3 x8$
$a = 0.333\,\mathrm{fm},\ \mathrm{T} \approx 74\,\mathrm{MeV}$