Hadronic vacuum polarization contribution to muon g-2 from four flavors of HISQ quarks

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(for the Fermilab Lattice, HPQCD, & MILC Collaborations)

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Participants

Fermilab Lattice Collaboration

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- Andreas Kronfeld (Fermilab)
- Ethan Neil (Colorado)
- Ruth Van de Water (Fermilab)

HPQCD

- Bipasha Chakraborty (JLAB)
- Daniel Hatton (Glasgow)
- Christine Davies (Glasgow)
- Jonna Koponen (INFN, Rome)
- Peter Lepage (Cornell)
- Andrew Lytle (Glasgow)
- Craig McNeile (Plymouth)

MILC Collaboration

- Carleton DeTar (Utah)
- Steve Gottlieb (Indiana)
- Jack Laiho (Syracuse)
- Yuzhi Liu (Indiana)
- Doug Toussaint (Arizona)
- Alejandro Vaquero (Utah)

Motivation

- Muon anomalous magnetic moment (g-2) provides sensitive probe of physics beyond the Standard Model:
  - Mediated by quantum-mechanical loops
  - Known to very high precision of 0.54ppm
- Measurement from BNL E821 disagrees with Standard-Model theory expectations by more than 3σ
- Muon g-2 Experiment at Fermilab aims to reduce the experimental error by a factor of four
  - Began running this month, and expect first results in Spring 2018!
- **Theory error must be reduced to a commensurate level to identify definitively whether any deviation observed between theory and experiment is due to new particles or forces**

> We are using *ab-initio* lattice-QCD to target the hadronic vacuum-polarization (HVP) contribution, which is the largest source of theory error

HPQCD work on g-2

- **Introduce "time moments" method to obtain precise determination of a_μ^{HVP} from zero-momentum vector-current correlation functions** [PRD89, no. 11, 114501 (2014)]:

  (1) Calculate derivatives of vacuum polarization function $\Pi(q^2)$ at $q^2=0$ from time moments of vector-current correlators

  (2) Use Taylor coefficients to construct [n,n] and [n,n-1] Padé approximants for renormalized vacuum polarization function

- Employ large set of four-flavor MILC HISQ ensembles with three lattice spacings, multiple spatial volumes, & physical light-quark masses to control systematic uncertainties

- **Obtain precise results for s-,c-, and b-quark connected contributions and bound on quark-disconnected contribution** [PRD 89, no. 11, 114501 (2014); PRD 91, no. 7, 074514 (2015); HPQCD+HadSpec, PRD 93, no. 7, 074509 (2016)]

- Uncertainties on light-quark connected contribution and complete leading-order a_μ^{HVP} ~2%, and limited by omission of isospin breaking, electromagnetism, and quark-disconnected contributions [HPQCD+RV,1601.03071]

Fermilab-HPQCD-MILC data

- Fermilab Lattice, HPQCD, & MILC have joined efforts to improve HPQCD's results and meet target experimental precision
- Have extended vector-current correlator data set from 1601.03071 with:
  - N_f=2+1+1 a~0.15 fm physical-mass ensemble with better quark masses & statistics
  - N_f=1+1+1+1 a~0.15 fm physical-mass ensemble with three valence-quark masses

$\approx a$ (fm)	$am^{\rm sea}_{l(u/d)}/am^{\rm sea}_s/am^{\rm sea}_c$	$Z_{V,\bar{s}s}$	M_{π_5} (GeV)	$\left(\frac{L}{a}\right)^3 \times \left(\frac{T}{a}\right)$	$N_{\rm conf.}$
0.15	0.01300/0.065/0.838	0.9881(10)	0.3020(20)	$16^3 \times 48$	9947
0.15	0.0064/0.064/0.828	0.9881(10)	0.2160(10)	$24^3 \times 48$	1000
0.15	0.00235/0.0647/0.831	0.9881(10)	0.1330(10)	$32^3 \times 48$	997
0.15	0.002426/0.0673/0.8447	0.9881(10)	0.1350(10)	$32^3 \times 48$	1902
0.15	0.001524/0.003328/0.0673/0.8447	0.9881(10)	~ 0.135	$32^3 \times 48$	1938
0.12	0.01020/0.0509/0.635	0.99220(40)	0.3010(20)	$24^3 \times 64$	1053
0.12	0.00507/0.0507/0.628	0.99220(40)	0.2180(10)	$24^3 \times 64$	1020
0.12	0.00507/0.0507/0.628	0.99220(40)	0.2170(10)	$32^3 \times 64$	1000
0.12	0.00507/0.0507/0.628	0.99220(40)	0.2160(10)	$40^3 \times 64$	331
0.12	0.00184/0.0507/0.628	0.99220(40)	0.1330(10)	$48^3 \times 64$	998
0.09	0.00740/0.037/0.440	0.99400(50)	0.3080(20)	$32^3 \times 96$	1000
0.09	0.00363/0.0363/0.430	0.99400(50)	0.2190(10)	$48^3 \times 96$	298

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Also updated Z_V values from HPQCD, 1703.05522

Noise-reduction strategy

✦ Reduce statistical errors in a_μ as in 1601.03071 by:

(1) Simultaneous fit of 4 combinations of (local, smeared) correlators G_{ij}

❖ Employ cosh parameterization that accounts for periodic temporal boundary conditions

(2) Replacing $G_{data}(t)$ with $G_{fit}(t)$ for $t > t^*$

❖ Correct for finite temporal extent by calculating $G_{fit}(t)$ using 2-point fit parameters in exponential parameterization and extending times in $G_{fit}(t)$ to 2 x T

$$G(t) = \begin{cases} G_{\rm data}(t) & t \leq t^* \\ G_{\rm fit}(t) & t > t^* \end{cases}$$

✦ Choose t^* such that value of a_μ^{HVP} comes primarily from data region ($t < t^*$), but before errors in a_μ^{HVP} begin increasing rapidly

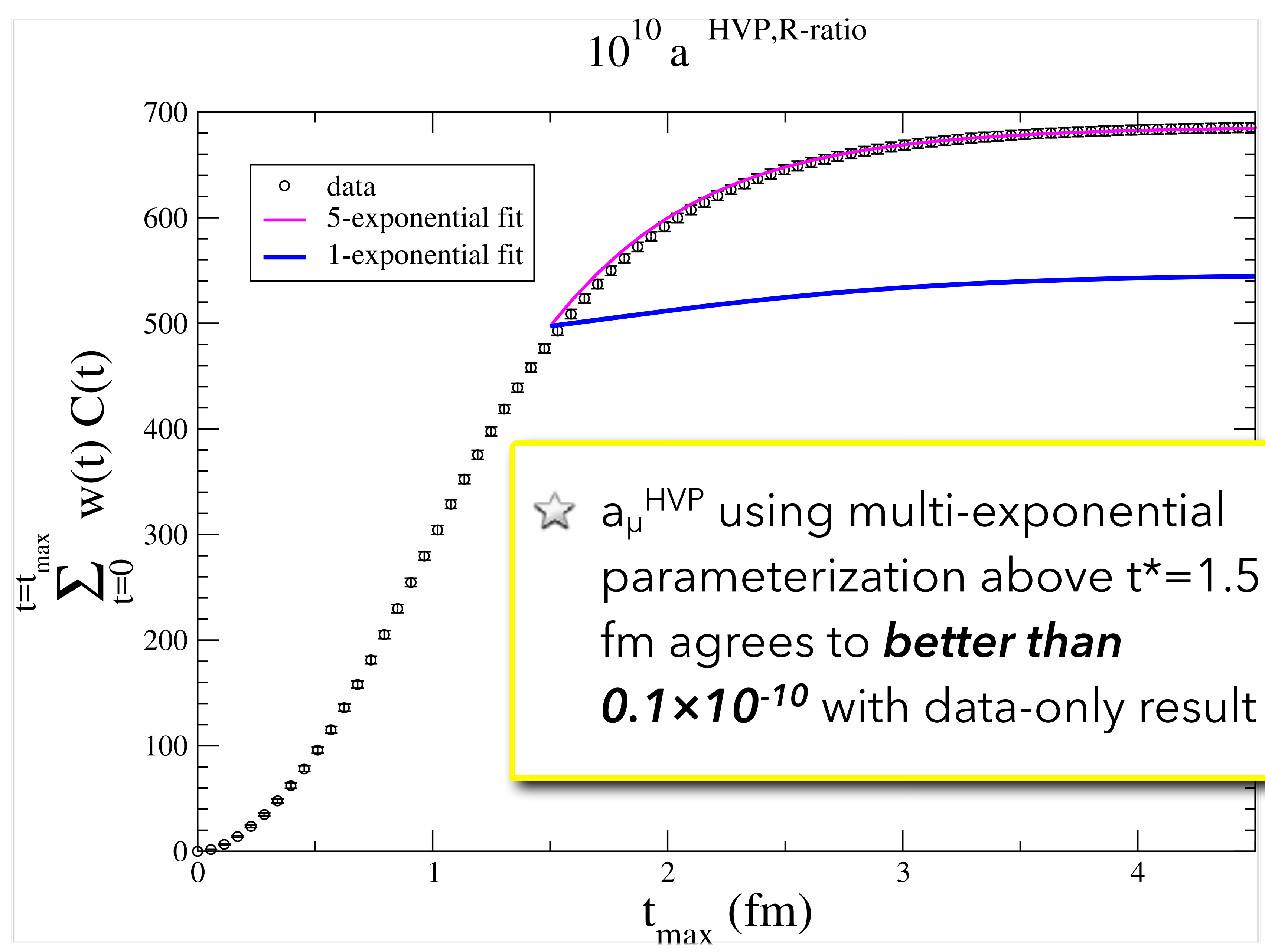
❖ With **t^* = 1.5 fm**, the data contribution is ≳ 80% for all ensembles

☆ *Check:* a_μ^{HVP} computed via time-momentum-representation weighted sum and via time-moments + [3,3] Padé approximants agree to < 0.1 x 10^{-10}

Test with e^+e^- (*"R-ratio"*) data

- Fit e^+e^- data to multi-exponential parameterization and compute a_μ using data below 1.5 fm and fit above 1.5 fm

➡ Method yields correct result provided use of multi-exponential parameterization that accurately describes data above transition time



☆ a_μ^{HVP} using multi-exponential parameterization above t*=1.5 fm agrees to ***better than 0.1×10⁻¹⁰*** with data-only result

☆ Thanks to F. Jegerlehner for compiling e^+e^- data in public alphaQED fortran package!

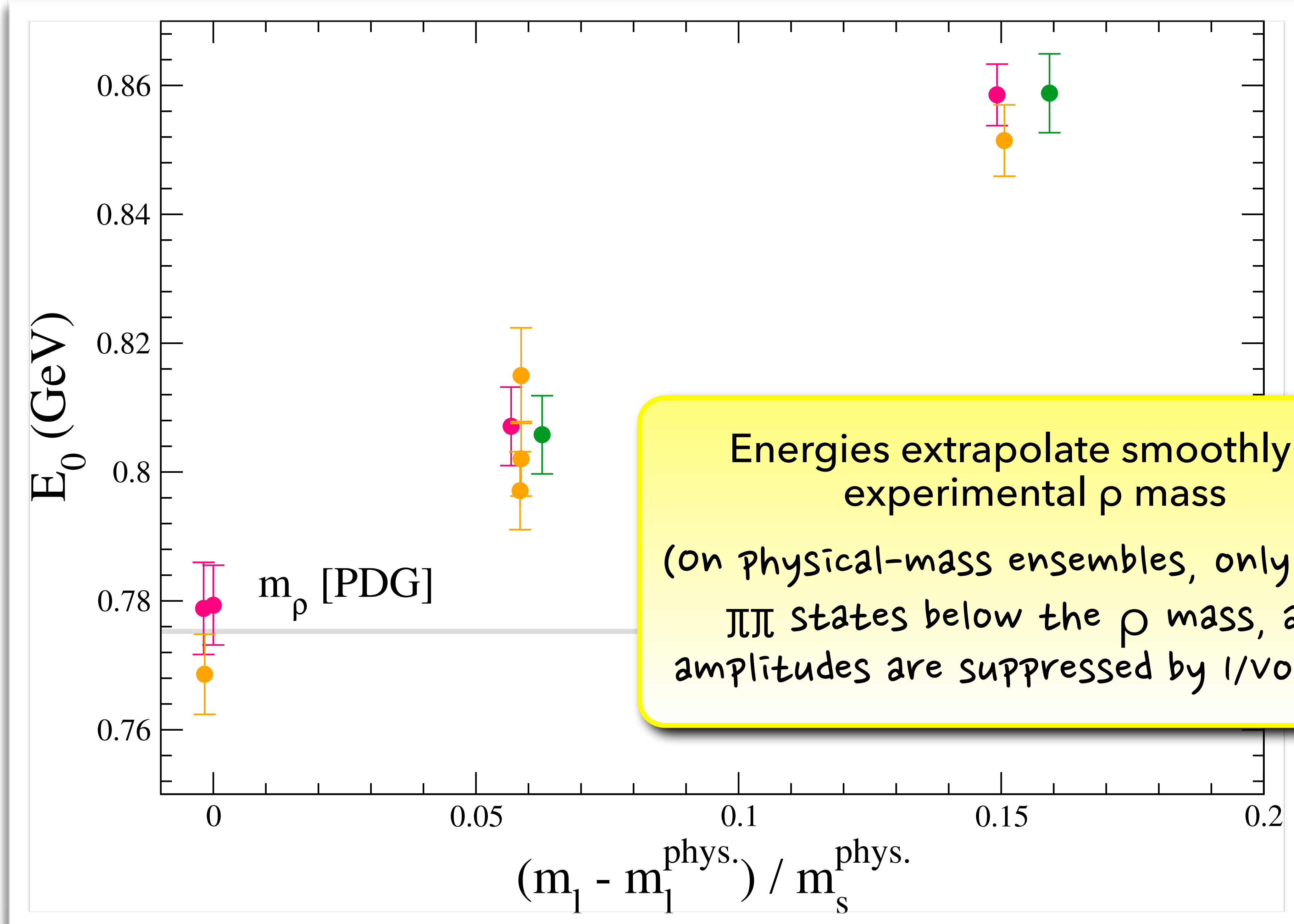
Correlator fits

Simultaneous fit of four combinations of (local, smeared) correlators

$$G_{ij}(t) = a^3 \sum_{k=0}^{N-1} b_i^{(k)}\, b_j^{(k)} \left(e^{-E^{(k)}t} + e^{-E^{(k)}(T-t)}\right) - (-1)^t a^3 \sum_{k=0}^{N-1} d_i^{(k)}\, d_j^{(k)} \left(e^{-\tilde{E}^{(k)}t} + e^{-\tilde{E}^{(k)}(T-t)}\right)$$

- Constrain energies & amplitudes with Gaussian priors
- Employ SVD cuts to reduce d.o.f. and improve reliability of correlation matrix
  - Conservative approach replaces eigenvalues of correlation matrix below SVD cut by the SVD cut times the maximum eigenvalue, thereby increasing fit error
- Choose number of states & fit range based on stability of E_0, A_0, E_1, & goodness-of-fit
  - Fitted ground-state energies & errors (mostly) insensitive to t_{max} ➡ **fit between (t_{min}, T-t_{min}) to ensure that fit describes correlator over entire lattice time extent**
  - t_{min}/a = [3,4,5] for a~[0.15, 0.12, 0.09] fm ➡ **t_{min}~0.45 fm for all lattice spacings**
  - **For all ensembles, obtain good χ^2/dof and stable fit results with $N_{states} \geq 3$**

Ground-state energies

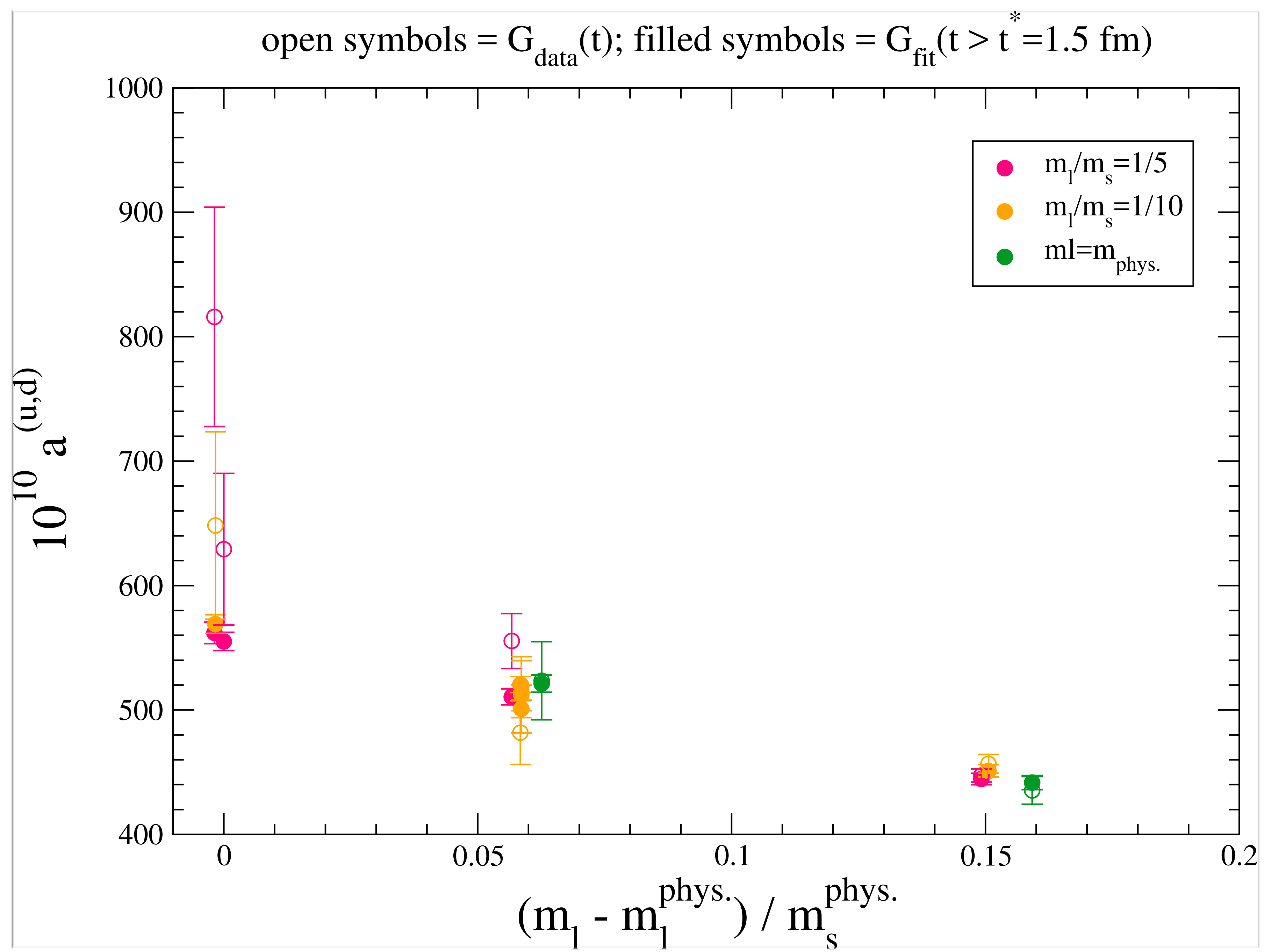


Energies extrapolate smoothly to experimental ρ mass

(on physical-mass ensembles, only a few ππ states below the ρ mass, and amplitudes are suppressed by 1/volume)

Check: comparison with a_μ from data

- a_μ^{HVP} computed with $G_{fit}(t)$ for $t > 1.5$ fm consistent with data within 1σ on 8/11 ensembles
- Use of fitted correlator above $t^*=1.5$ fm:



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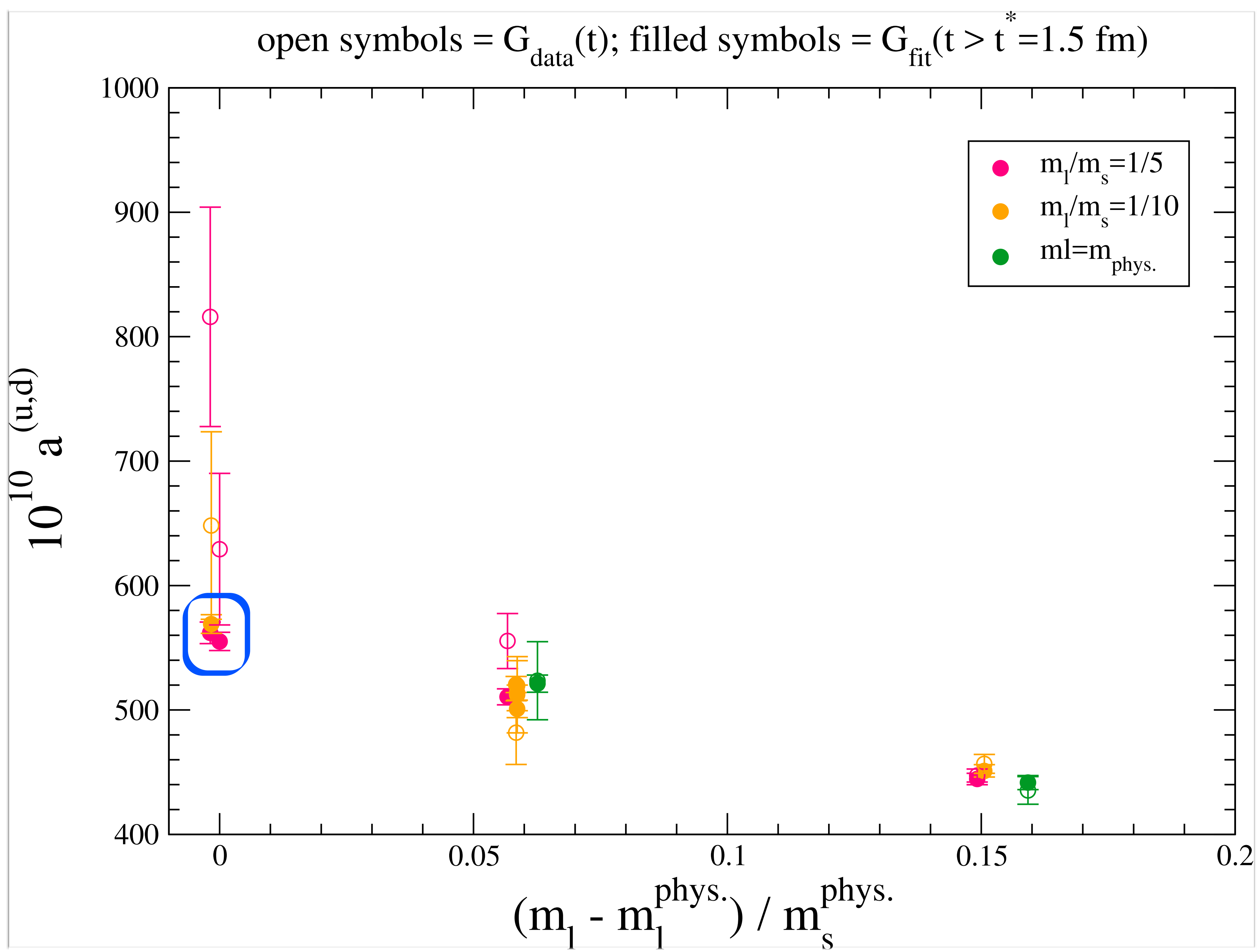
  - Brings physical-mass a_μ^{HVP} values into agreement at sub-percent level



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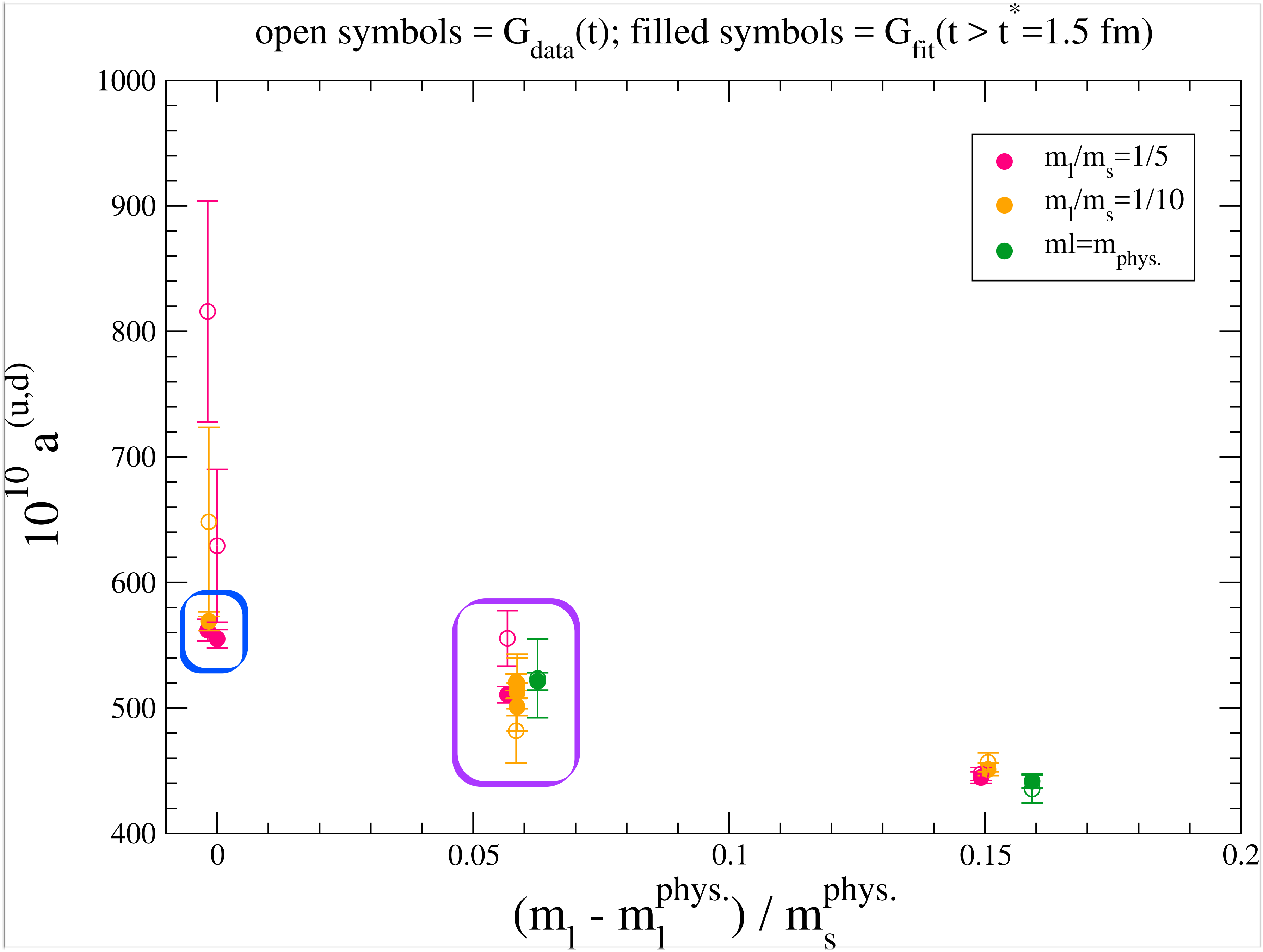
  - Brings coarse m_l=m_s/10 results on different spatial volumes even closer



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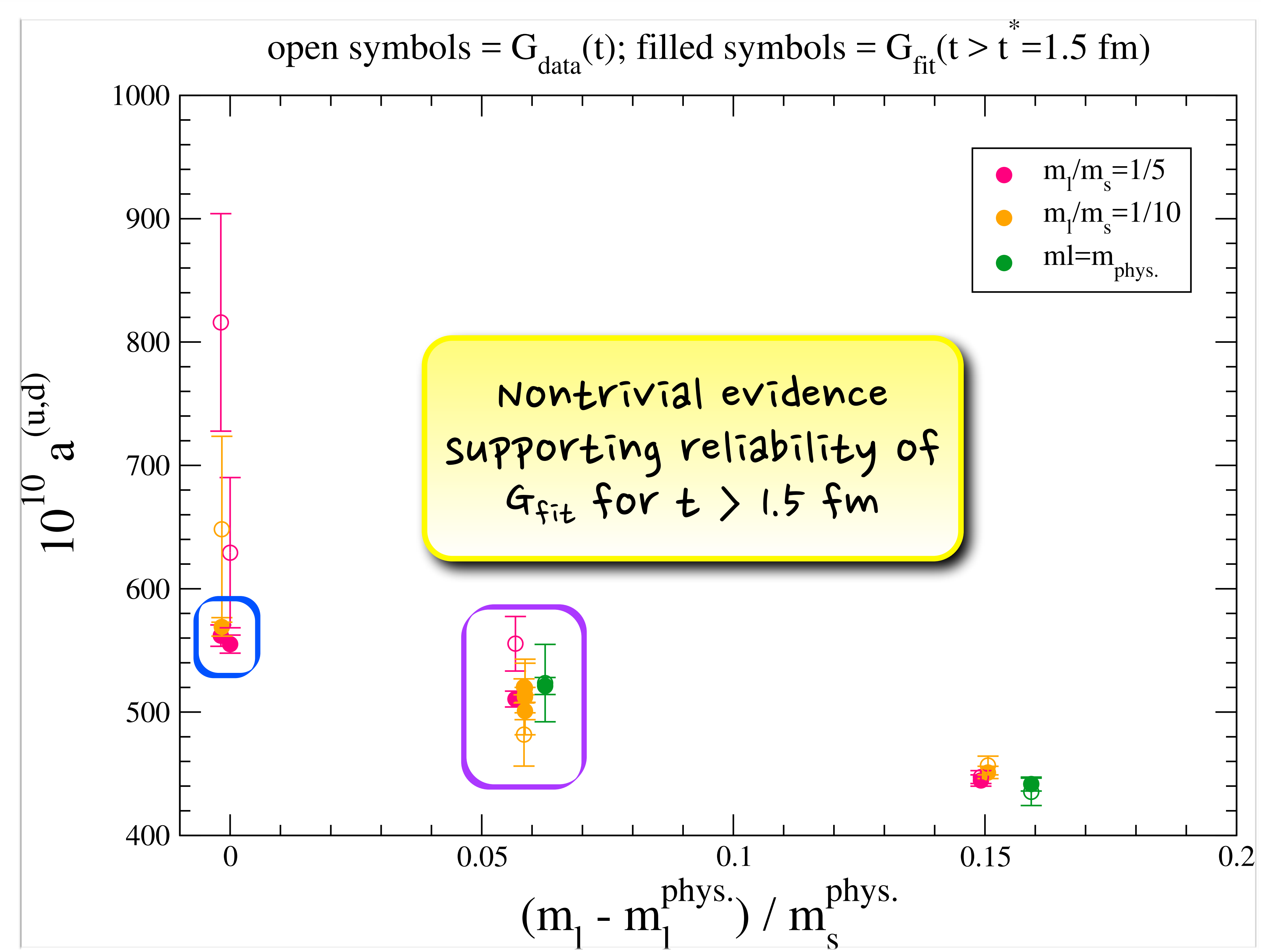


Lattice corrections to a_μ

(1) Rescale Π_{ij}s by $(m_\rho^{lat.}/m_\rho^{exp.})^{2i}$ and calculate a_μ^{HVP}

- Use E_0 values from 2-point correlator fits, which are consistent with experiment
- **Substantially reduces light-quark-mass dependence** (and lattice-spacing error)

(2) Correct a_μ^{HVP} for finite-volume and taste-breaking discretization effects

- Use extended chiral perturbation theory that includes π's, ρ's, and γ's [Jegerlehner & Szafron, EPJC71 1632 (2011)]
- Calculate 1-pion-loop contributions to a_μ^{HVP} to all orders in leading interactions that couple ρ^0-γ-$\pi^+\pi^-$ channels
- **Account for finite lattice spatial volume** by replacing momentum integral with sum (*assume infinite temporal extent*) and **account for taste breaking** by averaging contributions from all pion-taste pairings

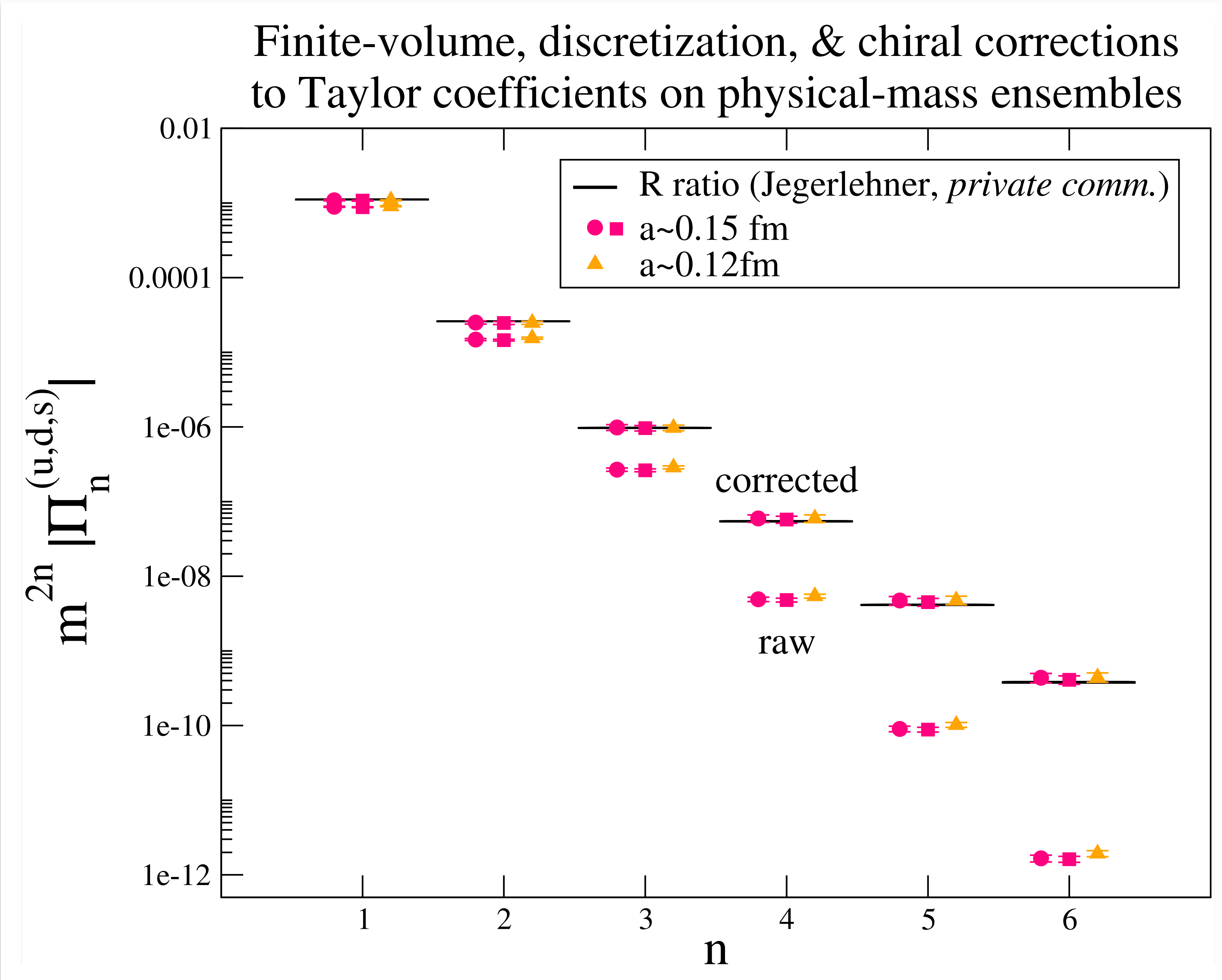
(3) Subtract lattice (infinite-volume) $\gamma \rightarrow \pi^+\pi^- \rightarrow \gamma$ vacuum polarization contribution and add back physical $\pi^+\pi^-$ contribution to a_μ^{HVP} (also estimated in 1-loop scalar QED)

Apply corrections (2) and (3) to a_μ^{HVP} rather than moments as in 1601.03071; take 10% error on $\Delta a_\mu^{FV+disc.}$ to account for higher-order terms in chiral expansion

Test of lattice corrections

- Compare Taylor coefficients on physical-mass ensembles with values obtained from experimental $e^+e^- \to$ hadrons data
- **Corrections bring sum of u/d/s/c/b-quark Πs into agreement with experiment**

➡ Evidence that scalar-QED calculation of finite-volume + discretization corrections is reliable ✔



Chiral-continuum fit

Same fit function & constraints as in 1601.03071

$$a_\mu = a_\mu^{\rm LO} \times \left(1 + c_\ell \frac{\delta m_\ell}{\Lambda} + c_s \frac{\delta m_s}{\Lambda} + \tilde{c}_\ell \frac{\delta m_\ell}{m_\ell} + c_{a^2} \frac{(a\Lambda)^2}{\pi^2}\right)$$

$$\delta m_f \equiv m_f - m_f^{\rm phys.}, \qquad \Lambda = 500 \text{ MeV}$$

Constraints

$$a_\mu^{\rm LO} = 600(200) \times 10^{-10}$$

$$c_\ell, c_{a^2} = 0(1), \quad c_s = 0.0(0.3), \quad \tilde{c}_\ell = 0.00(0.03)$$

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- c_s small because only enters through sea, and m_s well tuned

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- Account for residual generic and taste-breaking discretization errors

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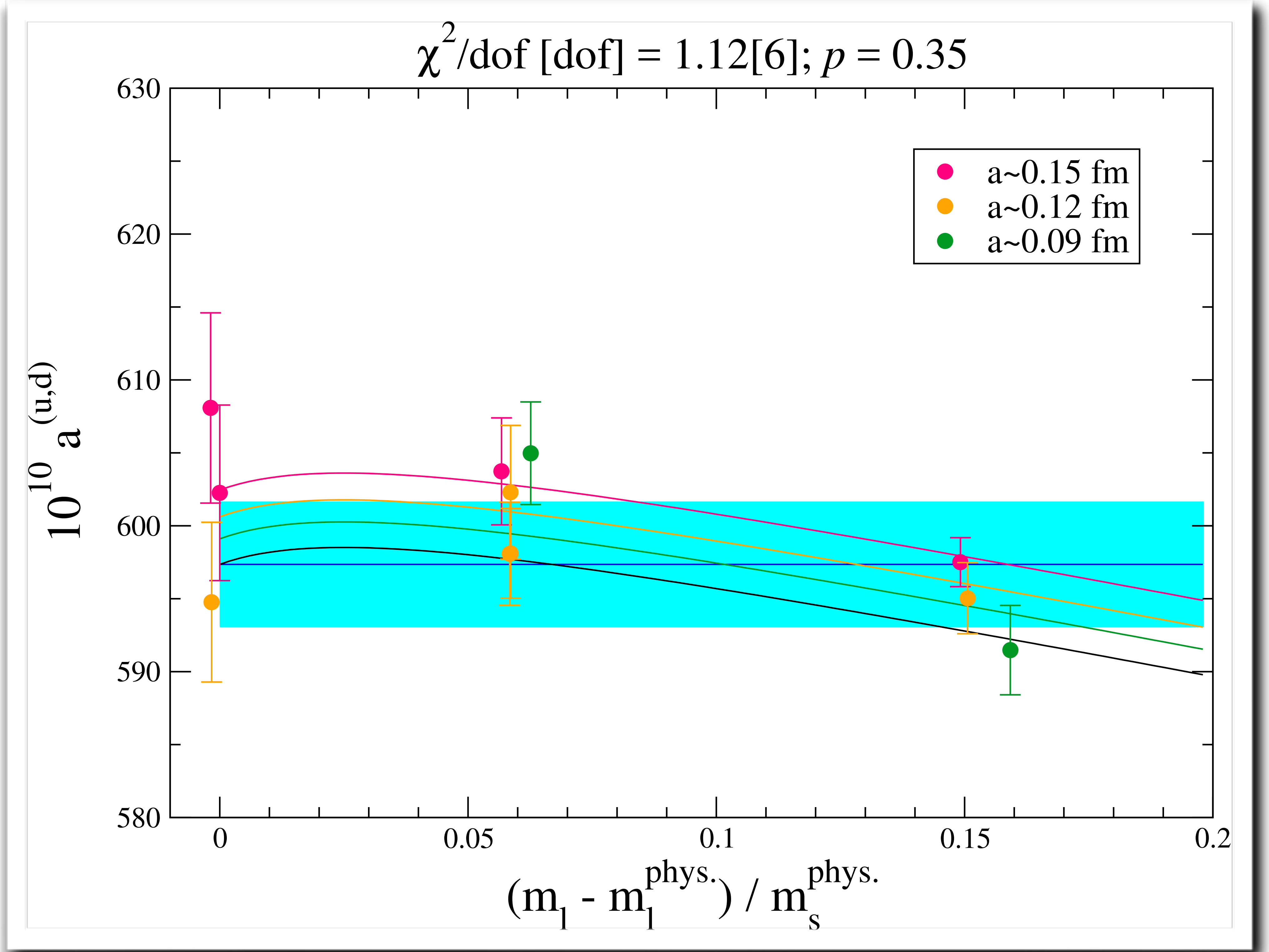
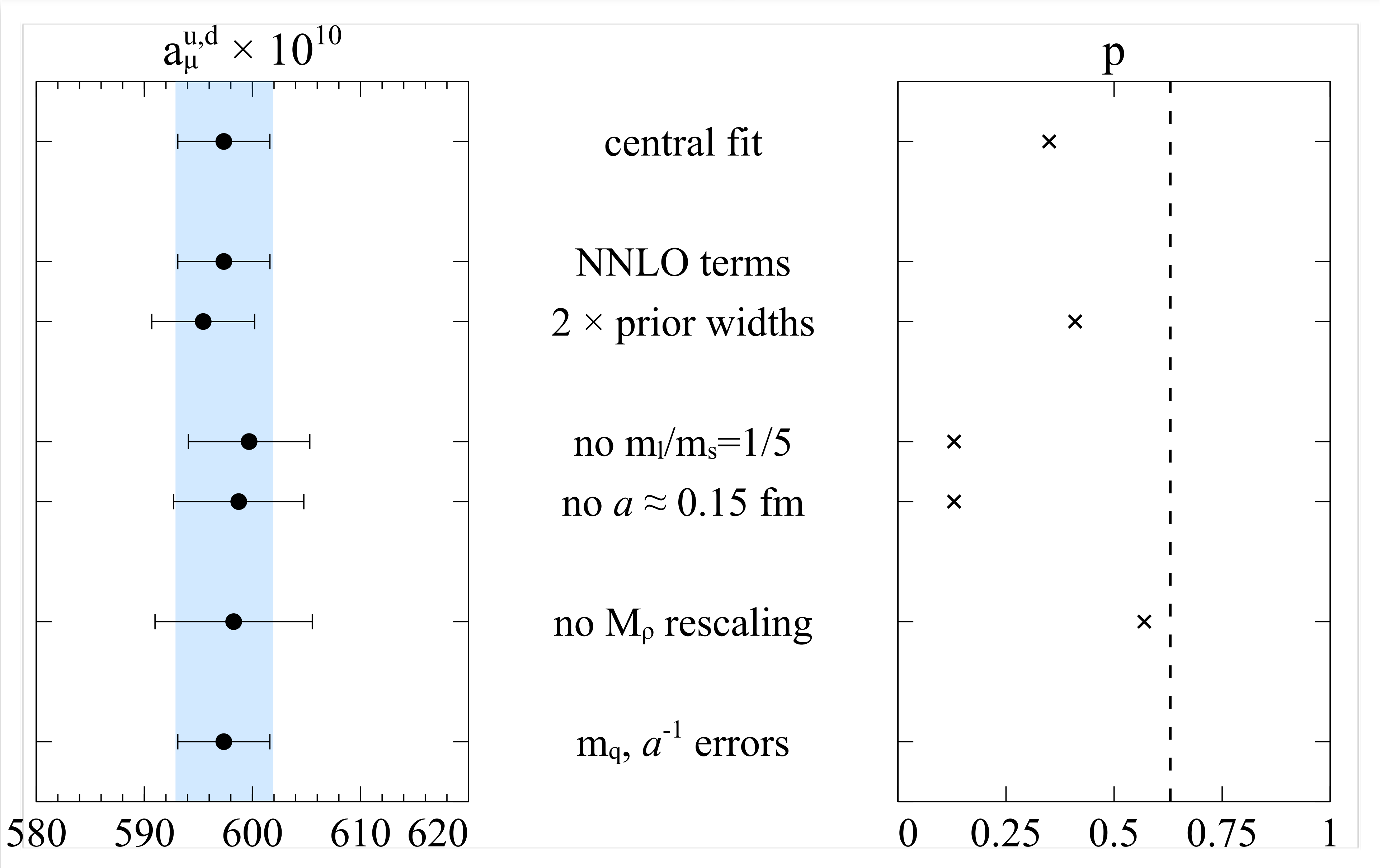
Light-quark-mass interpolation



Fit stability

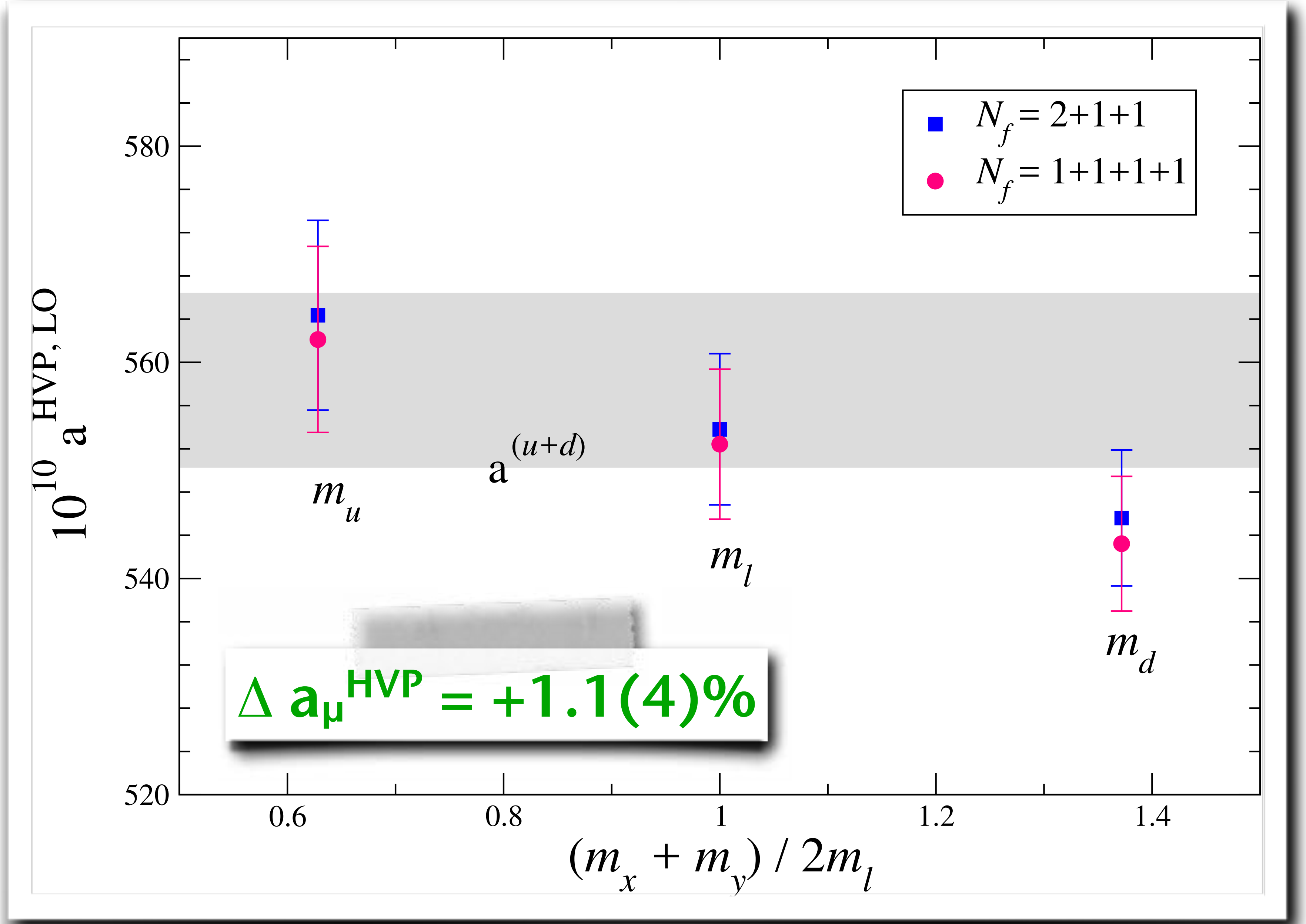
☆ Fit result stable with the addition of higher-order terms, increased prior widths, omission of data, M_ρ rescaling, …



New! Isospin-breaking correction

- Directly calculate correction to a_μ^{HVP} from $m_u \neq m_d$ on a~0.15 fm $N_f = 1+1+1+1$ (and $N_f = 2+1+1$) physical-mass ensemble

- Fix ratio m_u / m_d = 0.4582 to physical value from MILC quenched-QED analysis [1606.01228]
- Compute vector-current correlators with valence-quark masses m_u, $(m_u + m_d)/2$, & m_d
- Fit and analyze correlators together to preserve correlations



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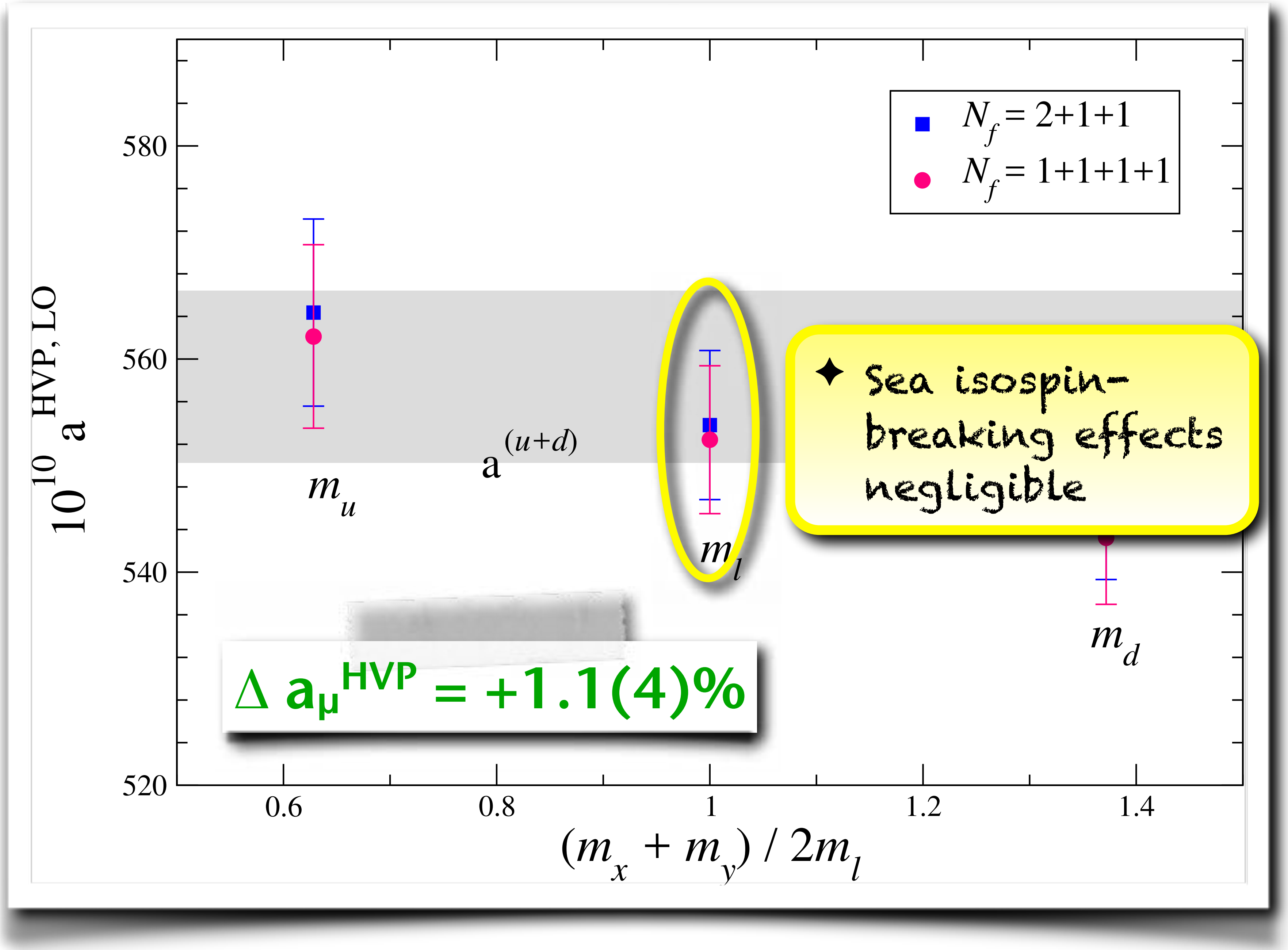

Error budget

- Largest uncertainty in light-quark-connected contribution from omission of quark electromagnetic charges ➡ take 1% QED error as in earlier analysis

	$a_\mu^{ud,\mathrm{HVP}}$ (%)	
	HPQCD + RV 1601.03071	Fermilab-HPQCD-MILC *Preliminary*
QED corrections	1.0	1.0
$\pi\pi$ states (t^*)	0.5	0.5
Statistics + 2pt fit	0.4	0.5
Isospin-breaking ($m_u \neq m_d$) corrections	1.0	0.4
Finite-volume & discretization corrections	0.7	0.4
Continuum ($a \to 0$) extrapolation	0.2	0.2
Chiral (m_l) extrapolation/interpolation	0.4	0.2
Current renormalization (Z_V)	0.2	0.2
Pion mass ($M_{\pi,5}$) uncertainty	–	0.1
Sea (m_s) adjustment	0.2	0.1
Experimental M_ρ	–	0.1
Padé approximants	0.4	0.0
Lattice-spacing (a^{-1}) uncertainty	<0.05	<0.00
Total	1.8%	1.4%

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Calculate isospin-breaking correction directly

	$a_\mu^{ud,\mathrm{HVP}}$ (%)	
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[illegible]	[illegible]	0.5
Statistics + 2pt fit	0.4	0.5
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Isospin-breaking ($m_u \neq m_d$) corrections	1.0	0.4
Finite-volume & discretization corrections	0.7	0.4
Continuum ($a \to 0$) extrapolation	[illegible]	[illegible]
Chiral (m_l) extrapolation/interpolation	[illegible]	[illegible]
Current renormalization (Z_V)	[illegible]	[illegible]
Pion mass ($M_{\pi,5}$) uncertainty	[illegible]	[illegible]
Sea (m_s) adjustment	0.2	0.1
Experimental M_ρ	–	0.1
Padé approximants	0.4	0.0
Lattice-spacing (a^{-1}) uncertainty	<0.05	<0.00
Total	1.8%	1.4%

Calculate isospin-breaking correction directly

Apply FV correction as total shift to a_μ rather than individual moments

Error budget

- Largest uncertainty in light-quark-connected contribution from omission of quark electromagnetic charges ➡ take 1% QED error as in earlier analysis

	$a_\mu^{ud,\mathrm{HVP}}$ (%)	
	HPQCD + RV 1601.03071	Fermilab-HPQCD-MILC *Preliminary*
QED corrections	1.0	1.0
[illegible]	[illegible]	0.5
Statistics + 2pt fit	0.4	0.5
Isospin-breaking ($m_u \neq m_d$) corrections	1.0	0.4
Finite-volume & discretization corrections	0.7	0.4
Continuum ($a \to 0$) extrapolation	[illegible]	[illegible]
Chiral (m_l) extrapolation/interpolation	[illegible]	[illegible]
Current renormalization (Z_V)	[illegible]	[illegible]
Pion mass ($M_{\pi,5}$) uncertainty	[illegible]	[illegible]
Sea (m_s) adjustment	[illegible]	[illegible]
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Padé approximants	0.4	0.0
Lattice spacing (a^{-1}) uncertainty	<0.05	<0.00
	1.8%	1.4%

Calculate isospin-breaking correction directly

Apply FV correction as total shift to a_μ rather than individual moments

Use [3,3] Padés

Work in progress

- Current and proposed running focused on reducing leading sources of error from

(1) Omission of electromagnetism,

- MILC will soon begin generating a dynamical QED+QCD ensemble with a~0.15 fm at the physical pion mass
- Will analyze these configurations immediately including nonzero valence-quark charges to obtain direct estimate of EM effects

(2) Omission of the quark-disconnected contribution, and

- Contributes roughly half of total uncertainty on $a_\mu^{HVP,LO}$
- Have added components for eigenvector deflation to MILC code and are testing and tuning parameters on a~0.15 fm physical-mass ensemble

(3) Finite spatial volume and staggered discretization effects

- Corrections largely from taste splittings between staggered pions in the sea, and become smaller and better controlled as the continuum limit is approached
- Generating vector-current correlators on physical-mass ensembles with a~0.09 & 0.06 fm (*root-mean-squared pion mass @ a~0.06 fm is ~140 MeV*)

Summary and outlook

- Analysis has begun from Fermilab Lattice-HPQCD-MILC effort to calculate hadronic vacuum polarization contribution to muon g-2
  - Key achievement is first direct lattice-QCD calculation of isospin-breaking effects at the physical pion mass

$$\Delta a_\mu^{\mathrm{HVP}, m_u \neq m_d} = +1.1(4)\%$$

PRELIMINARY

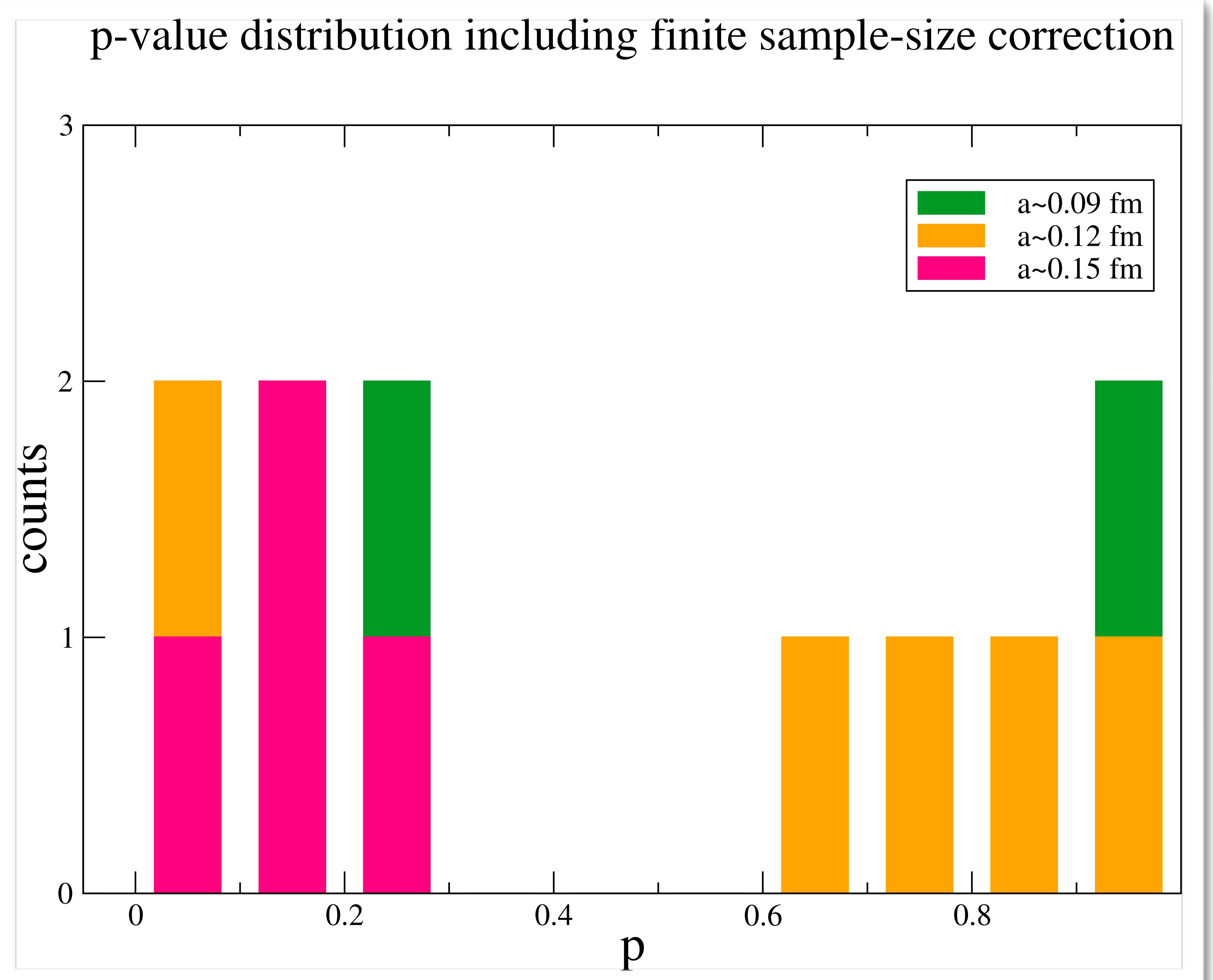
  - Compatible with RBC/UKQCD $\Delta a_\mu = +0.9\%$ from $M_\pi \sim 340$ MeV [1706.05293]
- Inclusion of isospin-breaking correction (*plus other incremental improvements*) reduces error on light-quark connected contribution to a_μ^{HVP} to 1.4%
  - Tests for stability of results, comparison with experimental $e^+e^- \to$ hadrons data, and other consistency checks substantiate methodology & error estimate
- Current and proposed running focus on reducing leading sources of error in HPQCD+RV result for $a_\mu^{\mathrm{HVP,LO}}$ from **omission of electromagnetism & quark-disconnected contribution**, and **finite spatial volume & staggered discretization effects**

> With direct determinations of these corrections / contributions in hand, plus data at finer lattice spacings, hope to obtain sub-per cent precision in the coming 1 or 2 years!

Extras

Goodness-of-fit

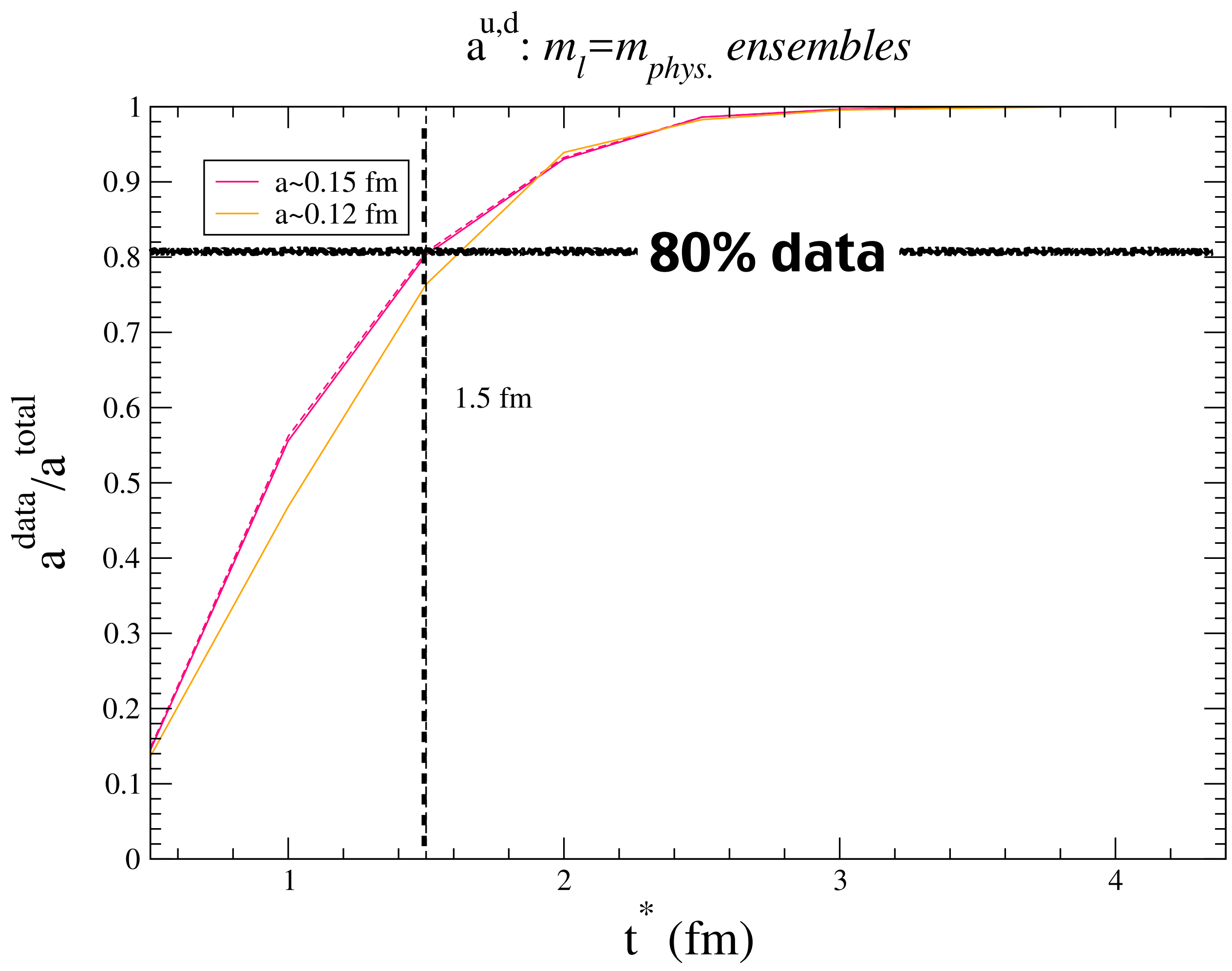
- Two-point fits provide good description of data as measured by $\chi^2_{data}/(N_{data}-N_{params})$ and confidence level
- p-value distribution reasonably uniform given small number of fits



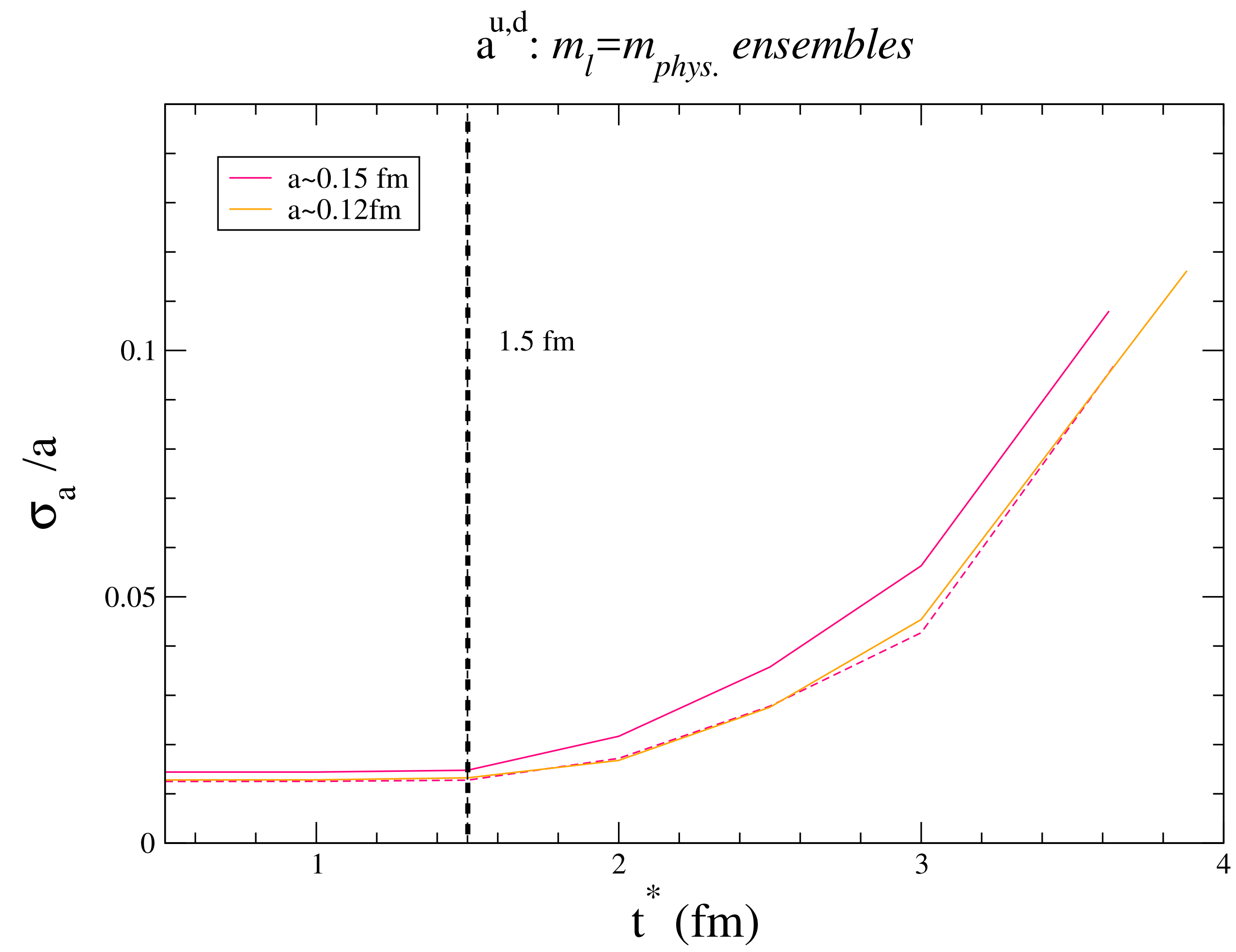
Selection of t*

- Choose t* such that value of a_μ^{HVP} comes primarily from data region (t< t*), but before errors in a_μ^{HVP} begin increasing rapidly
  - With **t* = 1.5 fm**, the data contribution is ≳ 80% for all ensembles

Data contribution



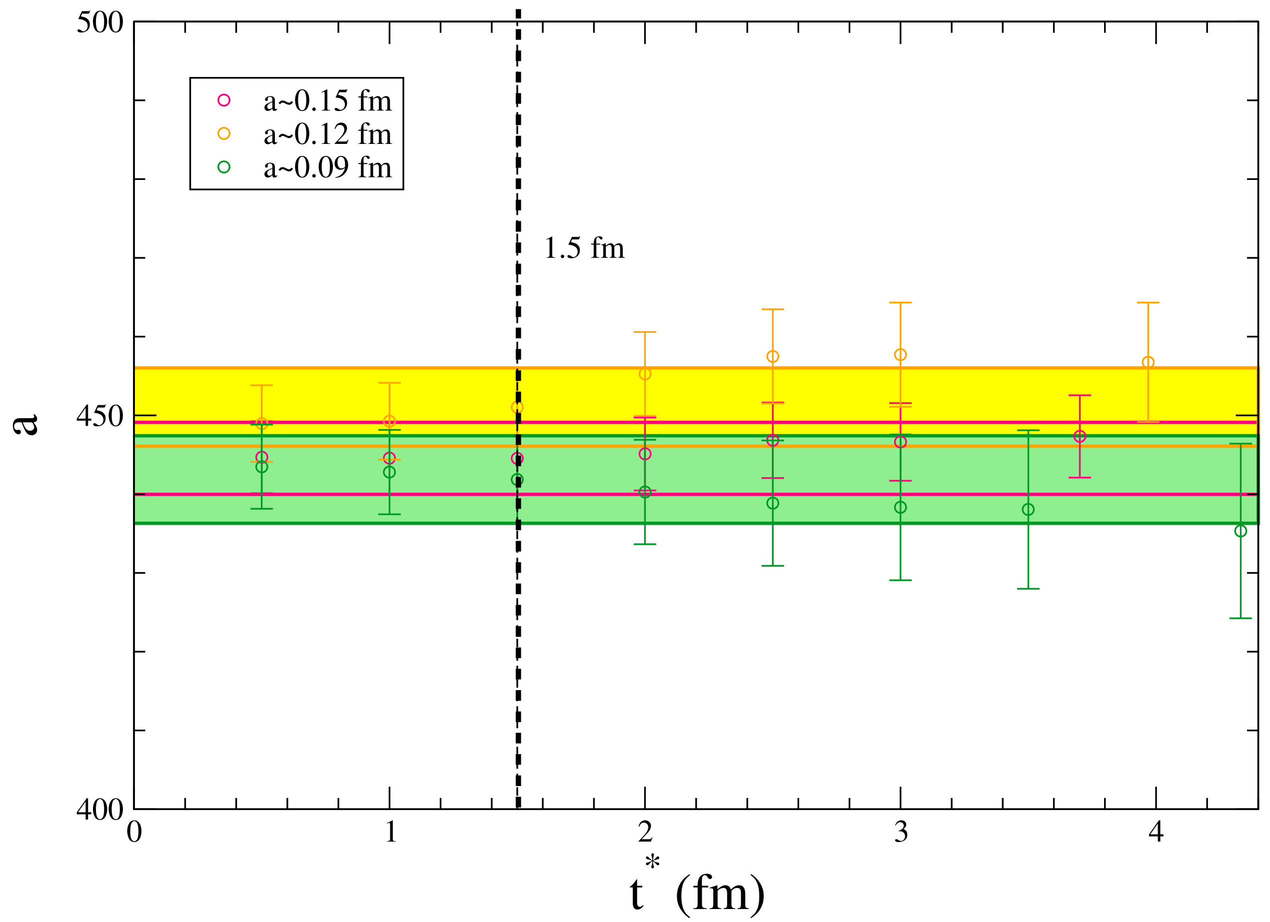
Relative error



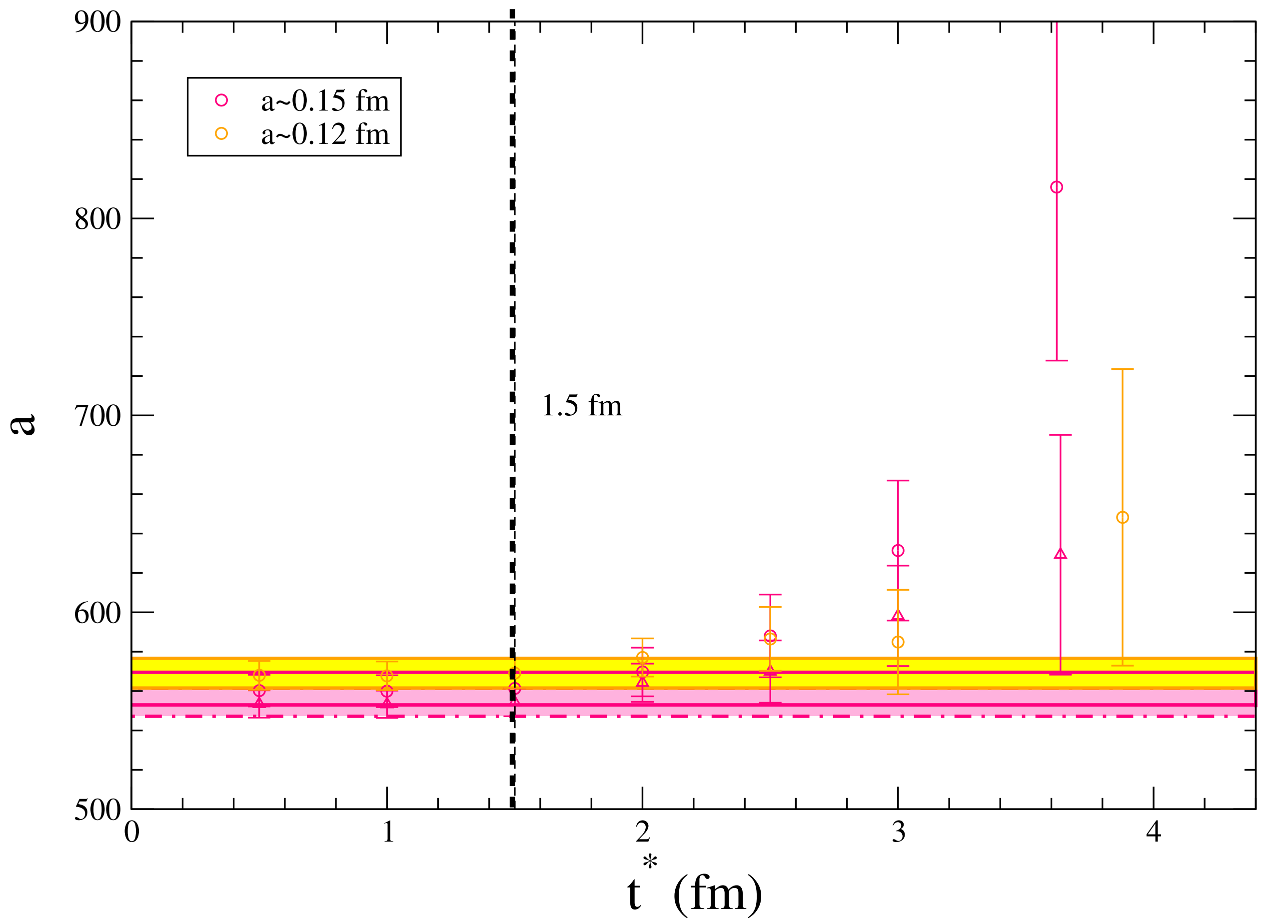
Noise-reduction check: a_μ vs. t^*

- a_μ^{HVP} independent of t^* from $t^* = 0.5$ fm (<20% data) to $t^* \sim 2.0$ fm (~95% data)

$m_l/m_s = 1/5$



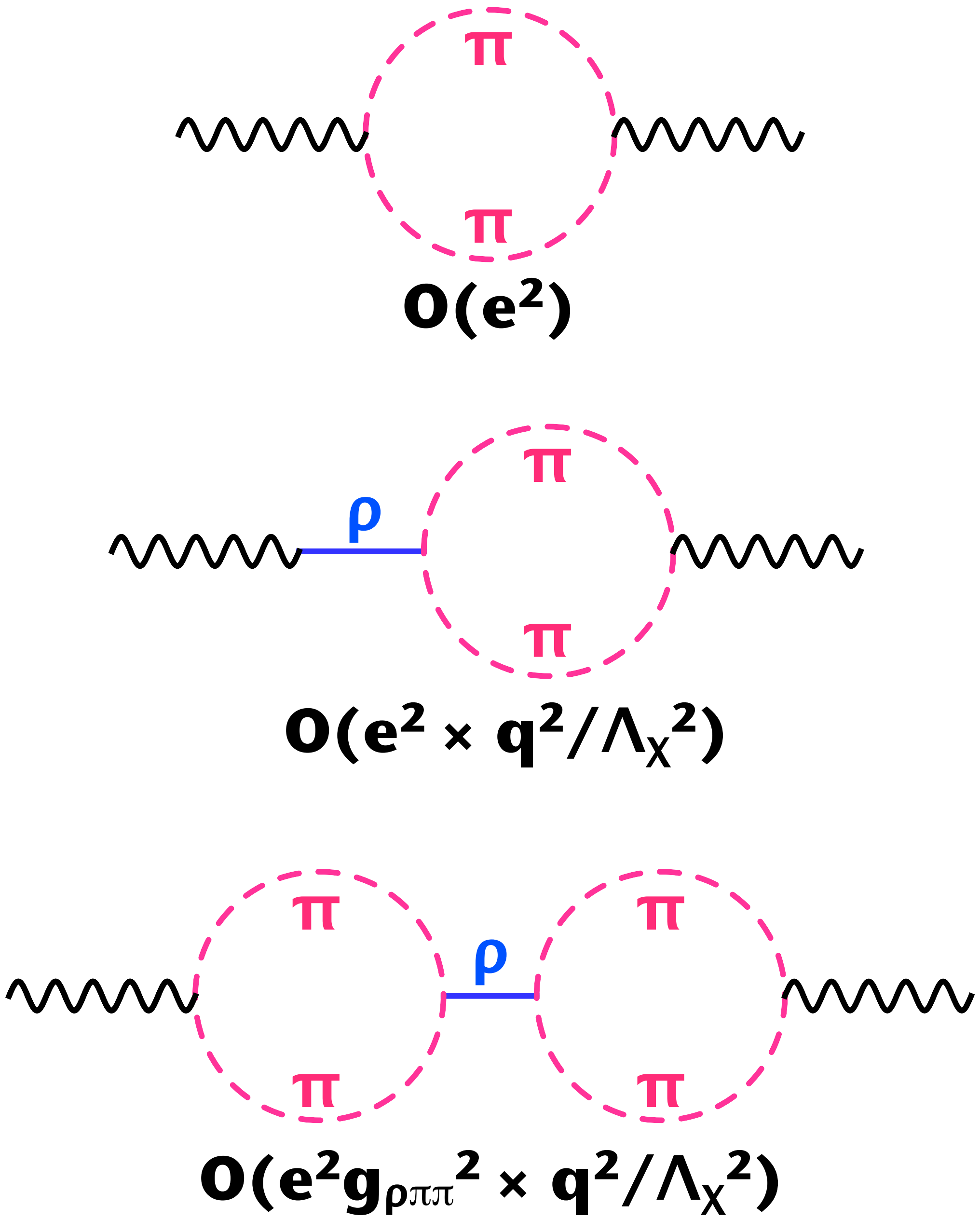
$m_l = m_{phys.}$



Calculation of lattice corrections

- **Use extended chiral perturbation theory that includes π's, ρ's, and γ's** [Jegerlehner & Szafron, EPJC71 1632 (2011)]
  - Focus on pion-loop diagrams sensitive to spatial volume & sea-pion masses
  - **Calculate contributions to a_μ^{HVP} to all orders in leading interactions that couple ρ^0-γ-$\pi^+\pi^-$ channels**
- Corrections given by difference between results in infinite volume / continuum, and in finite volume with lattice artifacts
  - Contribution from leading ππ bubble about 5× larger than from diagrams with ρ meson
  - **Corrections largely from taste splittings, and decrease with lattice spacing**
  - Take 10% uncertainty on corrections to account for higher-order terms in chiral expansion suppressed by $m_q/\Lambda_\chi, q^2/\Lambda_\chi^2$

(diagrams below + all iterations of these diagrams)



Summary of lattice corrections

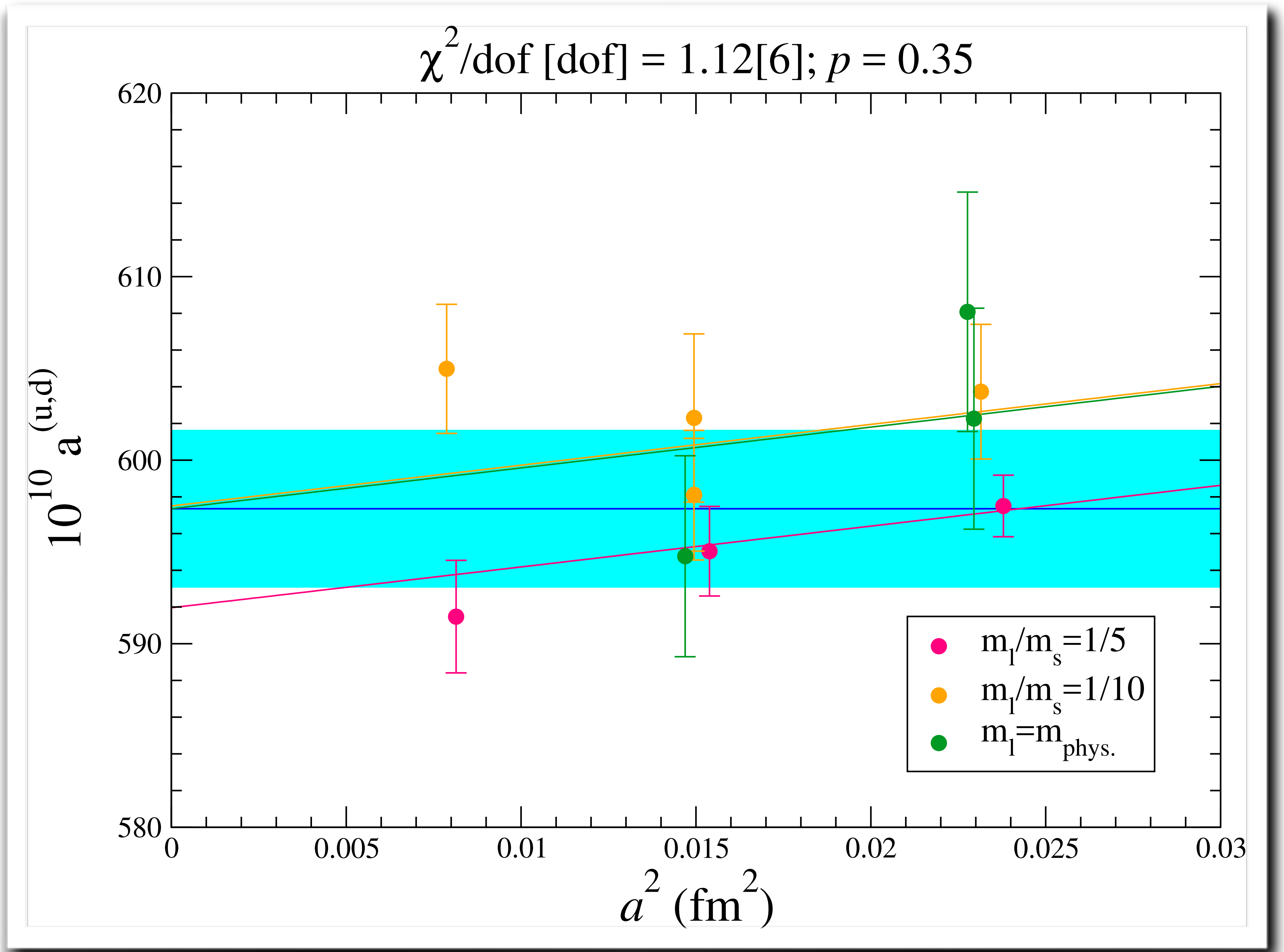
Ensemble	raw	+rescaling	$+\pi^+\pi^-$	+ FV,disc
	$10^{10} a_\mu^{\rm HVP}$			
l1648f211b580m01300m0650m838	444.5(4.6)	536.9(1.5)	590.6(1.5)	597.5(1.7)
l2448f211b580m0064m0640m828	510.2(6.3)	549.5(3.3)	588.3(3.3)	603.7(3.7)
l3248f211b580m00235m0647m831	561.3(8.3)	566.1(4.2)	559.8(4.3)	608.1(6.5)
l3248f211b580m002426m06730m8447	554.3(7.1)	559.7(3.5)	555.4(3.7)	602.3(6.0)
l2464f211b600m01020m0509m635	451.1(5.0)	536.5(2.4)	590.1(2.4)	595.0(2.4)
l2464f211b600m00507m0507m628	500.9(7.0)	549.3(4.4)	588.5(4.4)	602.3(4.6)
l3264f211b600m00507m0507m628	513.9(6.1)	547.1(2.8)	586.1(2.8)	598.1(3.1)
l4064f211b600m00507m0507m628	520.5(6.5)	547.9(3.3)	586.6(3.3)	598.1(3.5)
l4864f211b600m00184m0507m628	569.1(7.5)	560.2(3.4)	553.9(3.5)	594.8(5.5)
l3296f211b630m00740m0370m440	441.9(5.6)	533.9(3.0)	588.2(3.0)	591.5(3.1)
l4896f211b630m00363m0363m430	520.0(6.8)	558.4(3.4)	598.0(3.4)	605.0(3.5)

Finite-volume + discretization error

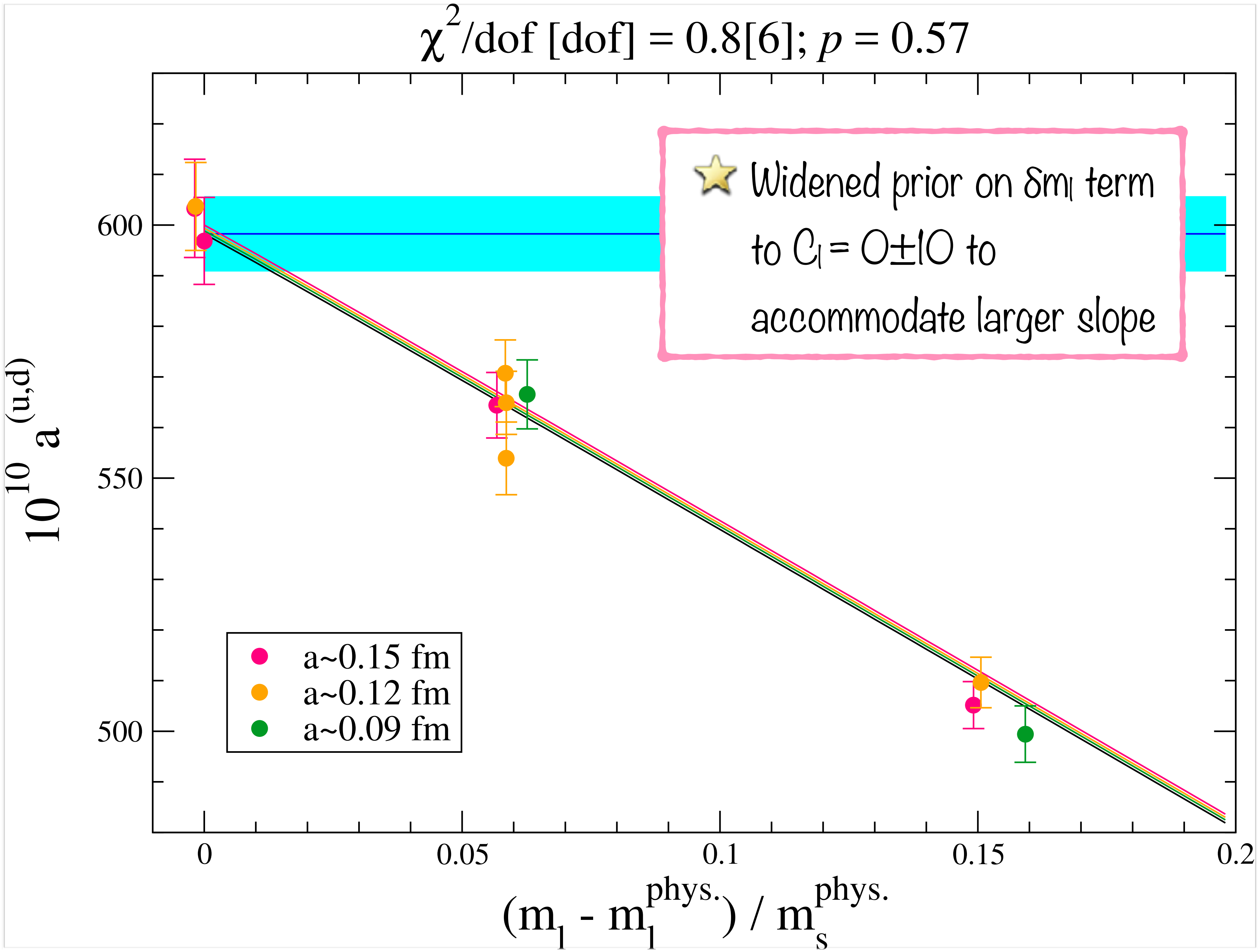
	$10^{10} \times \Delta a_\mu$		
Ensemble	$\pi\pi$	ρ	total
l1648f211b580m01300m0650m838	8.24	-1.33	6.92
l2448f211b580m0064m0640m828	18.61	-3.13	15.48
l3248f211b580m00235m0647m831	57.76	-9.46	48.29
l3248f211b580m002426m06730m8447	55.89	-9.04	46.85
l2464f211b600m01020m0509m635	5.93	-0.95	4.98
l2464f211b600m00507m0507m628	16.46	-2.7	13.76
l3264f211b600m00507m0507m628	14.46	-2.41	12.04
l4064f211b600m00507m0507m628	13.88	-2.36	11.51
l4864f211b600m00184m0507m628	48.8	-7.92	40.87
l3296f211b630m00740m0370m440	3.83	-0.6	3.23
l4896f211b630m00363m0363m430	8.42	-1.39	7.02

- Contributions from leading ππ bubble about 5× larger than from diagrams with ρ meson
- Finite-volume + discretization corrections largely from taste splittings between staggered pions in the sea, and become smaller as continuum limit is approached

Continuum extrapolation



Fit without M_ϱ rescaling



$\pi\pi$ states/t* uncertainty

- Correlator on physical-mass ensembles has not decayed to asymptotic $t \to \infty$ $\pi\pi$ ground state by the lattice temporal extent
- Calculate low-energy $\pi\pi$ contribution to a_μ^{HVP} from $t>t^*=1.5$ fm within chiral theory to be 3×10^{-10} and take as bound systematic error associated with $\pi\pi$ states below the ρ mass
  - Add estimated error in quadrature to physical a_μ^{HVP} after chiral-continuum fit
- Check estimate with data by changing t* to 0.5 fm on physical-mass ensembles, which doubles the estimated $\pi\pi$ contribution in the chiral theory
  - Observed shifts consistent with systematic error estimate

	$10^{10} a_\mu^{HVP}$		
Ensemble	$t^* = 1.5$ fm	$t^* = 0.5$ fm	$\lvert\Delta\rvert$
l3248f211b580m00235m0647m831	561.3(8.3)	560.2(8.1)	1.0(1.9)
l3248f211b580m002426m06730m8447	554.3(7.1)	553.4(6.9)	0.9(1.4)
l4896f211b630m00363m0363m430	520.0(6.8)	514.9(6.4)	5.1(2.0)