More on heavy tetraquarks in lattice QCD at almost physical pion mass

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The observed heavy hadron spectrum suggests a phenomenological binding mechanism from "good" diquark configurations for tetraquarks containing heavy quarks, e.g. $qq'\bar{b}\bar{b}$, $qq'\bar{c}\bar{b}$ and $qq'\bar{c}\bar{c}$.

Predictions:

- Deeper binding with heavier quarks, $\sim 1/m_Q$
- Deepest binding for pairs $\bar{Q}\bar{Q}$
- Binding set by the lighter of Q, Q' for $\bar{Q}\bar{Q}'$
- Deeper binding for lighter quarks in the qq' component

Since no disconnected diagrams need to be computed these tetraquark candidates and predictions are ideally suited for study using lattice QCD.

Caveat: In practice we use NRQCD and RHQ actions for the bottom and charm quarks.

Diquark-Diquark operator:

$$D(x) = (u_a^\alpha(x))^T (C\gamma_5)^{\alpha\beta} q_b^\beta(x) \ \times \ \bar{b}_a^\kappa(x)(C\gamma_i)^{\kappa\rho}(\bar{b}_b^\rho(x))^T \ ,$$

Dimeson-Dimeson operator:

$$M(x) = \ \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta} u_a^\beta(x) \ \bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho} d_b^\rho(x) - \bar{b}_a^\alpha(x)\gamma_5^{\alpha\beta} d_a^\beta(x) \ \bar{b}_b^\kappa(x)\gamma_i^{\kappa\rho} u_b^\rho(x) \ .$$

Compute the energies from the 2×2 GEVP

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix}, \quad F(t)\nu = \lambda(t) F(t_0)\nu \ ,$$

$$G_{\mathcal{O}_1\mathcal{O}_2} = \frac{C_{\mathcal{O}_1\mathcal{O}_2}(t)}{C_{PP}(t)C_{VV}(t)} \ , \ \lambda(t) = Ae^{-\Delta E(t-t_0)} \ .$$

Possibly ambiguous threshold identification, esp. if vol. eff. large

From a combined chiral and volume extrapolation we found:
$\Delta E_{ud\bar{b}\bar{b}} = 189(10)(3)$ MeV and $\Delta E_{ls\bar{b}\bar{b}} = 98(7)(3)$ MeV

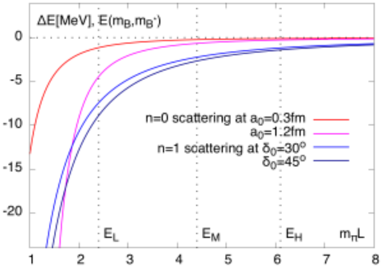
Back of an envelope ball park estimated finite volume effects

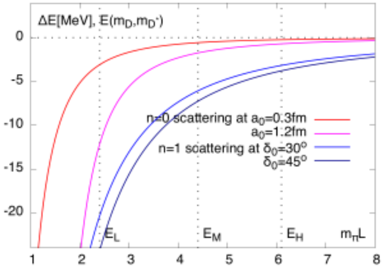
M. Lüscher, Commun. Math. Phys. 105, 153 (1986)

$$\Delta E_0 \approx -\frac{4\pi a_0}{\mu L^3}\left[1+c_1\frac{a_0}{L}+c_2\left(\frac{a_0}{L}\right)^2\right],\ \Delta E_1 \approx -\frac{12 tg(\delta_0)}{\mu L^2}\left[1+c_1' tg(\delta_0)+c_2' tg^2(\delta_0)\right]$$

For a system of two particles, e.g. the $\pi\pi$-system, with $I=0$ and $I=1$ explicit energy shifts due to finite volumes were derived

This is not exactly our system, still: estimate possible volume effects in a hypothetical BB- or DD-system.





BB: $\Delta E_0^V = 5, < 1$; $\Delta E_1^V = 10, 4, 2$ DD: $\Delta E_0^V = 12, 2, 1$; $\Delta E_1^V = 24, 8, 4$

New results: Heavy quark mass dependence and $ud\bar{c}\bar{b}$

For tetraquarks with $\bar{Q}\bar{Q}'$ the 2×2 GEVP can be extended to 3×3 by exploiting the second possible meson-meson threshold:

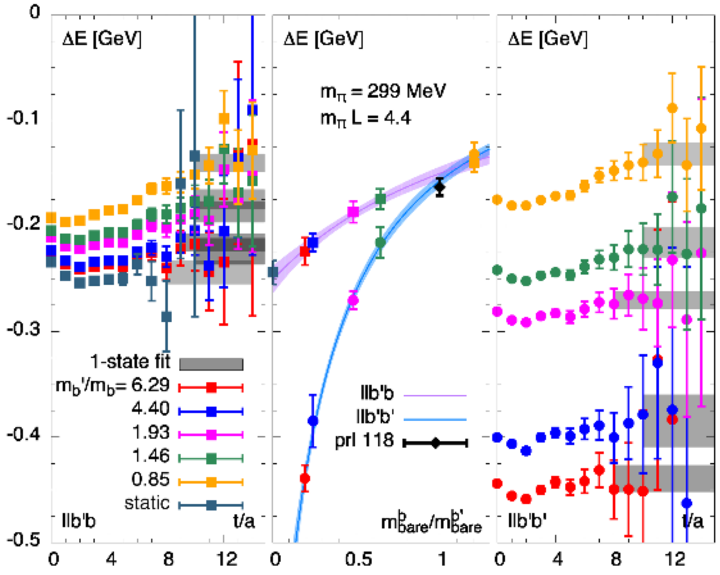
New flavor combination enables formulation of 3×3 GEVP:

$$F(t) = \begin{pmatrix} G_{DD} & G_{DM} \\ G_{MD} & G_{MM} \end{pmatrix} \Rightarrow F(t) = \begin{pmatrix} G_{DD} & G_{DM_{12}} & G_{DM_{21}} \\ G_{M_{12}D} & G_{M_{12}M_{12}} & G_{M_{12}M_{21}} \\ G_{M_{21}D} & G_{M_{21}M_{12}} & G_{M_{21}M_{21}} \end{pmatrix}$$

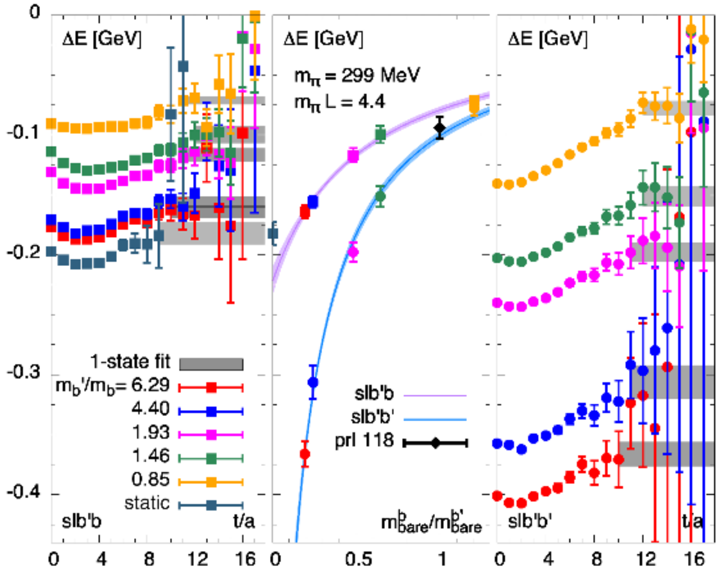
Clean(er) extraction of ground state, if threshold can be identified.

First test: **Post-dict previous results and check $1/m_Q$ dependence**

- All calculations at $m_\pi = 299$ MeV and $m_\pi L = 4.4$
- Unphysical bottom quarks b' in $qq'\bar{b}'\bar{b}$, $qq'\bar{b}'\bar{b}'$ tetraquarks
- $m_{b'}/m_b \approx 6.29, 4.40, 1.93, 1.46, 0.85$ - tuned via dispersion relation of spin-averaged mass mesons
- Physical point is interpolated, Ansatz: $A/(m_{Q'} + M)$



- $1/m_Q$ confirmed. ✓
- Good intercept with previous results. ✓



- $1/m_Q$ confirmed. ✓
- Reasonable intercept with previous results. ✓

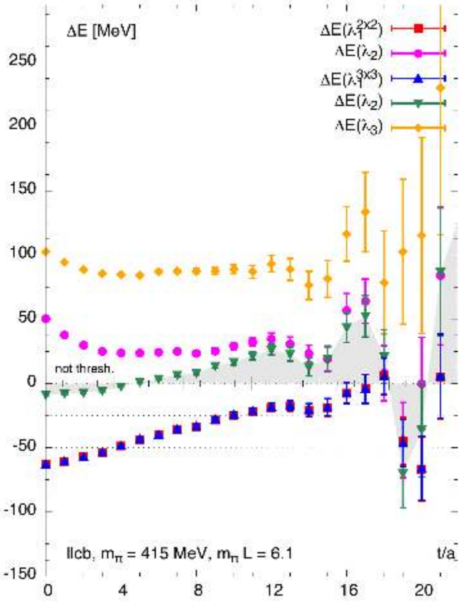
Towards $ud\bar{c}\bar{b}$ tetraquarks

Expectations

- Increased noise
- Binding around the electro-stable threshold, $\Delta E^p \approx 30 - 50$ MeV
- Significant volume effects for E_M and E_L, $\Delta E_M^V \approx (0.1 - 0.16)\Delta E^p$ and $\Delta E_L^V \approx (0.34 - 0.57)\Delta E^p$
- In particular: Significant shift of the threshold

Calculation

- Increased statistics, $N_{src}^{E_H} = 2 \to 8$, $N_{src}^{E_M} = 1 \to 8$, $N_{src}^{E_L} = 16 \to 48$, all Coulomb gauge-fixed wall propagators saved.
- Extensive use of the 3×3 GEVP
- Determination of ΔE from the difference of the ground state and threshold eigenvalues, if identified.

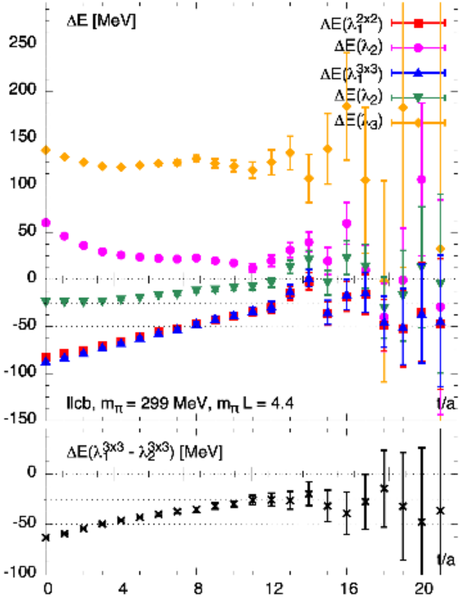


The second eigenvalue is not trending towards the threshold. It stays significantly above it.

We see the first eigenvalue trending towards the threshold.

Together this indicates the first eigenvalue is the threshold.

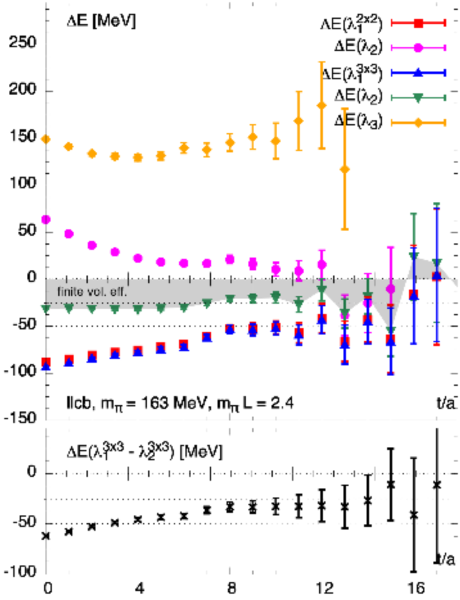
$\Rightarrow$ No binding of $ud\bar{c}\bar{b}$ at $m_\pi = 415$ MeV.



The first eigenvalue is always below the threshold (and below the second eigenvalue).

We see the second eigenvalue trending towards the threshold.

Binding should be calculated between $\Delta E(\lambda_1) - \Delta E(\lambda_2)$
$\Rightarrow$ Possible binding window of $\Delta E = 5 - 50$ MeV at $m_\pi = 299$ MeV.



The first eigenvalue is always below the threshold (and below the second eigenvalue).

We see the second eigenvalue trending towards the threshold, but staying below it.
We observe significant finite volume effects!

Binding must be calculated between $\Delta E(\lambda_1) - \Delta E(\lambda_2)$
$\Rightarrow$ Possible binding energy of $\Delta E = 12 - 50$ MeV at $m_\pi = 163$ MeV.

At $m_\pi = 299, 163$ MeV we can identify the threshold from the second eigenvalue of the 3x3 GEVP.

We find evidence of binding in the $ud\bar{c}\bar{b}$ channel at the level of $\Delta E = 12 - 50$ MeV, close to the electro-stable threshold.
$\Rightarrow$ Assuming $\Delta E \uparrow$ with $m_q \downarrow$, this is a lower bound at the physical point.

Extrapolate to physical point?
$\Rightarrow$ Needs proper estimate of volume effects!

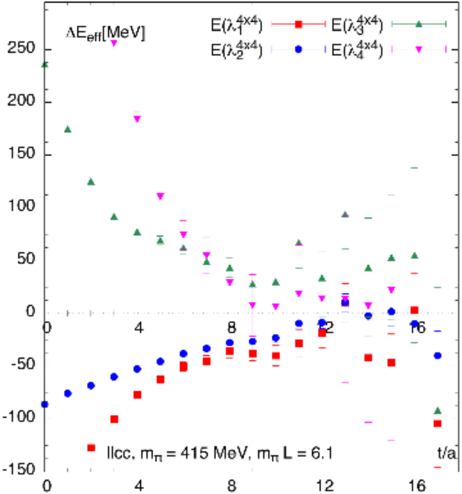
Perhaps there's something we can do?

R. A. Briceno, Phys. Rev. D 89, no. 7, 074507 (2014)

$$[\delta \mathcal{G}_j^V]_{Jm_J,lS;J'm_{J'},l'S'} = \frac{ik_j^* \delta_{SS'}}{8\pi E^*} n_j \Bigg[\delta_{JJ'} \delta_{m_J m_{J'}} \delta_{ll'} + i \sum_{l'',m''} \frac{(4\pi)^{3/2}}{k_j^{*l''+1}} c^{\mathbf{d}}_{l''m''}(k_j^{*2}; L) \times \sum_{m_l, m_{l'}, m_S} \langle lS, Jm_J | lm_l, Sm_S \rangle \langle l'm_{l'}, Sm_S | l'S, J'm_{J'} \rangle \int d\Omega \, Y^*_{l,m_l} Y^*_{l'',m''} Y_{l',m_{l'}} \Bigg],$$

Thinking ahead: $ud\bar{c}\bar{c}$ tetraquarks

We saw the binding is set by the lighter of $\bar{Q}, \bar{Q}'$ in the $ud\bar{c}\bar{b}$ case.
$\Rightarrow$ **Indication for a possibly bound $ud\bar{c}\bar{c}$ flavor combination?**



Calculation

- No NRQCD required $\Rightarrow$ access to many more operators!
- E.g. 4×4 GEVP with parity $(++)/(--)$ sub-operators
- So far only low statistics on E_H gathered.
- Degenerate eigenvalues? Threshold? Plateaus?
- All results very preliminary

Note: ΔE increases with decreasing light quark mass in this mechanism.

Conclusions

- Study of $qq'\bar{Q}\bar{Q}'$ and $qq'\bar{Q}'\bar{Q}'$ for unphysically heavy b quarks reveals $1/m_Q$ dependence and consistently post-dicts previous results

- Using an extended 3×3 GEVP and increased statistics, we find evidence of a bound $ud\bar{c}\bar{b}$ tetraquark. The binding is $\Delta E = 12 - 50$ MeV and close to the electro-stable threshold.
  $\Rightarrow$ More work required!

- Due to the absence of b quarks in the $ud\bar{c}\bar{c}$ channel, a larger GEVP can be implemented. Here, results are very preliminary, yet promising.