Mixed action with Borici-Creutz valence on Asqtad sea

Subhasish Basak[1] Jishnu Goswami [2] Dipankar chakraborty[2]

[1]NISER, India

[2]IIT Kanpur, India

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Minimally Doubled Fermions

Lattice action with two massless fermions –
Minimum number of fermions allowed by no-go theorem

- Three main variants:
  - Karsten-Wilczek [Karsten PLB **104**, 315; Wilczek PRL **59**, 2397]
  - Borici-Creutz [Creutz JHEP **0804**, 017; Borici PRD **78**, 074504]
  - Twisted-ordering [Creutz et.al. PRD **82**, 074502]
- Reduces number of massless species to two but breaks hypercubic symmetry

Borici-Creutz Dirac operator, where $\Gamma = \frac{1}{2}\sum_\mu \gamma_\mu$ and $\gamma'_\mu = \Gamma\gamma_\mu\Gamma$:

$$\mathcal{D}(p) = i\sum_\mu \left[\gamma_\mu \sin p_\mu + \gamma'_\mu \cos p_\mu\right] - 2i\Gamma$$

$\Rightarrow$ Two zeros of $\mathcal{D}(p)$ at $(0,0,0,0)$ and $(\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2})$.

Borici-Creutz action

Borici-Creutz action has a special direction in Euclidean space – major hypercube diagonal (line joining two zeros)

$\longrightarrow$ Breaks reflection symmetry of hypercube $\Rightarrow$ breaking of parity & time symmetry $\Rightarrow$ counter terms for renormalized theory

- Dimension-3 $\quad ic_3(g_0)\bar{\psi}\Gamma\psi$
- Dimension-4 $\quad c_4(g_0)\bar{\psi}\Gamma\sum_\mu D_\mu\psi$

Borici-Creutz Dirac operator with counter terms

$$\mathcal{D}(p) = i\sum_\mu \left[\gamma_\mu \sin p_\mu + \gamma'_\mu \cos p_\mu + c_4\Gamma \sin p_\mu\right] + i(c_3 - 2)\Gamma$$

1-loop lattice perturbation theory gives [Capitani et.al. JHEP **1009**, 027],

$$c_3(g_0) = 29.542 \cdot \frac{g_0^2\, C_F}{16\pi^2} + \mathcal{O}(g_0^4)$$

$$c_4(g_0) = 1.5277 \cdot \frac{g_0^2\, C_F}{16\pi^2} + \mathcal{O}(g_0^4)$$

Borici-Creutz action contd. ...

- Spinor $\psi(p)$ contains 2-degenerate flavors
- Point splitting – flavor fields at two different poles
[Creutz PoS LATTICE 2010, 078; Tiburzi PRD **82**, 034511]

$$
\begin{aligned}
d(p) &= \frac{\Gamma}{4}\sum_{\mu}\left[1-\sin p_\mu\right]\psi(p) \\
u(p) &= \frac{1}{4}\sum_{\mu}\left[1-\cos(\pi/2+p_\mu)\right]\psi(\pi/2+p)
\end{aligned}
$$

Full renormalized action in 4-dimension space time lattice ($\tilde{c}_3 = c_3 - 2$),

$$
\begin{aligned}
S = \sum_x \Big[\frac{1}{2}\sum_\mu \Big\{ & \bar{\psi}(x)(\gamma_\mu + c_4\Gamma + i\gamma'_\mu)U_\mu(x)\psi(x+\hat{\mu}) \\
& - \bar{\psi}(x+\hat{\mu})(\gamma_\mu - c_4\Gamma - i\gamma'_\mu)U^\dagger_\mu(x)\psi(x)\Big\} \\
& + \bar{\psi}(x)(m + i\tilde{c}_3\Gamma)\psi(x)\Big].
\end{aligned}
$$

Simulation details

Mixed action lattice QCD: Borici-Creutz valence quarks on MILC lattices with $N_f = 2 + 1$ Asqtad improved staggered sea quarks [MILC PRD **64**, 054506]

Lattice	$\beta = 10/g^2$	a (fm)	m_l/m_s	cfgs
$16^3 \times 48$	6.572	≈ 0.15	0.0097/0.0484	40
$20^3 \times 64$	6.760	≈ 0.12	0.01/0.05	40
$28^3 \times 96$	7.090	≈ 0.09	0.0062/0.031	30

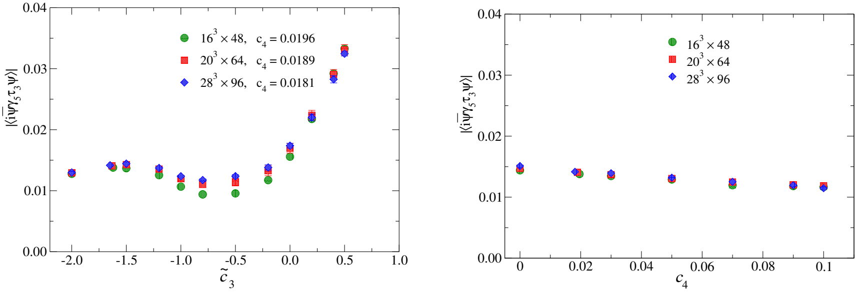
- Borici-Creutz valence quark propagators for $m = [0.0075 - 0.9]$, which restricts $m_\pi L > 4$.
- Strange mass tuned to set $s\bar{s}$-pseudoscalar to 686 MeV. Similar strange mass from $m_{\rm ps}/m_{\rm vec} = 0.673$.
- Random sources for quark propagators. Fits reported here are uncorrelated and errors from jack-knife analysis.

Determining $\tilde{c}_3$

Absence of hypercubic symmetry – exhibit parity-flavor breaking

- Parity condensate $|i\bar{\psi}\gamma_5\tau_3\psi|$ determines the size of parity breaking

  [Aoki NPB **314**, 79]

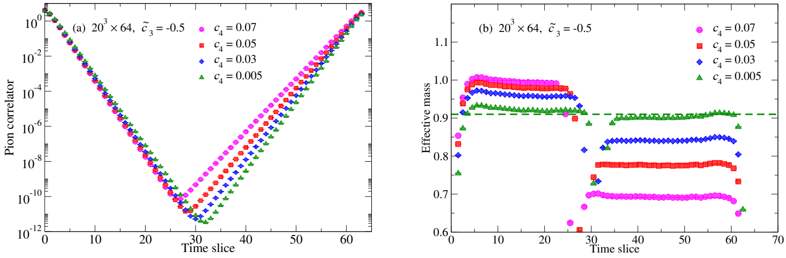


- $\tilde{c}_3$ alone seems to drive parity restoration.
- Minimal parity breaking around $\tilde{c}_3 = -0.5$, almost independent of a.

Determining c_4

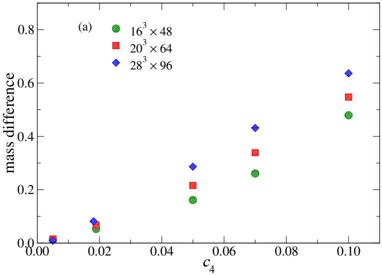
Borici-Creutz action has PT-symmetry – expect signs of T asymmetry

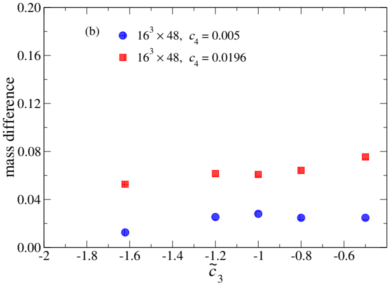
- c_4 is kinetic '*like*' term
- Non-degeneracy of forward & backward propagating meson states



- Asymmetry in pion and mass difference vanishes when c_4 below 0.005

Determining c_4

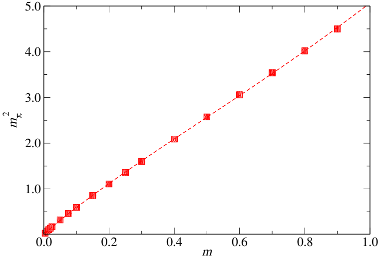


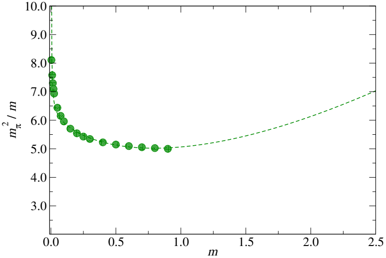


- $\tilde{c}_3$ has practically no role in T-symmetry restoration
- Smallness of c_4 suggests T-symmetry is only weakly broken
- Smallness of a dependence – use of observables not known to have significant cut-off effect

Meson spectrum: π

- π spectrum from both u and d fields give identical results





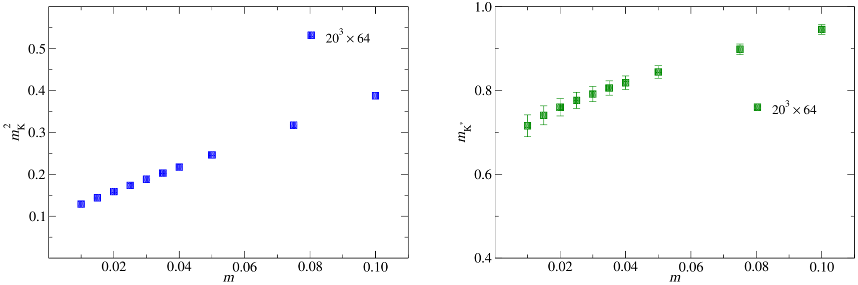
- 1-loop partially quenched chiral PT formula [Chen et.al. PRD **70**, 034502]

$$m_\pi^2 = C_1 m + C_{1L} m \log m + C_2 m^2 + C_{2L} m^2 \log m$$

- Partially quenched chiral logarithm due to mismatch of valence and sea quark masses.
- Quark mass chosen for $m_\pi \sim 400$ MeV

Meson spectrum: K, $K^\star$

- $m_s = 0.03$ [45.6 MeV on $20^3 \times 64$] gives $m^{\rm ps}_{s\bar{s}} = 682$ MeV, $m^{\rm vec}_{\phi} = 1150$ MeV [PDG: 1020 MeV]
- For $m_\pi \approx 400$ MeV: $m_K \approx 550$ MeV [PDG: 496 MeV], $m_{K^\star} \approx 1060$ MeV [PDG: 892 MeV]

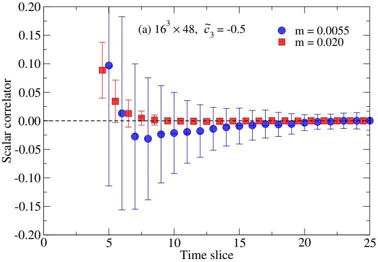


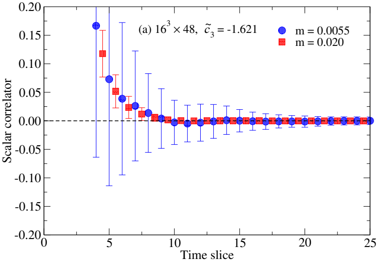
Scalar correlator

Unitarity violation in Mixed Action QCD

- Scalar meson $\bar{\psi}\psi$ is known to be sensitive to unitarity violation.
- Negative scalar correlator for $m_{\mathrm{val}} < m_{\mathrm{sea}}$

[Prelovesk et.al. PRD **70**, 094503]





- Negative scalar correlator only $\tilde{c}_3 = -0.5$ *i.e.* in presence of parity breaking no negative contribution.

$\Delta_{\rm mix}$

3 different mesons in Mixed action theory: composed of (*i*) two valence quarks, (*ii*) two sea quarks and (*iii*) a mix of valence and sea quarks.

- They undergo lattice spacing dependent mass renormalization.
- Mixed action χPT in LO has an $\mathcal{O}(a^2)$ dependent LEC $\Delta_{\rm mix}$.
- Degree of unitarity violation at finite a depends on size of $\Delta_{\rm mix}$.

In the LO, pseudoscalar meson masses for BC valence and Asqtad sea,

$$\begin{aligned} m^2_{v_1 v_2} &= B_v(m_{v_1} + m_{v_2}) \\ m^2_{s_1 s_2, t} &= B_0(m_{s_1} + m_{s_2}) + a^2\Delta_t \\ m^2_{vs} &= B_v m_v + B_0 m_s + a^2\tilde{\Delta}_{\rm mix} \end{aligned}$$

$a^2\Delta_t =$ taste splittings and $t = A, T, V, I$ & $a^2\Delta_5 = 0$.

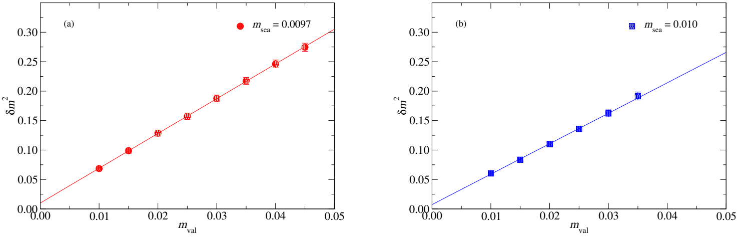
Δ_{mix}

The $a^2\tilde{\Delta}_{\text{mix}} = a^2\Delta_{\text{mix}} + a^2\Delta'_{\text{mix}}$ where, [Goltermaan 0912.4042; Chen et.al. PRD **79**, 117502]

$$a^2\Delta'_{\text{mix}} = \frac{1}{8}a^2\Delta_A + \frac{3}{16}a^2\Delta_T + \frac{1}{8}a^2\Delta_V + \frac{1}{32}a^2\Delta_I.$$

$a^2\tilde{\Delta}_{\text{mix}}$ from linear interpolation of δm^2 in bare valence mass m_{v}.

$$\delta m^2 \equiv m^2_{vs} - \frac{1}{2}m^2_{ss,5} = B_v m_v + a^2\tilde{\Delta}_{\text{mix}}.$$



$a^2\tilde{\Delta}_{\text{mix}} = 0.13(1)$ GeV^4 [0.15 fm] and $0.13(8)$ GeV^4 [0.12 fm]

Summary & Conclusion

- Borici-Creutz fermions (minimally doubled fermions) are known to preserve chiral symmetry for a degenerate quark doublet and are local.
- Can be helpful for $N_f = 2$ lattice simulations, relatively simpler and possibly faster than Ginsparg-Wilson fermions.
- Mixed action Lattice QCD with BC valence quarks and Asqtad sea quarks shows encouraging first results.
- Tuning of counter-term coefficients $\tilde{c}_3$ and c_4 require some care so as to properly restore the desired symmetries.
- Once properly tuned, all the known results of mixed action simulations are reproduced – partially quenched chiral log, negative scalar correlator and non-zero Δ_{mix}.
- Unitarity violation, as measured by $\tilde{\Delta}_{\mathrm{mix}}$, is comparable to the previous lattice mixed action simulations.
- Warrant further studies on hadron spectroscopy, particularly charm.