Computation of α_{qq} in QCD ($N_f = 2$) using Lattice Perturbation Theory

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Physical context

- QCD($N_f = 2+1$) is a good approximation of the full theory
- Discovery of new charm-states makes charm physics very interesting
- We want to evaluate dynamical charm effects on the strong coupling derived from the static force:

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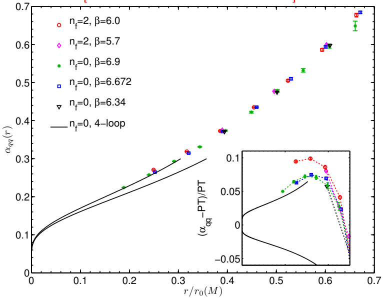
  1. QCD($N_f = 2$) at $m = m_c$
  2. QCD($N_f = 0$)
- Significant effects at $\mu = 1/r \approx 1.6$ GeV and above

[T. Korzec et al.: arXiv:1612.07634]



A few details of these simulations

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- Dynamical ensembles
  1. $\beta = 5.70, 5.88, 6.00$
  2. $r_0/a = 9.123(57), 11.946(55), 14.34(10)$
  3. $\kappa_c = 0.136698, 0.136509, 0.136335$
  4. $a\mu = 0.113200, 0.087626, 0.072557$
  5. $[LS, LT] = [32, 120], [48, 192], [48, 192]$

- Quenched ensembles
  1. $\beta = 6.34, 6.672, 6.90$
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- Wilson Loops are calculated following [M. Donnellan et al.: arXiv:1012.3037v3]
  1. Gauge Links are replaced by HYP2-smeared ones
  2. HYP2: $\alpha_1 = 1.0$, $\alpha_2 = 1.0$, $\alpha_3 = 0.5$

Main goals and strategy

1. Extract leading fermionic lattice artifacts in α_{qq} at $m = m_c$
   - Compute, up to order g^4 in perturbation theory, the fermionic contribution to the static force [A. Athenodorou and H. Panagopoulos: arXiv:hep-lat/0509039v1]
   - Then, one-loop lattice artifacts can be calculated following the approach described in [A. Athenodorou and R. Sommer: arXiv:1109.2303v1]
   - Main differences with these two previous works:
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2. Take the continuum limit of α_{qq} in QCD($N_f = 2$) with $m = m_c$ and QCD($N_f = 0$) from the simulations produced in [F. Knechtli et al.: arXiv:1706.04982]
   - Subtract leading fermionic lattice artifacts
   - Parametrize β-function and cutoff-effects [M. Dalla Brida et al.: arXiv:1607.06423v1]

Force between static quarks

- Definition of the static force in the continuum: $F(r) = \frac{dV(r)}{dr}$
- On a lattice, we can extract $V(Ra)$ from a Wilson Loop $W(R, T)$:

$$aV(Ra) = -\lim_{T\to\infty} \frac{1}{T} \log(W(R, T))$$

- Naive definition on a lattice ($r_{naive}/a = R - 1/2$)

$$F(r_{naive}) = \frac{1}{a}(V(Ra) - V(Ra - a))$$

- Improved definition ($r_I/a = R - 1/2 + \mathcal{O}(a)$) [R. Sommer: arXiv:hep-lat/9310022]

$$F(r_I) = \frac{1}{a}(V(Ra) - V(Ra - a)) = C_F \frac{g^2}{4\pi r_I^2} + \mathcal{O}(g^4 a^2)$$

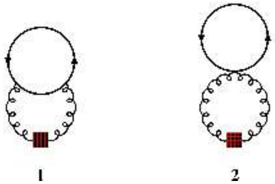
- We also use r_I calculated for HYP2-smearing [M. Donnellan et al.: arXiv:1012.3037v3]

Perturbative Static Force

- Wilson loops can be expanded in powers of g^2

$$W/N = 1 - W_1 g^2 - (W_{2g} + W_{2f})g^4 + \cdots$$

- W_{2f} is a fermionic contribution coming from two Feynman diagrams



- Perturbative expansion of the static potential

$$aV(Ra) = V_1(R)g^2 + V_2(R)g^4 + \cdots$$
$$V_2(R) = V_{2g}(R) + V_{2f}(R)$$

- Perturbative static force

$$a^2 F(r_I) = (V_1(R) - V_1(R-1))\, g^2 + (V_2(R) - V_2(R-1))g^4$$

How to extract the leading fermionic lattice artifacts

- We calculate 1-loop fermionic lattice artifacts for $F(r_I)$ (using clover improved TM Wilson fermions with $m = m_c$), comparing unsmeared and HYP-2 smeared links

  [A. Athenodorou and H. Panagopoulos: arXiv:hep-lat/0509039v1] [A. Athenodorou and R. Sommer: arXiv:1109.2303v1]

- Moving to the $\overline{\text{MS}}$ scheme, $F(r_i)$ is given by

$$F(r_I) = \frac{C_F\,\alpha_{\overline{MS}}(1/r_I)}{r_I^2}\left\{1 + f_1(z, a/r_I)\alpha_{\overline{MS}}(1/r_I) + \mathcal{O}(\alpha^2_{\overline{MS}})\right\}$$

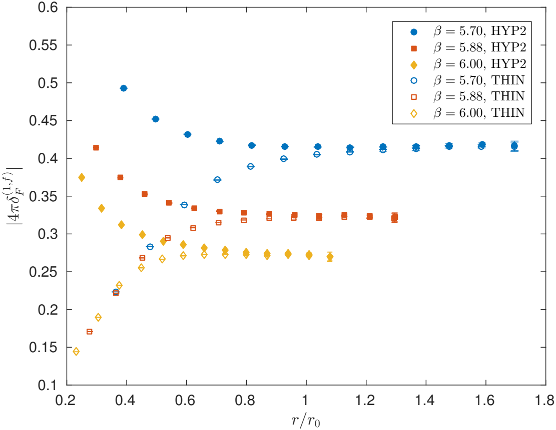
$$f_1(z, a/r_I) = f_{1,g}(a/r_I) + \sum_{i=1}^{N_f} f_{1,f}(z_i, a/r_I); \quad z_i = z = r_I m_c$$

- $f_{1,g}(0)$ and $f_{1,f}(z,0)$ are known functions
- $\frac{F - F_{cont}}{F_{cont}} = \left(\delta_F^{(1,g)}(a/r_I) + \sum_{i=1}^{N_f} \delta_F^{(1,f)}(z_i, a/r_I)\right) g^2_{\overline{MS}}(1/r_I) + \mathcal{O}(g^4_{\overline{MS}})$

$$4\pi\delta_F^{(1,f)}(z_i, a/r_I) = f_1(z, a/r_I) - f_1(z, 0)$$

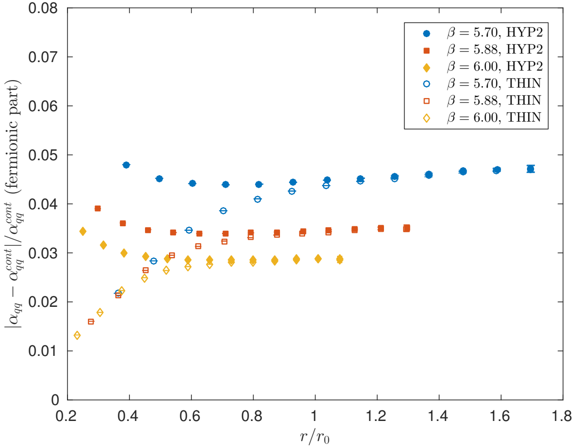
- Calculation is carried out using our computer package in Mathematica
- Diagrams are computed for a sequence of finite lattice sizes ($L = [16, 64]$, $\Delta k = 2\pi/L$) and then we take the limit $L \to \infty$

Study of $4\pi\delta_F^{(1,f)}$: Unsmeared Links vs HYP2-smeared Links



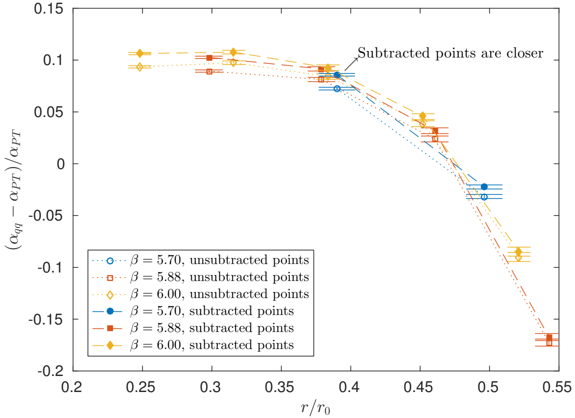
- HYP2-smeared links produce bigger lattice artifacts compared to unsmeared links at small distances in lattice units.
- Lattice artifacts are smaller when $a \to 0$ (as expected)

One-loop lattice artifacts of dynamical charm quarks in α_{qq}



- $\frac{F - F_{cont}}{F_{cont}} \approx \delta_F^{(1)} g_{MS}^2$
- 1-loop lattice artifacts of dynamical charm quarks are smaller than 5%

Subtract leading lattice artifacts from non-perturbative data



- α_{PT} is a 4-loop calculation of α_{qq} for $N_f = 0$
- 1-loop lattice artifacts are small $\Rightarrow$ visible effects only at short distances
- Subtracted points at similar distances are closer to each other

Step scaling function

- Studying the step scaling function $\sigma(u)$ can provide a tool to extract the continuum limit of α_{qq}

$$\sigma(u) = g^2_{qq}(fr)|_{g^2_{qq}(r)=u} \quad f = \text{ fixed scale factor}$$

- Definition of the β_{qq}-function

$$-r\frac{\partial g_{qq}(\frac{1}{r})}{\partial r} = \beta_{qq}$$

- Exact relation

$$\log(f) = -\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{dx}{\beta_{qq}(x)}$$

- We choose a convenient parametrization of the β_{qq}-function

$$\beta_{qq} = -\frac{g^3_{qq}}{P(g^2_{qq})}, \quad P(g^2_{qq}) = p_0 + p_1 g^2_{qq} + p_2 g^4_{qq} + \cdots$$

Parametrize $\sigma(u)$ and cutoff effects

- Parametrization of β_{qq} $\rightarrow$ Parametrization of $\sigma(u)$

$$\log(f) = -\frac{p_0}{2}\left[\frac{1}{\sigma(u)} - \frac{1}{u}\right] + \frac{p_1}{2}\log\left[\frac{\sigma(u)}{u}\right] + \sum_{n=1}^{n_{max}} \frac{p_{n+1}}{2n}\left[\sigma^n(u) - u^n\right]$$

- In order to parametrize the cutoff effects, we introduce a function $\rho(u)$

$$\rho(u) = \sum_{i=0}^{n_\rho - 1} \rho_i u^i$$

- Global-fit to find $p_0, p_1, \cdots$

$$\log(f) + \rho(u)\left(\frac{a}{r_0}\right)^2 = -\int_{\sqrt{u}}^{\sqrt{\Sigma(u,a/r_0)}} \frac{dx}{\beta_{qq}(x)}, \quad \lim_{a\to 0} \Sigma(u, a/r_0) = \sigma(u)$$

- Find α_{qq} for QCD($N_f = 2$) and QCD($N_f = 0$) in the continuum from the definition of β_{qq} $\Rightarrow$ We need to solve an ODE
- Ansatz: initial condition = one value of g_{qq} coming from our finest lattice
- Using improved distances we cannot choose a fixed integer f $\Rightarrow$ We choose $f \in [2.07, 2.10]$

Results of the best-fits

1. QCD($N_f = 0$)
   - Good agreement for different types of best-fits
   - 6 parameters for β_{qq}, 1 for $\rho \Rightarrow \frac{\chi^2}{N_{dof}} = \frac{9.17}{8}$
   - $p_0 = 16.74(94)$, $p_1 = -3.73(55)$, $p_2 = 0.69(11)$
     $p_3 = -0.0469(89)$, $p_4 = 0.00143(31)$, $p_5 = 0.0000156(39)$

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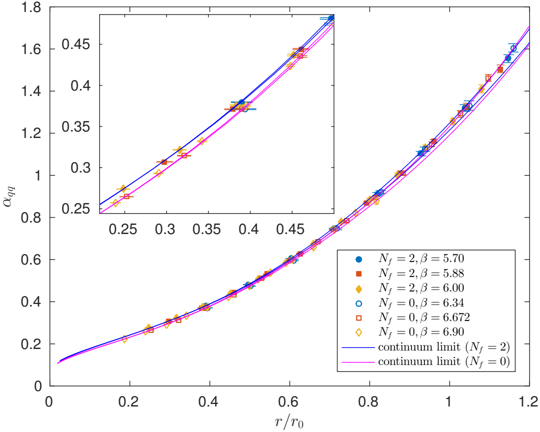
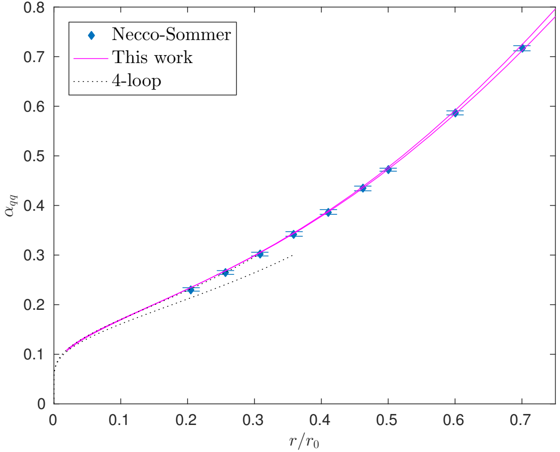
     $p_3 = -0.0469(89)$, $p_4 = 0.00143(31)$, $p_5 = 0.0000156(39)$
2. QCD($N_f = 2$) at $m = m_c$
   - Good agreement for different types of best-fits
   - Best-fit parameters obtained subtracting or not the leading lattice artifacts are compatible
   - Relative errors are a bit smaller subtracting one-loop lattice artifacts, so we show the results obtained with the subtracted points
   - 5 parameters for β_{qq}, 1 for $\rho \Rightarrow \frac{\chi^2}{N_{dof}} = \frac{8.00}{9}$
   - $p_0 = 15.25(77)$, $p_1 = -2.11(33)$, $p_2 = 0.314(46)$
     $p_3 = -0.0132(25)$, $p_4 = 0.000187(43)$

QCD($N_f = 0$) vs QCD($N_f = 2$) with $m = m_c$



- Bands for the continuum limits orginate from the errors on the data and their correlation is taken into account
- Dynamical charm effects on α_{qq} look significant at $r/r_0 \lesssim 0.5$

QCD($N_f = 0$): comparison with PT and previous works



- Good agreement with perturbation theory for $\alpha_{qq} \lesssim 0.25$
- Good agreement with [S. Necco and R. Sommer: arXiv:hep-lat/0108008v1] for $\alpha_{qq} \gtrsim 0.25$

Conclusions

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1. 1-loop lattice artifacts of dynamical charm quarks using HYP2-smearing
   - These effects are small (as found in [A. Athenodorou and R. Sommer: arXiv:1109.2303v1] for unsmeared links), but HYP2-smeared links produce bigger lattice artifacts compared to unsmeared links at small distances in lattice units.
2. Continuum limit of α_{qq} studying the step scaling function
   - QCD($N_f = 0$): good agreement with PT and previous works
   - Significant dynamical charm effects on α_{qq} at $r/r_0 \lesssim 0.5$

Outlook

1. Calculate 1-loop fermionic lattice artifacts in α_{qq} for different masses using HYP-smearing
2. Systematic study of α_{qq} using different quark-masses

Thank you very much for your attention.