Weakly coupled conformal gauge theories on the lattice

Daniel Nogradi

in collaboration with

Zoltan Fodor, Kieran Holland

Julius Kuti, Chik Him Wong

We study conformal gauge theories close to the *upper* end of the conformal window

These have a perturbatively accessible IR fixed point

Close to *upper* end of conformal window: small g_*^2, Banks-Zaks works

No doubt they are conformal, we can even calculate everything reliably in perturbation theory

Close to *lower* end of conformal window: things are uncertain because of non-perturbative dynamics, coupling is large

Why?

Mainly to test lattice methods/tools

Close to *lower* end of the conformal window lattice groups sometimes disagree

By studying the weakly coupled CFT case we know what we should get and the methods should be able to reproduce it (hopefully)

If we learn something, go back to the more difficult cases (close to *lower* end of conformal window)

Focus on one method: finite volume running coupling, discrete β-function

Outline

- Running coupling in periodic finite volume (very briefly)
- Two models: $N_f = 14$ fundamental, $N_f = 3$ sextet (both $SU(3)$)
- Numerical results
- Comments on 5-loop $\overline{\text{MS}}$ running
- Did we learn something?

Running coupling

Gradient flow on T^4

$$g^2(t,L) = \frac{128\pi^2}{3(N^2-1)\,(1+\delta(t,L))}\langle t^2 E(t)\rangle$$

$\delta(t,L)$ is known, $c = \sqrt{8t}/L$ constant

Single scale $\mu = 1/L$

Step scaling: finite change $L \to sL$ with $s = 3/2$.

Gauge fields periodic, fermions massless anti-periodic in all 4-directions

Perturbative β-functions

5-loop β-function in $\overline{\text{MS}}$

$$\mu^2 \frac{dg^2}{d\mu^2} = -L^2 \frac{dg^2}{dL^2} = \beta(g^2) = \sum_{i=1}^{5} b_i \frac{g^{2i+2}}{(16\pi^2)^i}$$

Solve for g_*^2: $\beta(g_*^2) = 0$

Close to *upper* end of conformal window g_*^2 should be small

Perturbative series expected to converge

Model 1: $SU(3)$ with $N_f = 14$ fundamental

Upper end of conformal window: $N_f = 16.5$, should be close

$$b_1 = -\frac{5}{3} \qquad b_2 = \frac{226}{3} \qquad b_3 = \frac{70547}{54}$$

$$b_4 = -15506.48 \qquad b_5 = -668754.5$$

$$g_*^2(2-loop) = 3.494 \qquad g_*^2(3-loop) = 2.696$$

$$g_*^2(4-loop) = 2.810 \qquad g_*^2(5-loop) = 2.926$$

Model 2: $SU(3)$ with $N_f = 3$ sextet

Upper end of conformal window: $N_f = 3.3$, should be (very) close

$$b_1 = -1 \qquad b_2 = 148 \qquad b_3 = \frac{3493}{2}$$

$$b_4 = -22834.071 \qquad b_5 = -2365262.5$$

$$g_*^2(2-loop) = 1.067 \qquad g_*^2(3-loop) = 0.993$$

$$g_*^2(4-loop) = 0.999 \qquad g_*^2(5-loop) = 1.002$$

We should be able to reproduce these

(modulo scheme dependence) from lattice!

Numerical simulation

Periodic gauge field, fermions anti-periodic in 4 directions, $m = 0$

Staggered discretization, stout-improved

Symanzik tree-level improved gauge action

Symanzik tree-level improved gradient flow

Clover discretization for observable E

SSC (in our 1406.0827 terminology)

Numerical simulation

Staggered: need rooting trick for both models $\rightarrow$ RHMC

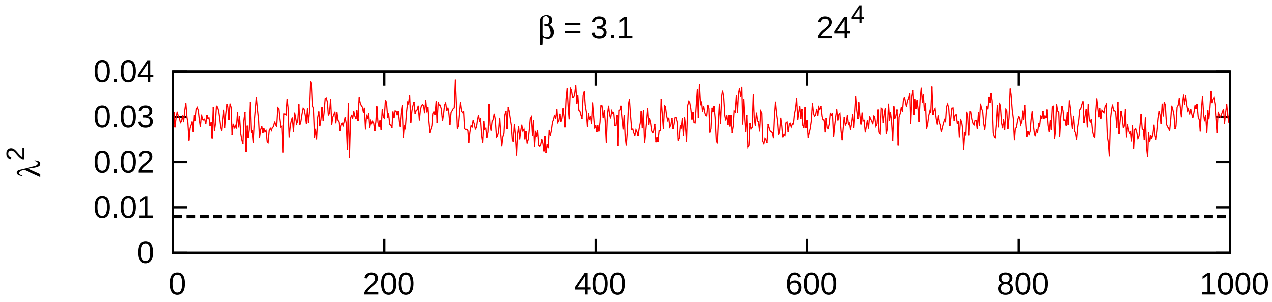
Since $m = 0$, lower bound on Dirac spectrum needs to be set for Remez algorithm

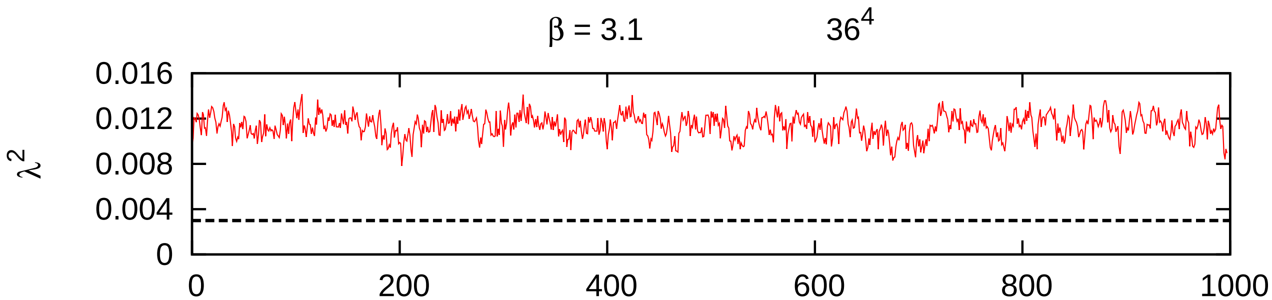
There is a gap because of anti-periodic boundary conditions for fermions

Gap needs to be measured

Numerical simulation

Examples: $N_f = 14$ fundamental $\qquad (N_f = 3$ sextet similar$)$



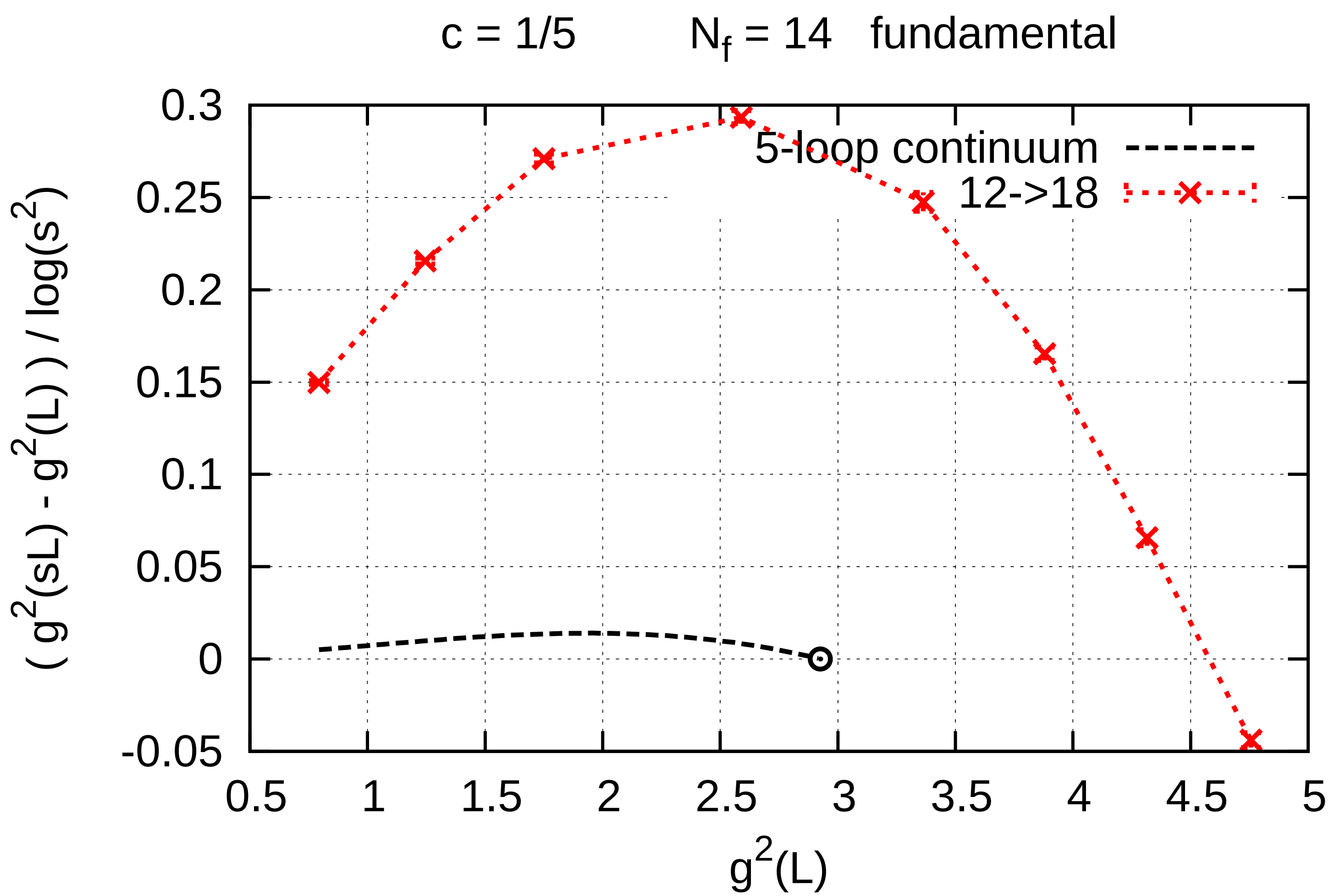


Lowest β, every 10^{th} measured

Dashed line: lower bound set for Remez

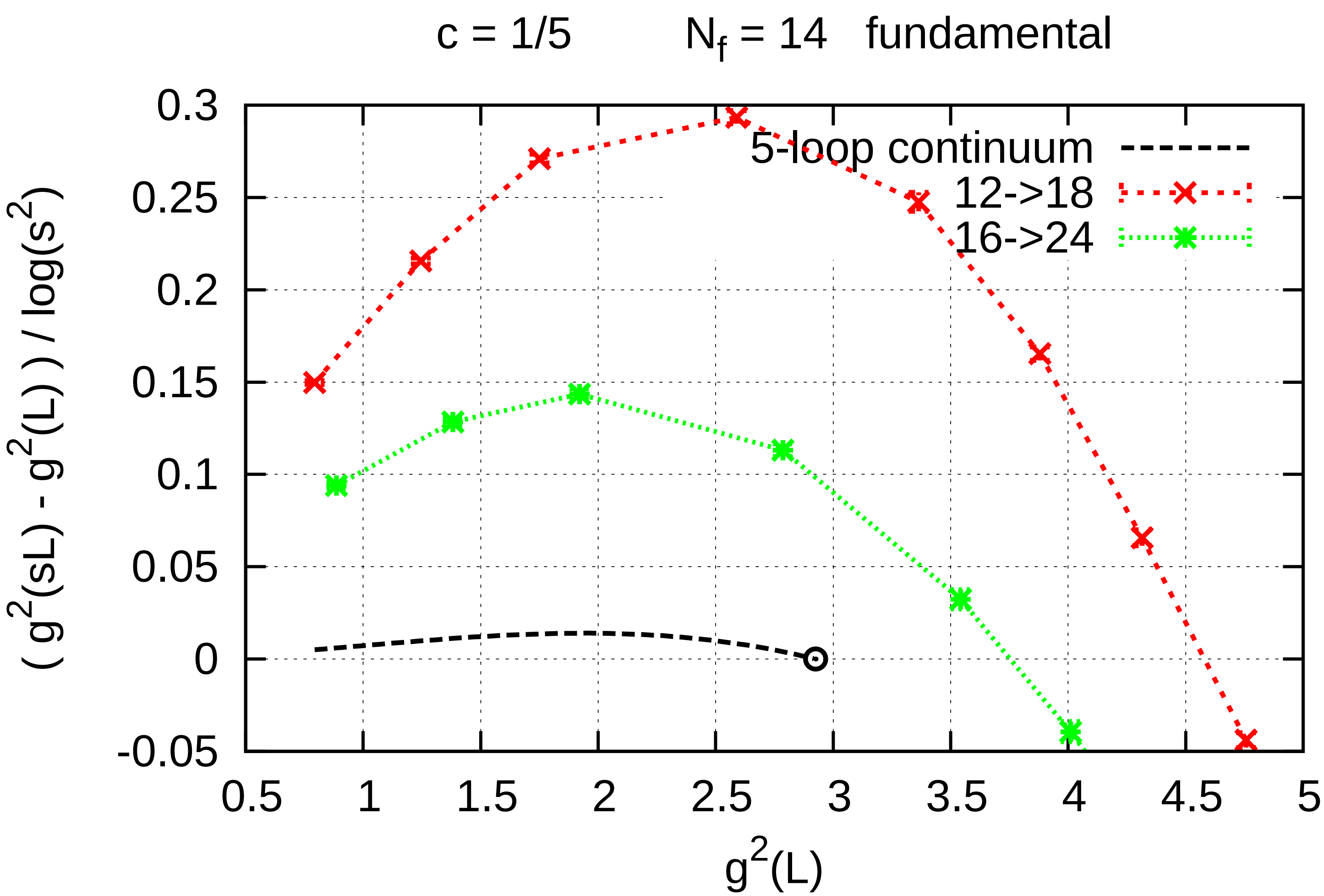
Results

Results for $N_f = 14$ fundamental



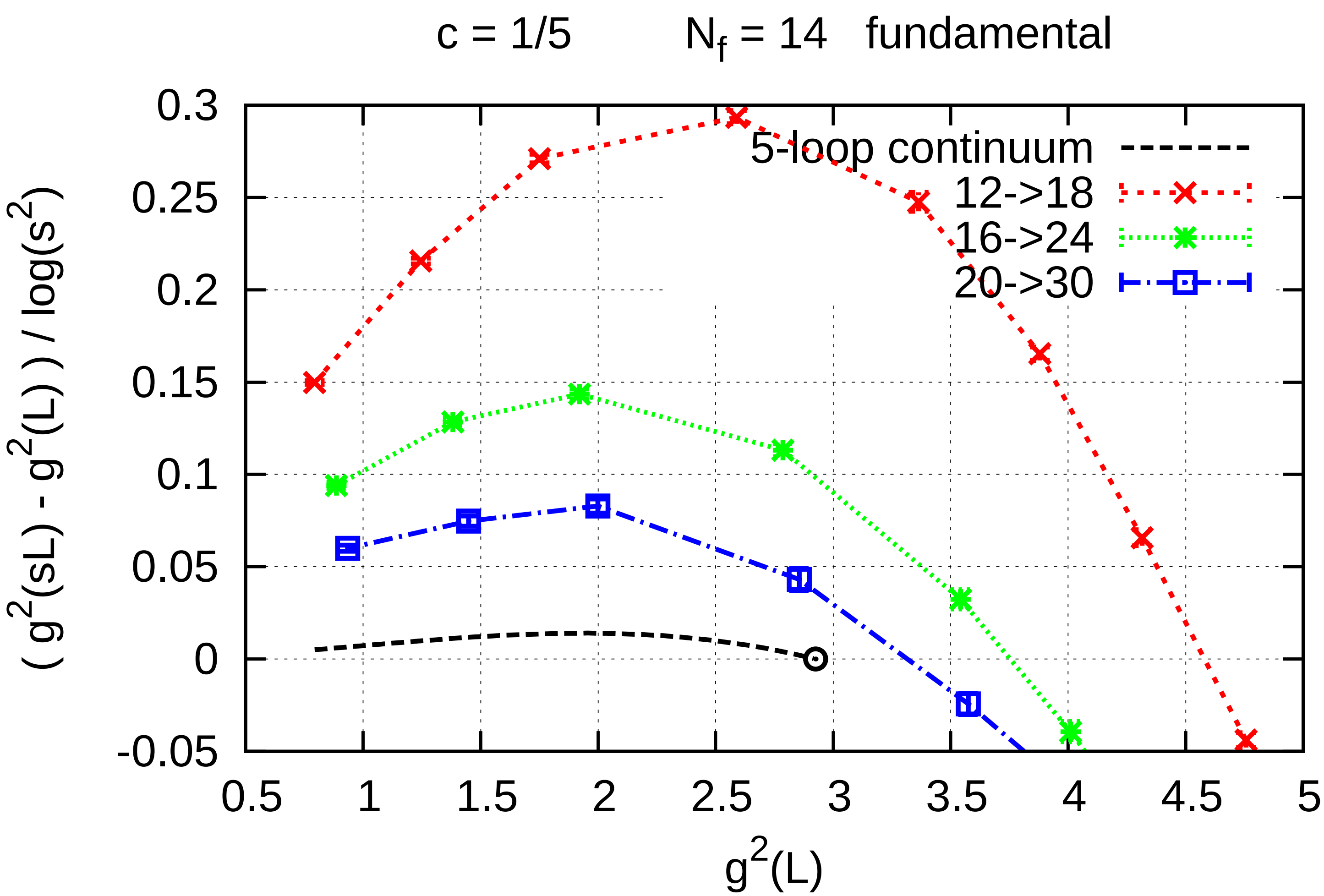
Maximum of 5-loop β-function: $\simeq 0.014$

Results for $N_f = 14$ fundamental



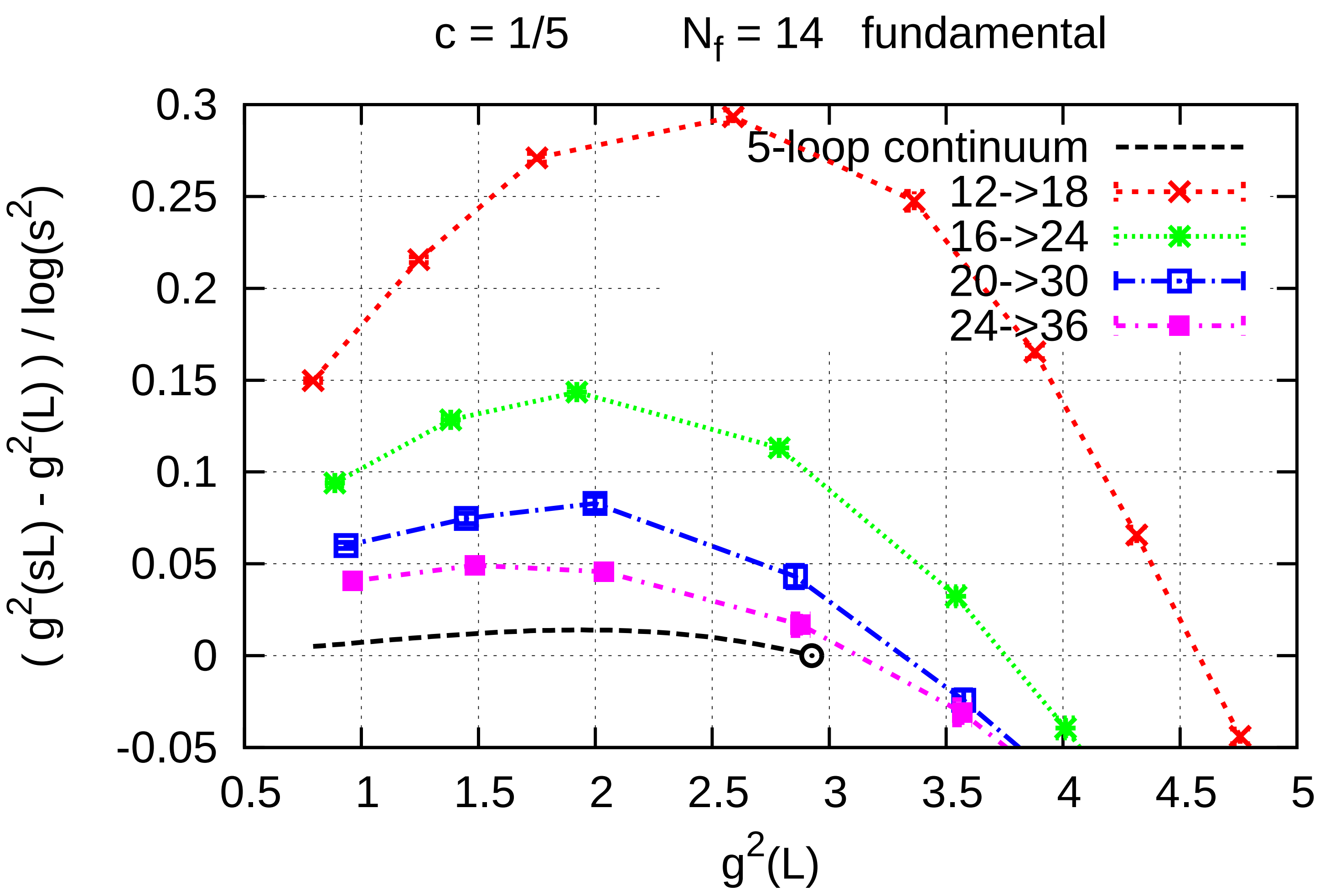
Maximum of 5-loop β-function: $\simeq 0.014$

Results for $N_f = 14$ fundamental



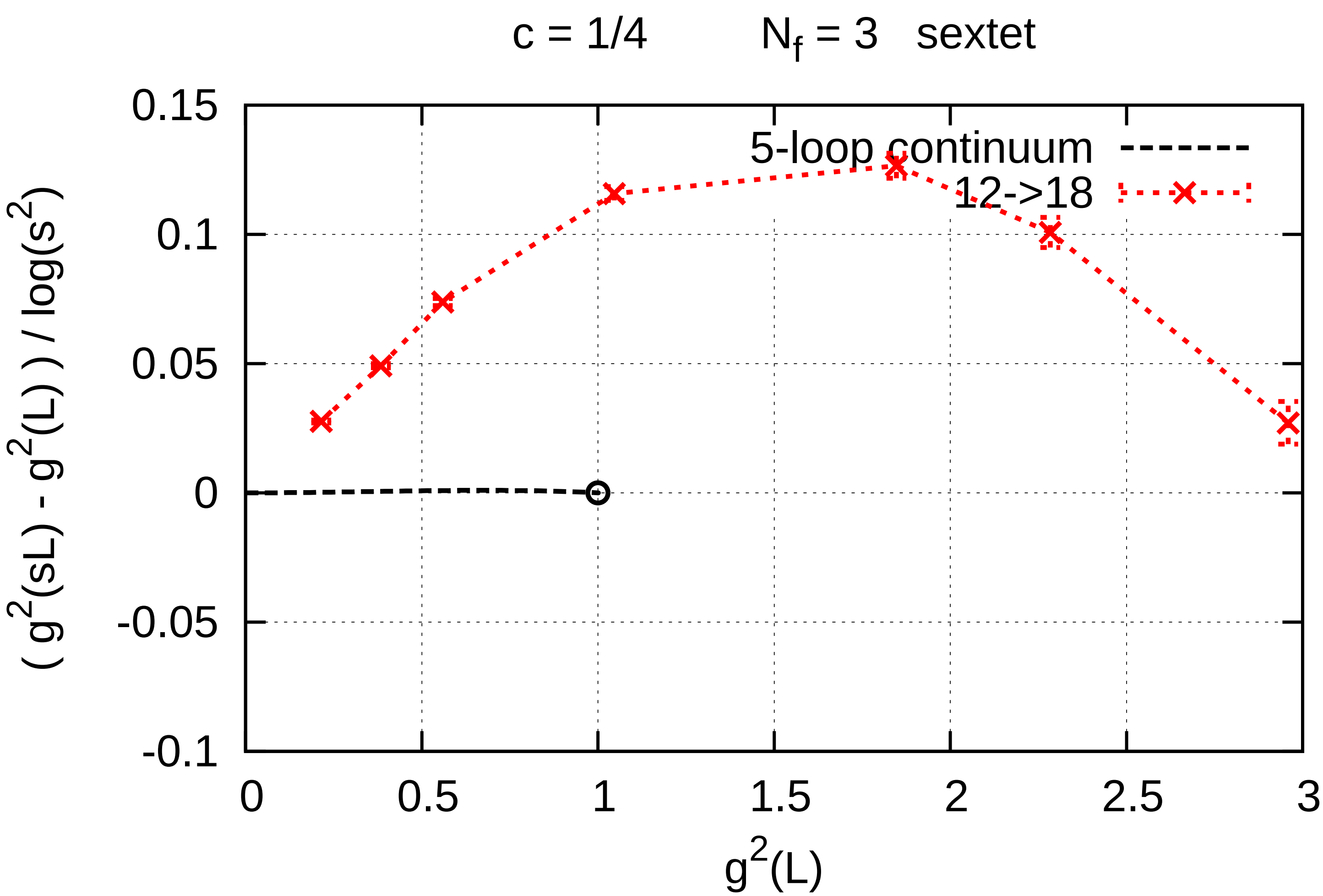
Maximum of 5-loop β-function: $\simeq 0.014$

Results for $N_f = 14$ fundamental



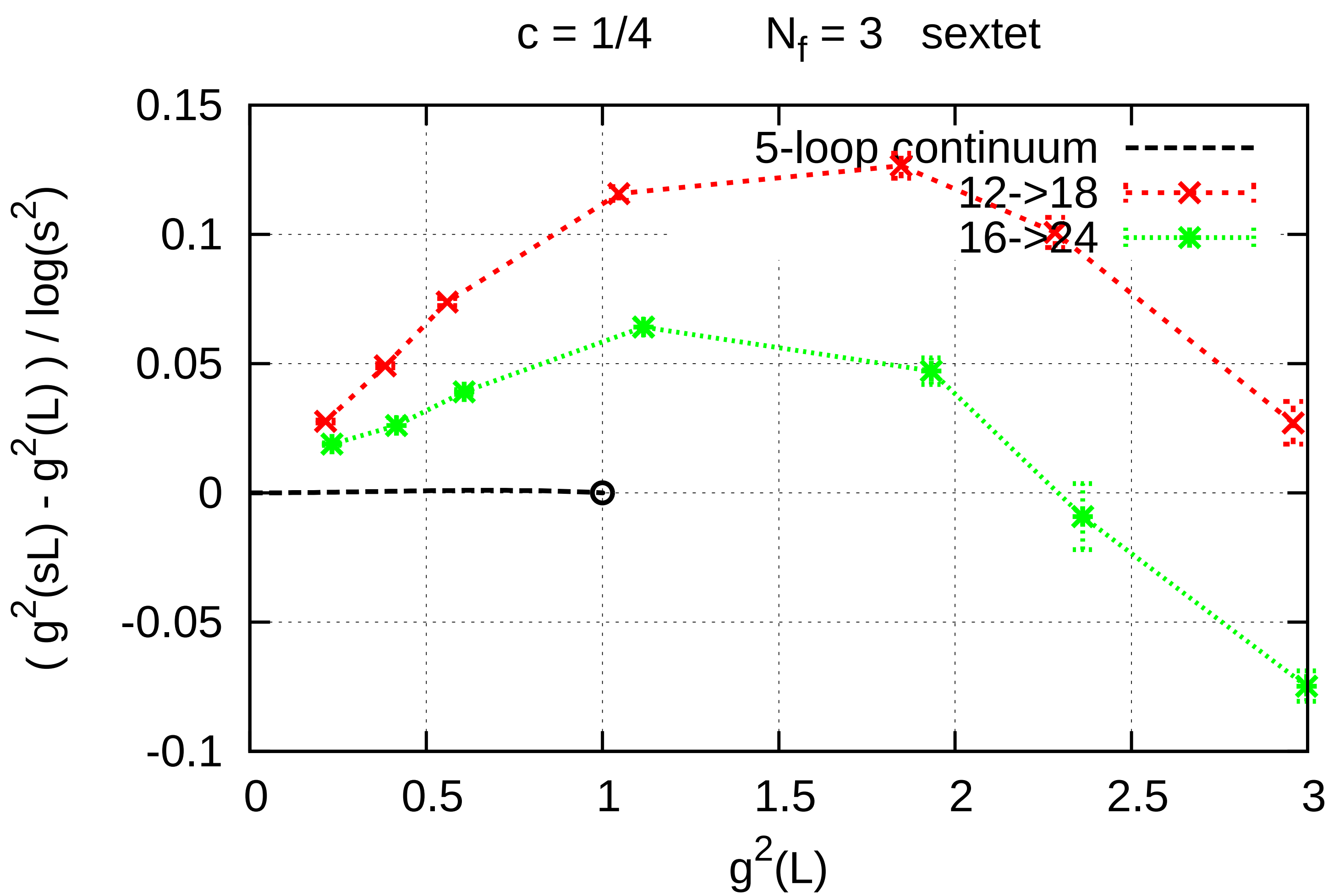
Maximum of 5-loop β-function: $\simeq 0.014$

Results for $N_f = 3$ sextet



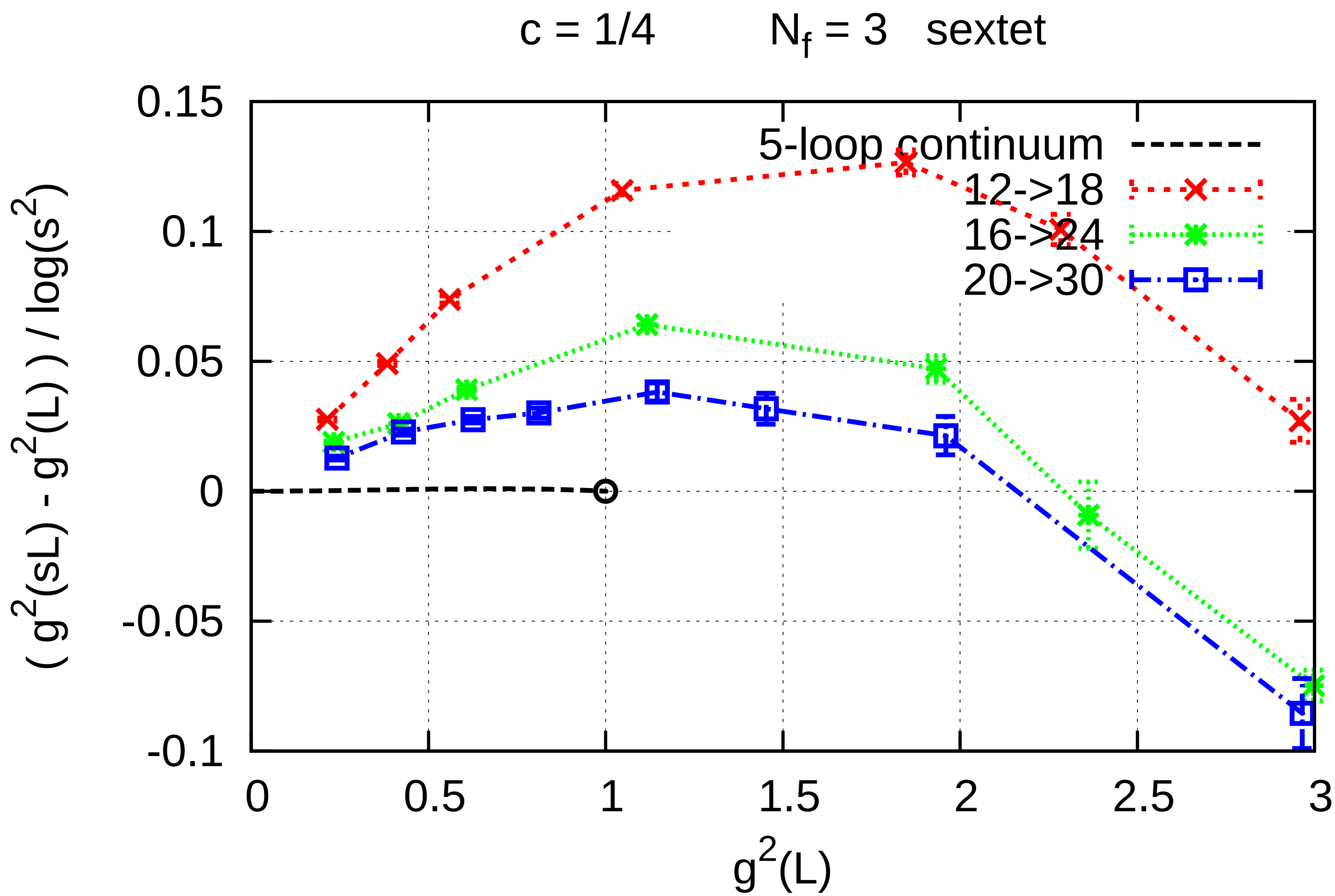
Maximum of 5-loop β-function: $\simeq 0.001$

Results for $N_f = 3$ sextet



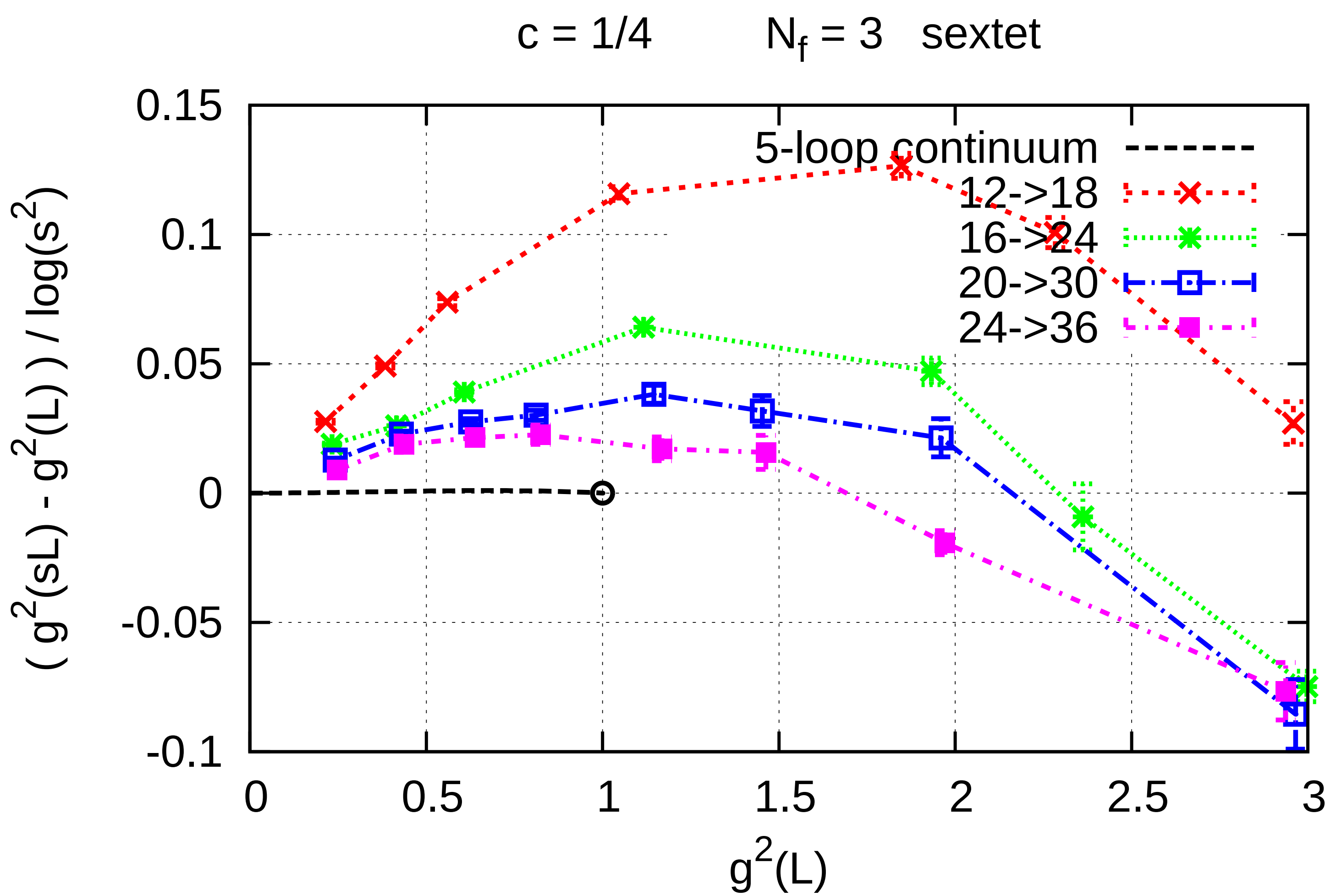
Maximum of 5-loop β-function: $\simeq 0.001$

Results for $N_f = 3$ sextet



Maximum of 5-loop β-function: $\simeq 0.001$

Results for $N_f = 3$ sextet



Maximum of 5-loop β-function: $\simeq 0.001$

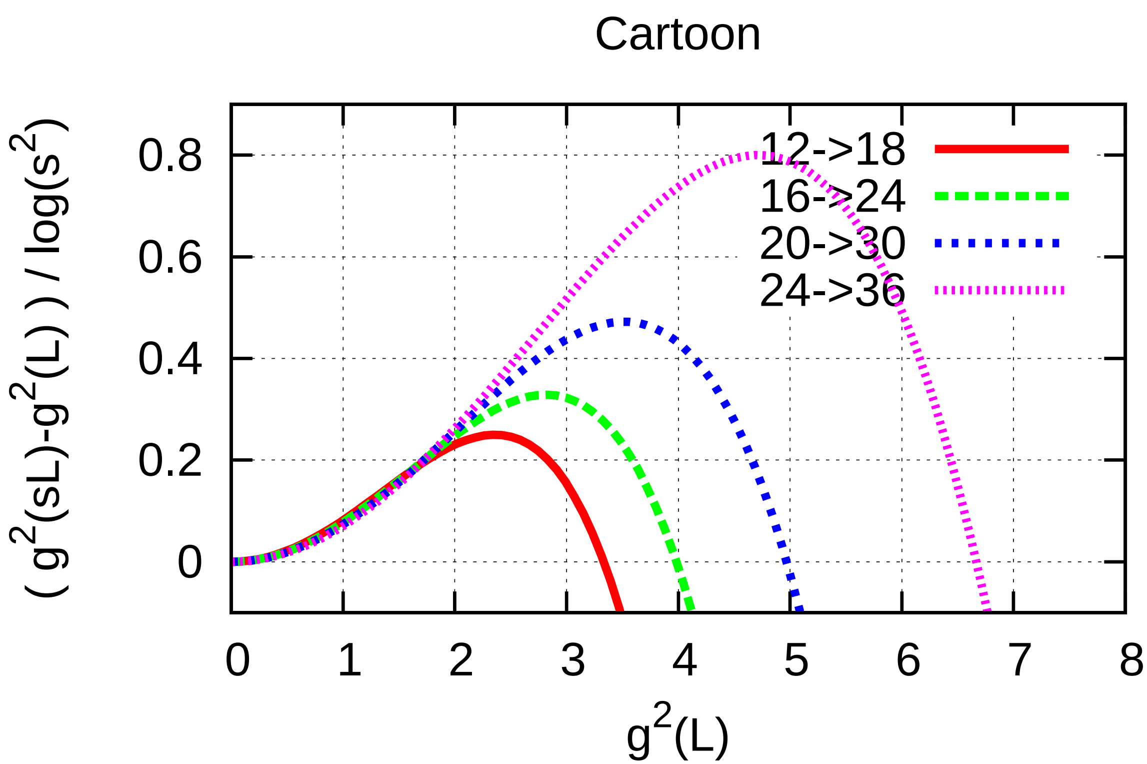
Results - notes

- Even at finite lattice volume pairs, the discrete β-function has a zero

- Could mean nothing, could disappear in the continuum (have seen it before)

Results - notes

Sketch for disappearing finite volume zeros (cartoon)



Continuum: maybe no IRFP if $g_*^2(L/a)$ keeps growing with increasing L/a

Difficult to conclude in such cases

Results - notes

Back to our lattice data for the two models:

- But! $g_*^2(12) > g_*^2(16) > g_*^2(20) > g_*^2(24)$
- Decreasing towards continuum
- Assume trend doesn't change
- At very small g^2 continuum β-function positive for sure

$\rightarrow$ IR fixed point in the continuum

Results - notes

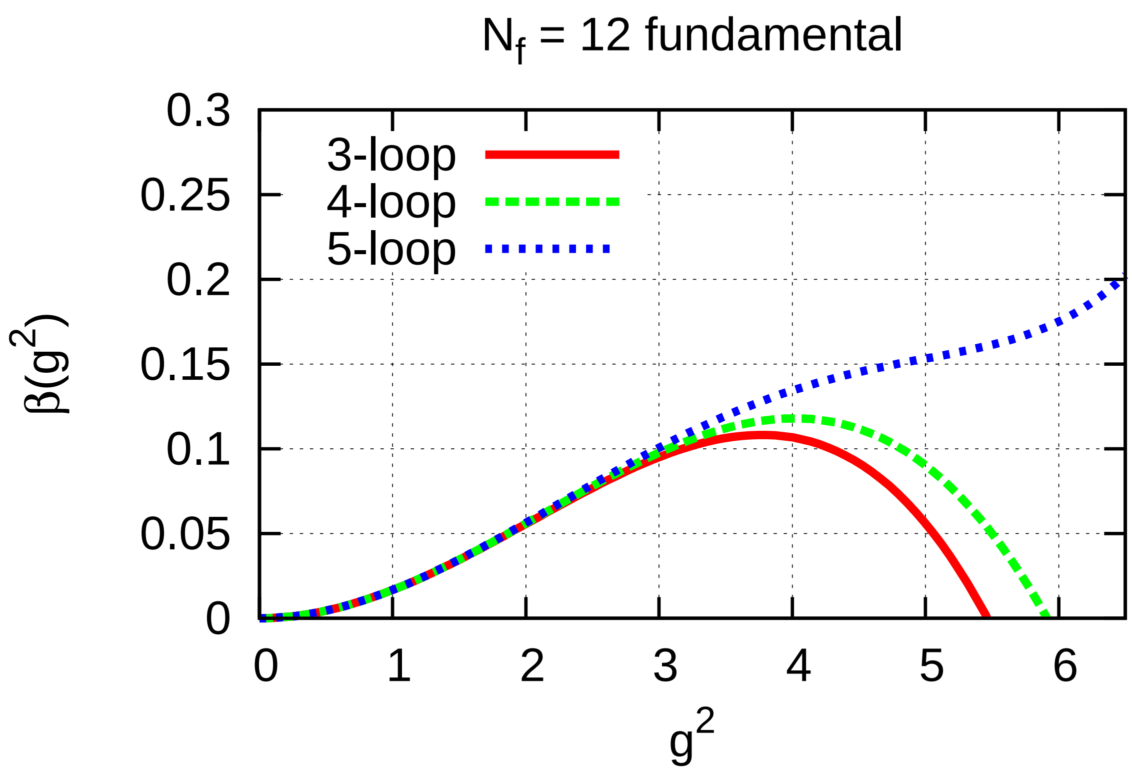
Actual continuum extrapolation difficult because:

- Errors large (especially $N_f = 3$ sextet)
- Very small absolute value for β-function
- Not obvious how to extrapolate $g_*^2(L/a)$ as $L/a \to \infty$

But existence of IR fixed point in continuum is guaranteed

Results - notes - disappearing zeros

Something similar happens for $N_f = 12$ fundamental and the loop expansion

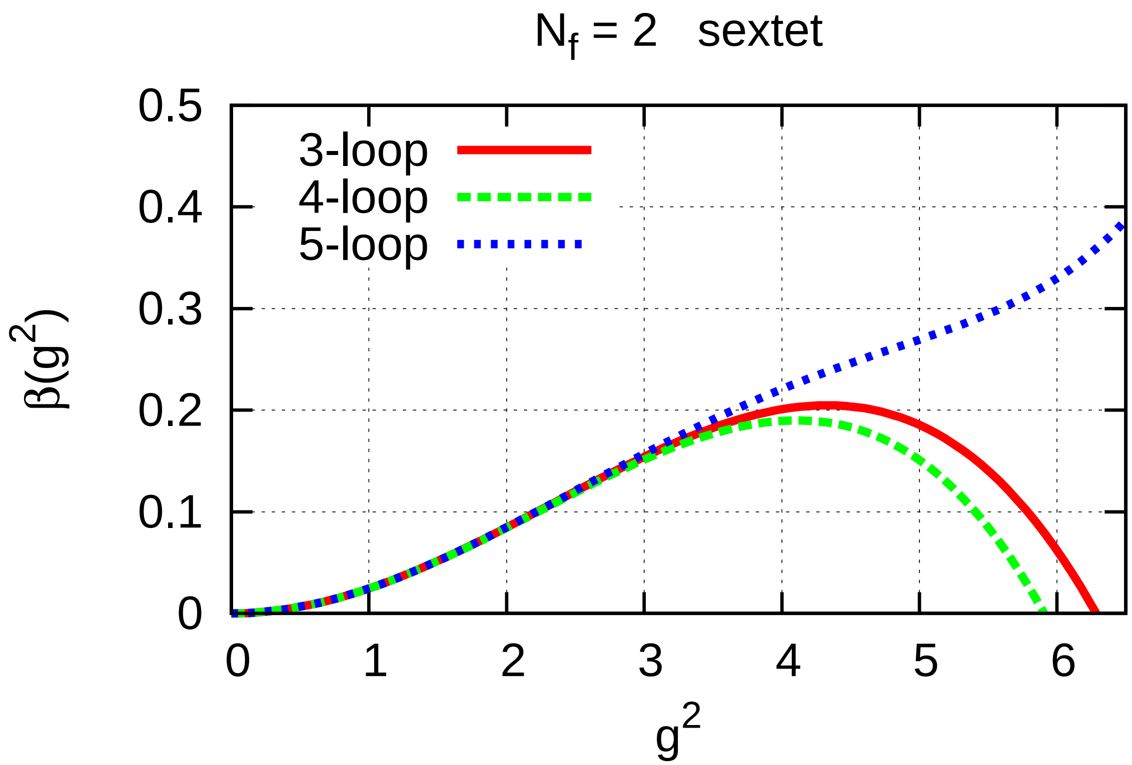


All-loops: ??

Difficult to conclude in such cases

Results - notes - disappearing zeros

Something similar happens for $N_f = 2$ sextet
and the loop expansion



All-loops: ??

Difficult to conclude in such cases

Did we learn something?

- Continuum extrapolation difficult (tiny β-function)

- If $g_*^2(L/a)$ decreasing as L/a increases + assuming this doesn't change $\rightarrow$ unambiguous sign of IR fixed point in continuum

Thank you for your attention!