Progress in computing parton distribution functions from the quasi-PDF approach

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Outline of the talk

1. Introduction
2. Quasi-PDFs
   - Basics
   - Renormalization
     - ⋆ Renormalization prescription
     - ⋆ Helicity/transversity
     - ⋆ Unpolarized
     - ⋆ Conversion RI'→ $\overline{\mathrm{MS}}$
3. Results
   - Renormalization for B55
   - Physical point – bare and renorm.
4. Conclusions and prospects

Based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", arXiv: 1706.00265 [hep-lat]
- M. Constantinou, H. Panagopoulos, "Perturbative Renormalization of quasi-PDFs", arXiv: 1705.11193 [hep-lat]
- preliminary unpublished results
- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "A Lattice Calculation of Parton Distributions", arXiv: 1504.07455 [hep-lat], Phys. Rev. D92 (2015) 014502
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, "New Lattice Results for Parton Distributions", arXiv: 1610.03689 [hep-lat]

PDFs – why is it difficult on the lattice?

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs have non-perturbative nature ⇒ LATTICE?
- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian – problem for the lattice!

$$q(x) = \frac{1}{2\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle N|\overline{\psi}(\xi^-)\Gamma\mathcal{A}(\xi^-,0)\psi(0)|N\rangle,$$

where: $\xi^- = \frac{\xi^0-\xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-,0)$ is the Wilson line from 0 to ξ^-.

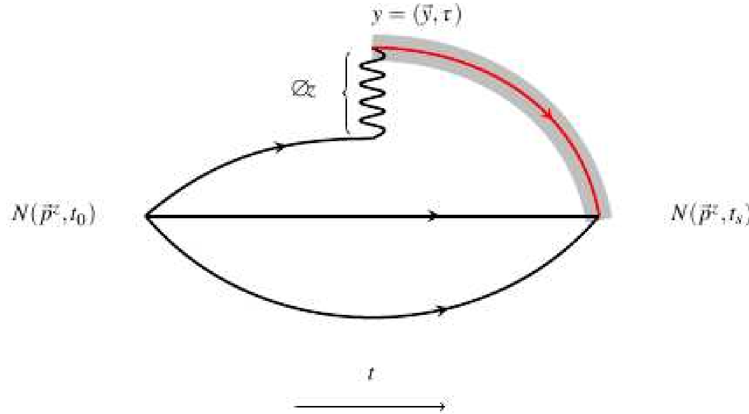
- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!
- Accessible on the lattice – moments of the distributions, but ...

See also the plenary talk by L. del Debbio!

Quasi-PDFs

- Quasi-PDF approach:
  X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002
- Compute a **quasi distribution** $\tilde{q}$, which is purely spatial and uses nucleons with finite momentum:

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi}\, e^{-ixP_3z} \langle N|\overline{\psi}(z)\gamma^z \mathcal{A}(z,0)\psi(0)|N\rangle.$$



- z – distance in any *spatial* direction z,
- P_3 – momentum boost in this direction.
- Differs from light-front PDFs by $\mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{P_3^2}, \frac{M_N^2}{P_3^2}\right)$.
- On the lattice, one is limited to finite values of P_3.
- Thus, the proposal is to calculate $\tilde{q}$ at finite P_3 and relate them to the usual distributions via perturbation theory.

Renormalization

Proposed renormalization programme described in:
C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, "A complete non-perturbative renormalization prescription for quasi-PDFs", arXiv: 1706.00265 [hep-lat].

First proposed and discussed in the talks of Martha Constantinou:
1. 7th Workshop of the APS Topical Group on Hadronic Physics, Washington D.C., February 2, 2017
2. Parton Distributions and Lattice Calculations in the LHC era, Oxford, March 23, 2017

Important insights also from the lattice perturbative paper:
M. Constantinou, H. Panagopoulos, "Perturbative Renormalization of quasi-PDFs", arXiv: 1705.11193 [hep-lat]
$\rightarrow$ discovered mixing between the vector and scalar matrix elements (unpolarized PDF). **This perturbative analysis is very important guidance to non-perturbative renormalization!**

Renormalization

Features of the proposed renormalization programme:

1. Removes the linear divergence that resums into a multiplicative exponential factor, $\exp\left(-\delta m|z|/a + c|z|\right)$
   δm – strength of the divergence, operator independent,
   c – arbitrary scale (fixed by the renormalization prescription).
2. Takes away the logarithmic divergence with respect to the regulator, $\log(a\mu)$, where μ is the renormalization scale.
3. Applies the necessary finite renormalization related to the lattice regularization.
4. Unpolarized – eliminates the mixing between the vector operator and the twist-3 scalar operator; the two may be disentangled by the construction of a 2×2 mixing matrix.

Non-perturbative renormalization scheme: RI'-MOM.
Considered flavour non-singlet operators: $\mathcal{O}_\Gamma = \overline{u}(x)\Gamma\mathcal{P}e^{ig\int_0^z d\zeta A(\zeta)}d(x+z\hat{\mu})$,
where $\Gamma = \gamma_\mu,\ \gamma_\mu\gamma_5,\ \gamma_\mu\gamma_\nu$.

Helicity/transversity case (no mixing)

RI'-MOM renormalization conditions:
for the operator:

$$Z_q^{-1}\, Z_{\mathcal{O}}(z)\, \frac{1}{12} \mathrm{Tr}\left[\mathcal{V}(p,z)\left(\mathcal{V}^{\mathrm{Born}}(p,z)\right)^{-1}\right]\Big|_{p^2=\bar{\mu}_0^2} = 1\,,$$

for the quark field:

$$Z_q = \frac{1}{12}\mathrm{Tr}\left[(S(p))^{-1}\, S^{\mathrm{Born}}(p)\right]\Big|_{p^2=\bar{\mu}_0^2}\,.$$

- momentum p entering the vertex function is set to the RI$'$ renormalization scale $\bar{\mu}_0$, chosen such that p_3 is the same as the nucleon boost P_3,
- $\mathcal{V}(p,z)$ – amputated vertex function of the operator,
- $\mathcal{V}^{\mathrm{Born}}$ – its tree-level value, $\mathcal{V}^{\mathrm{Born}}(p,z)=i\gamma_3\gamma_5\, e^{ipz}$ for helicity,
- $S(p)$ – fermion propagator ($S^{\mathrm{Born}}(p)$ at tree-level).

Helicity/transversity case (no mixing)

- The vertex functions $\mathcal{V}(p)$ contain the same linear divergence as the nucleon matrix elements.
- This is crucial, as it allows the extraction of the exponential together with the multiplicative Z-factor
- $Z_\mathcal{O}$ can be factorized as $Z_\mathcal{O}(z) = \overline{\mathcal{Z}}_\mathcal{O}(z)\; e^{+\delta m|z|/a - c|z|}$, where $\overline{\mathcal{Z}}_\mathcal{O}$ is the multiplicative Z-factor of the operator.
- Note that the exponential comes with a different sign compared to the nucleon matrix element ($Z_\mathcal{O}$ is related to the inverse of the vertex function).
- Consequently, the above renormalization condition handles all the divergences which are present in the matrix element under consideration.
- In the absence of a Wilson line ($z{=}0$), the renormalization functions reduce to the local currents, free of any power divergence, e.g. for helicity $Z_\mathcal{O}(z{=}0) \equiv Z_A$.

- Special treatment required – mixing with the scalar operator!
  [M. Constantinou, H. Panagopoulos, arXiv:1705.11193]
- Operators:

$$\mathcal{O}_S = \overline{u}(x)\,\hat{1}\,\mathcal{P}\,e^{i\,g\,\int_0^z A(\zeta)d\zeta}\,d(x+z\hat{\mu})\,,$$

$$\mathcal{O}_V = \overline{u}(x)\,\gamma_\mu\,\mathcal{P}\,e^{i\,g\,\int_0^z A(\zeta)d\zeta}\,d(x+z\hat{\mu})\,.$$

- One must compute the mixing matrix:

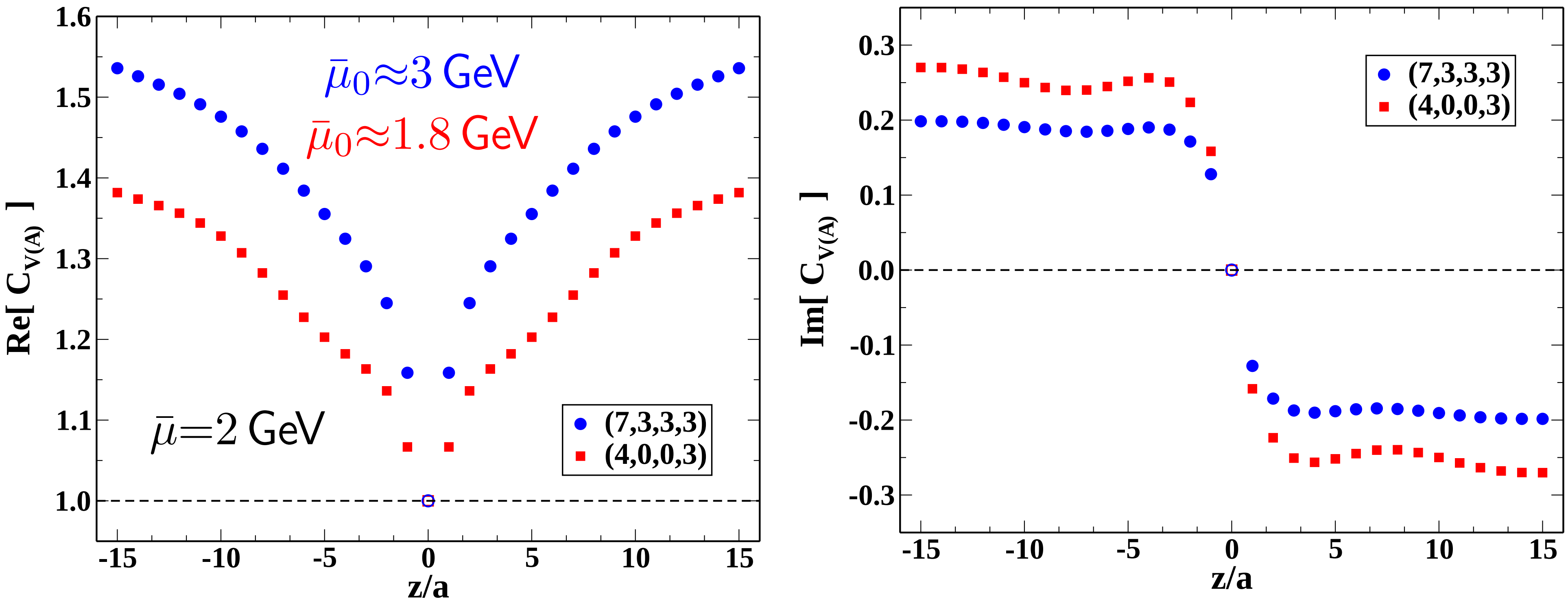
$$\begin{pmatrix}\mathcal{O}_V^R(P_3,z)\\ \mathcal{O}_S^R(P_3,z)\end{pmatrix} = \begin{pmatrix} Z_{VV}(z) & Z_{VS}(z)\\ Z_{SV}(z) & Z_{SS}(z)\end{pmatrix}\begin{pmatrix}\mathcal{O}_V(P_3,z)\\ \mathcal{O}_S(P_3,z)\end{pmatrix}.$$

- Thus, the renormalized matrix elements for the unpolarized quasi-PDFs are:

$$h_V^R(P_3,z) = Z_{VV}\;h_V(P_3,z) + Z_{VS}\;h_S(P_3,z)\,.$$

- Similar RI'-MOM conditions to find the mixing matrix.

- Z-factors expressed in the RI' scheme can be converted to the $\overline{\mathrm{MS}}$ scheme.
- 1-loop conversion found in: [M. Constantinou, H. Panagopoulos, arXiv:1705.11193]
- 1-loop anomalous dimension, $\gamma_{\mathcal{O}} = -3\frac{g^2 C_f}{16\pi^2}$, is scheme-independent and also independent of the Dirac structure.



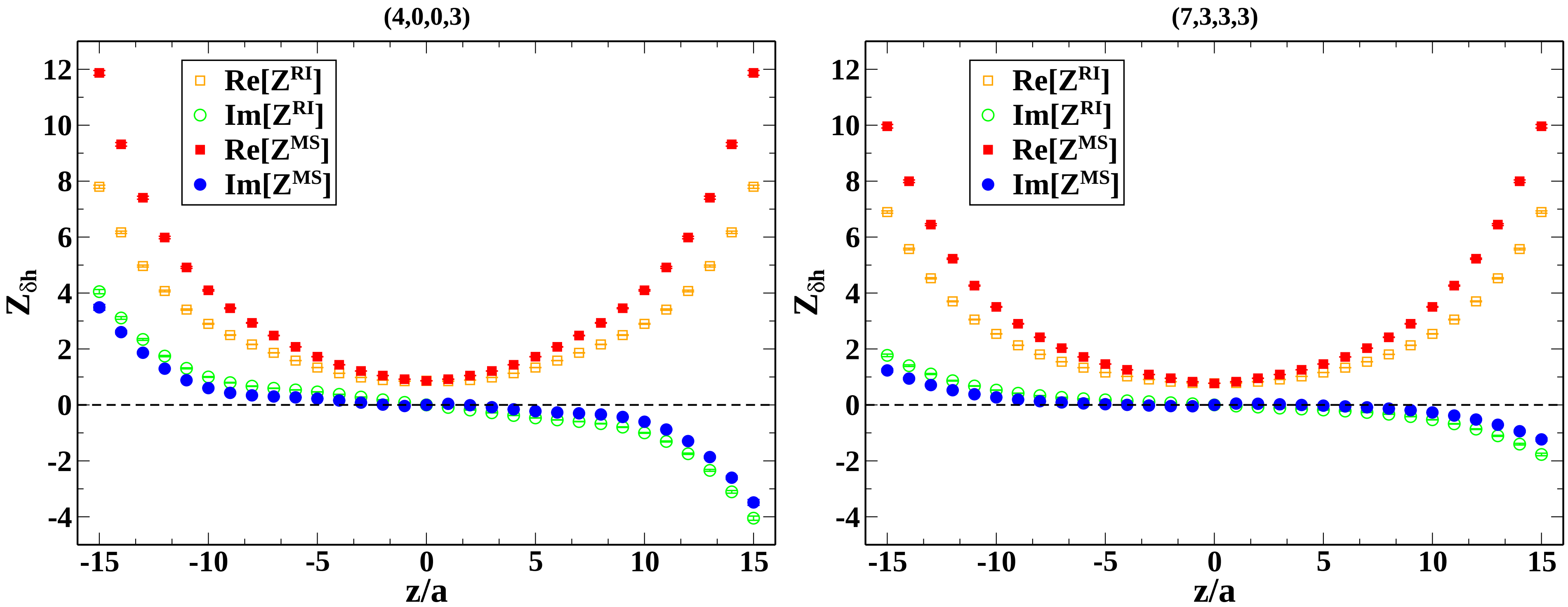
Lattice setup

Two setups:

1. $N_f = 2 + 1 + 1$ twisted mass fermions, Iwasaki gauge action, $\beta = 1.95$, $L/a = 32$, $a \approx 0.0815$ fm, $m_\pi \approx 370$ MeV.
2. $N_f = 2$ twisted mass fermions + clover term, Iwasaki gauge action, $\beta = 2.1$, $L/a = 48$, $a \approx 0.093$ fm, $m_\pi \approx 135$ MeV.

Renormalization – helicity

$P_3 = 6\pi/32 \approx 1.43$ GeV

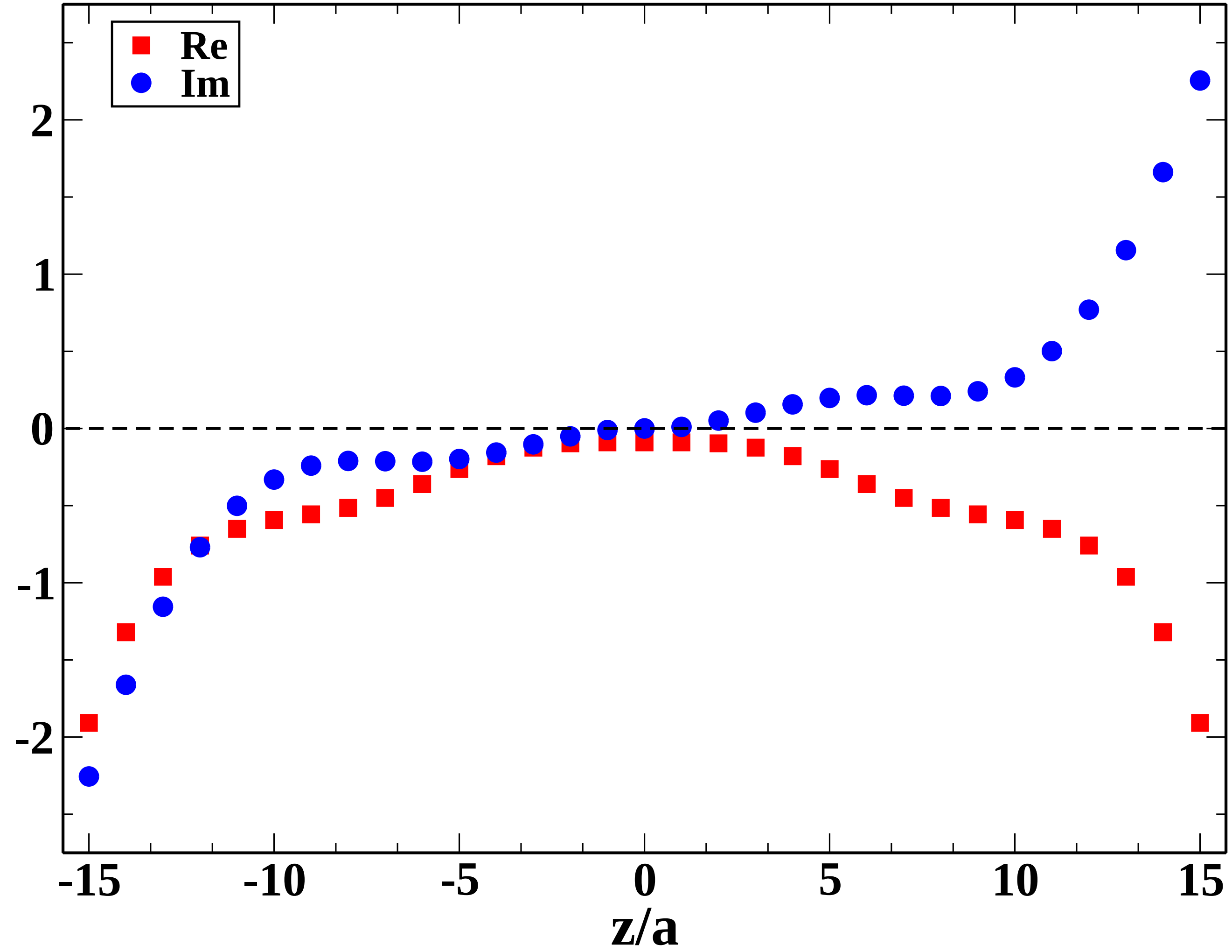


$\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{4}{2}+\frac{1}{4}, 0, 0, 3\right)$ $\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{7}{2}+\frac{1}{4}, 3, 3, 3\right)$

$z = 0: \quad Z_A \approx 0.86$ $z = 0: \quad Z_A \approx 0.77$

[C. Alexandrou, M. Constantinou, H. Panagopoulos, Phys. Rev. D95 (2017) 034505]: $Z_A = 0.7556(5)$

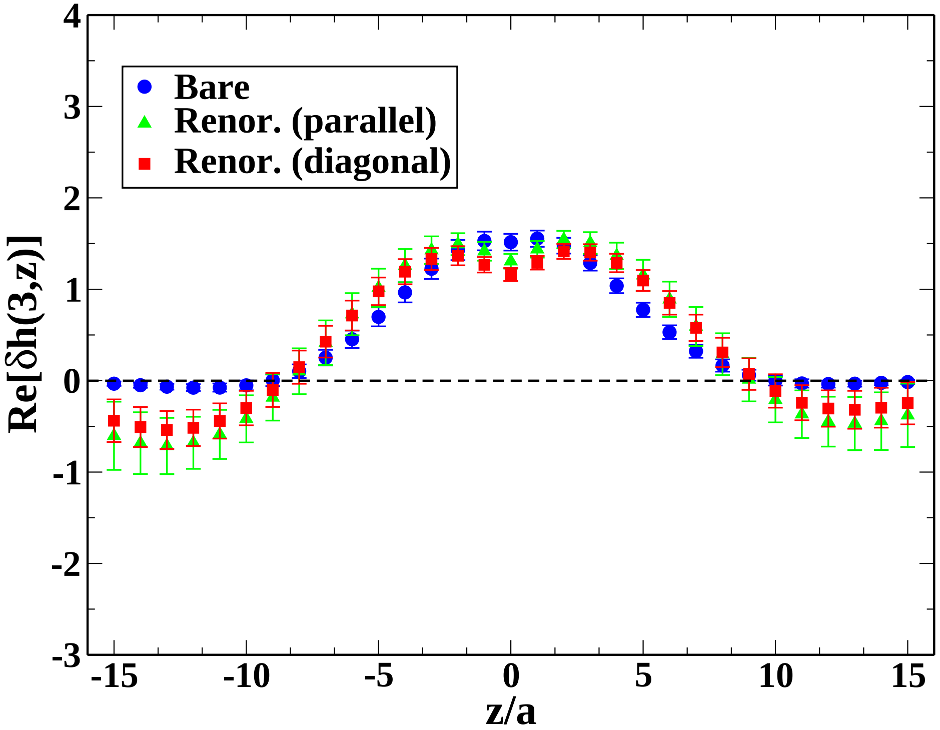
Difference of $Z_{\delta m}^{\overline{\mathrm{MS}}}$ between the "parallel" and "diagonal" cases for the RI' momentum $\bar{\mu}_0$

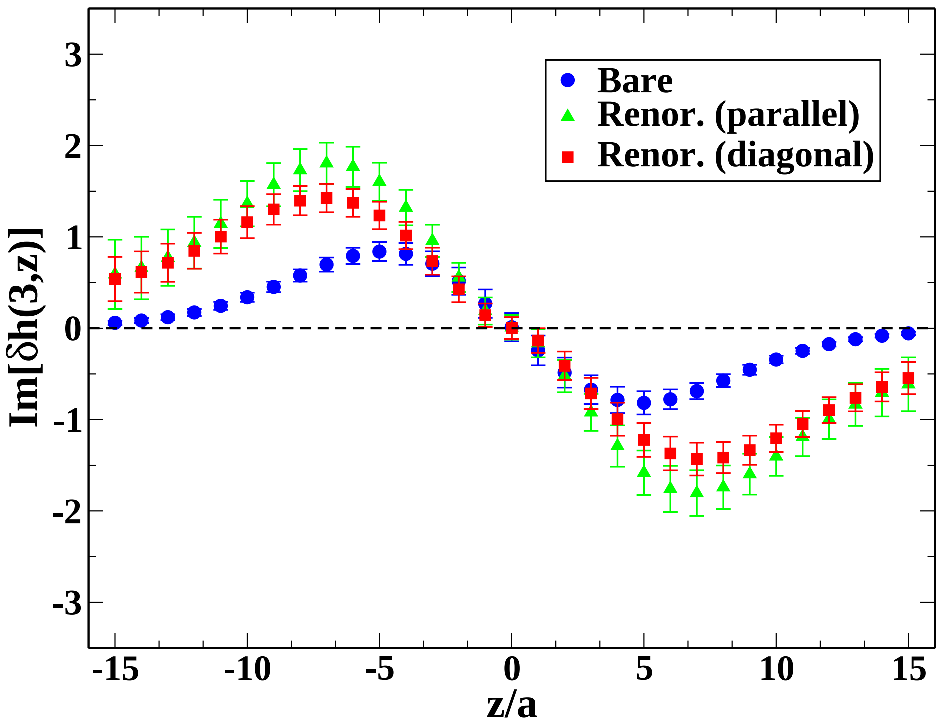


Renormalization – helicity

Comparison of bare and renormalized matrix elements for the "parallel" and "diagonal" cases: $\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{4}{2}+\frac{1}{4}, 0, 0, 3\right)$ and $\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{7}{2}+\frac{1}{4}, 3, 3, 3\right)$

$P_3 = 6\pi/32 \approx 1.43$ GeV



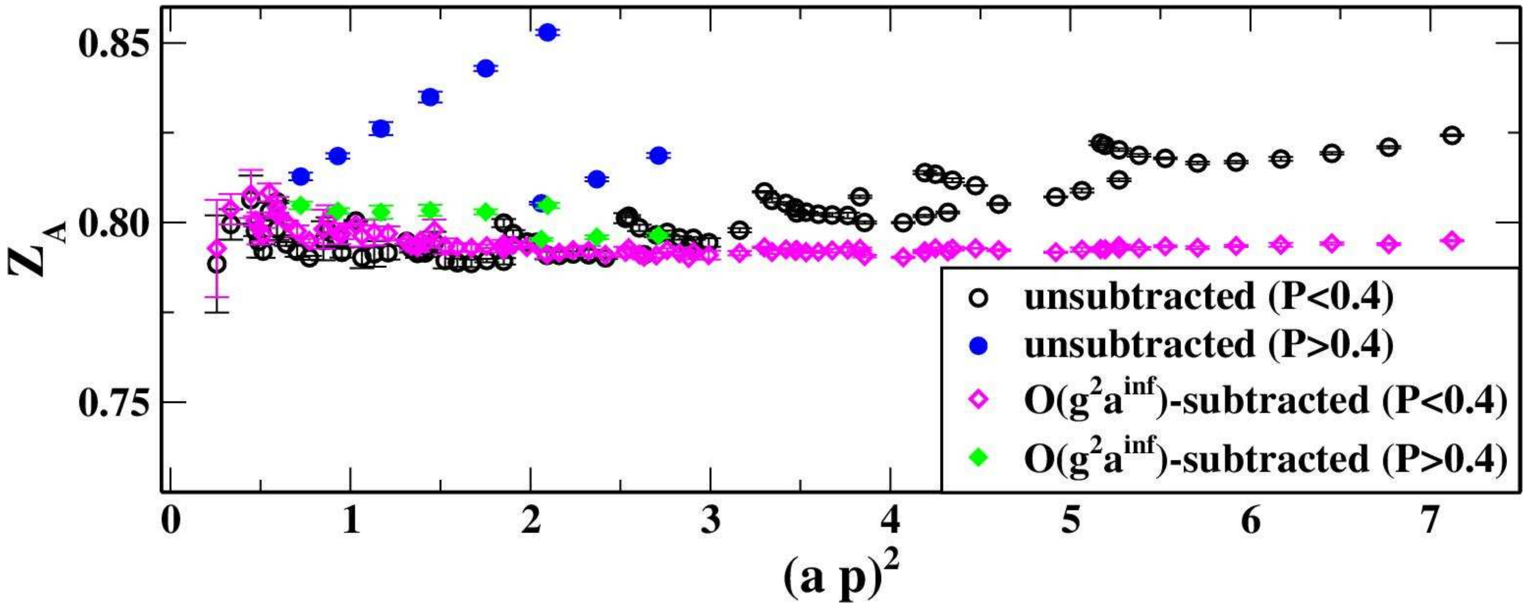


$$\mathrm{Re}\left[\delta h^{ren}\right] = \mathrm{Re}\left[Z_{\delta m}^{\overline{\mathrm{MS}}}\right] \mathrm{Re}\left[\delta h^{bare}\right] - \mathrm{Im}\left[Z_{\delta m}^{\overline{\mathrm{MS}}}\right] \mathrm{Im}\left[\delta h^{bare}\right]$$

$$\mathrm{Im}\left[\delta h^{ren}\right] = \mathrm{Re}\left[Z_{\delta m}^{\overline{\mathrm{MS}}}\right] \mathrm{Im}\left[\delta h^{bare}\right] + \mathrm{Im}\left[Z_{\delta m}^{\overline{\mathrm{MS}}}\right] \mathrm{Re}\left[\delta h^{bare}\right]$$

Example for Z_A:

[C. Alexandrou, M. Constantinou, H. Panagopoulos, Phys. Rev. D95 (2017) 034505]

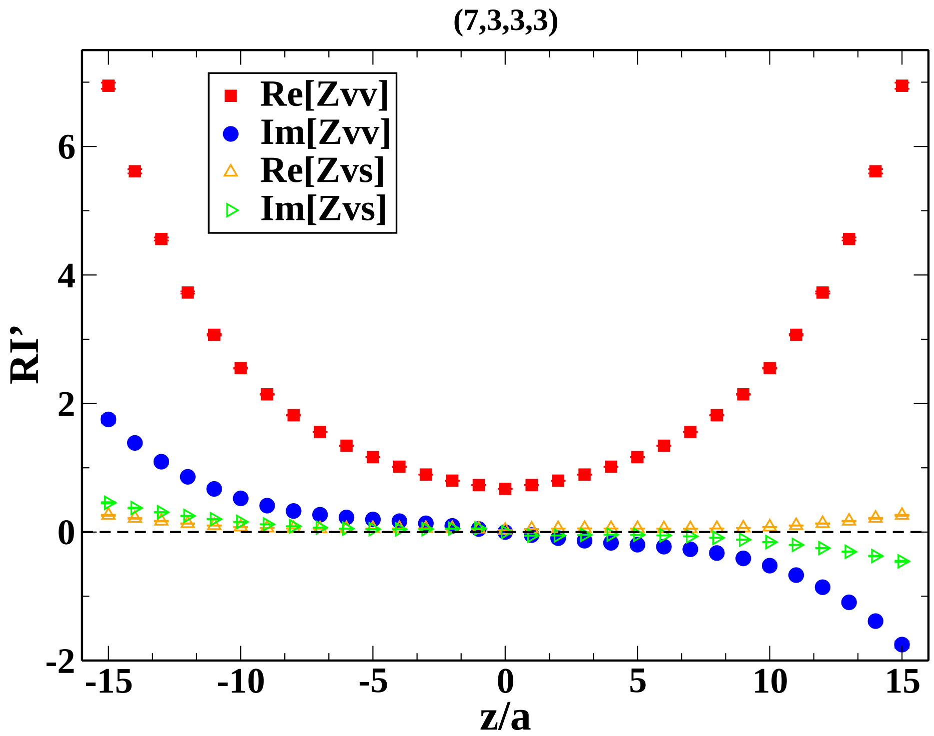


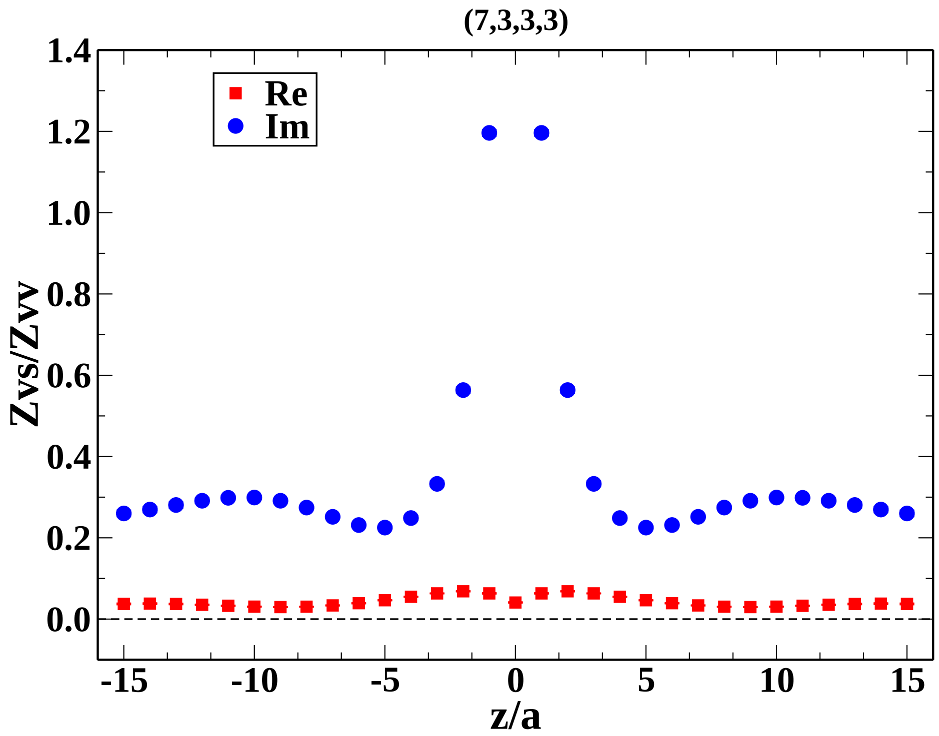
$\hat{P} \equiv \frac{\sum_\rho \bar{\mu}_{0\rho}^4}{(\sum_\rho \bar{\mu}_{0\rho}^2)^2}$ equals 0.5 for $\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{4}{2}+\frac{1}{4}, 0, 0, 3\right)$

and 0.25 for $\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{7}{2}+\frac{1}{4}, 3, 3, 3\right)$.

Renormalization – unpolarized (mixing)

$P_3 = 6\pi/32 \approx 1.43$ GeV

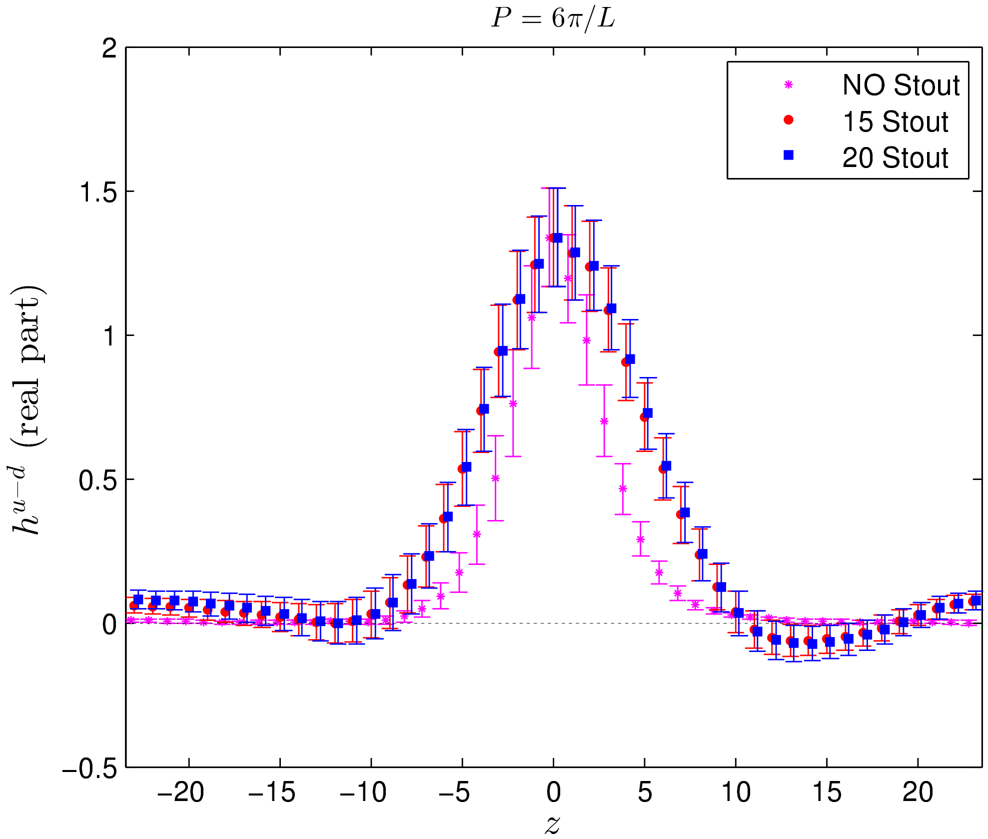


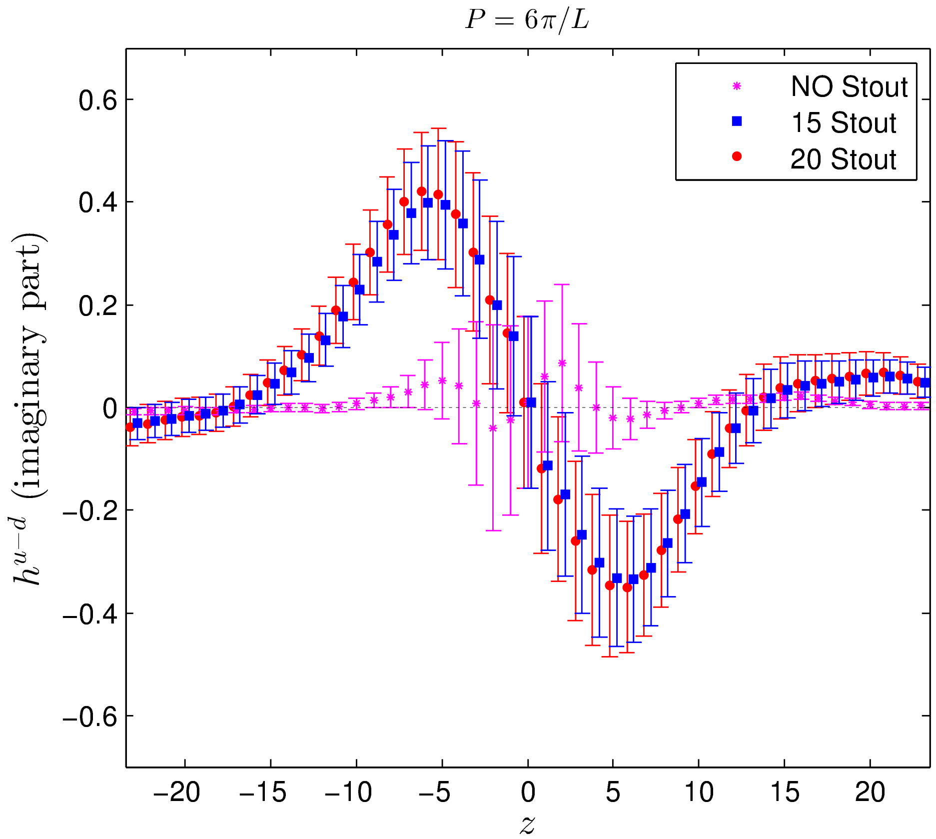


$\bar{\mu}_0 = \frac{2\pi}{32}\left(\frac{7}{2}+\frac{1}{4}, 3, 3, 3\right)$

$z = 0: \qquad Z_V \approx 0.67$

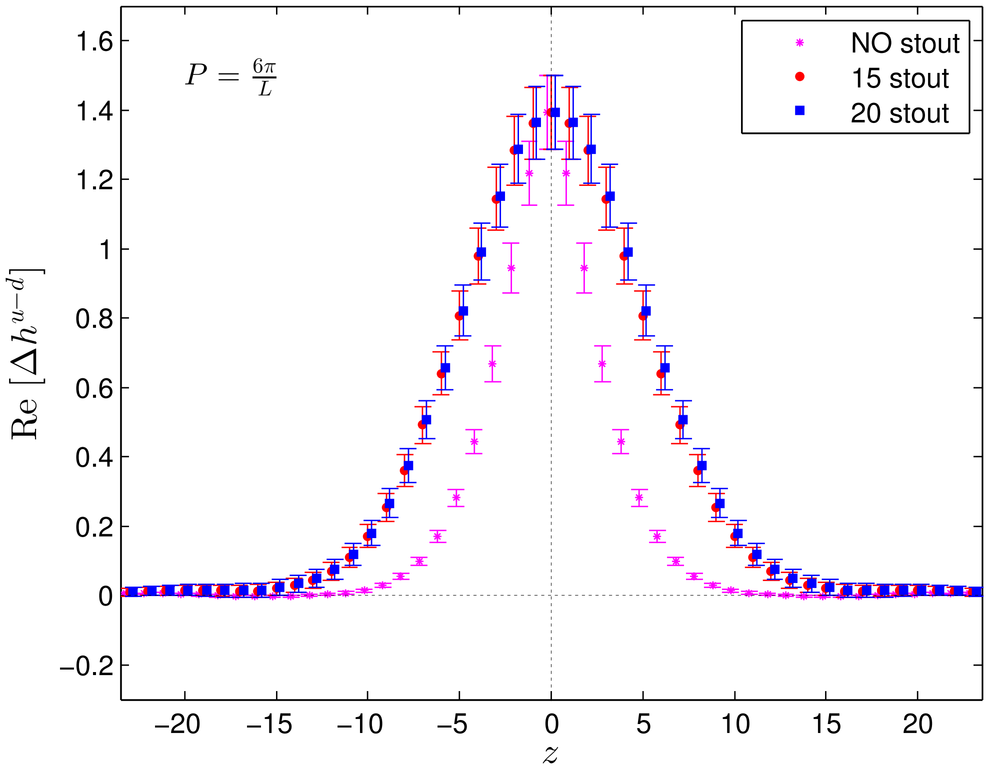
[C. Alexandrou, M. Constantinou, H. Panagopoulos, Phys. Rev. D95 (2017) 034505]: $Z_V = 0.6298(5)$

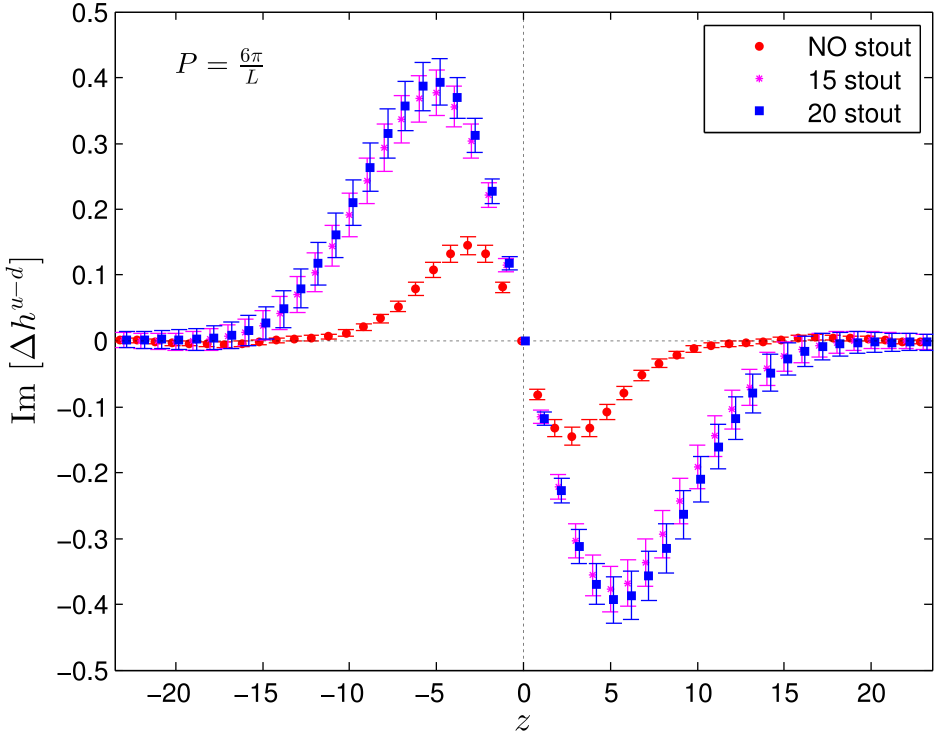




$P_3 = 6\pi/48 \approx 0.83$ GeV

See also poster of A. Scapellato later today!





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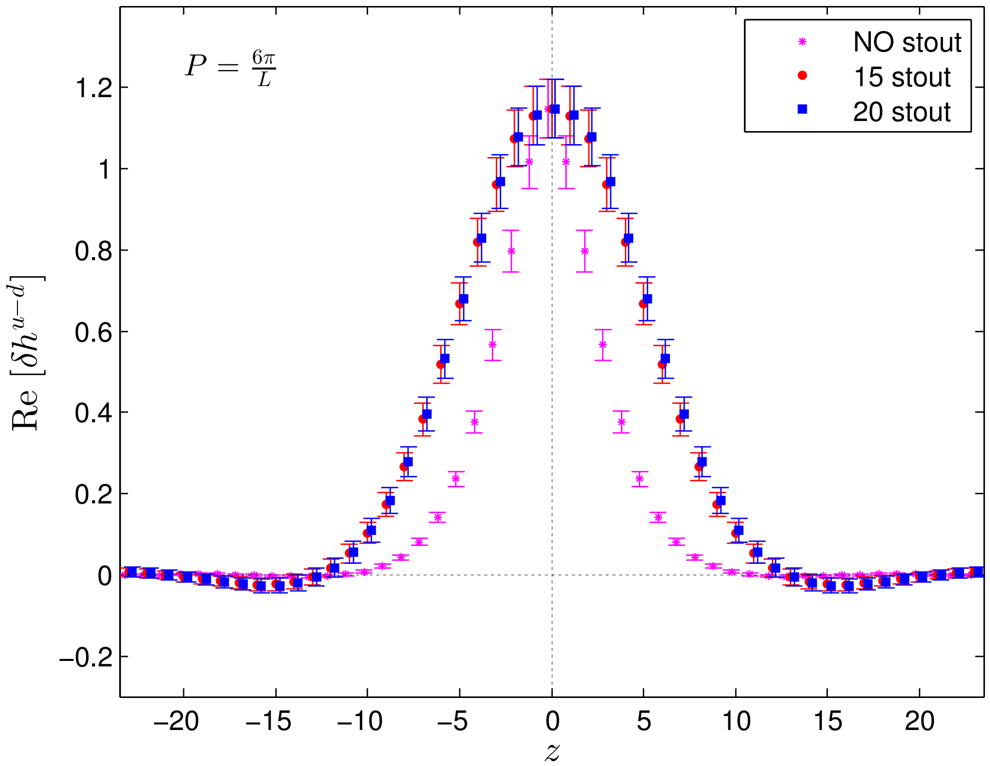

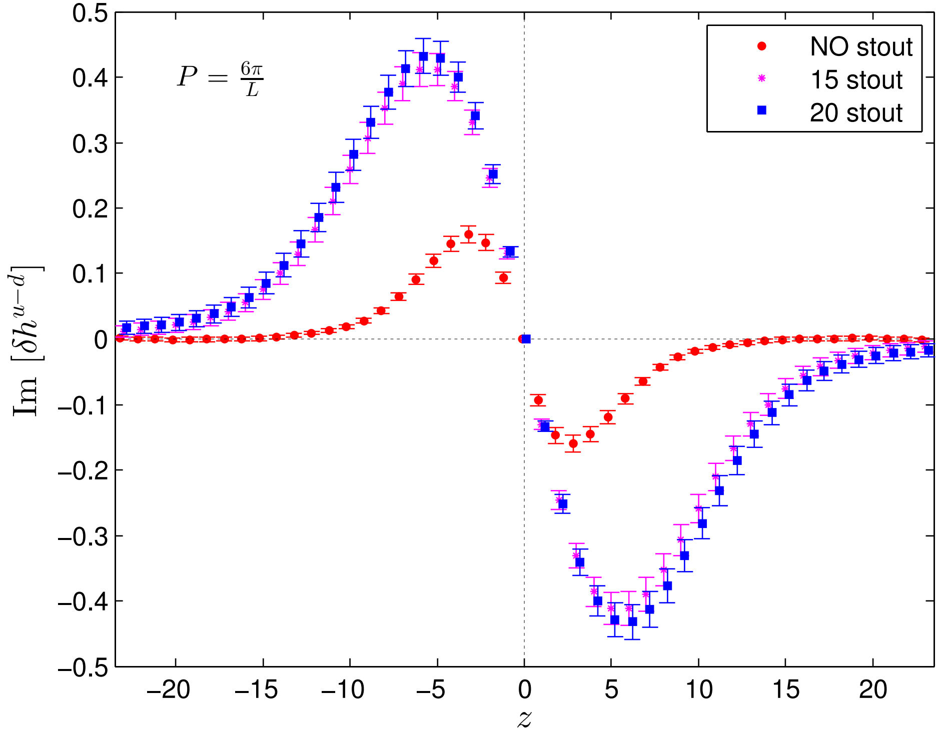

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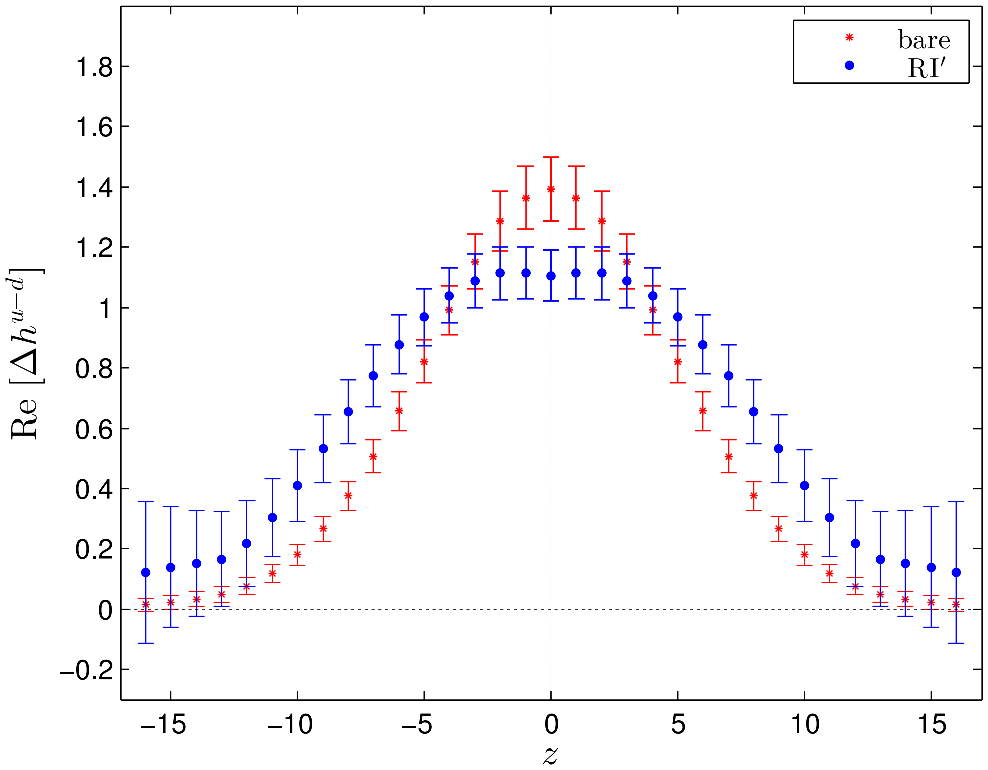

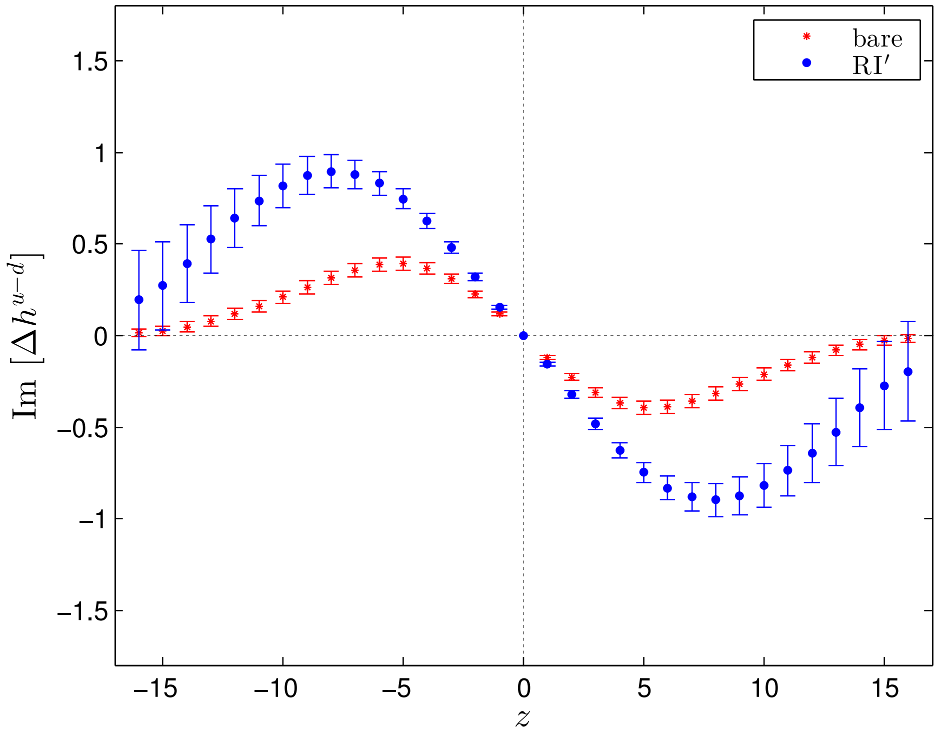

$P_3 = 6\pi/48 \approx 0.83$ GeV, $\bar{\mu}_0 = \frac{2\pi}{48}\left(\frac{9}{2}+\frac{1}{4}, 3, 3, 3\right)$ (RI' scale around 2 GeV)

Higher momentum computation $P_3 = 12\pi/48 \approx 1.67$ GeV in progress.

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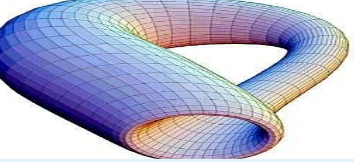
Conclusions

- We have presented a **full renormalization prescription** to handle all divergences present in matrix elements for quasi-PDFs:
  - ⋆ standard logarithmic divergence renormalized with $\overline{\mathcal{Z}}_{\mathcal{O}}$,
  - ⋆ power divergence renormalized with $e^{\delta m|z|/a - c|z|}$.
- For unpolarized, **mixing between vector and scalar matrix elements** – needs computation of a mixing matrix.
- **Hypercubic artefacts in Z-factors** – needs subtraction of $\mathcal{O}(a^\infty g^2)$ effects.
- For conversion to $\overline{MS}$, one needs to take care of **truncation effects in the conversion factor** – likely that 1-loop is not enough.
- Also, we have presented preliminary bare and renormalized results at the physical point – statistics constantly increasing, in particular at a higher momentum.



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Thank you for your attention!