Quasi parton distributions and the gradient flow

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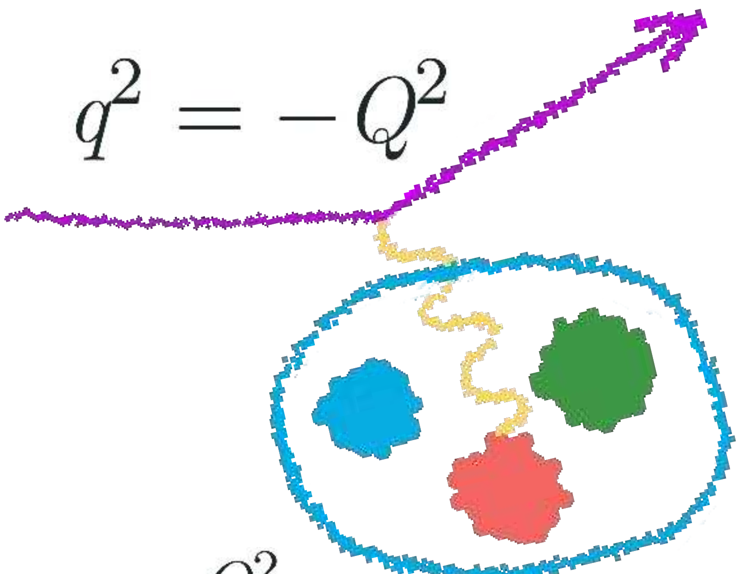
JHEP 03 (2017) 116
With Kostas Orginos

HOW FAST DO PARTONS TRAVEL?

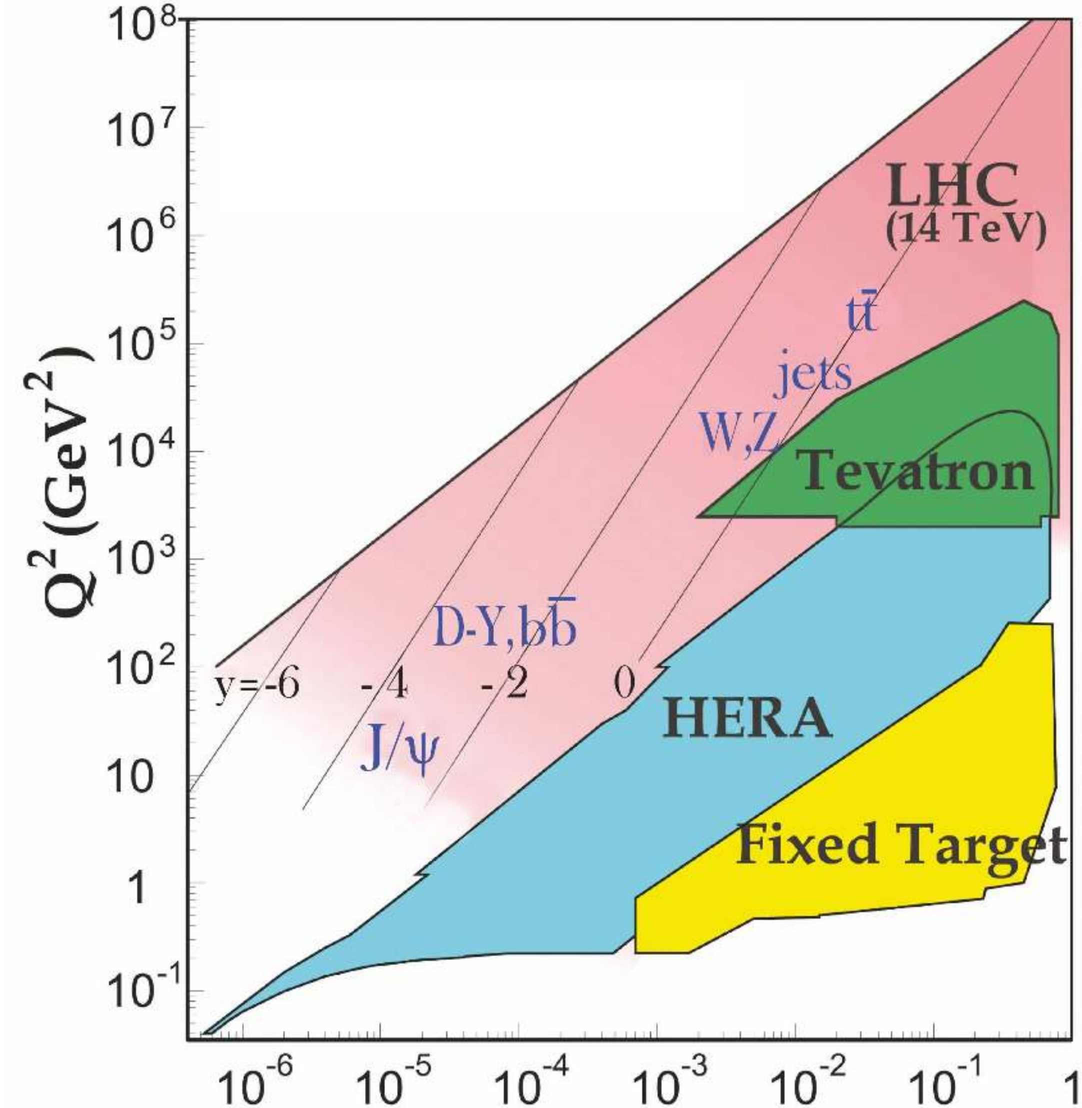
How is the momentum of a fast-moving nucleon distributed amongst its constituents?

EXPERIMENTAL EXTRACTION

$$q^2 = -Q^2$$



$$x = \frac{Q^2}{2P \cdot q}$$



From PDG 2016

QUASI DISTRIBUTIONS

X. Ji, PRL 110 (2013) 262002
X. Ji, Sci.Ch. PMA 57 (2014) 1407

Defined as

$$q(x,P_z) = \int \frac{\mathrm{d}z}{4\pi} e^{ixzP_z} \langle P | \overline{\psi}(z) \Gamma e^{-ig_0 \int_0^z \mathrm{d}z' A(z')} \psi(0) | P \rangle_C$$

Compare to

$$f(x) = \int \frac{\mathrm{d}\omega^-}{4\pi} e^{-ix\omega^- P^+} \langle P | \overline{\psi}(0,\omega^-,\mathbf{0}) \Gamma e^{-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A(0,y^-,\mathbf{0})} \psi(0) | P \rangle_C$$

Related to light-front PDFs via

$$q(x,\mu^2,P^z) = \int_x^1 \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) f(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}, \frac{M^2}{(P^z)^2}\right)$$

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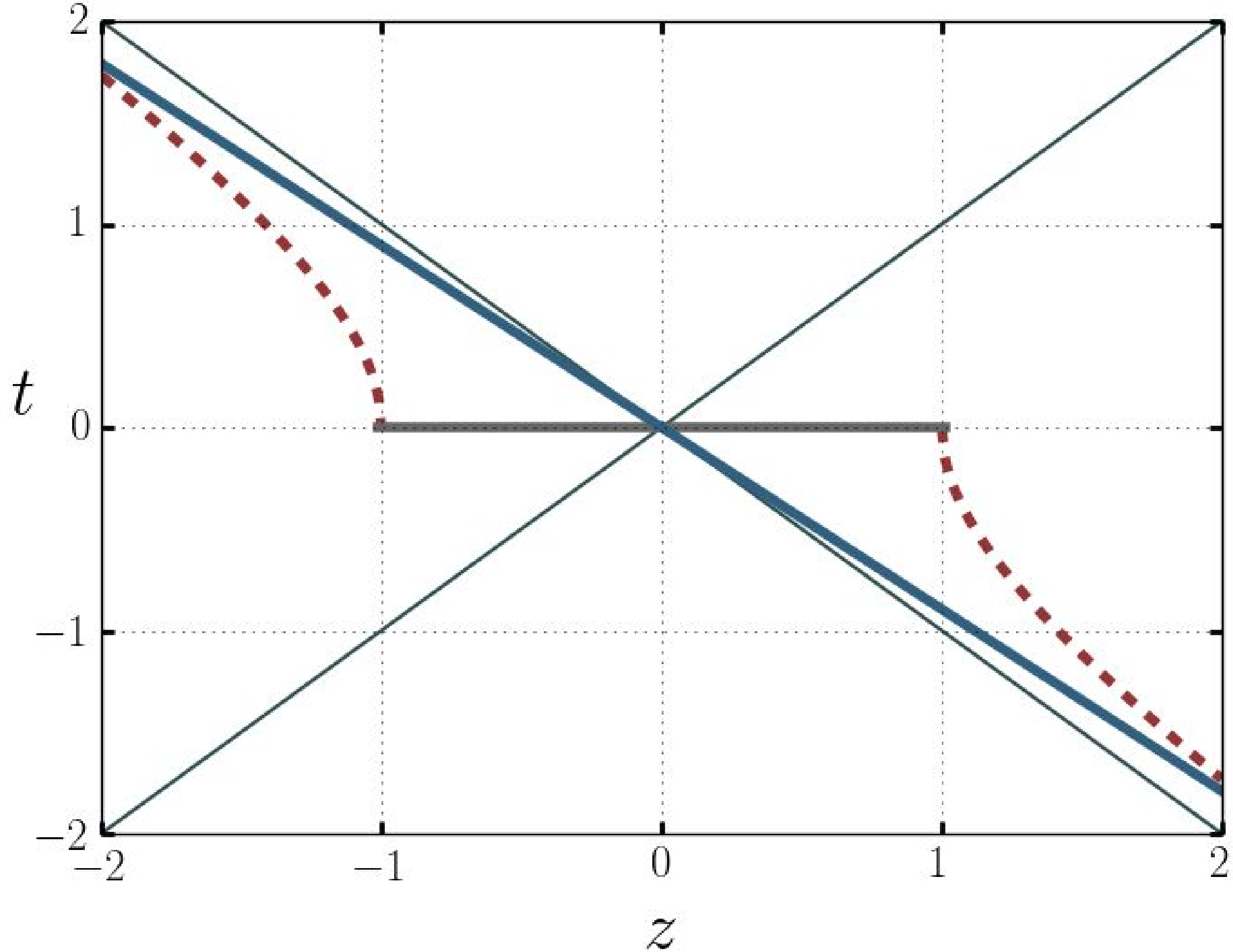


A MORE GENERAL FRAMEWORK

Define lattice “cross-sections”

Y.-Q. Ma & J.-W. Qiu, arXiv:1404.6860

$$\lim_{a\to 0} \sigma\left(x, a, P^z\right) = \widetilde{\sigma}\left(x, \widetilde{\mu}, P^z\right)$$

Quasi distributions - lattice “cross-section” from which one can extract PDFs

$$\widetilde{\sigma}\left(x, \widetilde{\mu}, P^z\right) = \sum_{\alpha} H_{\alpha}\left(x, \frac{\widetilde{\mu}}{P^z}, \frac{\widetilde{\mu}}{\mu}\right) \otimes f_{\alpha}(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\widetilde{\mu}^2}\right)$$

See also:

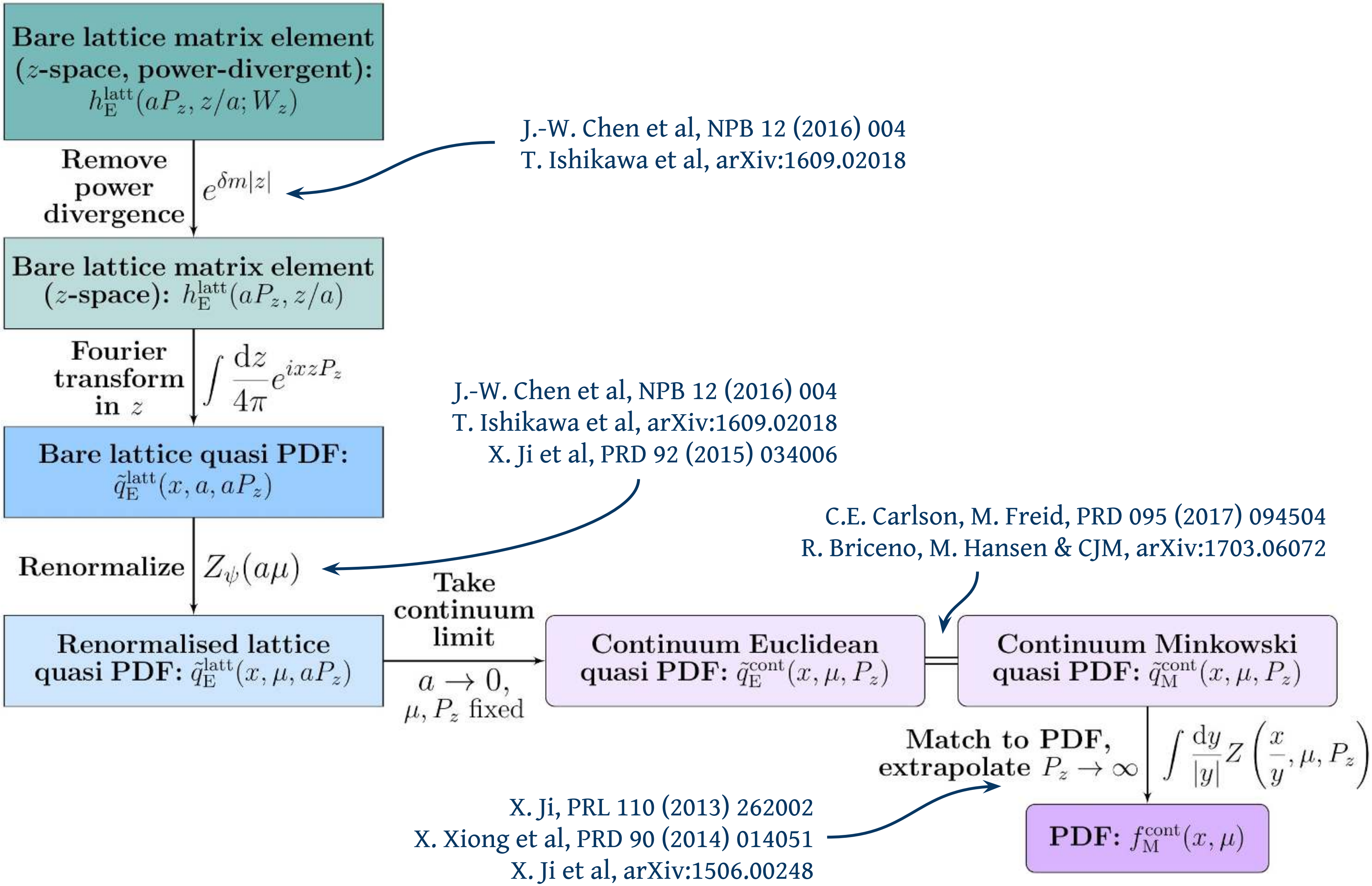
K.-F. Liu, PRD 62 (2000) 074501
W. Detmold & C.J.D. Lin, PRD 73 (2006) 014501
A. Radyushkin, PLB (2017) 02 019

GENERAL PROCEDURE

Bare lattice matrix element (z-space, power-divergent):
$h_{\mathrm{E}}^{\mathrm{latt}}(aP_z, z/a; W_z)$

PDF: $f_{\mathrm{M}}^{\mathrm{cont}}(x, \mu)$

GENERAL PROCEDURE



GENERAL PROCEDURE: GENERAL CHALLENGES

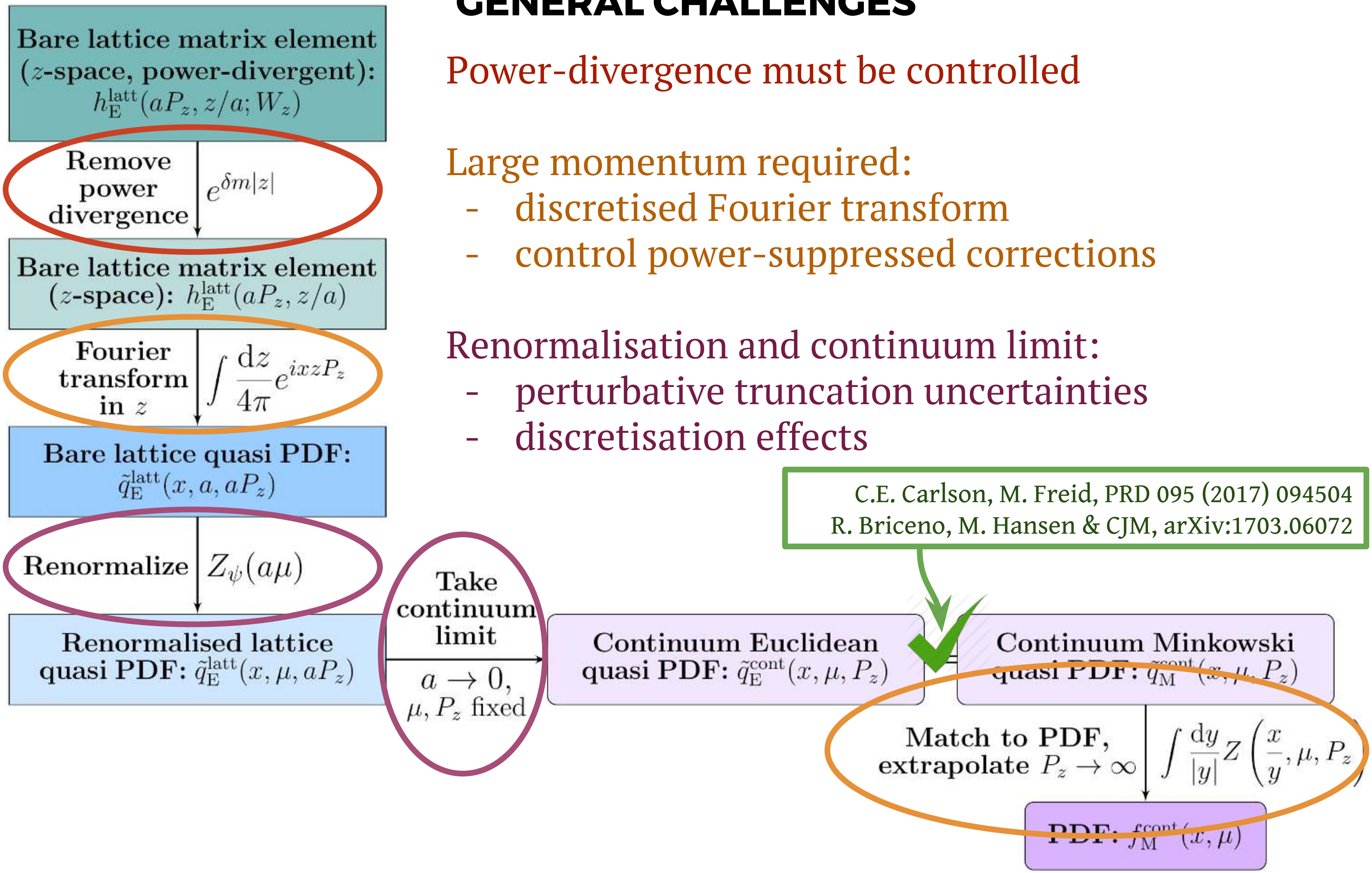
Power-divergence must be controlled

Large momentum required:

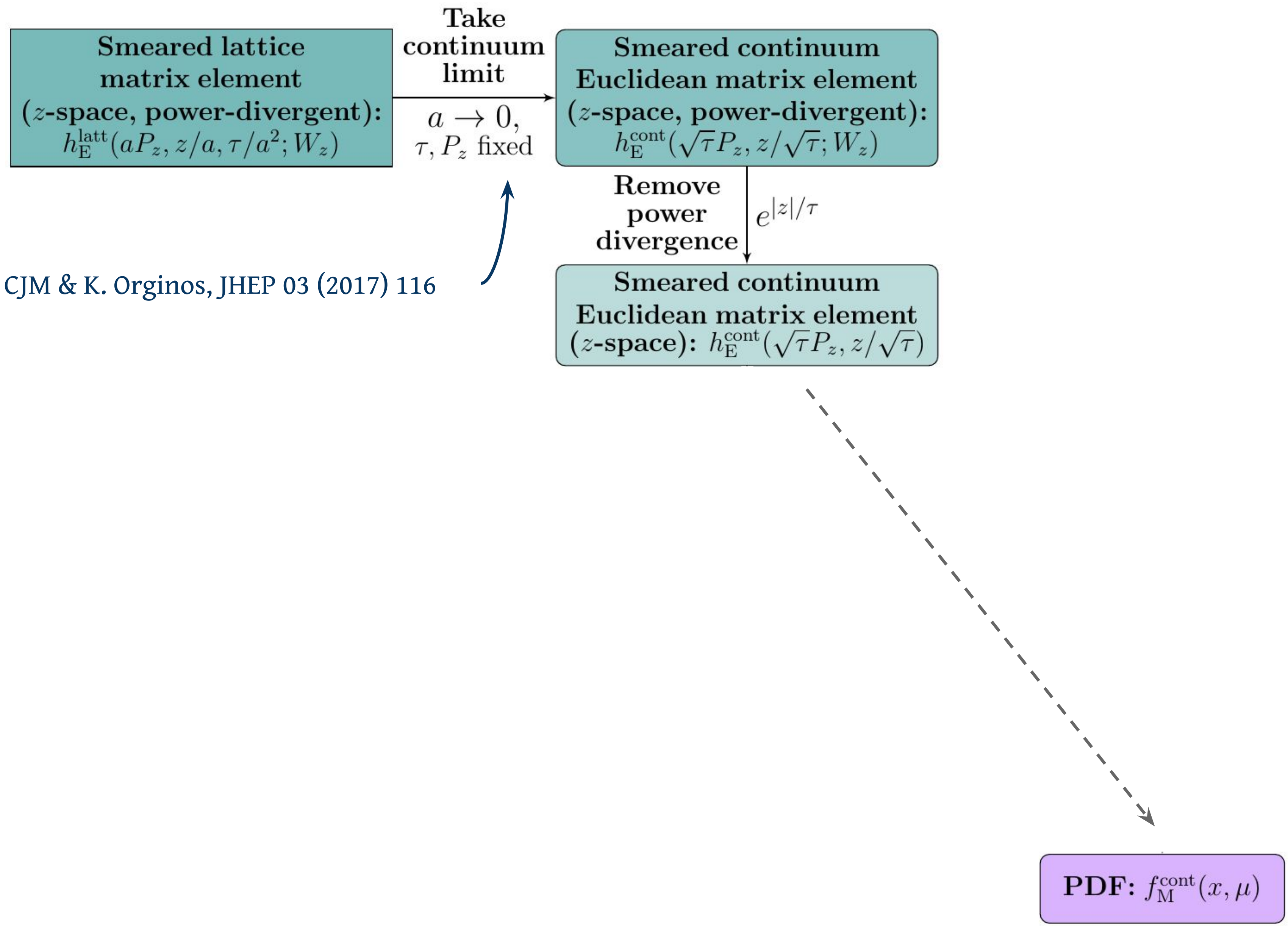
- discretised Fourier transform
- control power-suppressed corrections

Renormalisation and continuum limit:

- perturbative truncation uncertainties
- discretisation effects



GRADIENT FLOW PROCEDURE



SMEARED QUASI DISTRIBUTIONS

CJM & K. Orginos, JHEP 03 (2017) 116

Defined as

$$q(x, \sqrt{\tau}P^z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_N) = \int \frac{\mathrm{d}z}{4\pi} e^{ixz\,k^z} \langle P|\overline{\chi}(z,\tau)\gamma^z e^{-ig\int_0^z \mathrm{d}z' B^z(z',\tau)}\chi(0,\tau)|P\rangle_C$$

Related to light-front PDFs via

$$q(x, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}P^z) = \int_{-1}^{1} \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \sqrt{\tau}\mu, \sqrt{\tau}P^z\right) f(y,\mu^2) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\rm QCD}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right)$$

Here "ringed" fermions remove need for wavefunction renormalisation

H. Makino and H. Suzuki, PTEP (2014) 063B02
K. Hieda and H. Suzuki, MPLA 31 (2016) 1650214

$$\chi(x,\tau) = \sqrt{\frac{-2\dim(R)N_f}{(4\pi)^2\tau^2\left\langle\overline{\psi}(x,\tau)\overleftrightarrow{\not{D}}\psi(x,\tau)\right\rangle}}\psi(x,\tau)$$

RELATION TO PDFs

CJM & K. Orginos, JHEP 03 (2017) 116

Introduce Mellin moments

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = \int_{-\infty}^{\infty} \mathrm{d}x\, x^{n-1} q(x, \sqrt{\tau}P^z, \sqrt{\tau}\Lambda_{\mathrm{QCD}}, \sqrt{\tau}M_N)$$

In limit of vanishing separation z

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = \frac{c_n(\sqrt{\tau}P^z)}{2P^z}\left\langle P^z \middle| \chi(z,\tau)\gamma_z(iD_z)^{n-1}\frac{\lambda^a}{2}\chi(0,\tau) \middle| P^z \right\rangle_C$$

Unlike PDFs these matrix elements are not twist-2, but related via

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\mathrm{QCD}}}{P^z}, \frac{M_N}{P^z}\right) = C(\sqrt{\tau}\mu, \sqrt{\tau}P^z)\,a_n(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}}, \frac{\Lambda_{\mathrm{QCD}}^2}{(P^z)^2}\right)$$

RELATION TO PDFs

CJM & K. Orginos, JHEP 03 (2017) 116

Introduce a kernel with Mellin moments

$$\left[C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)\right]^{-1} = \int_{-\infty}^{\infty} dx\, x^{n-1} Z(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z)$$

Then, assuming one corrects for target mass effects and that

$$\Lambda_{\rm QCD}, M_N \ll P_z \ll \tau^{-1/2}$$

This leads to

$$q(x, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}P^z) = \int_{-1}^{1} \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y}, \sqrt{\tau}\mu, \sqrt{\tau}P^z\right) f(y, \mu^2) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\rm QCD}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right)$$

DGLAP EQUATION

Introduce Mellin moments

CJM & K. Orginos, JHEP 03 (2017) 116

$$b_n\left(\sqrt{\tau}P^z, \frac{\Lambda_{\rm QCD}}{P^z}, \frac{M_N}{P^z}\right) = \int_{-\infty}^{\infty} {\rm d}x\, x^{n-1} q(x, \sqrt{\tau}P^z, \sqrt{\tau}\Lambda_{\rm QCD}, \sqrt{\tau}M_N)$$

and apply small-flow time expansion

$$b_n^{(s)}(\sqrt{\tau}\Lambda_{\rm QCD}) = C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z)a^{(n)}(\mu) + \mathcal{O}\left(\sqrt{\tau}\Lambda_{\rm QCD}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

such that

$$\left[\mu\frac{\rm d}{{\rm d}\mu} + \frac{\alpha_s(\mu)}{\pi}\gamma^{(n)}\right] C_n^{(0)}(\sqrt{\tau}\mu, \sqrt{\tau}P_z) = 0 + \mathcal{O}(\sqrt{\tau}\Lambda_{\rm QCD})$$

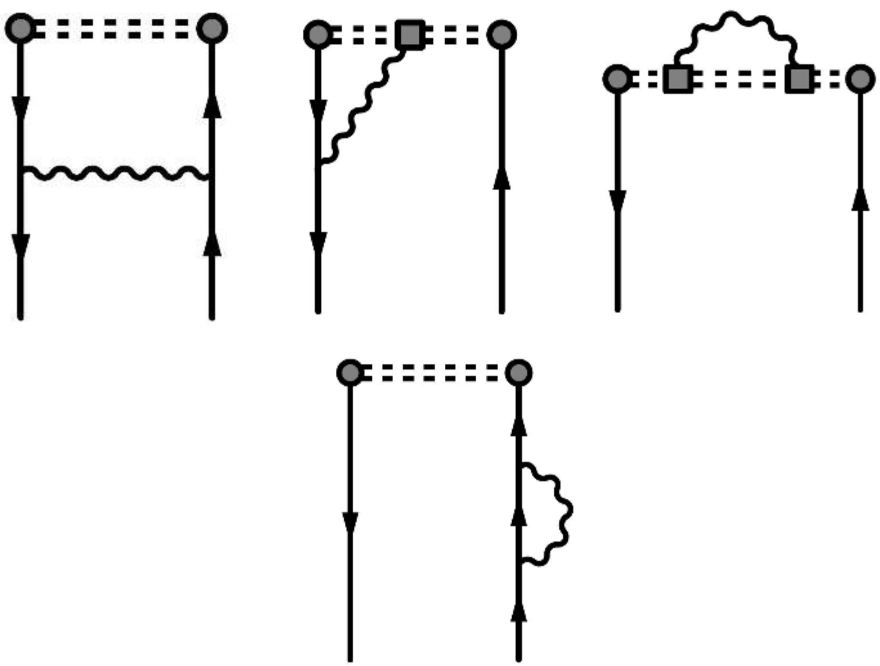
Then the matching kernel satisfies

$$\mu\frac{d}{d\mu} Z\left(x, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) = \frac{\alpha_s(\mu)}{\pi}\int_x^{\infty}\frac{dy}{y} Z\left(y, \sqrt{\tau}\mu, \sqrt{\tau}P_z\right) P\left(\frac{x}{y}\right)$$

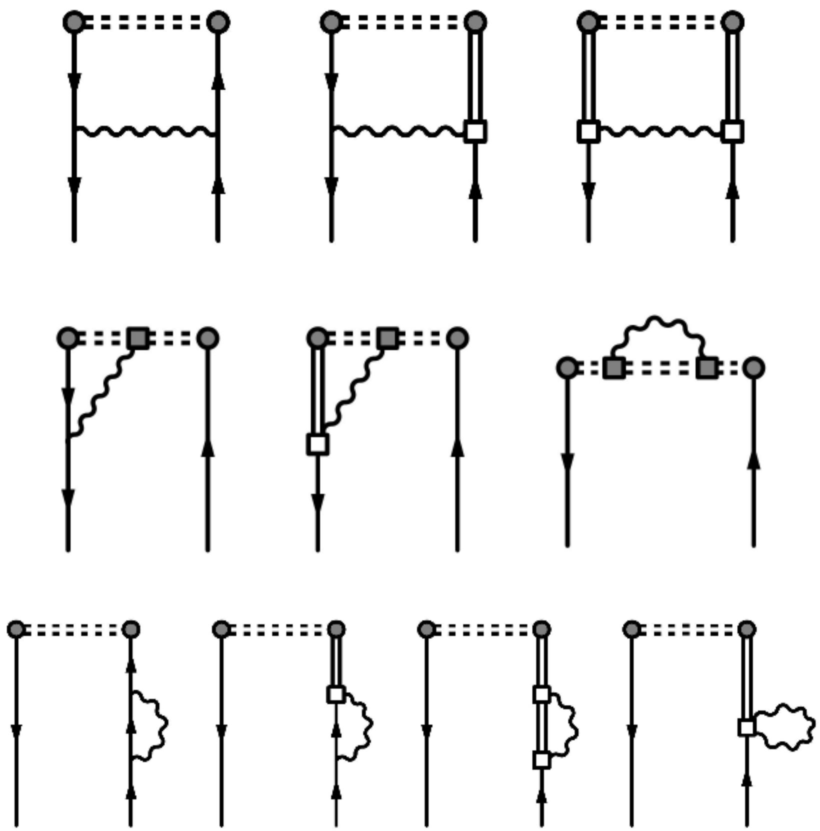
MATCHING IN PERTURBATION THEORY

Axial gauge simplest. But gradient flow requires generalised Feynman gauge.

Quasi distribution



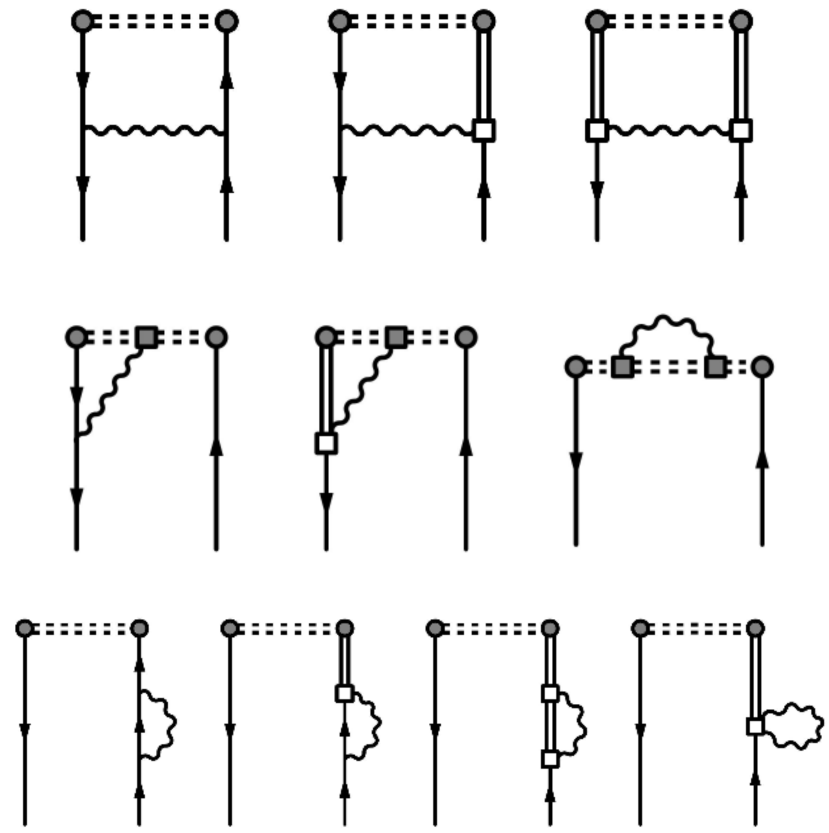
Smeared quasi distribution



X. Xiong *et al.*, PRD 90 (2014) 014051
Ji et al., PRD 92 (2015) 014039

MATCHING IN PERTURBATION THEORY

In principle, one can calculate these diagrams directly.



But integrals not solvable analytically

- usual techniques fail

(or at least, all the ones I can think of)

Simplified in coordinate space:

- infrared behaviour identical
- set zero external mass, momentum
- reduces number of diagrams
- simplifies remaining integrals

Determine finite parts numerically

MATCHING IN PERTURBATION THEORY

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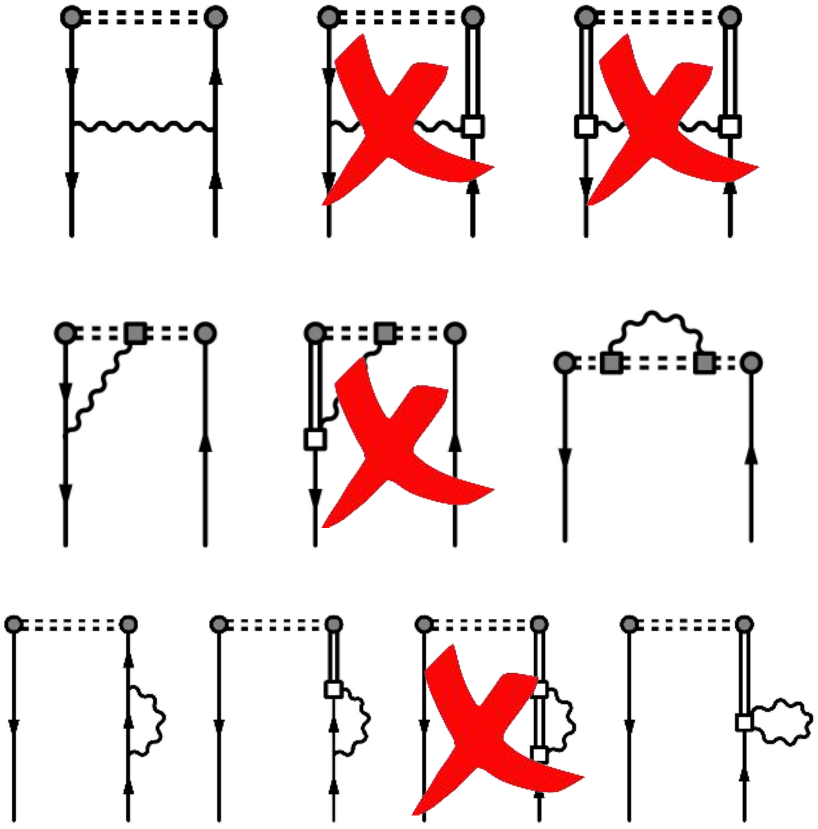

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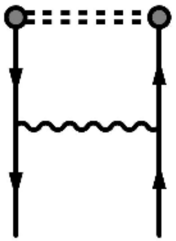
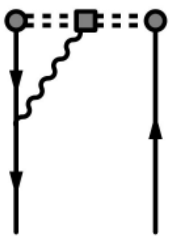
RESULTS

In coordinate space, in units of $-\frac{g_0^2}{(4\pi)^2} C_2(R)\gamma_\mu$

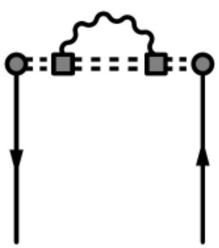


$$= \left[\frac{1}{\epsilon} - \mathrm{Ei}\left(-\bar{z}^2\right) + \frac{3}{\bar{z}^4}\left(e^{-\bar{z}^2} - 1\right) + \frac{1}{\bar{z}^2}\left(4 - e^{-\bar{z}^2}\right) - 3 + \gamma_{\mathrm{E}} + \log\left(\pi\mu^2 z^2\right)\right]$$



$$= \left[\mathrm{Ei}_1\left(\bar{z}^2\right) + \frac{1}{\bar{z}^2}\left(1 - e^{-\bar{z}^2}\right) + \gamma_{\mathrm{E}} - 1 + \log\left(\bar{z}^2\right)\right]$$

$$\boxed{\bar{z} = \frac{z^2}{8\tau}}$$

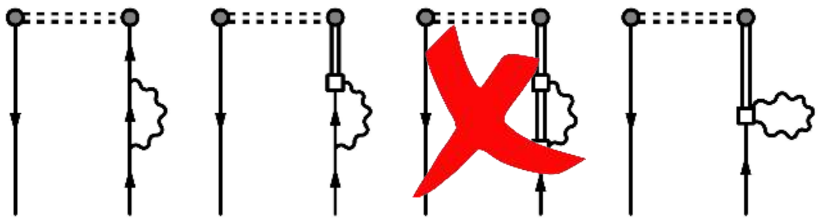


$$= 2\left[\mathrm{Ei}\left(-\bar{z}^2\right) + \sqrt{\pi}\,\bar{z}\,\mathrm{erf}\left(\bar{z}\right) + \left(e^{-\bar{z}^2} - 1\right) - \gamma_{\mathrm{E}} - \log\left(\bar{z}^2\right)\right]$$

Power-divergence

RESULTS

Include the wavefunction-type diagrams



Local vector current limit:

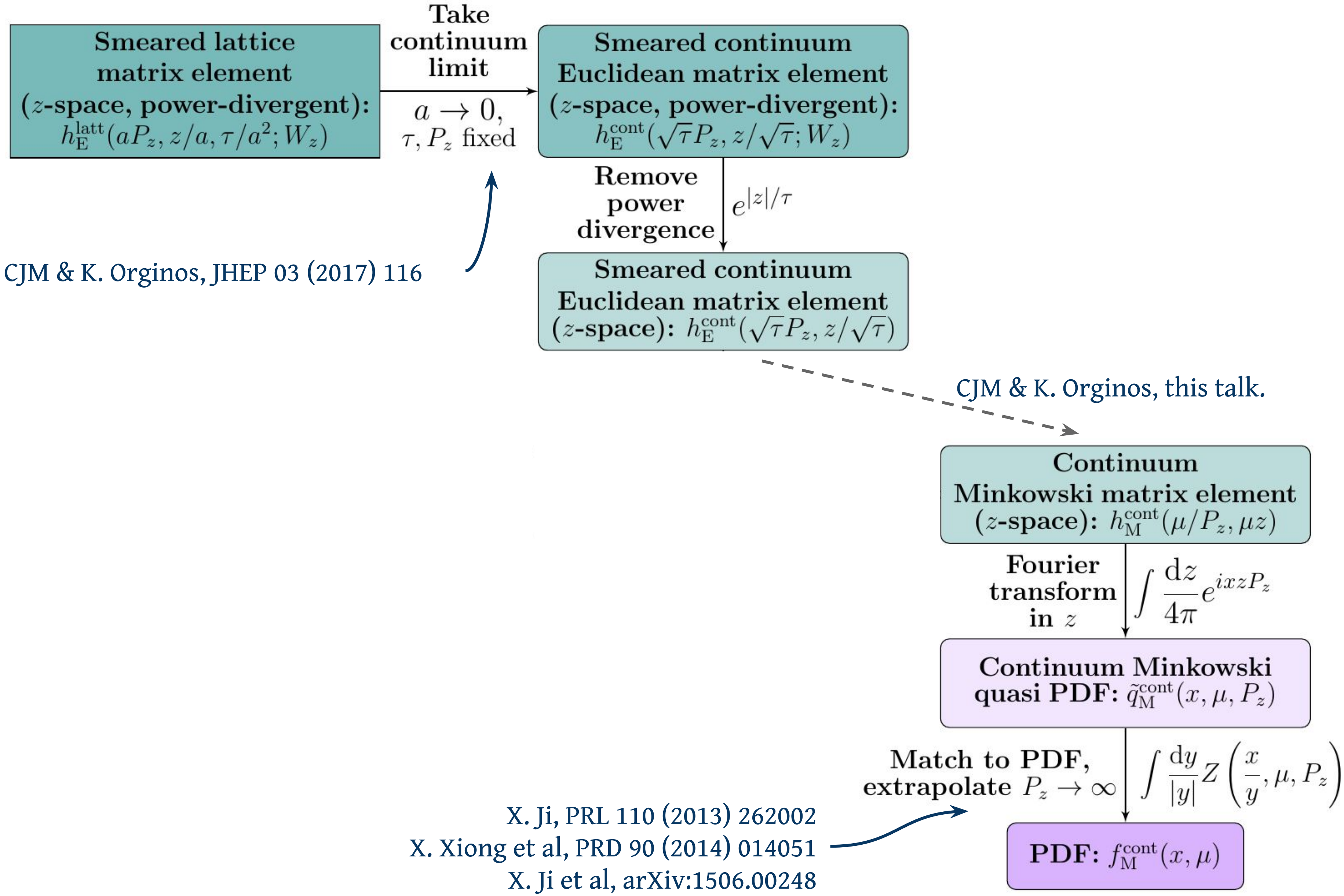
$$\lim_{\bar{z}\to 0} h^{(1)}(\bar{z}) = 0$$

Small flow-time limit:

$$\lim_{\bar{z}\to \infty} h^{(1)}(\bar{z}) = h^{(1)}_{\overline{MS}}$$

Full momentum-dependent result in progress.

GRADIENT FLOW PROCEDURE



PDFs FROM FIRST PRINCIPLES: QUASI DISTRIBUTIONS

Quasi distributions

One current challenge: the continuum limit

THE GRADIENT FLOW

Matrix elements finite at fixed flow time

SMEARED QUASI DISTRIBUTIONS

Finite continuum distributions

Matching in perturbation theory ~ almost complete

Nonperturbative study of systematics underway

THANK YOU

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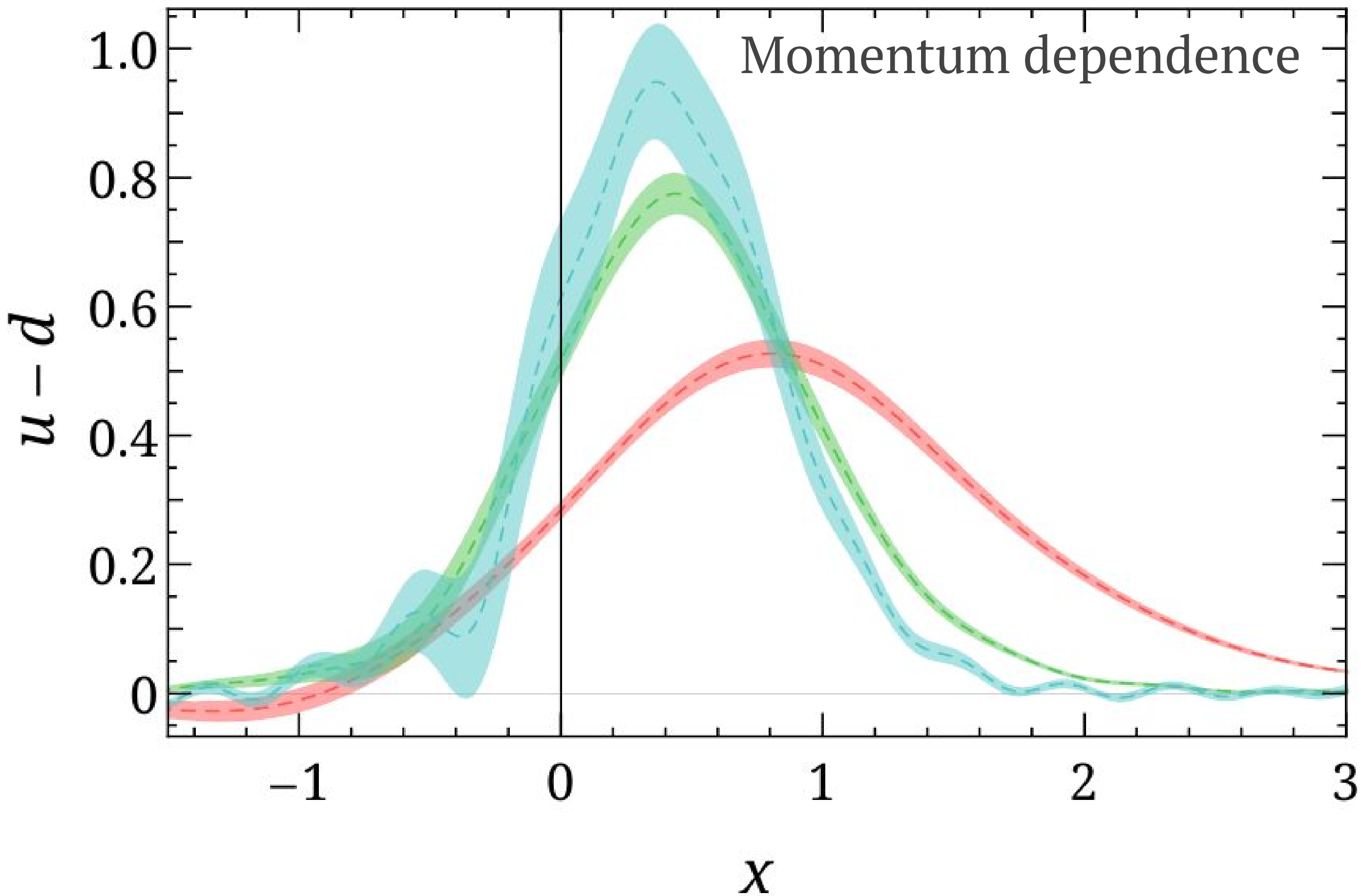
QUASI DISTRIBUTIONS

From J.-W. Chen et al., NPB 911 (2016) 246



See also:

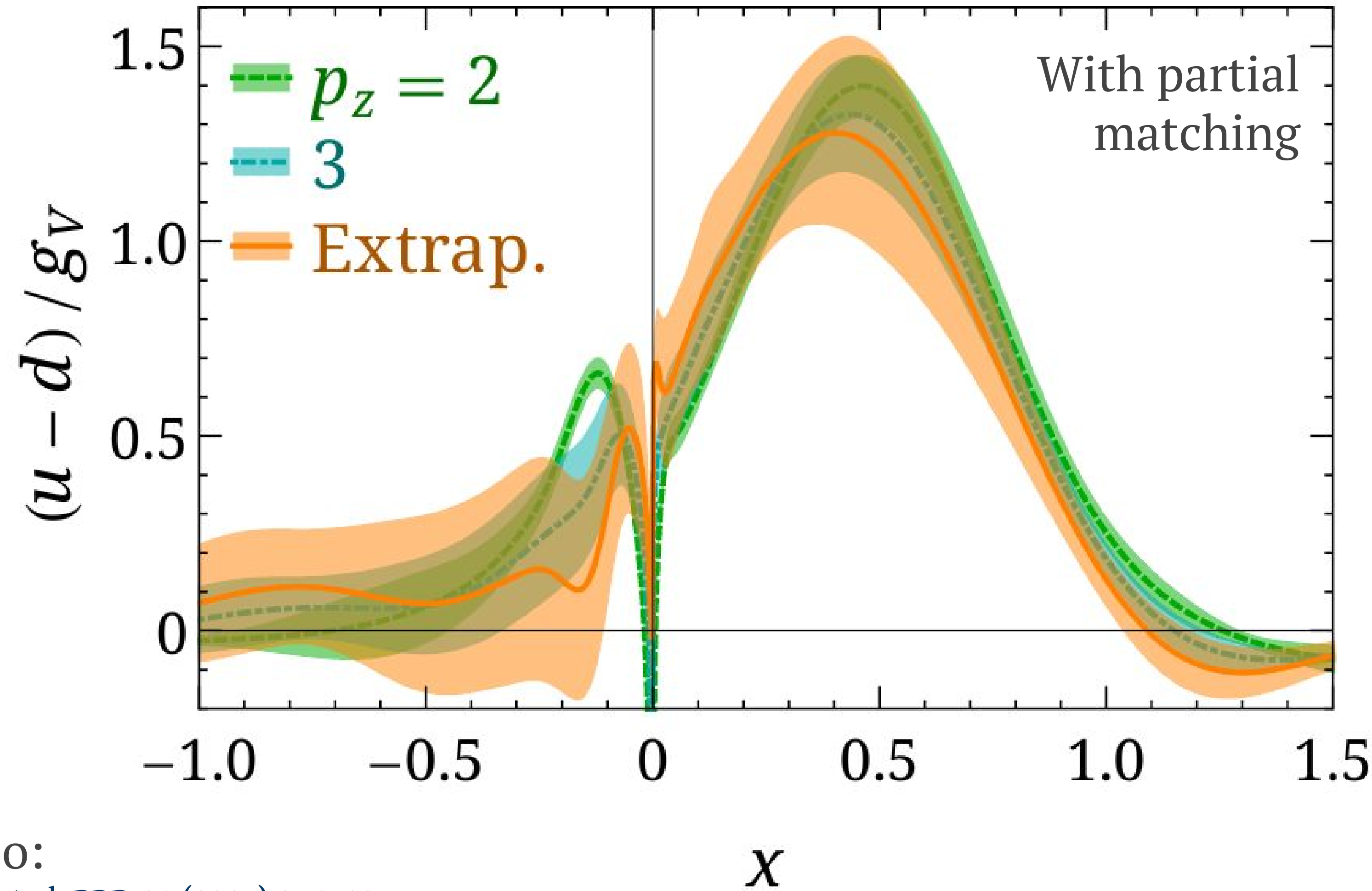
H.-W. Lin et al, PRD 91 (2015) 054510

C. Alexandrou et al., PRD 92 (2015) 014502

J.-H. Zhang et al., arXiv:1702.00008

QUASI DISTRIBUTIONS

From J.-W. Chen et al., NPB 911 (2016) 246



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H.-W. Lin et al, PRD 91 (2015) 054510

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QCD

QCD

$$\frac{\partial}{\partial \tau} B_\mu(\tau, x) = D_\nu \Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu] \Big) \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

$$\frac{\partial}{\partial \tau} \chi(\tau, x) = D^F_\mu D^F_\mu \chi(\tau, x) \qquad D^F_\mu = \partial_\mu + B_\mu$$

Exact solution not possible (even with Dirichlet boundary conditions)

$$B_\mu(\tau, x) = \int \mathrm{d}^4 y \Big\{ K_\tau(x-y)_{\mu\nu} A_\nu(y) + \int_0^\tau \mathrm{d}\sigma K_{\tau-\sigma}(x-y)_{\mu\nu} R_\nu(\sigma, y) \Big\}$$

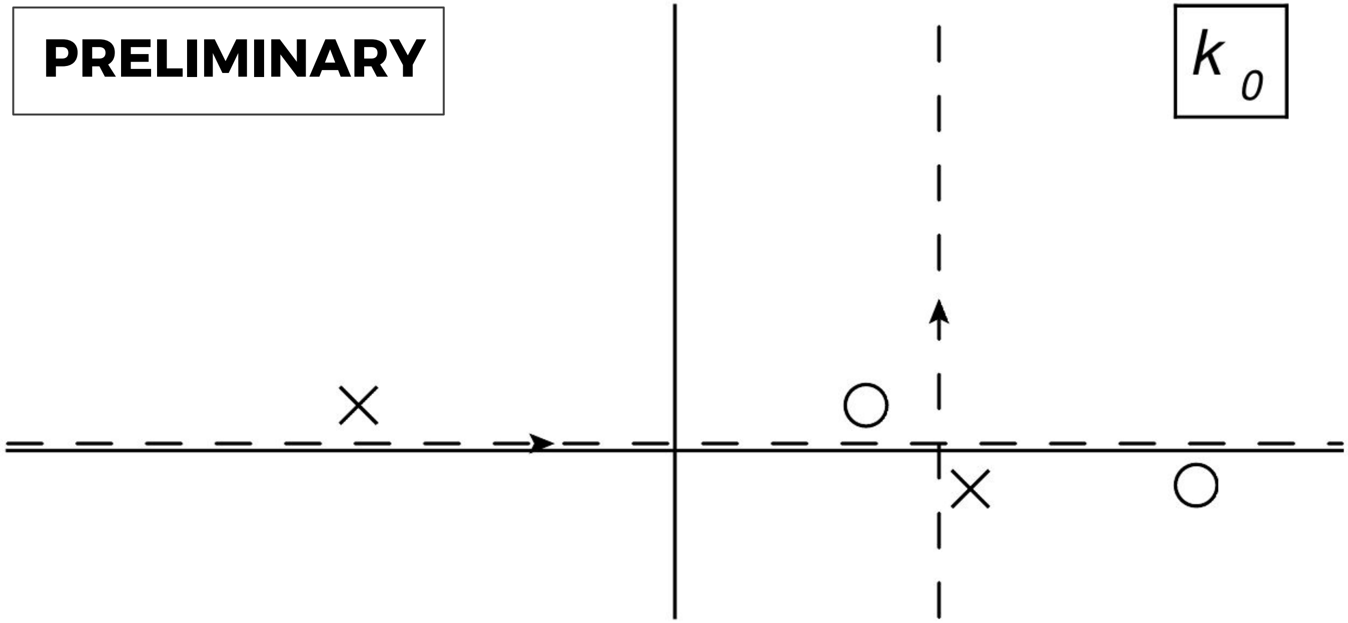
$$K_\tau(x)_{\mu\nu} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{e^{ipx}}{p^2} \Big\{ (\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-\tau p^2} + p_\mu p_\nu \Big\}$$

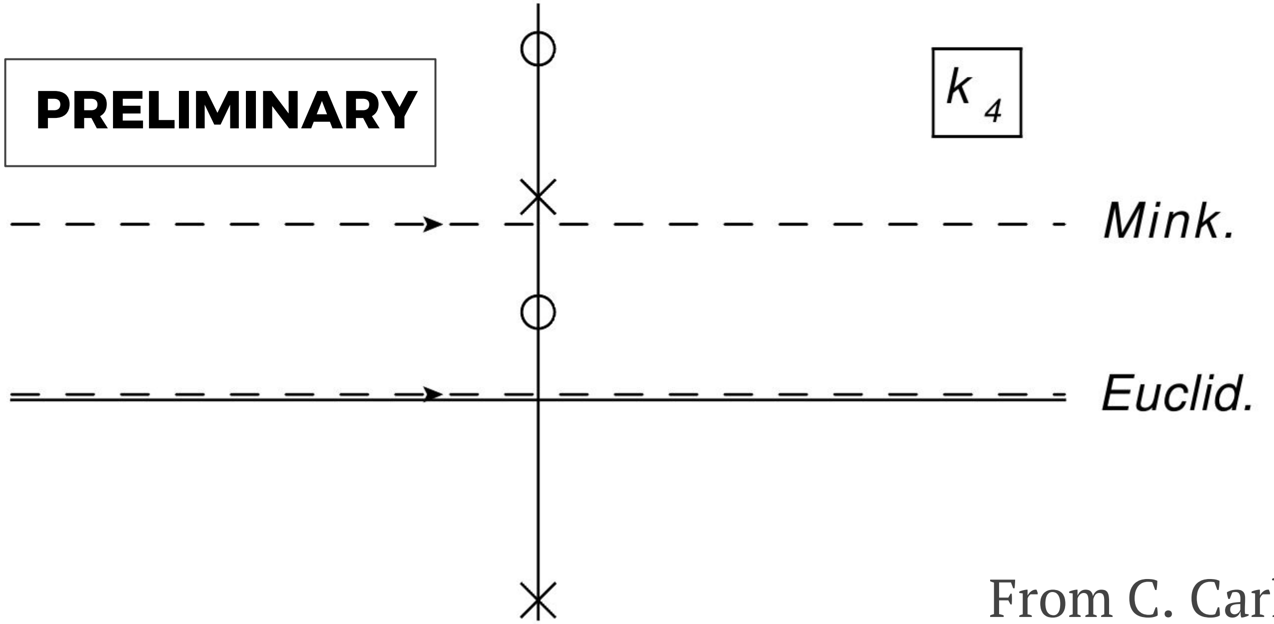
$$R_\mu(\tau, x) = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] - [B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Smearing radius $s_{\mathrm{rms}} = \sqrt{8\tau}$

Interactions occur at non-zero flow time: generalised BRST symmetry guarantees renormalised correlation functions remain finite.

Lüscher & Weisz, JHEP 1102 (2011) 51
Luscher, JHEP 04 (2013) 123





From C. Carlson & M. Freid (2017)
“Lattice corrections to the quark quasidistribution at one-loop”

EXPERIMENTAL EXTRACTION

Process	Subprocess	Partons	x range
$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$10^{-4} \lesssim x \lesssim 0.1$
$e^+ p \to \bar{\nu} X$	$W^+ \{d, s\} \to \{u, c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \to e^\pm c\bar{c}X, e^\pm b\bar{b}X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	c, b, g	$10^{-4} \lesssim x \lesssim 0.01$
$e^\pm p \to$ jet$+X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p}, pp \to$ jet$+X$	$gg, qg, qq \to 2j$	g, q	$0.00005 \lesssim x \lesssim 0.5$
$p\bar{p} \to (W^\pm \to \ell^\pm\nu) X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
$pp \to (W^\pm \to \ell^\pm\nu) X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 0.001$
$p\bar{p}(pp) \to (Z \to \ell^+\ell^-)X$	$uu, dd, ..(u\bar{u}, ..) \to Z$	$u, d, ..(g)$	$x \gtrsim 0.001$
$pp \to W^- c,\ \ W^+\bar{c}$	$gs \to W^- c$	$s, \bar{s}$	$x \sim 0.01$
$pp \to (\gamma^* \to \ell^+\ell^-)X$	$u\bar{u}, d\bar{d}, .. \to \gamma^*$	$\bar{q}, g$	$x \gtrsim 10^{-5}$
$pp \to b\bar{b}X,\quad t\bar{t}X$	$gg \to b\bar{b},\quad t\bar{t}$	g	$x \gtrsim 10^{-5}, 10^{-2}$
$pp \to$ exclusive $J/\psi,\ \Upsilon$	$\gamma^*(gg) \to J/\psi,\ \Upsilon$	g	$x \gtrsim 10^{-5}, 10^{-4}$
$pp \to \gamma X$	$gq \to \gamma q,\ g\bar{q} \to \gamma\bar{q}$	g	$x \gtrsim 0.005$

From PDG 2016