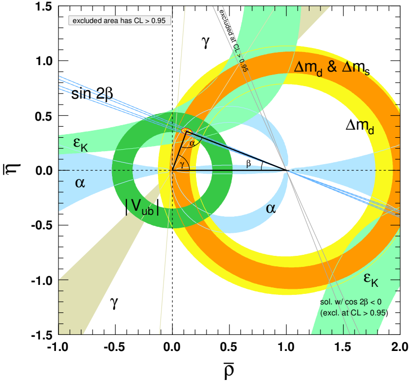
$B \to \pi \ell \nu$ with Möbius Domain Wall Fermions

Brian Colquhoun
w/ Shoji Hashimoto, Takashi Kaneko
for the JLQCD Collaboration

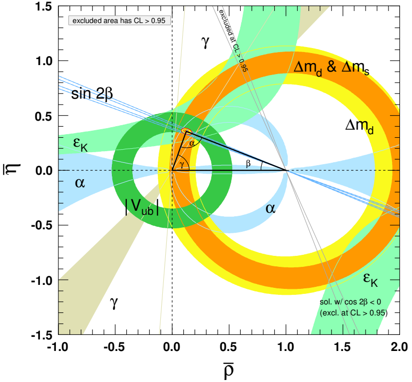
KEK

June 19 2017



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\bar{B}_s\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

Motivation



This talk

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\bar{B}_s\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

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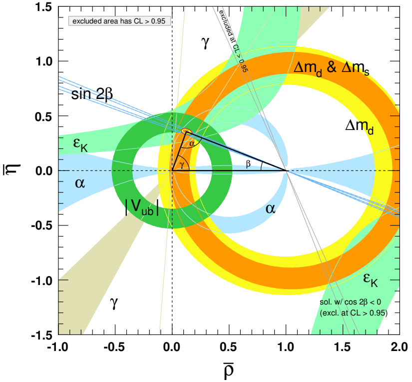

This talk

V_{ud} $\pi \to \ell\nu$	V_{us} $K \to \ell\nu$ $K \to \pi\ell\nu$	V_{ub} $B \to \pi\ell\nu$
V_{cd} $D \to \ell\nu$ $D \to \pi\ell\nu$	V_{cs} $D_s \to \ell\nu$ $D \to K\ell\nu$	V_{cb} $B \to D\ell\nu$
V_{td} $\langle B_d \vert \bar{B}_s \rangle$	V_{ts} $\langle B_s \vert \bar{B}_s \rangle$	V_{tb}

T. Kaneko, Weak Decays: Tues, 17:10

Möbius Domain Wall Fermions

- Relativistic action ✓
- Chiral Symmetry ✓
- MDWF for all quarks ✓
- Extrapolate to m_b

This talk → V_{ub}

V_{ud} $\pi \to \ell\nu$	V_{us} $K \to \ell\nu$ $K \to \pi\ell\nu$	V_{ub} $B \to \pi\ell\nu$
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T. Kaneko, Weak Decays: Tues, 17:10

Background

CKM element V_{ub} relates to the differential decay rate:

$$\frac{\mathrm{d}\Gamma\left(B \to \pi \ell \nu\right)}{\mathrm{d}q^2} = \frac{G_F^2 \left|V_{ub}\right|^2}{24\pi^3} \left|p_\pi\right|^3 \left|f_+(q^2)\right|^2$$

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Experiment

CKM

Lattice

Semileptonic Decays

For pseudoscalar to pseudoscalar decays: :

$$
\begin{aligned}
\langle \pi(k_\pi) | V^\mu | B(p_B) \rangle &= f_+(q^2) \left[(p_B + k_\pi)^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] \\
&\quad + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu,
\end{aligned}
$$

- p_B and k_π: B and π four-momenta
- $q^\mu = p_B^\mu - k_\pi^\mu$: four-momentum transfer
- Constraint: $f_0(0) = f_+(0)$

Semileptonic Decays

In the context of HQET, a useful parametrisation is:

[Burdman et al. (1994)]

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = 2\sqrt{m_B}\left[f_1\left(v\cdot k_\pi\right)v^\mu + f_2\left(v\cdot k_\pi\right)\frac{k^\mu}{v\cdot k_\pi}\right]$$

- $v^\mu = \dfrac{p_B}{m_B}$: heavy quark velocity
- $E_\pi = v\cdot k_\pi = \dfrac{m_B^2 + m_\pi^2 - q^2}{2m_B}$

Matrix Elements

Working in the B meson rest frame:

$$f_1(v\cdot k) + f_2(v\cdot k) = \frac{\langle \pi(k_\pi)|V^0|B(p_B)\rangle}{2\sqrt{m_B}};$$

$$f_2(v\cdot k) = \frac{\langle \pi(k_\pi)|V^i|B(p_B)\rangle}{2\sqrt{m_B}}\frac{v\cdot k_\pi}{k_\pi^i}$$

Pole dominance model:

$$\lim_{v\cdot k_\pi \to 0} f_2(v\cdot k) = g\frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\frac{v\cdot k_\pi}{v\cdot k_\pi + \Delta_B},$$

with,

$g:\ B^*B\pi$ coupling; $\quad \Delta_B:\ m_{B^*} - m_B$.

Semileptonic Decays

These two form factor definitions are related by:

$$
\begin{aligned}
f_+\left(q^2\right) =& \sqrt{m_B}\left[\frac{f_2\left(v\cdot k\right)}{v\cdot k_\pi}+\frac{f_1\left(v\cdot k\right)}{m_B}\right];\\
f_0\left(q^2\right) =& \frac{2}{\sqrt{m_B}}\frac{m_B^2}{m_B^2-m_\pi^2}\Bigg[f_1\left(v\cdot k\right)+f_2\left(v\cdot k\right)\\
&-\frac{v\cdot k_\pi}{m_B}\left(f_1\left(v\cdot k\right)+\frac{m_\pi^2}{\left(v\cdot k_\pi\right)^2}f_2\left(v\cdot k\right)\right)\Bigg]
\end{aligned}
$$

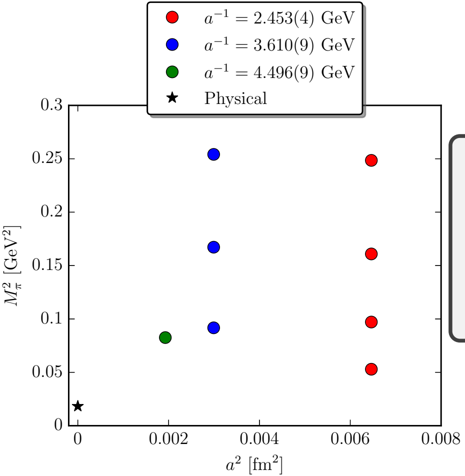
These two form factor definitions are related by:

In the heavy quark limit, then:

$$f_+\left(q^2\right) \sim \sqrt{m_B} f_2\left(v \cdot k\right);$$
$$f_0\left(q^2\right) \sim \frac{f_1\left(v \cdot k\right) + f_2\left(v \cdot k\right)}{\sqrt{m_B}}$$

Lattice Calculation

Gauge Configurations



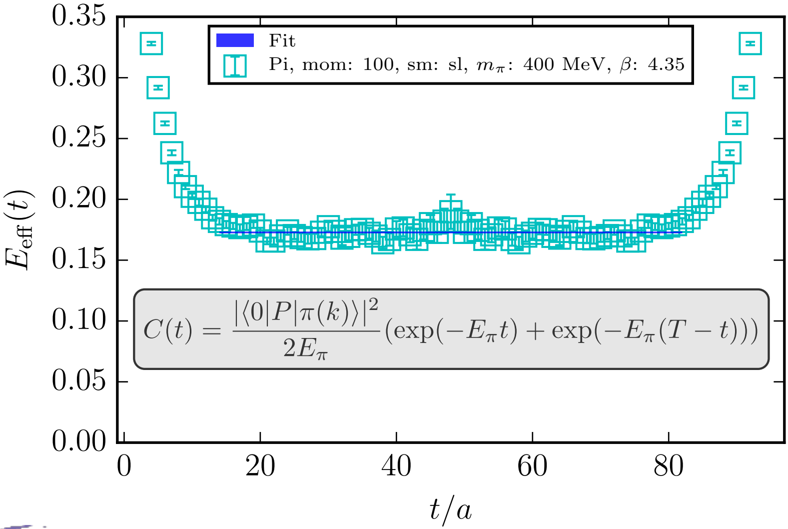
$N_f = 2 + 1$
$m_\pi \approx 300, 400, 500$ MeV
$\beta = 4.17, 4.35, 4.47$
$a^{-1} \approx 2.453, 3.610, 4.496$ GeV
$L^3 \times T = 32^3 \times 64, 48^3 \times 96, 64^3 \times 128$

Heavy Quarks

β	a [fm]	am_c	$1.25^2 \times am_c$	$1.25^4 \times am_c$
4.17	0.0804	0.44037	0.68808	-
4.35	0.0547	0.27287	0.42636	0.66619
4.47	0.0439	0.210476	0.328869	0.5138574

- Möbius Domain Wall Fermions
- m_h up to $2.44m_c$: m_c, $1.25^2 m_c$, $1.25^4 m_c$
  - $m_{H_l} \approx 1.95$ GeV, 2.55 GeV, 3.40 GeV
- Smeared sources with $\mathbb{Z}_2$ noise; local/smeared sink
- Time sources: 1, 2 or 4

Correlators: 2-point



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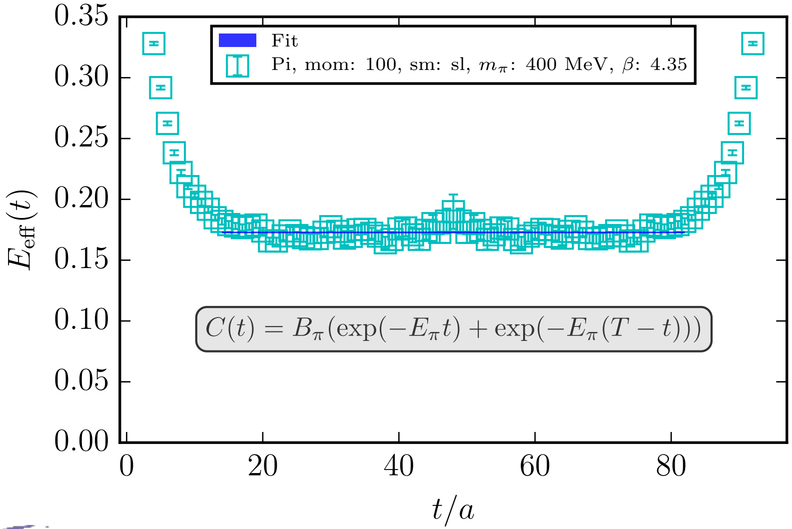

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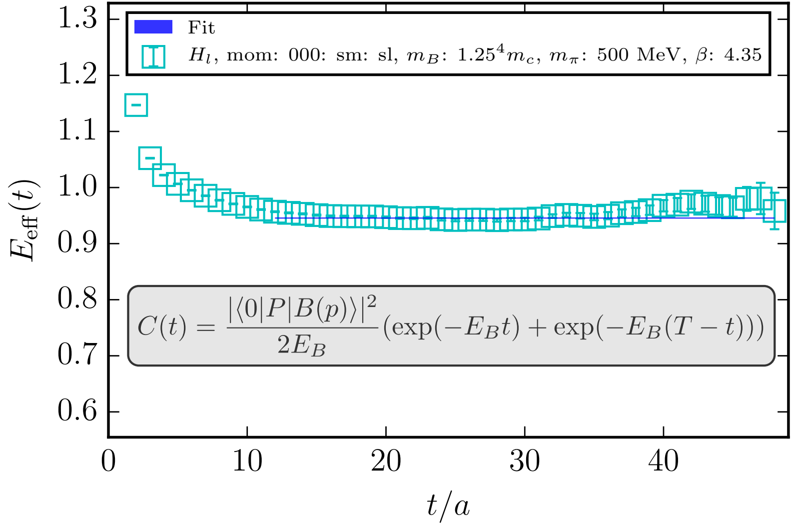
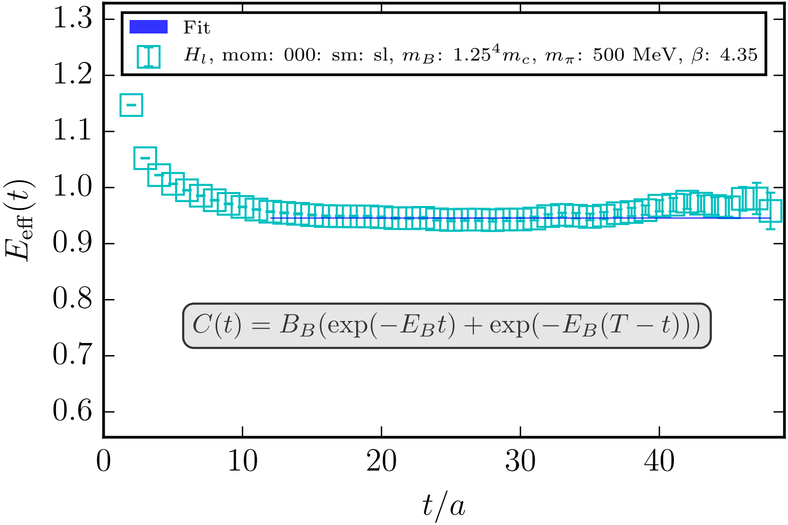
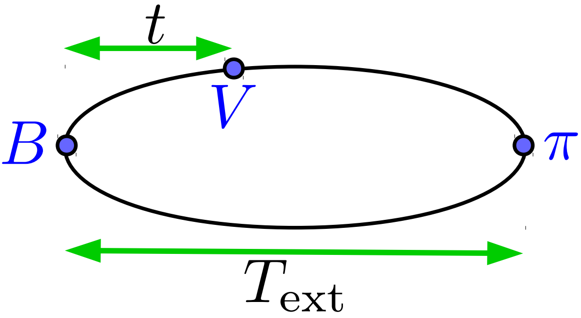


Correlators: 2-point

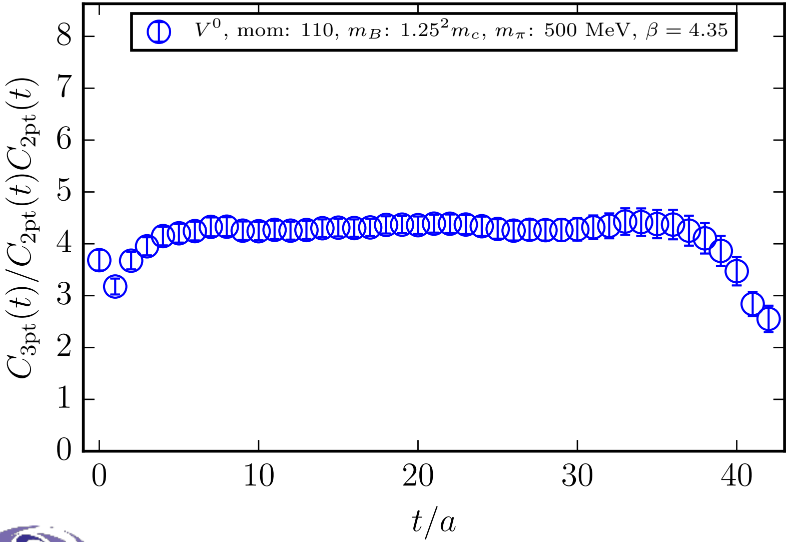


Correlators: 3-point

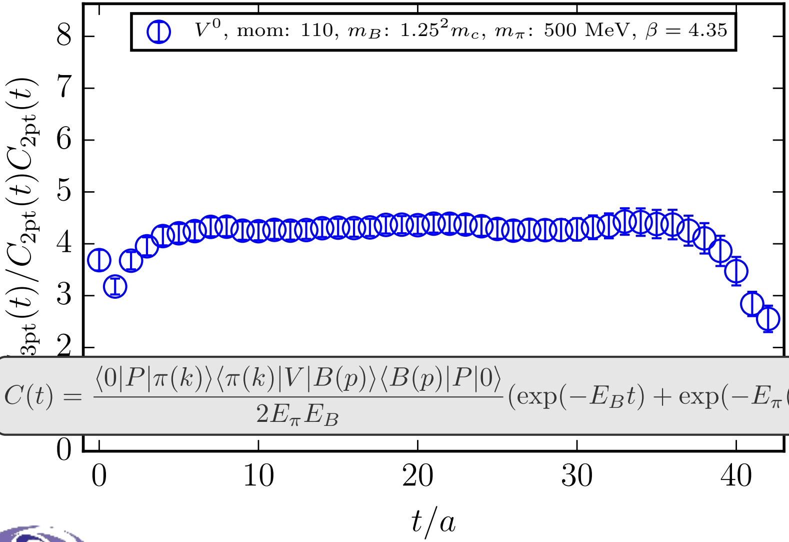
- B and π mesons are separated by time T_{ext}
- Operators V^μ are inserted at $0 \leq t \leq T_{\text{ext}}$



Correlators: 3-point

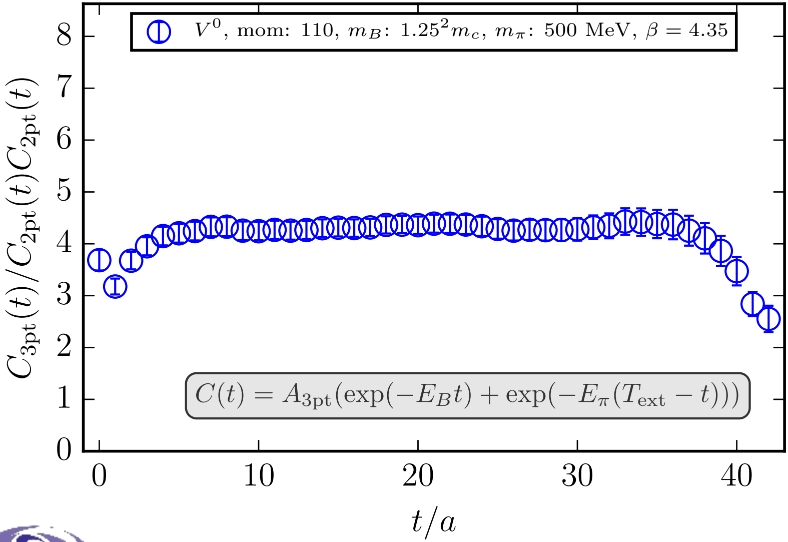


Correlators: 3-point



$$C(t) = \frac{\langle 0|P|\pi(k)\rangle\langle\pi(k)|V|B(p)\rangle\langle B(p)|P|0\rangle}{2E_\pi E_B}(\exp(-E_B t) + \exp(-E_\pi(T_{\text{ext}} - t)))$$

Correlators: 3-point



Extracting 3-point Matrix Elements

$$\langle \pi(k)|V|B(p)\rangle = 2Z_V\sqrt{E_\pi E_B}\frac{A_{3\mathrm{pt}}}{\sqrt{B_\pi B_B}}$$

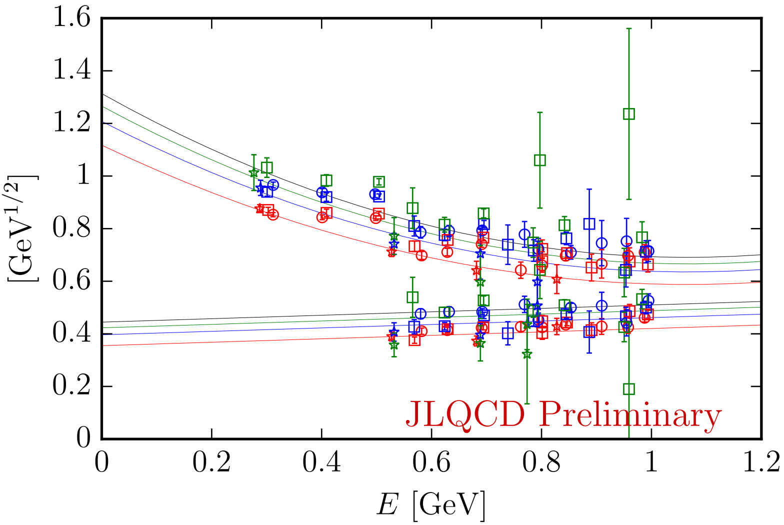
Z_V is determined nonperturbatively. For $m_h > m_c$ we determine:

$$Z_{Vbb}^{-1} = \langle B|V^0|B\rangle$$

and $Z_V = \sqrt{Z_{Vll}Z_{Vbb}}$. Z_{Vll} is determined from short-distance current correlators [Tomii et al. (2016) 1604.08702].

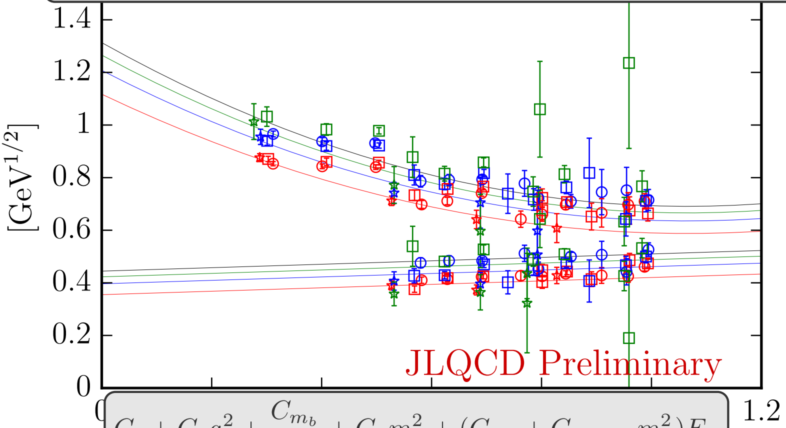
Results

$f_1(v \cdot k_\pi)$ and $f_1(v \cdot k_\pi)$



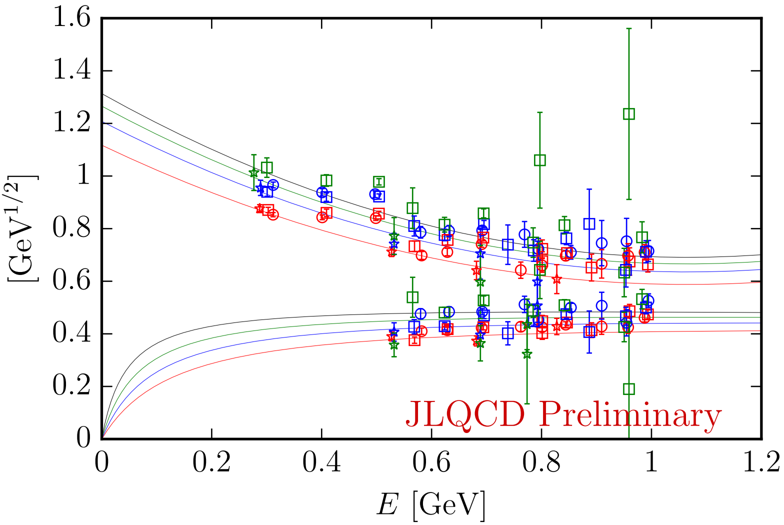
$f_1(v \cdot k_\pi)$ and $f_1(v \cdot k_\pi)$

$$C_0\left(1 + C_{E_\pi}E_\pi + C_{E_\pi{}^2}E_\pi^2\right)\left(1 + C_{a^2}a^2\right)\left(1 + C_{m_\pi}m_\pi^2\right)\left(1 + \frac{C_{m_b}}{m_b}\right)$$



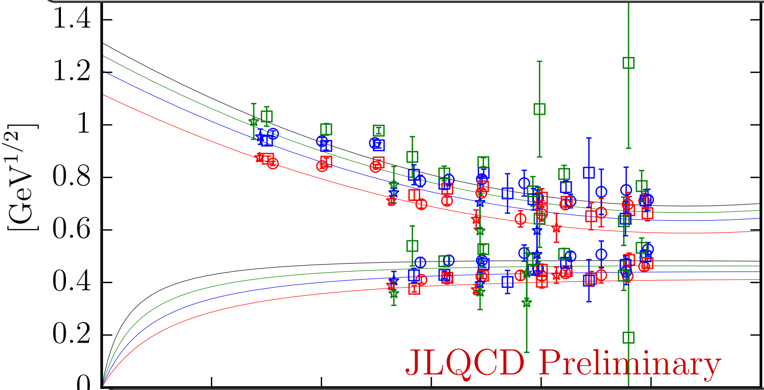
$$C_0 + C_a a^2 + \frac{C_{m_b}}{m_b} + C_\pi m_\pi^2 + (C_{E_\pi} + C_{E_\pi,m_\pi}m_\pi^2)E_\pi$$

Pole Dominance



Pole Dominance

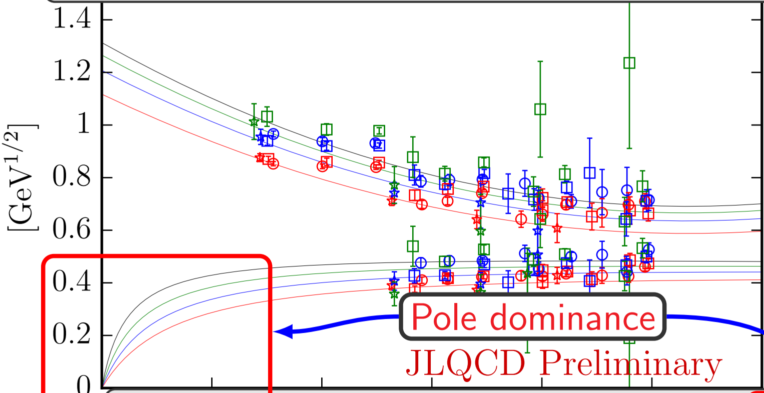
$$C_0\left(1+C_{E_\pi}E_\pi+C_{E_\pi{}^2}E_\pi^2\right)\left(1+C_{a^2}a^2\right)\left(1+C_{m_\pi}m_\pi^2\right)\left(1+\frac{C_{m_b}}{m_b}\right)$$



$$\left[C_0+C_a a^2+\frac{C_{m_b}}{m_b}+C_\pi m_\pi^2+(C_{E_\pi}+C_{E_\pi,m_\pi}m_\pi^2)E_\pi\right]\frac{E_\pi}{E_\pi+\Delta_B};$$

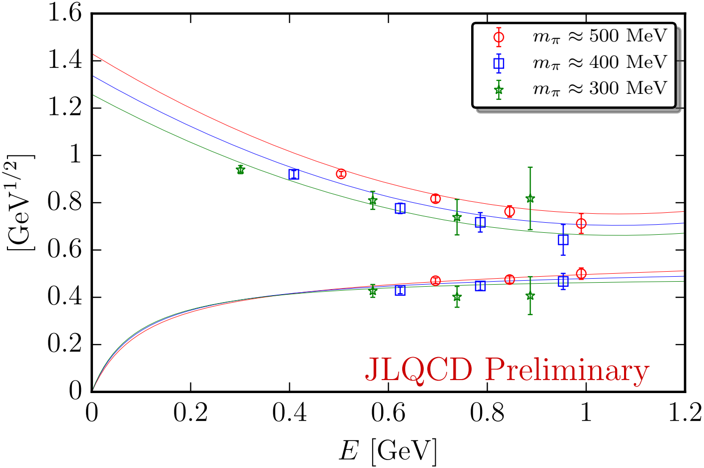
Pole Dominance

$$C_0\left(1+C_{E_\pi}E_\pi+C_{E_\pi{}^2}E_\pi^2\right)\left(1+C_{a^2}a^2\right)\left(1+C_{m_\pi}m_\pi^2\right)\left(1+\frac{C_{m_b}}{m_b}\right)$$

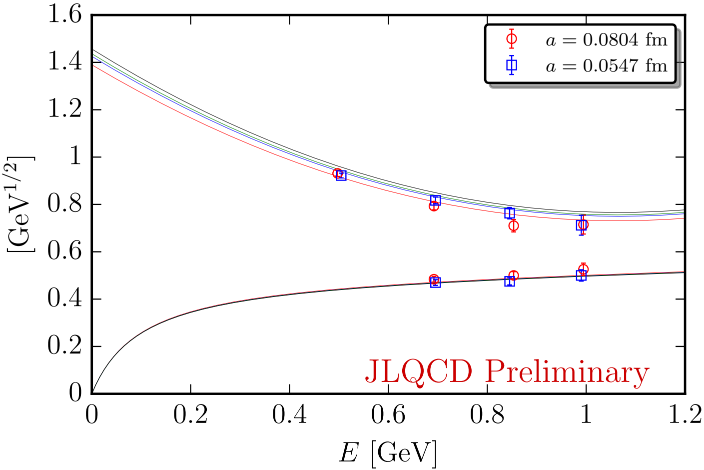


$$\left[C_0+C_a a^2+\frac{C_{m_b}}{m_b}+C_\pi m_\pi^2+\left(C_{E_\pi}+C_{E_\pi,m_\pi}m_\pi^2\right)E_\pi\right]\frac{E_\pi}{E_\pi+\Delta_B};$$

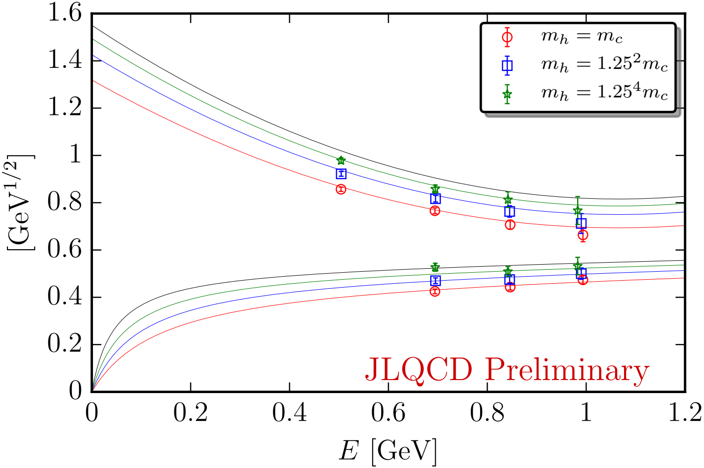
m_π dependence



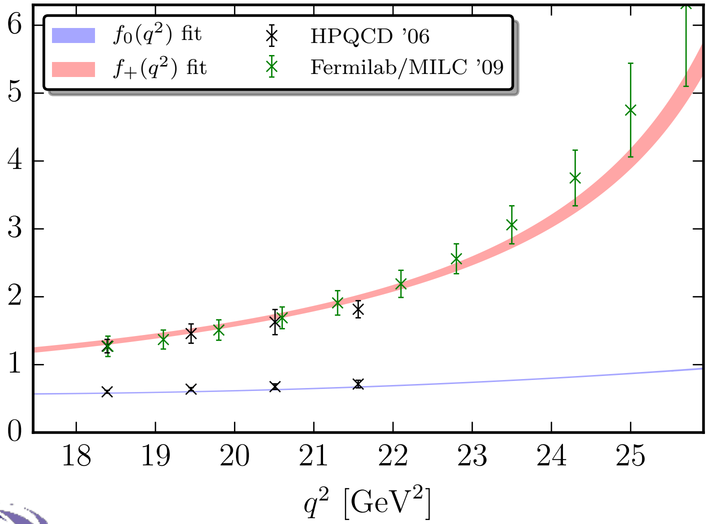
a^2 dependence



m_h dependence



Fitted Form Factors



Outlook

- Still generating data:
  - $m_\pi \approx 230\ \mathrm{MeV}$
  - We can reach $am_h \lessapprox 0.8$ (from f_{H_c}, Fahy Latt '16)
    - $\implies m_h \approx 3.5\ \mathrm{GeV}$ on finest set
  - Additional time sources for statistics
    - Particularly for $m_{\pi,\mathrm{lattice}} \to m_{\pi,\mathrm{physical}}$ and $m_h \to m_b$
- Continued analysis:
  - Determination of V_{ub} from differential decay rates
  - z-expansion
  - $\lim_{v\cdot k_\pi \to 0} f_2\,(v\cdot k) = g\frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\frac{v\cdot k_\pi}{v\cdot k_\pi + \Delta_B}.$
  - $f_0(q^2_{\mathrm{max}}) \lim_{v k_\pi \to 0} = \frac{2}{\sqrt{m_B}}[f_1(0) + f_2(0)] = \frac{f_B}{f_\pi}$

Summary

- $B \to \pi \ell \nu$ calculation with Möbius Domain Wall Fermions
  - m_h up to $2.44 m_c$
  - Lattices from $a^{-1} = 2.453$ GeV to 4.496 GeV
  - m_π down to 300 MeV; 230 MeV in progress at $a^{-1} = 2.453$ GeV
- We find mild deprendence on m_π^2 and a^2
  - Particularly for spatial vector matrix element
- m_B dependence larger but under control:
  - We expect to reach $m_h \approx 3.5$ GeV on finest lattice
- With further statistics, we will calculate V_{ub} over summer

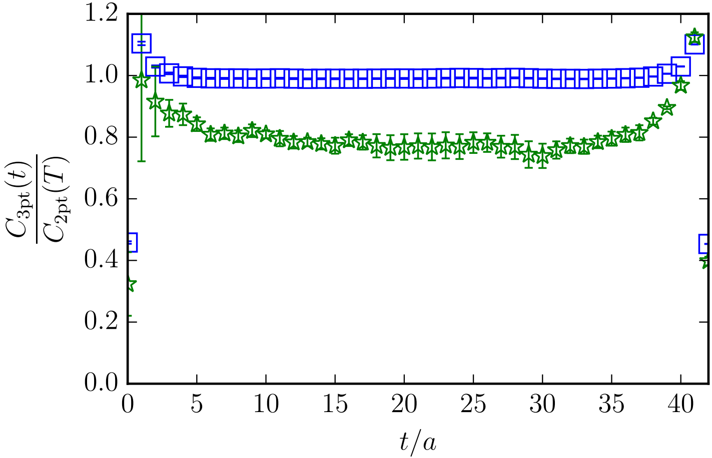
Thank you

Lattice2017

Thank you

Backup Slides

Z_{Vbb}^{-1}


1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
40
$\frac{C_{3\text{pt}}(t)}{C_{2\text{pt}}(T)}$
t/a

$f_0(q^2_{\mathrm{max}})$ Comparison