Higgs-Yukawa model on the lattice

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Collaborators

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Outline

- Strategy and tools.
- Finite-temperature phase transition with a dimension-6 operator
- Finite-size scaling of the Higgs-Yukawa model near the Gaussian fixed point.

The model

- O(4) scalar theory, two flavours of overlap fermions.
- Degenerate chiral Yukawa couplings.
- The lattice scalar action

$$S_B[\Phi] = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\mu} + \Phi_{x-\mu}\right] + \sum_x \left(\Phi_x^\dagger \Phi_x + \hat{\lambda} \left[\Phi_x^\dagger \Phi_x - 1\right]^2 + \hat{\lambda}_6 \left[\Phi_x^\dagger \Phi_x\right]^3 \right)$$

with

$$\varphi = \sqrt{2\kappa} \begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3 \end{pmatrix}, \quad m_0^2 = \frac{1 - 2\hat{\lambda} - 8\kappa}{\kappa}, \quad \lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad \lambda_6 = \frac{\hat{\lambda}_6}{8\kappa^3}.$$

Strategy and tools

- Phase structure is probed with the vev,

$$\hat{v} = a\varphi_c = \langle \hat{m} \rangle = \left\langle \frac{1}{V} \left| \sum_x \Phi_x^0 \right| \right\rangle .$$

- The constraint effective potential is a useful tool,

Fukuda and Kyriakopoulos, 1985

$$\mathrm{e}^{-VU(\hat{v})} \sim \int \mathcal{D}\varphi \mathcal{D}\bar{\psi} \mathcal{D}\psi \; \delta \left(\varphi_0^0 - \varphi_c \right) \; \mathrm{e}^{-S[\varphi, \bar{\psi}, \psi]},$$

$$\text{where } \varphi_0^0 = \frac{1}{V} \int d^4x \; \varphi^0 .$$

- Analytically calculated in lattice perturbation theory.
- Numerically extracted by histogramming $\hat{m}$ from lattice data.

Part I

$\lambda_6 \neq 0$

What we try to find

- Second-order non-thermal phase transition
  ➡ Stay close to the continuum limit.

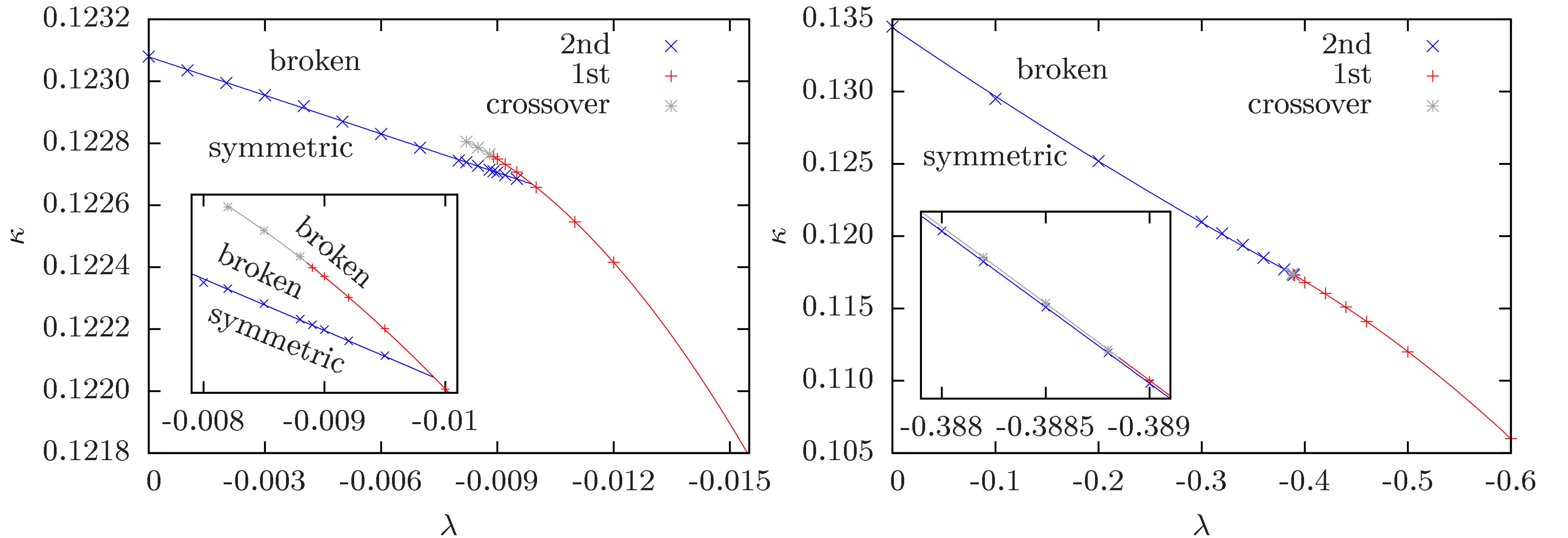
D.Y.-J. Chu *et al*., PLB744, 2015

- First-order thermal phase transition
  ➡ Electroweak baryongenesis.

- Phenomenological viability
  ➡ $av = v/\Lambda << 1$ and $v/M_H \sim 2$.

Recap: non-thermal phase structure

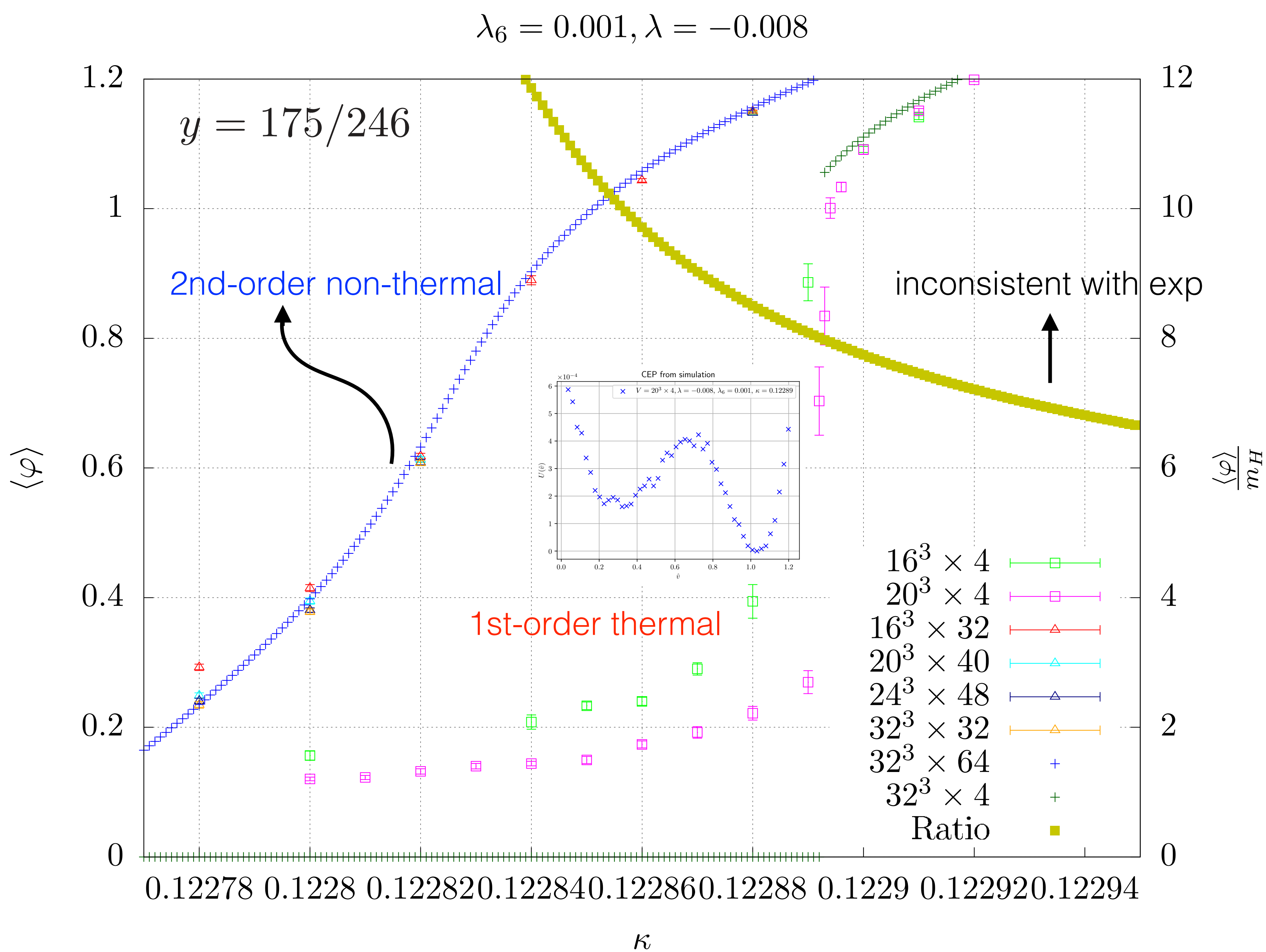
zero-T



(a) $\lambda_6 = 0.001$ (b) $\lambda_6 = 0.1$

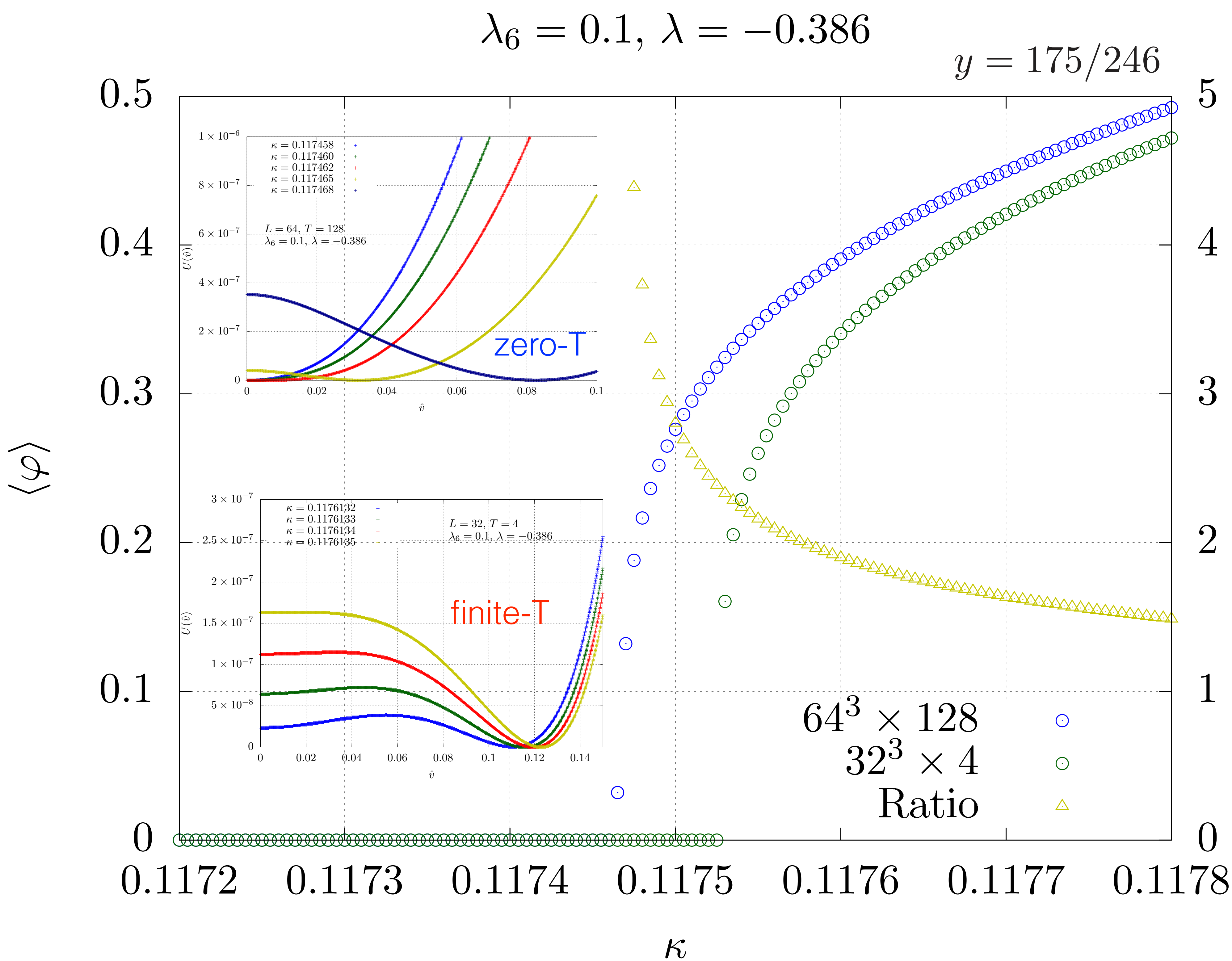
D.Y.-J. Chu *et al.*, PLB744, 2015

Weak coupling

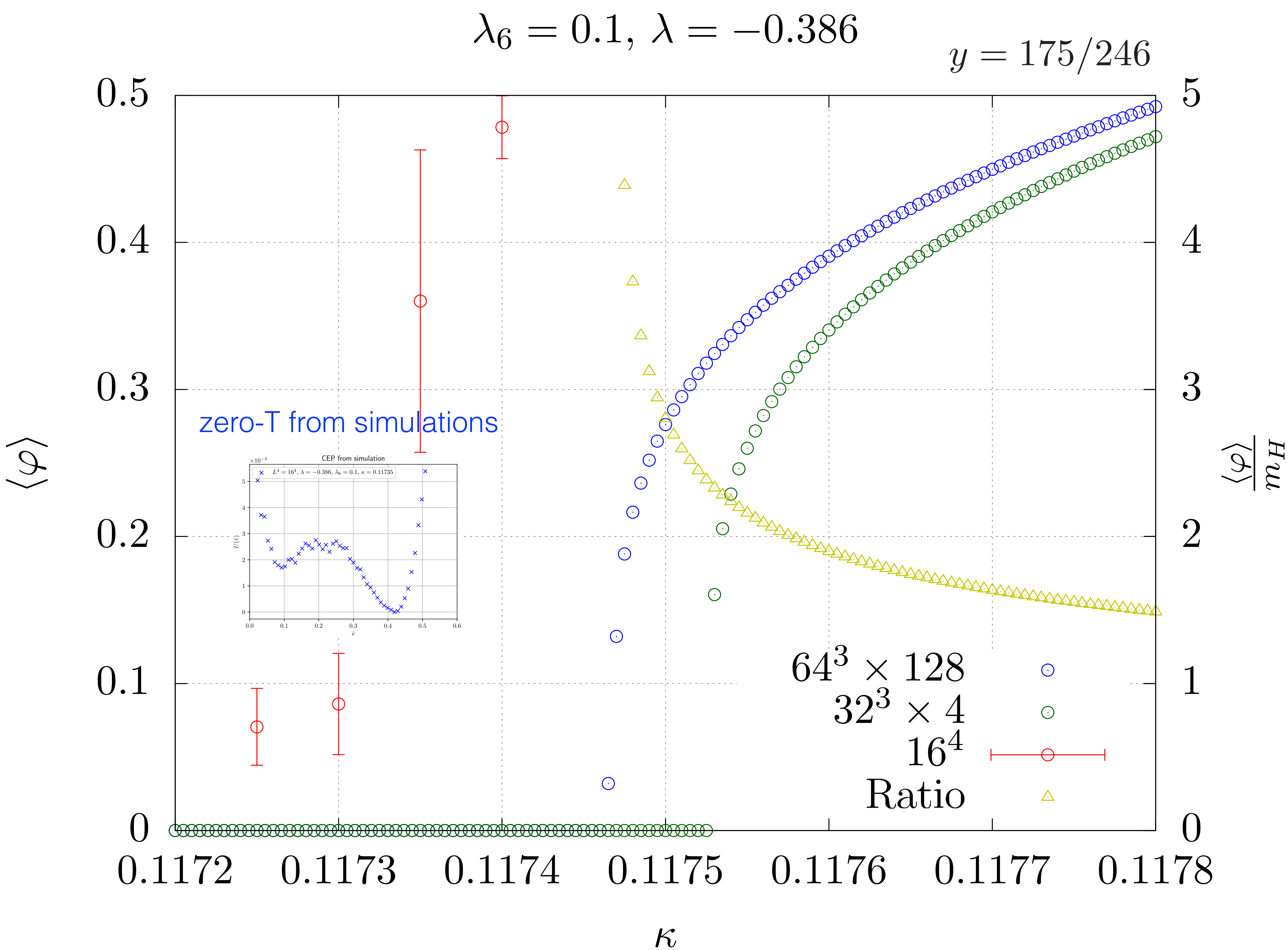


Non-thermal phase structure studied in D.Y.-J. Chu *et al.*, PLB744, 2015

Stronger coupling

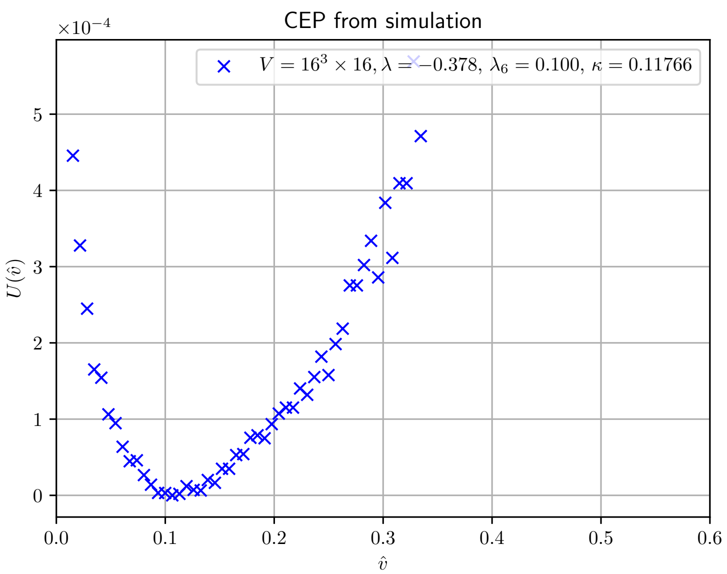


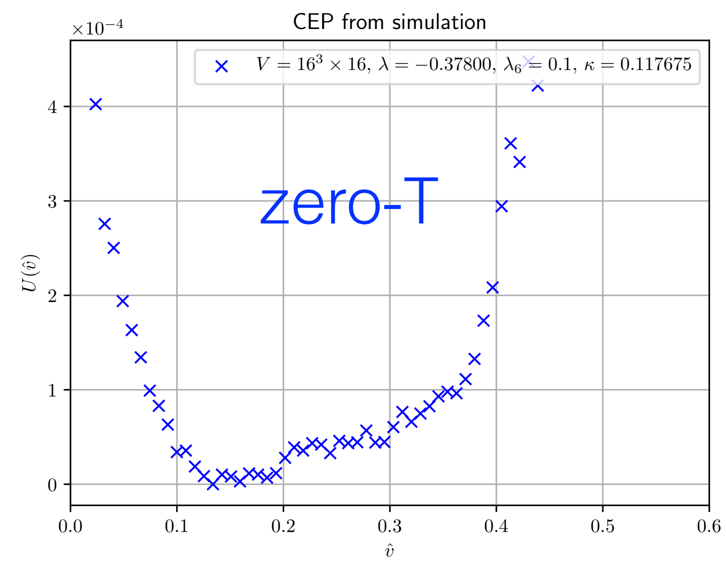
Stronger coupling

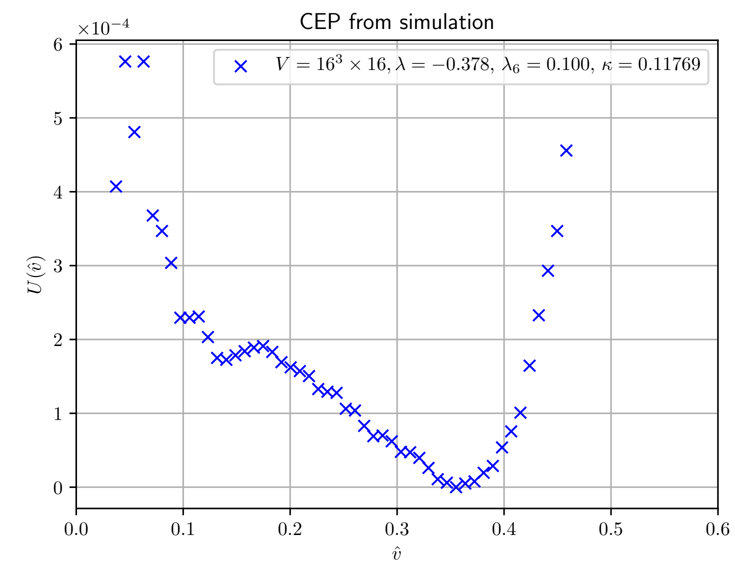


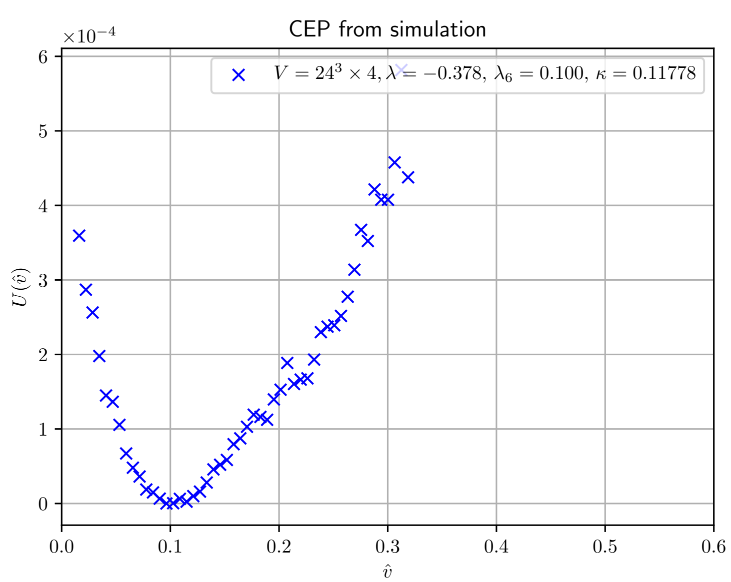
Stronger coupling

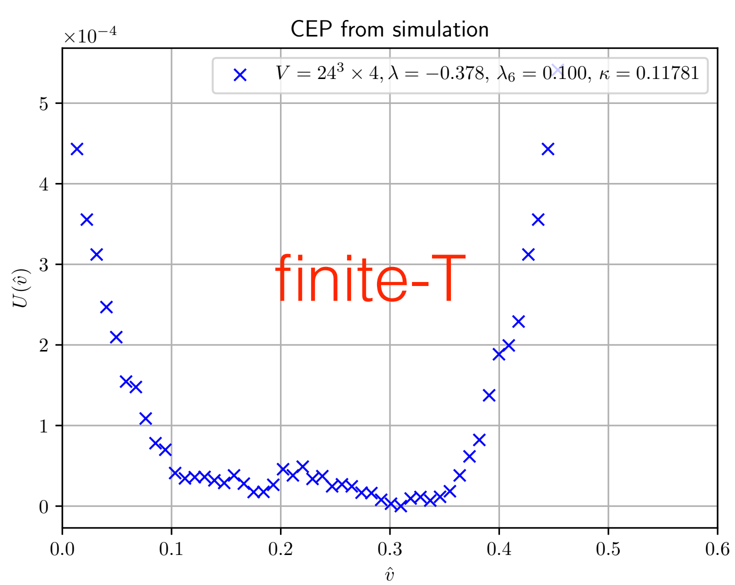
$$y = 175/246\ ,\ \ \lambda_6 = 0.1 \text{ and } \lambda = -0.378$$

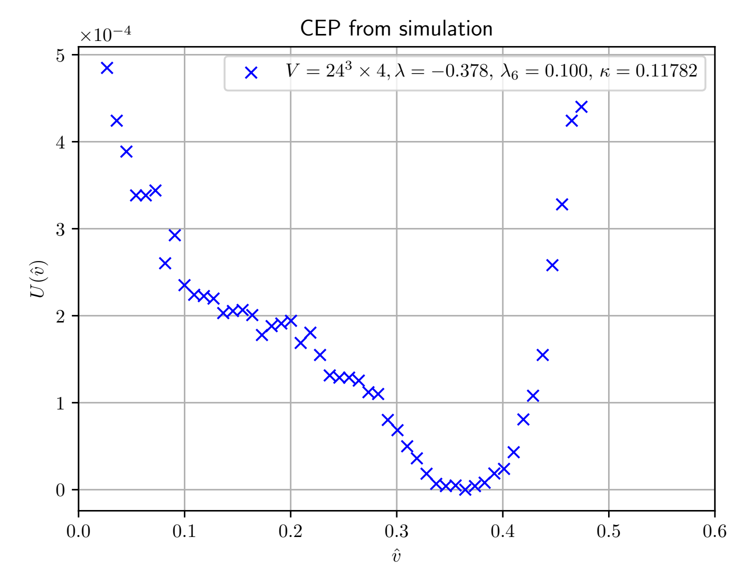












Keep scanning!

Part II

$$\lambda_6 = 0$$

What we try to do

- Explore the non-thermal phase structure in detail.
- Explore the triviality of the Higgs-Yukawa model.
  - ★ Resort to the technique of finite-size scaling.
  - ★ Need to understand mean-field scaling behaviour better.
    - → Logarithmic corrections.

Finite-size scaling

$$
\begin{aligned}
M_b\left[m_b^2, \lambda_b, Y_b; a, L\right] Z_\phi^{-D_M/2}(a,l) &= M\left[m^2(l), \lambda(l), Y(l); l, L\right] \\
\text{(RG running)} \quad &= M\left[m^2(L)L^2, \lambda(L), Y(L); L, L\right] \zeta_M(l,L) \\
\text{(Naive rescaling)} \quad &= M\left[m^2(L)L^2, \lambda(L), Y(L); 1, 1\right] \zeta_M(l,L) L^{-D_M} \\
\text{(Fixed point)} \quad &\xrightarrow{\text{F.P.}} M\left[m^2(L)L^2, \lambda_*, Y_*; 1, 1\right] \zeta_M(l,L) L^{-D_M} \\
&= f_s\left[m^2(L)L^2\right] \bar{\zeta}(L)
\end{aligned}
$$

Exact functional form is unknown in general.

★ FSS does not work in the same way near the Gaussian FP.

E. Brezin, J. Phys. 43, 1982

➡ *Need* to include logarithmic corrections.

E. Brezin and J. Zinn-Justin, NPB 257, 1985

FSS for H-Y model near the Gaussian FP

D.Y.-J. Chu *et al.*, arXiv:1611.00466 (LATTICE 2016)

- Derived the FSS function (f_s)
  - ➡ Valid to the order of the leading logarithms.
- Moments of the scalar zero modes can be expressed in

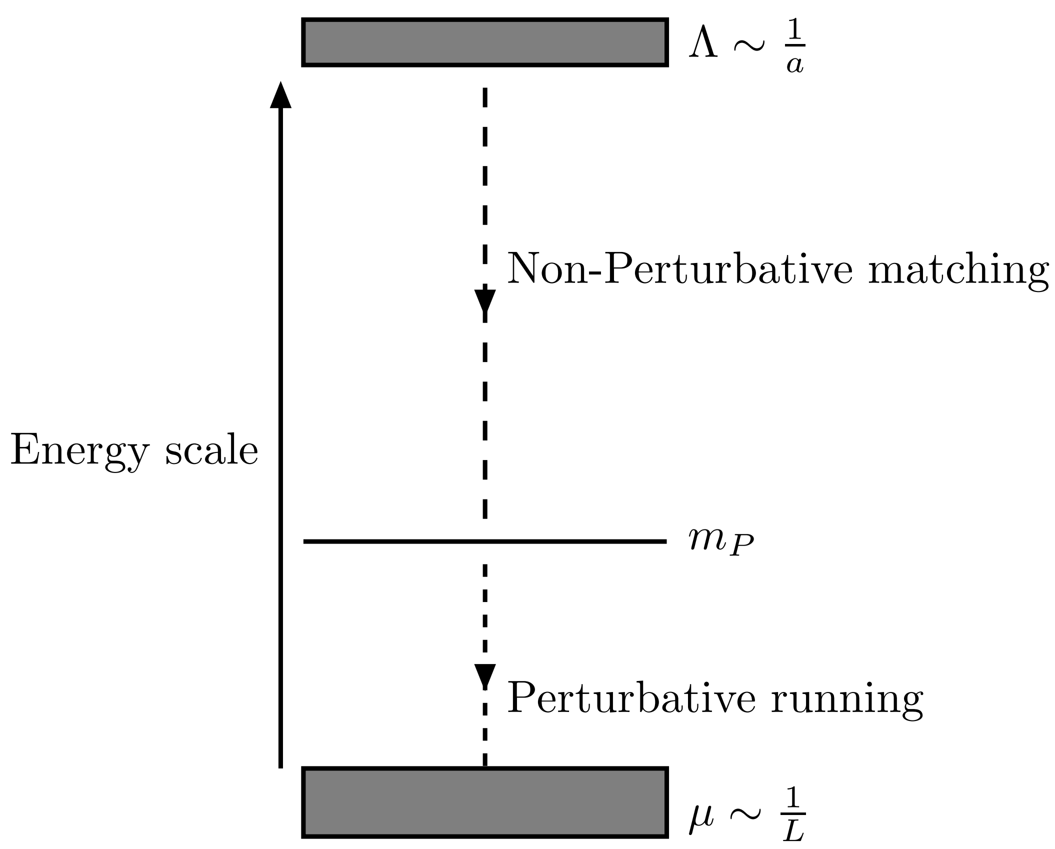
$$\bar{\varphi}_0 = \frac{\pi}{8}\exp\left(\frac{z^2}{32}\right)\sqrt{|z|}\left[I_{-1/4}\left(\frac{z^2}{32}\right) - \mathrm{Sgn}(z)\, I_{1/4}\left(\frac{z^2}{32}\right)\right],$$

$$\bar{\varphi}_1 = \frac{\sqrt{\pi}}{8}\exp\left(\frac{z^2}{16}\right)\left[1 - \mathrm{Sgn}(z)\,\mathrm{Erf}\left(\frac{|z|}{4}\right)\right],\quad \bar{\varphi}_{n+2} = -2\frac{\mathrm{d}}{\mathrm{d}z}\bar{\varphi}_n$$

- ➡ The scaling variable $z = \sqrt{s}\,\overbrace{\hat{M}^2(L^{-1})\hat{L}^2\lambda(L^{-1})^{-1/2}}^{\text{Logarithms}}$

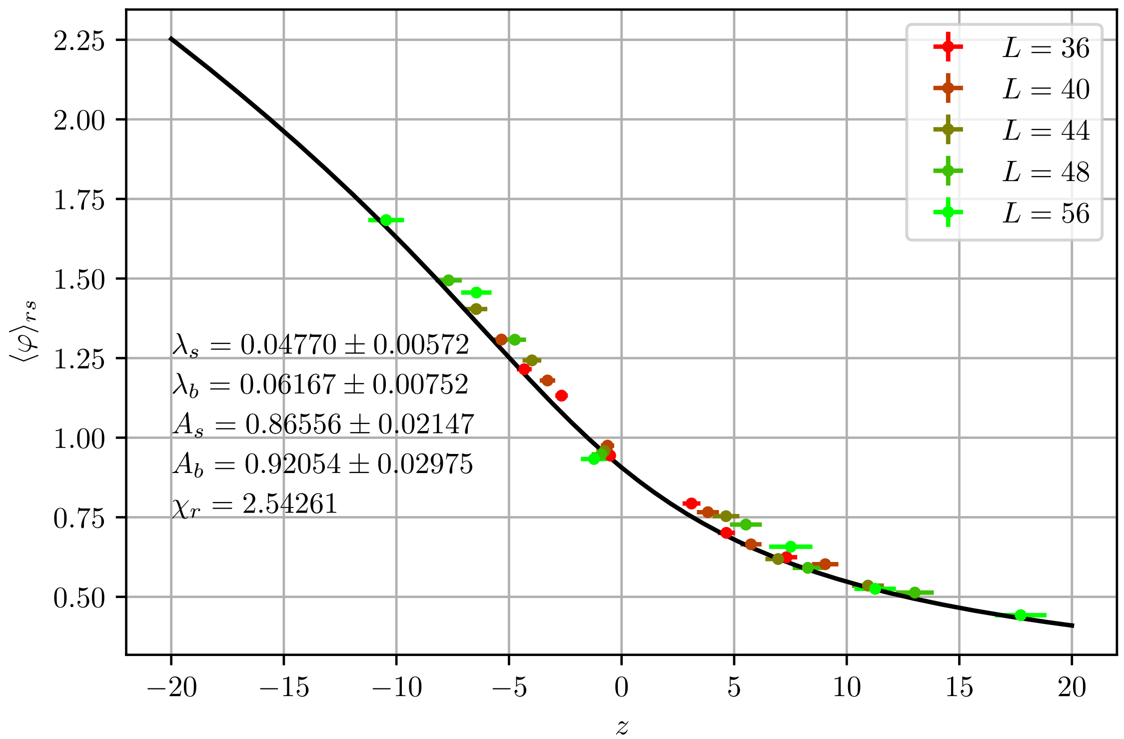
Test with O(4) scalar model

- Same scaling formulae but different logarithms.
- On-shell renormalisation scheme.

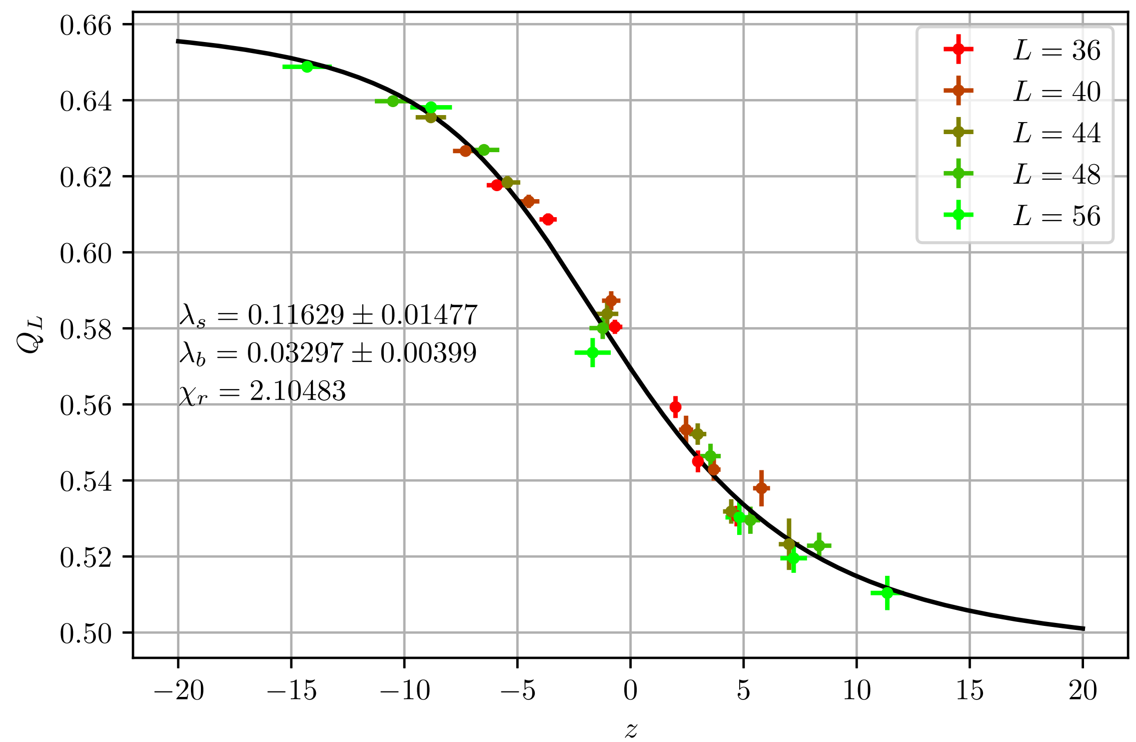


Numerical results, O(4) scalar model

$$\lambda_0 = 0.15$$



magnetisation



Binders cumulant

What we found

- Reasonable fits.

➡ Confidence in the fit (scaling) functions.

- No statistical significance of the logarithmic corrections

➡ Improving data (better statistics).

➡ Same quality of data to discern logs in the H-Y model?