Higgs compositeness in Sp(2N) gauge theories
Part III: Scale Setting and Topology

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Recap

- Beginning a study of Sp($2N$) gauge theory
- Intend to study theories of phenomenological relevance
- Currently developing a toolset to do this
- Have studied pure gauge theories

In this talk

- Dynamical fermions and resymplecticisation
- Scale setting
- Topological charge and susceptibility

Hybrid Monte Carlo

- Wilson gauge action, Wilson fermion action
- Evolve motion under the gauge force
  - Force: $F_G^A(x,\mu) = \frac{\beta}{N}\frac{1}{T_F}\mathrm{Re}\ \mathrm{tr}_c[iT_F^A\,U(x,\mu)\,V^\dagger(x,\mu)]$
  - Momenta: $\pi(x,\mu) = i\pi^A(x,\mu)\,T_F^A$
  - Equations of motion: $\frac{\mathrm{d}}{\mathrm{d}\tau}\,U(x,\mu) = \pi(x,\mu)\,U(x,\mu)$; $\frac{\mathrm{d}}{\mathrm{d}\tau}\pi(x,\mu) = F(x,\mu)$
- Naïve update:

$$U'(x,\mu) = e^{i\pi^A(x,\mu)\,T_F^A\mathrm{d}\tau}\,U(x,\mu)$$
$$\simeq (1 + i\pi^A(x,\mu)\,T_F^A\delta\tau)\,U(x,\mu)$$

- Improved update is used instead

Resymplecticisation

- Machine precision effects cause an unbounded simulation to leave the group manifold
- In SU(N) we reunitarise to avoid this
- In Sp($2N$) we must instead *resymplecticise*
- Represent a Sp(4) matrix as

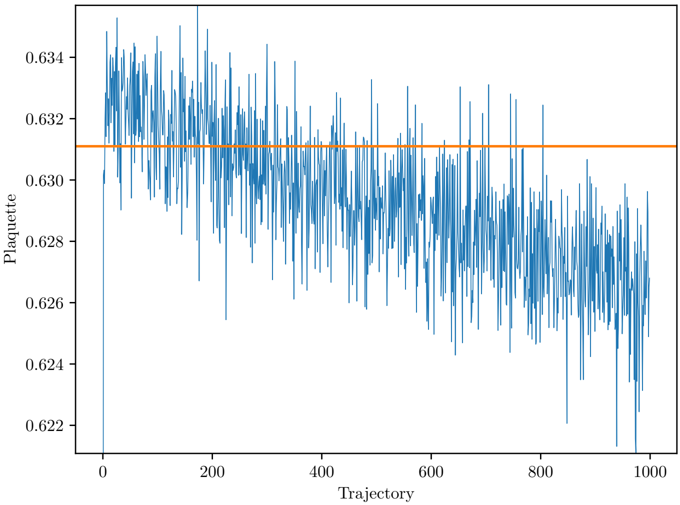
$$\begin{aligned} Q(x,\mu) = {} & Q_0(x,\mu) \otimes \mathbb{I}_2 + Q_1(x,\mu) \otimes e_1 \\ & + Q_2(x,\mu) \otimes e_2 + Q_3(x,\mu) \otimes e_3 \end{aligned}$$

$$\mathbb{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad e_1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$e_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad e_3 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

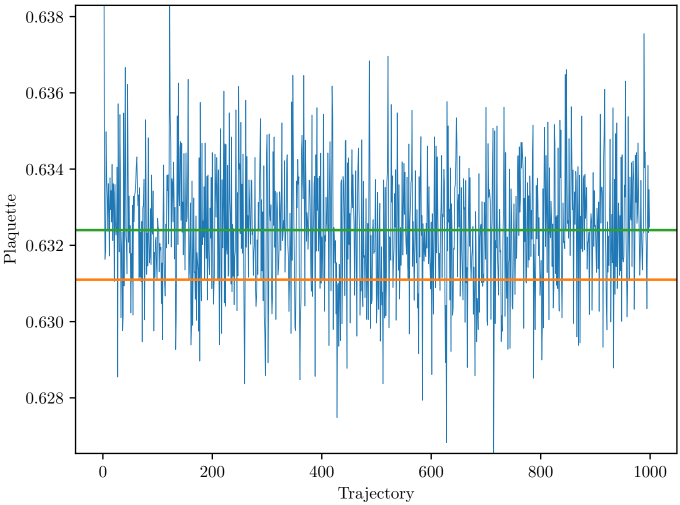
- Take updated gauge field $U(x,\mu)$, project onto this basis and normalise

Without resymplecticisation



Orange: Holland, Pepe, Wiese (2004)

With resymplecticisation



Orange: Holland, Pepe, Wiese (2004)

The gradient flow scale t_0

- In the continuum, define

$$\frac{\mathrm{d}B_\mu(t,x)}{\mathrm{d}t} = D_\mu G_{\mu\nu}(t,x)$$

- Integrate this numerically in t from $t=0$, $B_\mu(0,x) = A_\mu(x)$
- Define $E(t) = -\frac{1}{2}\,\mathrm{tr}\,G_{\mu\nu}G_{\mu\nu}$ and $\mathcal{E}(t) = t^2\langle E(t)\rangle$
- Pick a reference value $\mathcal{E}_0$
- Impose $\mathcal{E}(t=t_0) = \mathcal{E}_0$
- Smearing radius $\sqrt{8t}$; impose $\sqrt{8t} < \frac{L}{2a}$

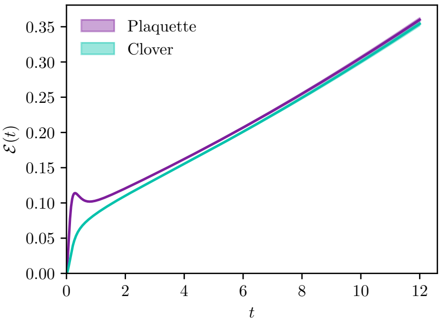
The w_0 scale

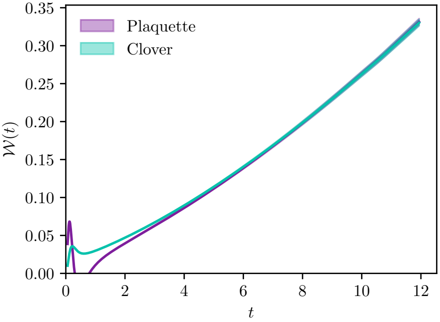
- t_0 can show a strong β- and m-dependence
- Consider $\mathcal{W}(t) = t\frac{\mathrm{d}\mathcal{E}(t)}{\mathrm{d}t}$
- Again, impose $\mathcal{W}(t = w_0^2) = \mathcal{W}_0$
- In QCD, conventionally $\mathcal{E}_0 = \mathcal{W}_0 = 0.3$

Lattice practicalities

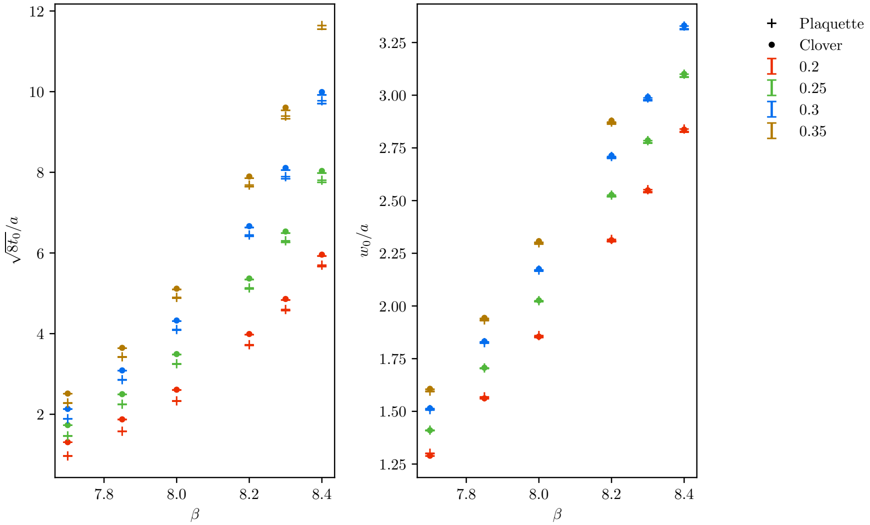
- Flow is integrated via fourth-order Runge–Kutta
- E relies on a discretisation of $G_{\mu\nu}$. Options:
  - Plaquette
  - Symmetrised four-plaquette clover
- All methods agree in continuum limit
- Relative discrepancy decreases with increasing t
- Large divergence at small t—choose $\mathcal{E}_0$, $\mathcal{W}_0$ to avoid this

Pure gauge: $\beta = 8.4$

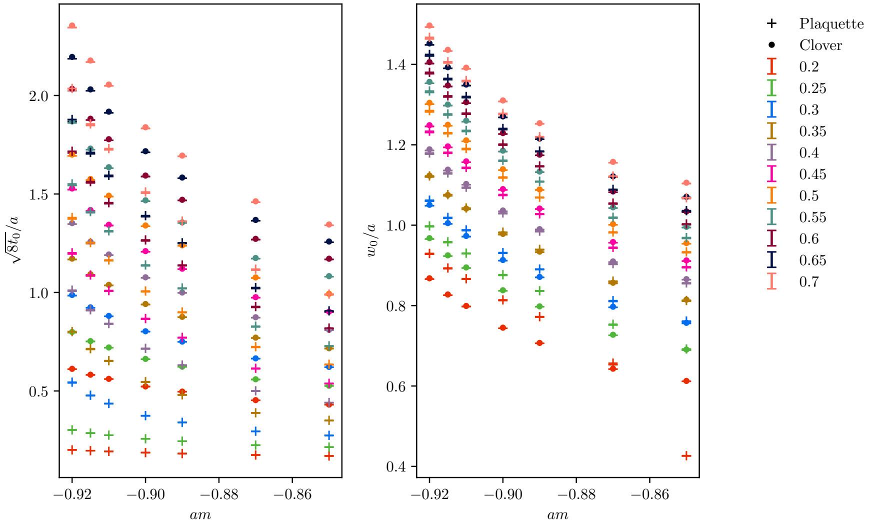




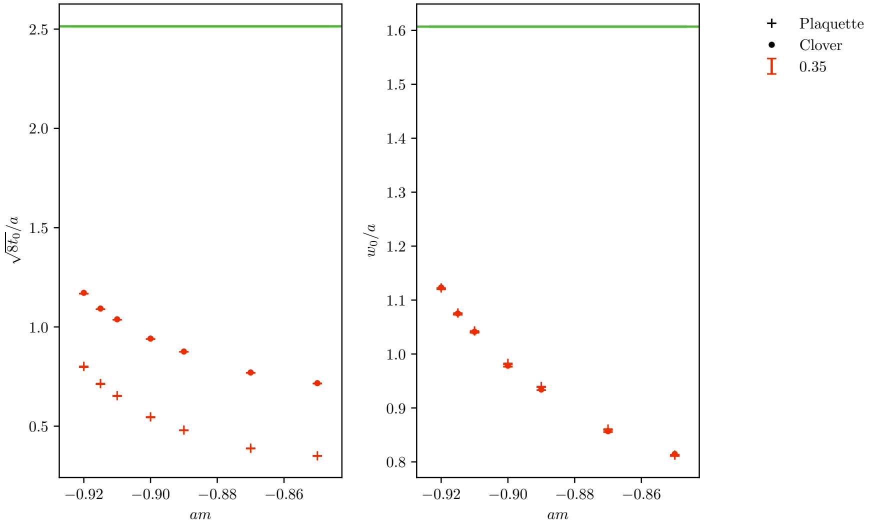
Pure gauge: t_0 and w_0 ($V = 24^4$)



$N_f = 2$: t_0 and w_0 ($\beta = 6.9$, $V = 32 \times 16^3$)



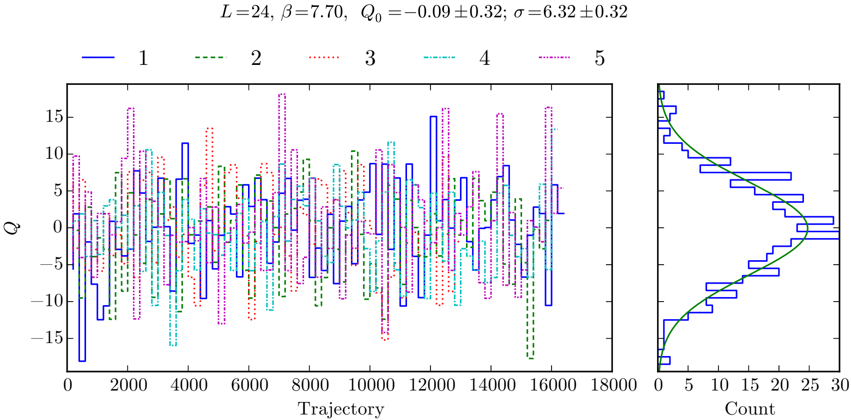
Comparison at $\mathcal{E}_0 = \mathcal{W}_0 = 0.35$



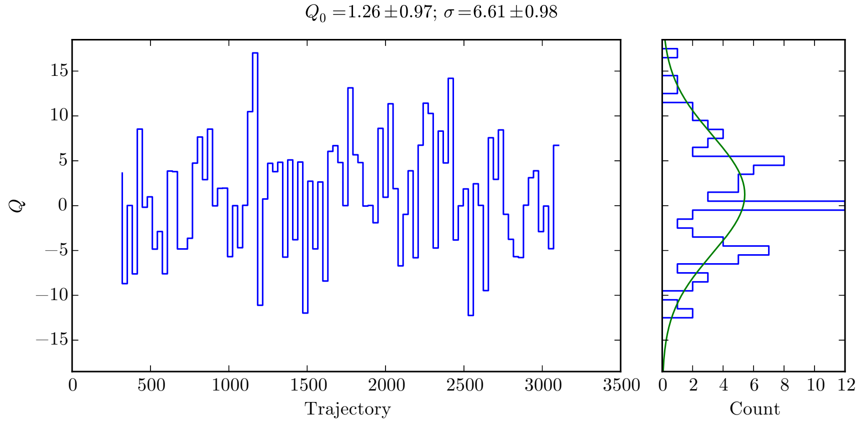
Topological charge

- $Q = \sum_x q(x) = \frac{1}{32\pi^2} \sum_x \epsilon_{\mu\nu\rho\sigma} \operatorname{tr}(U_{\mu\nu}(x) U_{\rho\sigma}(x))$
- Moves slowly or freezes at large volume, β
- $Q \in \mathbb{Z}$ for finite volume
- Should be Gaussian distributed about $Q = 0$
- Badly affected by UV noise; remove this with the gradient flow
- No signs of freezing seen
- Measure topological susceptibility as $\chi = \langle Q^2 \rangle / V$

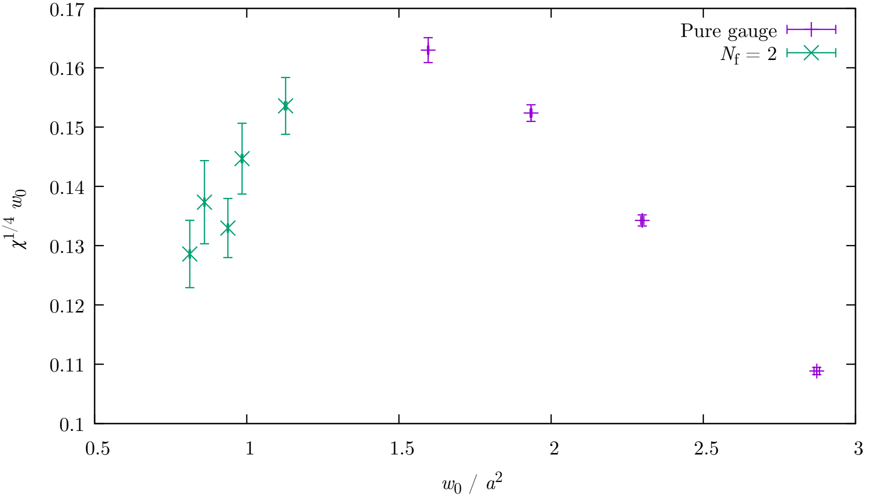
Q for pure gauge; $L = 24$, $\beta = 7.7$



Q for $N_{\mathrm{f}} = 2$; $L = 16$, $\beta = 6.9$, $m = -0.92$



Topological susceptibility



Conclusions

- We reproject to Sp(4) after each update to remain within the group
- We have measured the scales t_0 and w_0 on both pure gauge and dynamical ensembles
- We see that dynamical simulations at $N_{\mathrm{f}} = 2$, $\beta = 6.9$, are more strongly coupled than the strongest-coupled pure gauge ensemble
- We observe no issues with the topological charge, modulo limited statistics