Light-by-light forward scattering amplitudes in Lattice QCD

Antoine Gérardin

In collaboration with J. Green, O. Gryniuk, G. von Hippel, H. Meyer, V. Pascalutsa and H. Wittig

Institut für Kernphysik, Johannes Gutenberg-Universität Mainz

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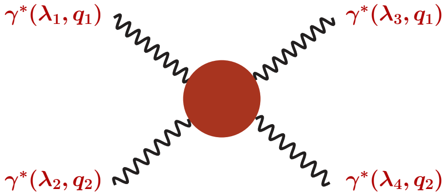
Motivations

- Light-by-light forward scattering amplitudes



$$\gamma^*(\lambda_1, q_1)\,\gamma^*(\lambda_2, q_2) \to \gamma^*(\lambda_3, q_1)\,\gamma^*(\lambda_4, q_2)$$

↪ Use phenomenology to describe the amplitudes computed on the lattice [Green et. al '15]

- Relevant for the $(g-2)_\mu$

↪ The hadronic light-by-light (HLbL) contribution dominates the theoretical error (with the HVP)

↪ There are two different approaches to compute the HLbL contribution to the $(g-2)_\mu$

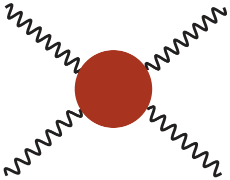
1) Direct calculation on the lattice [RBC-UKQCD, Mainz group]
   - ▶ The hadronic part involve the HLbL four-point correlation function

2) Models / dispersive approach : [de Rafael '94, Colangelo et. al '14]
   - ▶ The pion pole contribution can be evaluated on the lattice. Expected to give the dominant contribution
   - ▶ Contributions from other resonances rely on the knowledge of associated transition form factors (TFF)

     ↪ HLbL amplitudes can be used to constrain the TFFs
   - ▶ Only a few states are expected to contribute significantly to a_μ^{HLbL} → can be checked on the lattice

Light-by-light scattering amplitudes

- Forward scattering amplitudes $M_{\lambda_3\lambda_4\lambda_1\lambda_2}$: $\gamma^*(\lambda_1, q_1)\ \gamma^*(\lambda_2, q_2)\ \rightarrow\ \gamma^*(\lambda_3, q_1)\ \gamma^*(\lambda_4, q_2)$



- ▶ 81 helicity amplitudes ($\lambda_i = 0, \pm 1$)

$$\mathcal{M}_{\lambda_1'\lambda_2'\lambda_1\lambda_2} = \mathcal{M}_{\mu\nu\rho\sigma}\ \epsilon^{*\mu}(\lambda_1')\ \epsilon^{*\nu}(\lambda_2')\ \epsilon^{\rho}(\lambda_1)\ \epsilon^{\sigma}(\lambda_2)$$

- ▶ Photons virtualities : $Q_1^2 = -q_1^2 > 0$ and $Q_2^2 = -q_2^2 > 0$
- ▶ Cross symmetric variable : $\nu = q_1 \cdot q_2$

- Using parity and time invariance : only 8 independent amplitudes

$$(M_{++,++} + M_{+-,+-})\,,\ M_{++,--}\,,\ M_{00,00}\,,\ M_{+0,+0}\,,\ M_{0+,0+}\,,\ (M_{++,00} + M_{0+,-0})\,,$$
$$(M_{++,++} - M_{+-,+-})\,,\ (M_{++,00} - M_{0+,-0})$$

↪ Either even or odd with respect to ν

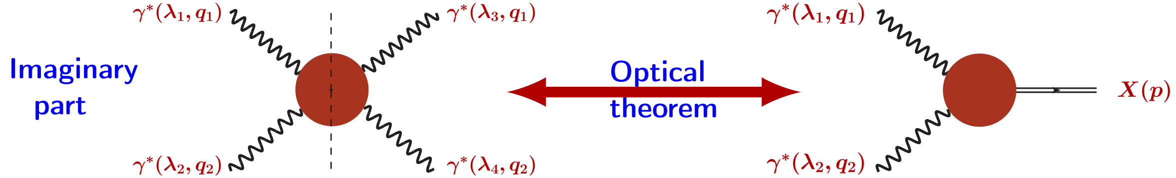
↪ The eight amplitudes have been computed on the lattice for different values of ν, Q_1^2, Q_2^2

- Relate the forward amplitudes to two-photon fusion cross sections using the optical theorem [Pascalutsa et. al '12]

↪ Eight independent dispersion relations for M_{TT}, M_{TT}^t, M_{TT}^a, M_{TL}, M_{LT}, M_{TL}^a, M_{TL}^t and M_{LL}

Dispersion relations

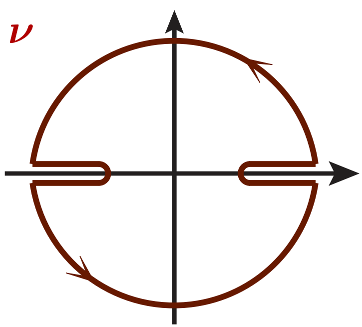
1) Optical theorem



$\hookrightarrow$ X can be any C-even final state

$$W_{\lambda_3\lambda_4,\lambda_1\lambda_2} = \text{Abs}\, M_{\lambda_3\lambda_4,\lambda_1\lambda_2} = \frac{1}{2}\int \mathrm{d}\Gamma_X (2\pi)^4 \delta(q_1 + 1_2 - p_X) \mathcal{M}_{\lambda_1\lambda_2}(q_1, q_2, p_X)\, \mathcal{M}^*_{\lambda_3\lambda_4}(q_1, q_2, p_X)$$

2) Dispersion relations [Pascalutsa et. al '12]



Once-subtracted sum rules : cross-symmetric variable $\nu = q_1 \cdot q_2$

$$M_{\text{even}}(\nu) = M_{\text{even}}(0) + \frac{2\nu^2}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{1}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} W_{\text{even}}(\nu')$$

$$M_{\text{odd}}(\nu) = \nu M_{\text{odd}}(\nu) + \frac{2\nu^3}{\pi}\int_{\nu_0}^{\infty} d\nu' \frac{1}{\nu'(\nu'^2 - \nu^2 - i\epsilon)} W_{\text{odd}}(\nu')$$

3) Higher mass singularities are suppressed with ν^2 :

$\hookrightarrow$ Only a few states X are necessary to saturate the sum rules and reproduce the lattice data

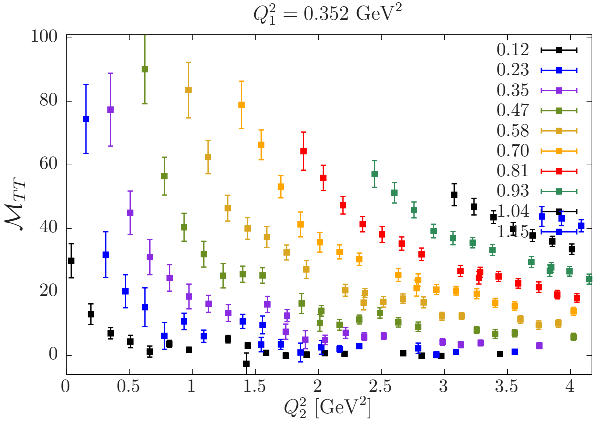
Description of the lattice data using phenomenology

$\rightarrow$ For each of the eight amplitudes, we have a dispersion relation :

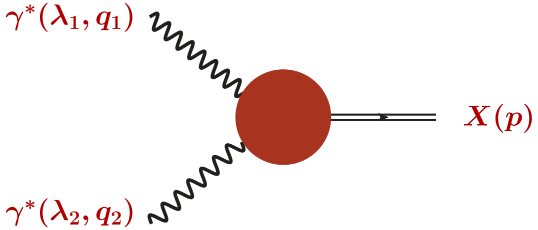
$$\overline{M}_{\alpha}(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'} \, \sigma_{\alpha}/\tau_{\alpha}(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

Lattice calculation

$\hookrightarrow$ 4-pt correlation function



$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \rightarrow \mathrm{X}(p_{\mathrm{X}})$ fusion cross sections

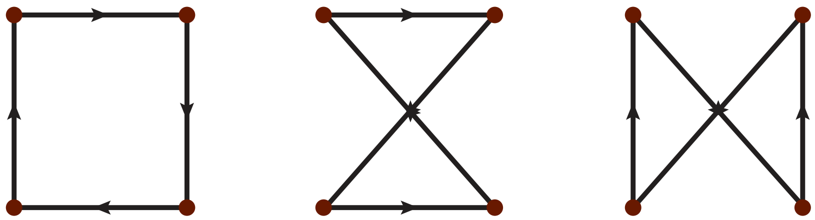


$\hookrightarrow$ X : any C-even states contribute

$\hookrightarrow$ Main contribution is expected from mesons :

Pseudoscalars (0^{-+}) Axial-vectors (1^{++})
Scalar (0^{++}) Tensors (2^{++})

Lattice calculation



- Four-point correlation function

$$\Pi^E_{\mu\nu\rho\sigma}(Q_1, Q_2) = \sum_{X_1, X_2, X_4} \langle J^c_\mu(X_1)\, J^c_\nu(X_2)\, J^l_\rho(0)\, J^c_\sigma(X_4) \rangle_E \; e^{-iQ_1(X_2-X_1)} \; e^{iQ_2 X_4} \; + \; \text{contact terms}$$

$\hookrightarrow$ $J_\mu(x) = \frac{2}{3}\overline{u}(x)\gamma_\mu u(x) - \frac{1}{3}\overline{d}(x)\gamma_\mu d(x)$

$\hookrightarrow$ 3 conserved vector currents ($Z^c_V = 1$)

$\hookrightarrow$ 1 local vector current (Z^l_V, computed non-perturbatively)

- Only the fully-connected diagrams are included so far

- Five CLS ensembles ($N_f = 2$, $\mathcal{O}(a)$-improved Wilson-Clover)

$\hookrightarrow$ 4 ensembles at $a \approx 0.065$ fm and m_π down to 180 MeV

$\hookrightarrow$ 1 ensemble at $a \approx 0.048$ fm

Description of the lattice data using phenomenology

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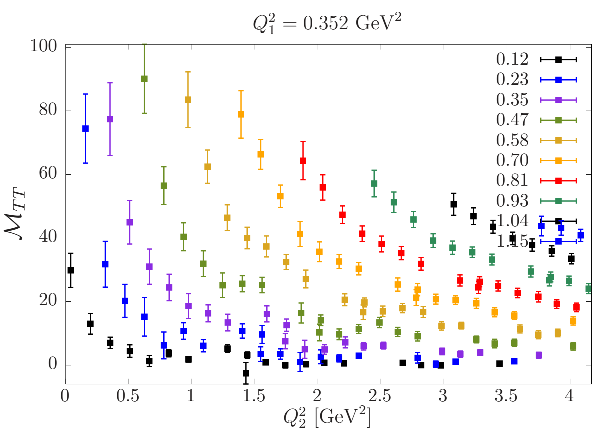
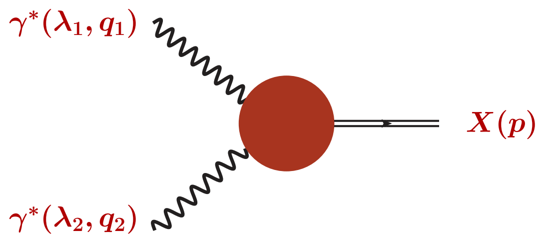
$$\overline{M}_{\alpha}(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'} \, \sigma_{\alpha}/\tau_{\alpha}(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

Lattice calculation

$\hookrightarrow$ 4-pt correlation function



$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \rightarrow \mathrm{X}(p_{\mathrm{X}})$ fusion cross sections



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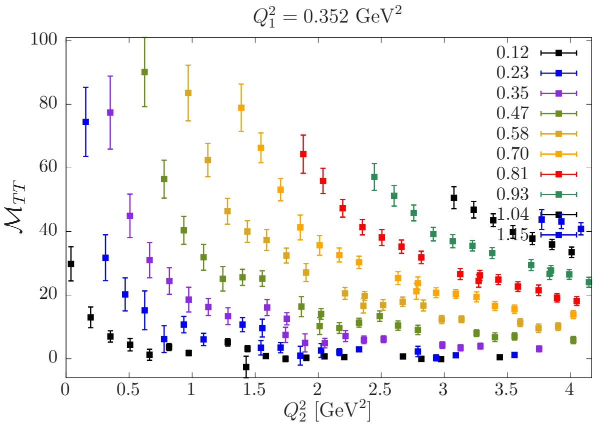
Lattice calculation

↪ 4-pt correlation function



$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \mathrm{X}(p_\mathrm{X})$ fusion cross sections

↪ Consider only one particle in each channel

↪ $N_f = 2$: no η meson

↪ Isospin symmetry + large-N_c approximation : isovector only with an overall factor $34/9$

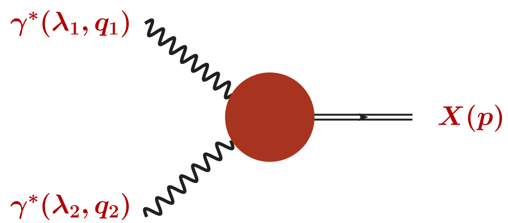
	Isovector	Isoscalar	Isoscalar
0^{-+}	π	η'	η
0^{++}	$a_0(980)$	$f_0(980)$	$f_0(600)$
1^{++}	$a_1(1260)$	$f_1(1285)$	$f_1(1420)$
2^{++}	$a_2(1320)$	$f_2(1270)$	$f_2'(1525)$

Contributions to the eight independent amplitudes

$$\overline{M}(\nu) = \frac{4\nu^2}{\pi} \int_{\nu_0}^{\infty} d\nu' \, \frac{\sqrt{X'} \, \sigma/\tau(\nu')}{\nu'(\nu'^2 - \nu^2 - i\epsilon)}$$

Amplitude	Pseudoscalar	Scalar	Axial	Tensor	Scalar QED
M_{TT}	$\sigma_0/2$	$\sigma_0/2$	$\sigma_0/2$	$\frac{\sigma_0+\sigma_2}{2}$	σ_{TT}
M_{TT}^t	$-\sigma_0$	σ_0	$-\sigma_0$	σ_0	τ_{TT}
M_{TT}^a	$\sigma_0/2$	$\sigma_0/2$	$\sigma_0/2$	$\frac{\sigma_0-\sigma_2}{2}$	τ_{TT}^a
M_{TL}	$\times$	$\times$	σ_{TL}	σ_{TL}	σ_{TL}
M_{LT}	$\times$	$\times$	σ_{LT}	σ_{LT}	σ_{LT}
M_{TL}^t	$\times$	τ_{TL}	τ_{TL}	τ_{TL}	τ_{TL}
M_{TL}^a	$\times$	τ_{TL}	$-\tau_{TL}$	τ_{TL}^a	τ_{TL}^a
M_{LL}	$\times$	σ_{LL}	$\times$	σ_{LL}	σ_{LL}

Two-photon fusion cross sections : modelisation



- Example : contribution of the pseudoscalar to the amplitude M_{TT}

$$\sigma_{TT} = 8\pi^2 \delta(s - m_P^2) \frac{2\sqrt{X}}{m_P^2} \times \frac{\Gamma_{\gamma\gamma}}{m_P} \times \left[\frac{F_{\mathcal{P}\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F_{\mathcal{P}\gamma^*\gamma^*}(0,0)} \right]^2$$

- Similar results for other mesons (assume Breit-Wigner shape for resonances)

- We assume a constant mass shift in the spectrum (scalar, axial, tensor)

$$m_X = m_X^{\rm phys} + (m_\rho^{\rm lat} - m_\rho^{\rm phys})$$

- The two-photons decay width $\Gamma_{\gamma\gamma} = \frac{\pi\alpha^2}{4} m_S \left[F^T_{\mathcal{S}\gamma^*\gamma^*}(0,0) \right]^2$ is taken from experiment

All the non-perturbative information is encoded into the meson transition form factors

Assumptions on form factors

- Pseudoscalar meson
  ↪ experimental data are available only when at least one photon is on-shell
  ↪ $F_{\mathcal{P}\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ has been computed on the lattice [Gerardin et. al '16]

- Scalar mesons
  ↪ can be produced by two transverse photons (T) or two longitudinal photons (L) : $F^T_{\mathcal{S}\gamma^*\gamma^*}$ and $F^L_{\mathcal{S}\gamma^*\gamma^*}$
  ↪ $F^T_{\mathcal{S}\gamma^*\gamma^*}$ has been measured experimentally in the region $Q^2 < 30\ \mathrm{GeV}^2$ for the $f_0(980)$ meson [Masuda '15]
  ↪ results are compatible with a monopole form factor with $M_S = 0.800(50)\ \mathrm{MeV}$

$$\frac{F^T_{\mathcal{S}\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F^T_{\mathcal{S}\gamma^*\gamma^*}(0,0)} = \frac{1}{(1+Q_1^2/M_S^2)(1+Q_2^2/M_S^2)}$$

↪ <u>In the following, M_S is considered as a free parameter</u>

Assumptions on form factors

- Axial mesons

↪ Two form factors $F^{(0)}_{\mathcal{A}\gamma^*\gamma^*}$ and $F^{(1)}_{\mathcal{A}\gamma^*\gamma^*}$ ($\Lambda = 0, 1$ corresponds to the two helicity states of the axial meson)

↪ Quark model inspired parametrisation [N. Cahn '87]

$$F^{(0)}_{\mathcal{A}\gamma^*\gamma^*}(Q_1^2, Q_2^2) = m_A^2 A(Q_1^2, Q_2^2)\,,$$
$$F^{(1)}_{\mathcal{A}\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{\nu}{X}\left(\nu + Q_2^2\right)\, m_A^2 A(Q_1^2, Q_2^2)\,,$$
$$F^{(1)}_{\mathcal{A}\gamma^*\gamma^*}(Q_2^2, Q_1^2) = -\frac{\nu}{X}\left(\nu + Q_1^2\right)\, m_A^2 A(Q_1^2, Q_2^2)$$

in which $2\nu = m_A^2 + Q_1^2 + Q_2^2$ with m_A the meson mass,

$$\frac{A(Q_1^2, 0)}{A(0,0)} = \frac{1}{(1 + Q_1^2/M_A^2)^2}\,,$$

↪ Single virtuel case : L3 Collaboration in the region $Q^2 < 5~\mathrm{GeV}^2$ [Achard '01 '07]

↪ $M_A = 1040(78)~\mathrm{MeV}$ for the $f_1(1285)$ meson

↪ One free fit parameter : M_A

Assumptions on form factors

- Tensor mesons

↪ Amplitudes are described by four form factors $F^{(\Lambda)}_{\mathcal{T}\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ with $\Lambda = (0,T),\ (0,L),\ 1,\ 2$

↪ The single-virtual form factors for helicities $\Lambda = (0,T), 1, 2$ have also been measured experimentally in the region $Q^2 < 30\ \mathrm{GeV}^2$ by the Belle Collaboration [Masuda '15]

↪ data are compatible with a dipole form factor [Danilkin '16]

$$\frac{F^{(\Lambda)}_{\mathcal{T}\gamma^*\gamma^*}(Q_1^2, Q_2^2)}{F^{(\Lambda)}_{\mathcal{T}\gamma^*\gamma^*}(0,0)} = \frac{1}{(1 + Q_1^2/M^2_{T,(\Lambda)})^2 (1 + Q_2^2/M^2_{T,(\Lambda)})^2}$$

↪ Four free fit parameters : $M_{T,(\Lambda)}$

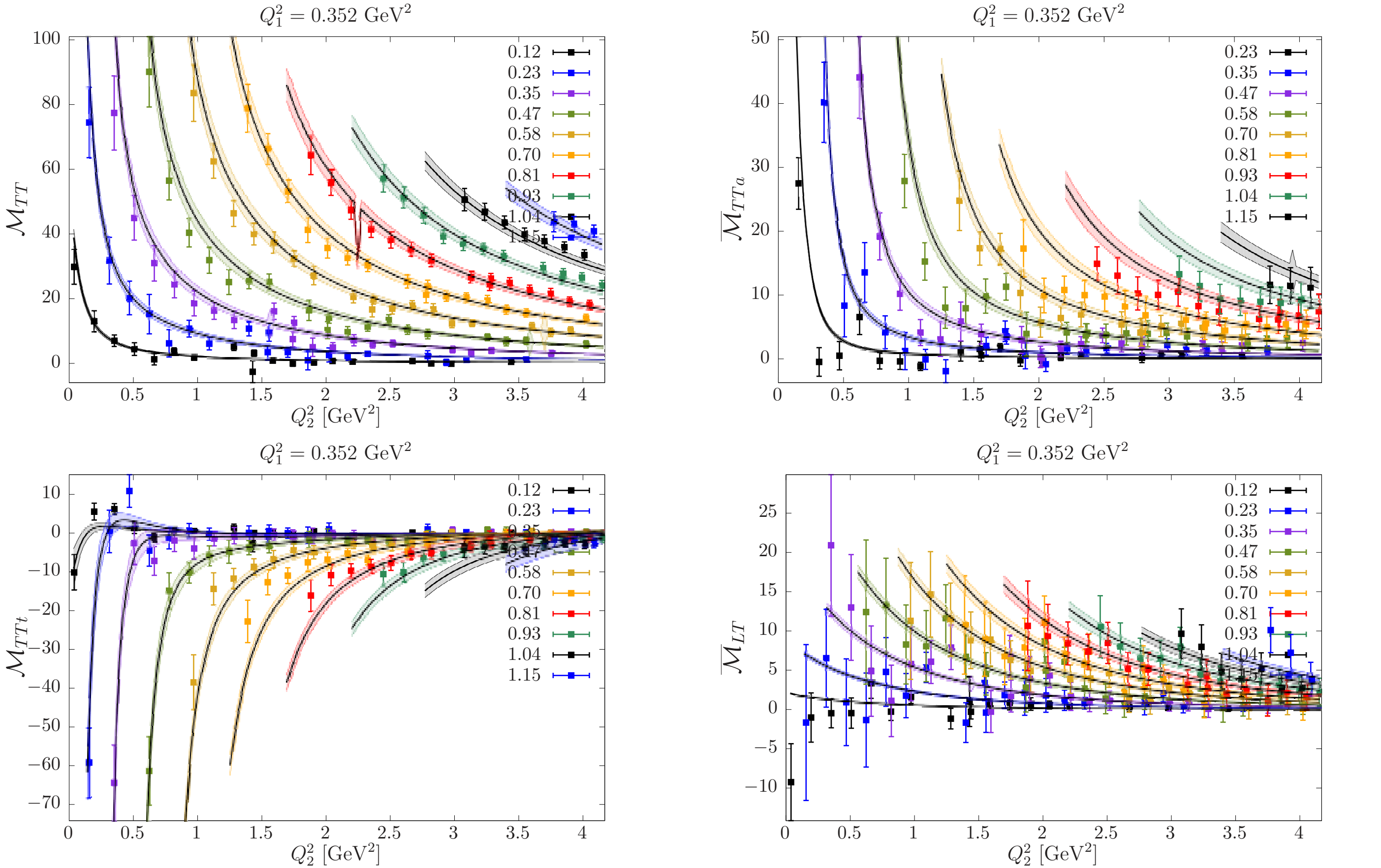
- Scalar QED

$\gamma^*\gamma^* \to \pi^+\pi^-$ evaluated using scalar QED dressed with monopole form factors

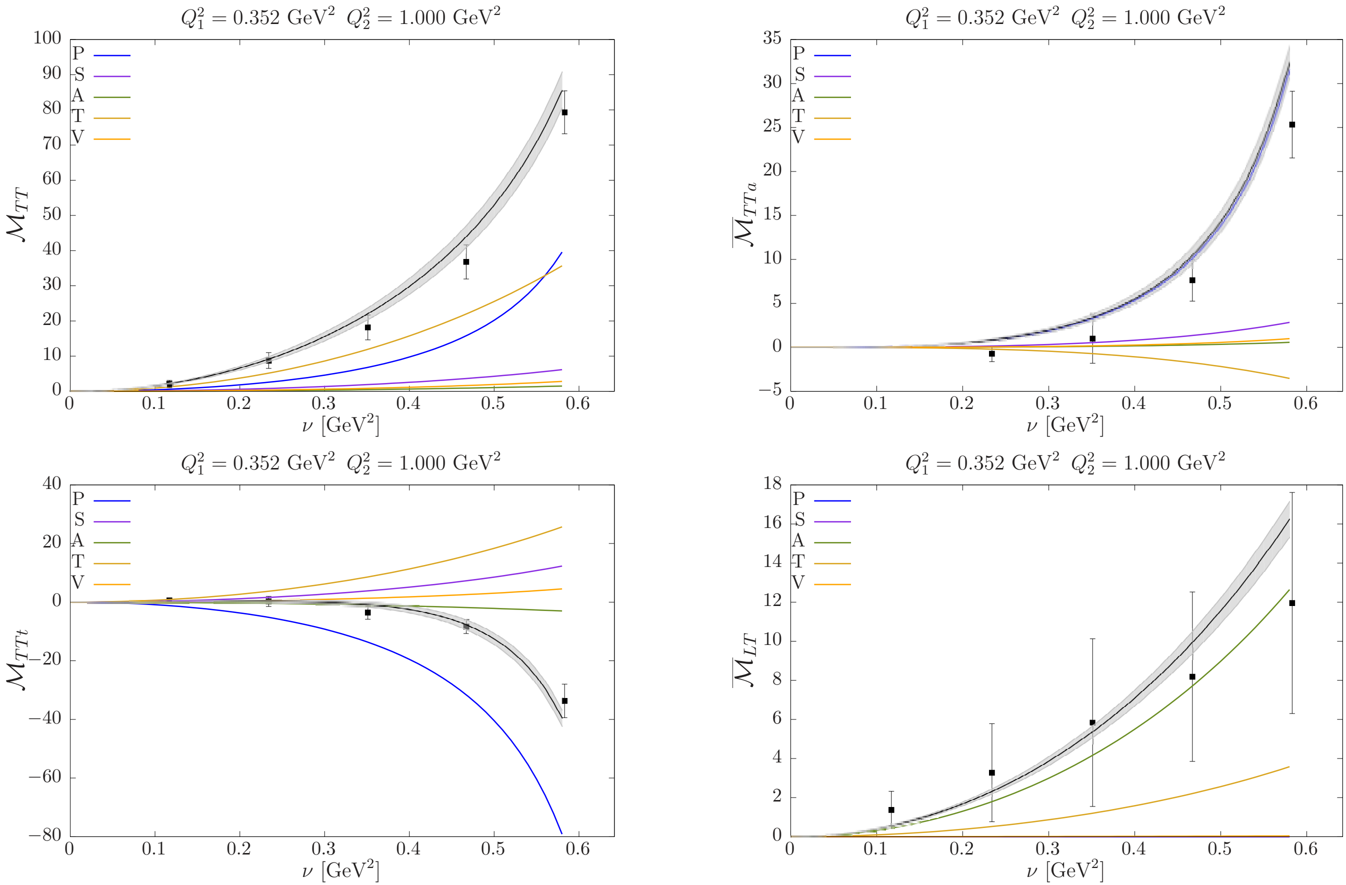
↪ Monopole mass set to the (lattice) rho mass

Preliminary results : F7 - dependance on ν and Q_2^2

- Each plot correspond to a fixed Q_1^2
- Different colours correspond to different values of $\nu = Q_1^2 \cdot Q_2^2$



Preliminary results : F7 - contributions from different channels



Preliminary results : monopole and dipole masses

Monopole FF

$$F_X(Q_1^2, Q_2^2) = \frac{F_X(0,0)}{\left(1 + Q_1^2/\Lambda_X^2\right)\left(1 + Q_2^2/\Lambda_X^2\right)}$$

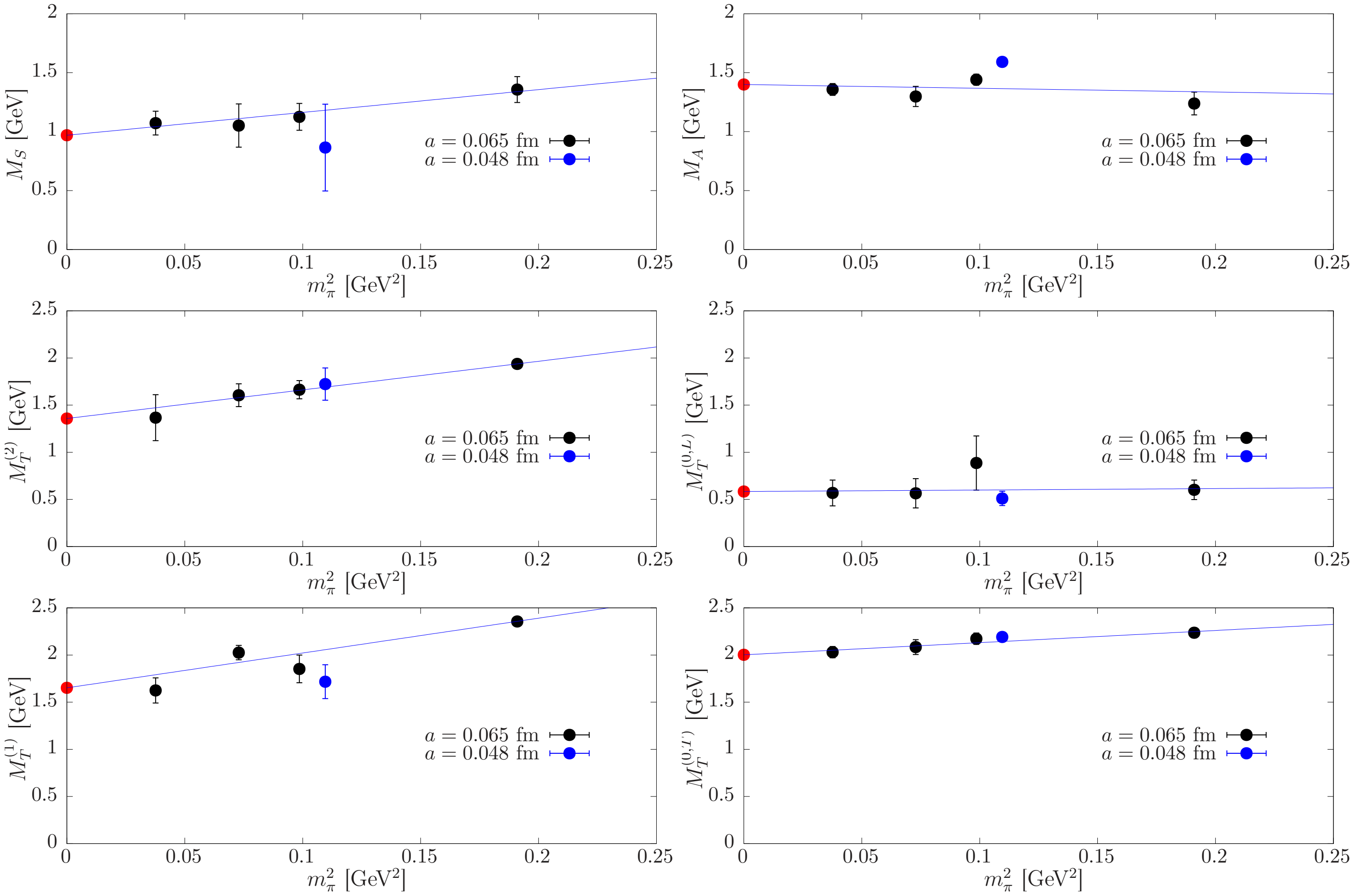
Dipole FF

$$F_X(Q_1^2, Q_2^2) = \frac{F_X(0,0)}{\left(1 + Q_1^2/\Lambda_X^2\right)^2\left(1 + Q_2^2/\Lambda_X^2\right)^2}$$

- Global fit of the eight amplitudes [Preliminary results]

	M_S [GeV]	M_A [GeV]	$M_T^{(2)}$ [GeV]	$M_T^{(0,T)}$ [GeV]	$M_T^{(1)}$ [GeV]	$M_T^{(0,L)}$ [GeV]	χ^2/d.o.f
E5	1.38(11)	1.26(10)	1.93(3)	2.24(5)	2.36(4)	0.60(10)	4.22(59)
F6	1.12(14)	1.44(5)	1.66(9)	2.17(5)	1.85(14)	0.89(28)	1.15(20)
F7	1.04(18)	1.29(8)	1.61(12)	2.08(7)	2.03(7)	0.57(16)	1.19(18)
G8	1.07(10)	1.36(5)	1.37(24)	2.03(6)	1.63(13)	0.73(14)	1.13(13)
N6	0.86(37)	1.59(3)	1.72(17)	2.19(4)	1.72(18)	0.51(8)	1.35(27)

Monopole and dipole masses : chiral extrapolations (preliminary, stat error only)



Preliminary results : monopole and dipole masses

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- $M_S \approx 1.0$ GeV : slightly above the experimental result from the Belle Collaboration ($M_S = 796(54)$ MeV for the isoscalar scalar meson [Masuda '15])
- $M_A \approx 1.4$ GeV to be compared with the experimental value by the L3 Collaboration $M_A = 1040(80)$ MeV for the isoscalar meson $f_1(1285)$ [Achard '01 '07].
- $M_T^{(2)} \approx 1.4$ GeV, $M_{0,T}^{(1)} \approx 1.6$ GeV and $M_{(1)}^{(0,T)} \approx 2.0$ GeV, above the experimental values for the $f_2(1270)$ mesons obtained by fitting the single-virtual form factor [Masuda '15, Danilkin '16].

Conclusion

- The eight forward light-by-light amplitudes have been computed on the lattice
- They are well described by the cross sections $\gamma^*\gamma^* \to$ a few resonances via dispersive sum rules.
- Allows us to put constraints on form factors used to estimate the HLbL contribution to the $(g-2)_\mu$
- Only the fully-connected contributions are included so far