

NONPERTURBATIVELY RENORMALIZED



BJORKEN-X HADRONIC DISTRIBUTION



HUEY-WEN LIN

# Lattice Parton Physics Project (LP3)

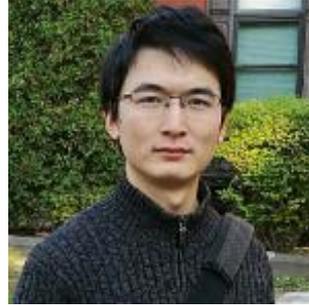
<https://www.pa.msu.edu/~hwlin/LP3/>



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## International collaborators



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(SJTU)



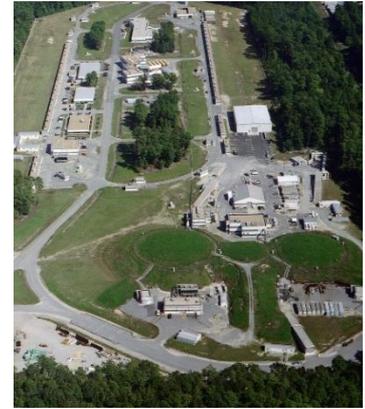
Jian-Hui Zhang  
(Regensburg)

Based on the work done in 1706.01295 (LP3)

# Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

∞ Many ongoing/planned experiments  
(BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

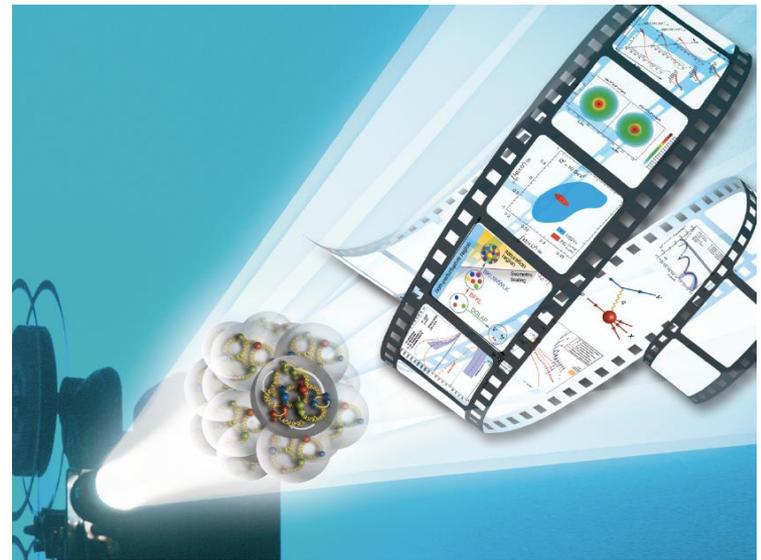


**Electron Ion Collider:  
The Next QCD Frontier**

**Imaging of the proton**

*How are the **sea** quarks and gluons,  
and their spins, distributed in space and  
momentum inside the nucleon?*

**EIC White Paper, 1212.1701**



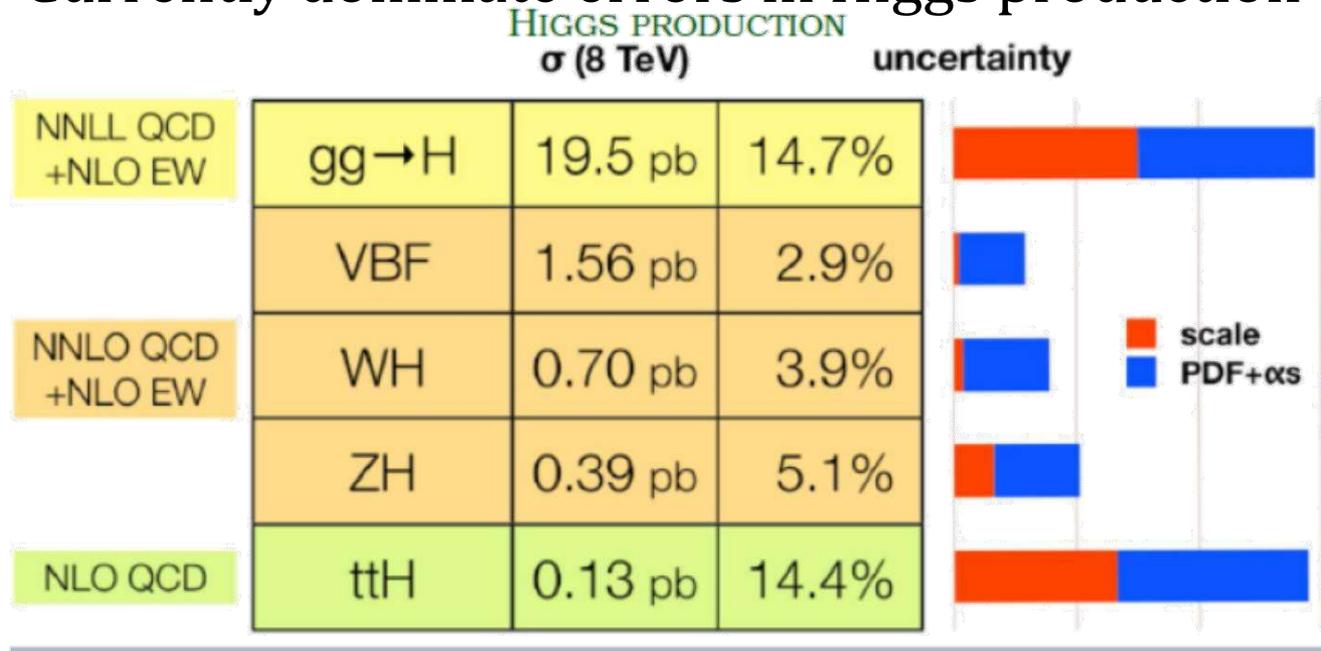
# Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

↪ Many ongoing/planned experiments  
(BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

§ Important inputs to discern new physics at LHC

↪ Currently dominate errors in Higgs production



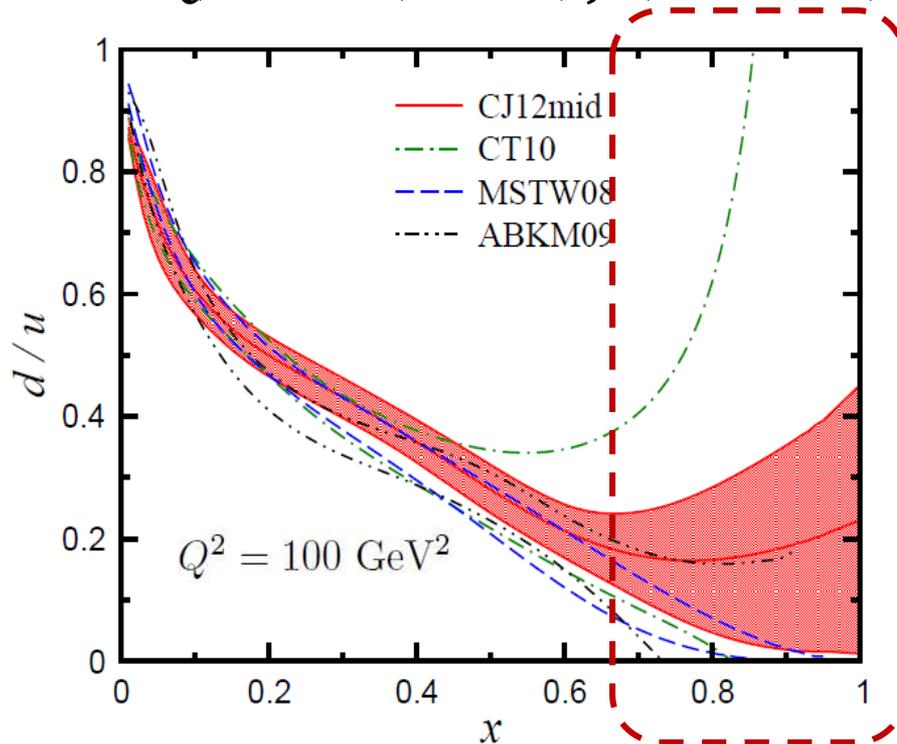
(J. Campbell, HCP2012)

# Global Analysis

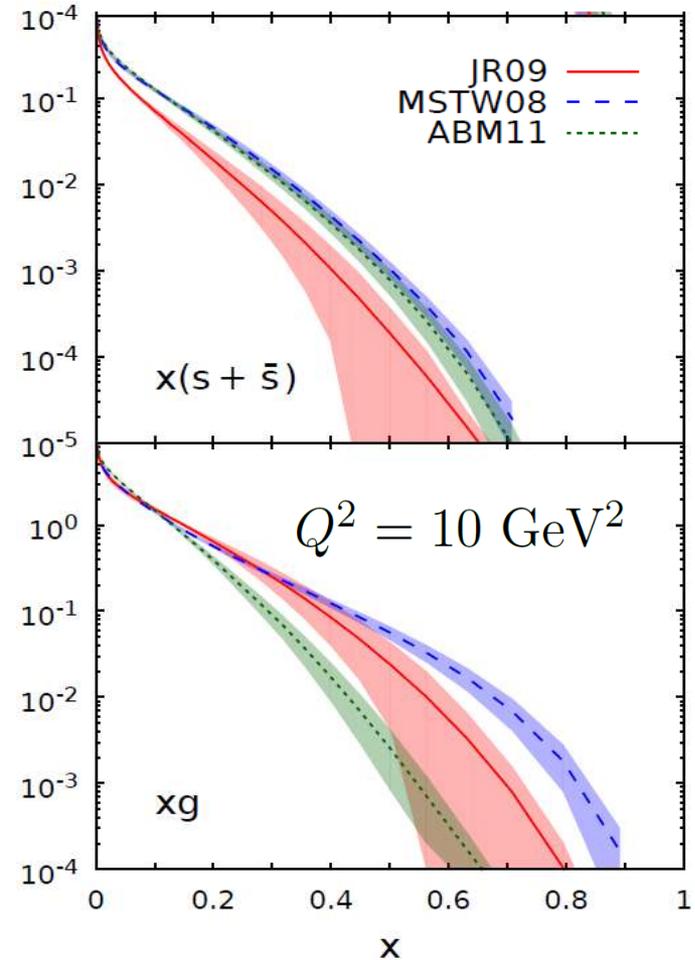
§ Discrepancies appear when data is scarce

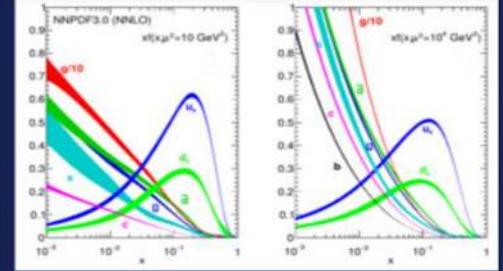
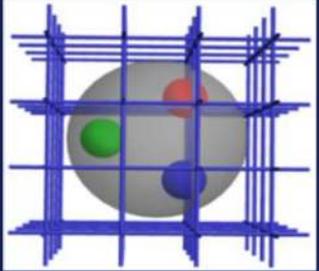
§ Many groups have tackled the analysis

∞ CTEQ, MSTW, ABM, JR, NNPDF, etc.



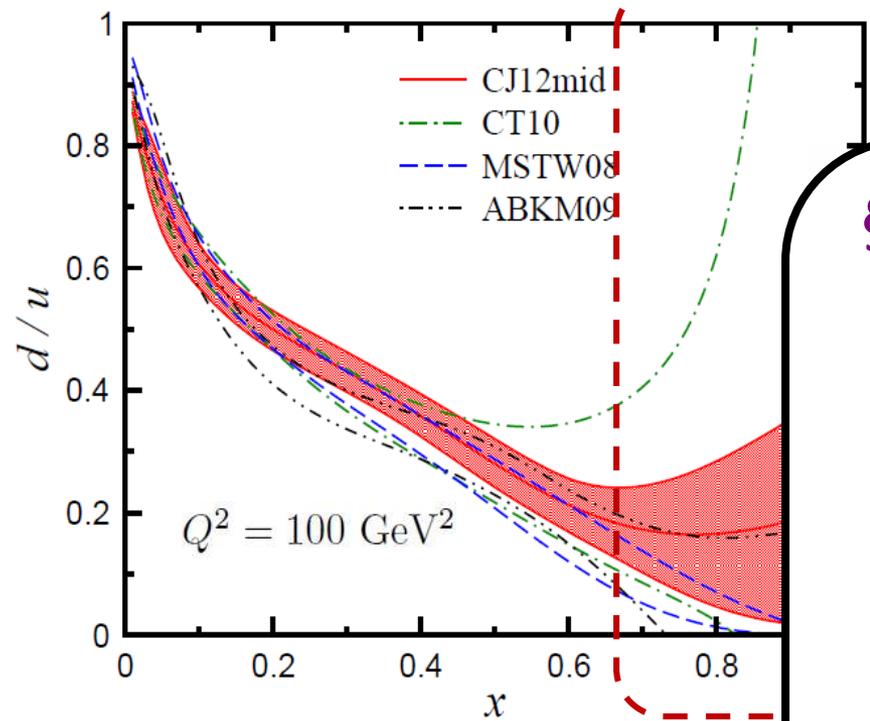
Jimenez-Delgado, Melnitchouk, Owens,  
J.Phys. G40 (2013) 09310





# Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017)

22-24 March 2017, Oxford, UK



§ A first joint workshop with global-fitting community to address key LQCD inputs

- ⌘ <http://www.physics.ox.ac.uk/confs/PDFlattice2017>
- ⌘ Whitepaper will study the needed precision of lattice PDFs in the large- $x$  region

Jimenez-Delgado, Melnitchouk, O...  
J.Phys. G40 (2013) 09310

# *A Promising New Direction*

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)  
+Many talks before mine today



# PDFs on the Lattice

Long existing obstacles!

§ Lattice calculations rely on operator product expansion,  
only provide moments  $\langle x^n \rangle$

$$\langle x^n \rangle_q = \int_{-1}^1 dx x^n q(x)$$

§ For higher moments, all ops mix with lower-dimension ops

∞ No practical proposal to overcome this

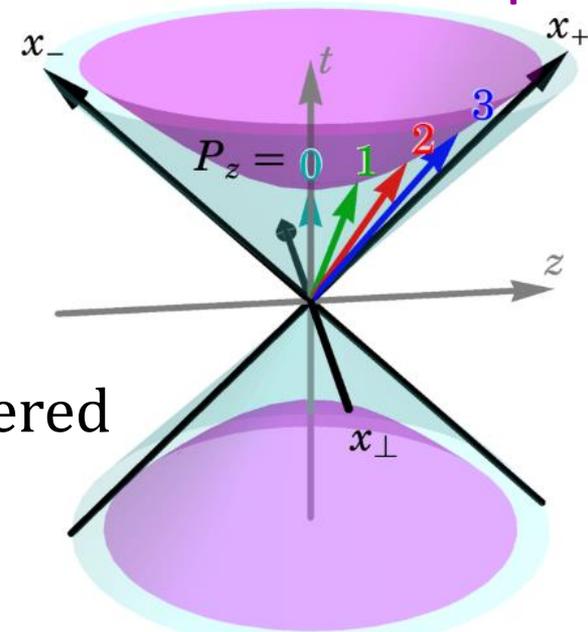
New Strategy (LaMET):

§ Calculate finite-momentum boosted  
quark distribution

∞ In  $P_z \rightarrow \infty$  limit, parton distribution is recovered

∞ For finite  $P_z$ , corrections are needed

§ Feasible with today's resources!

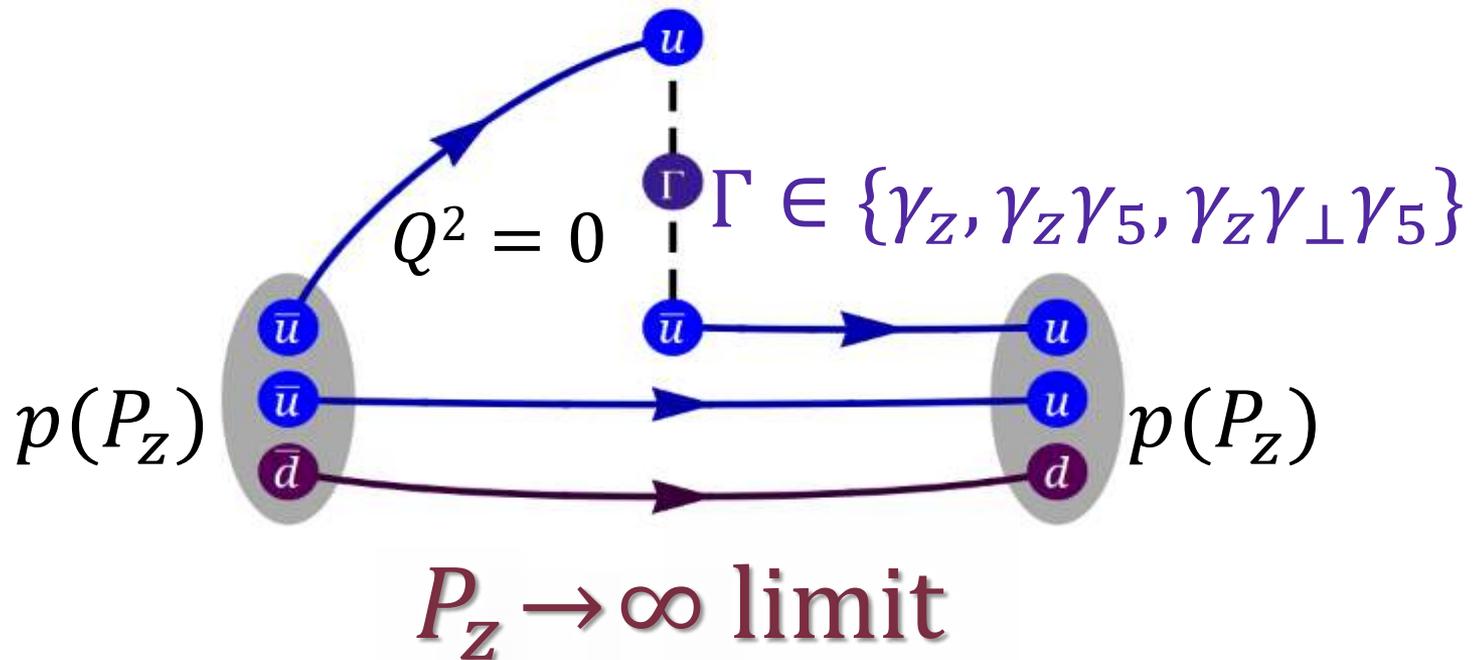


Xiangdong Ji, Phys. Rev. Lett. 111,  
039103 (2013)

# Parton Distribution Functions

## Large-Momentum Effective Theory for PDFs

$$\tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \langle P | \bar{\psi}(z) \Gamma \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) | P \rangle$$



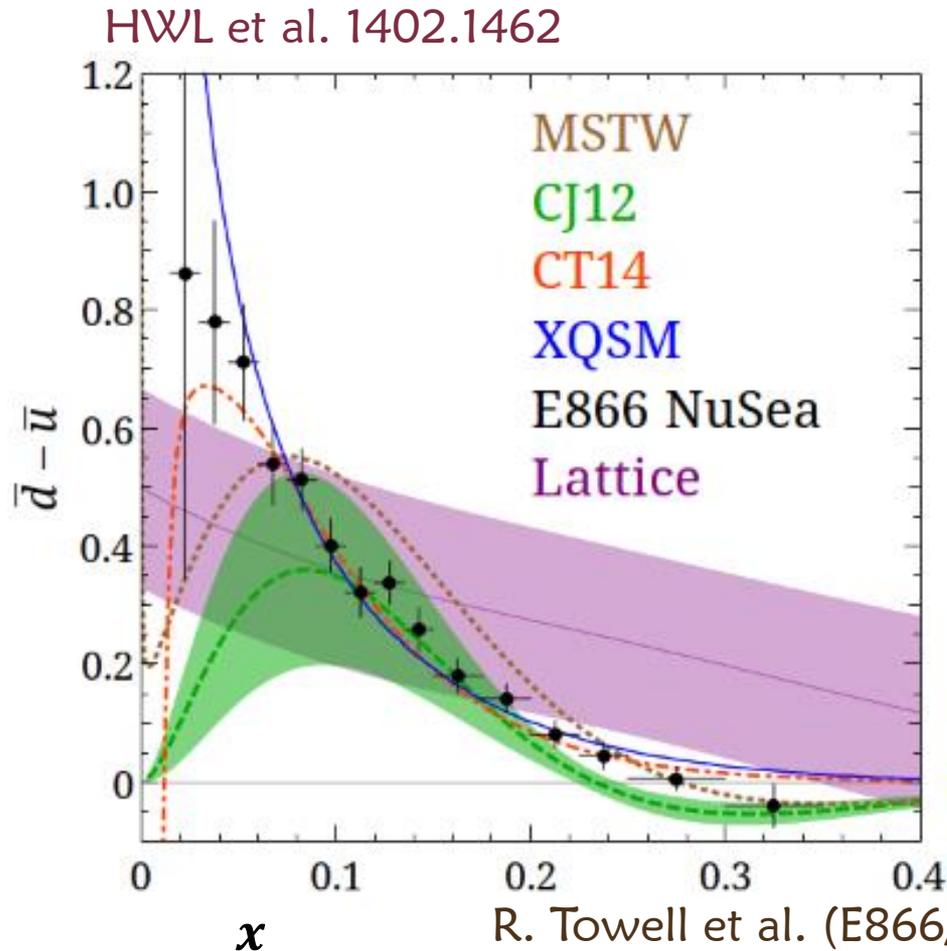
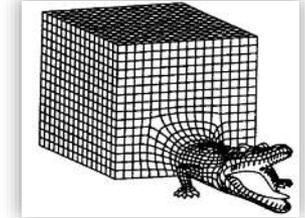
$$q(x, \mu) = \tilde{q}(x, \mu, P_z) + \mathcal{O}(\alpha_s) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\Lambda_{\text{QCD}}^2/P_z^2)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

# Sea Flavor Asymmetry

§ First time in LQCD history to study antiquark distribution!

$$\approx M_\pi \approx 310 \text{ MeV}$$



$$\bar{q}(x) = -q(-x)$$

Lost resolution in  
small- $x$  region

Future improvement:  
larger lattice volume

$$\int dx (\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

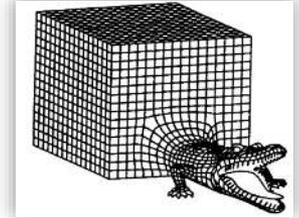
Experiment	$x$ range	$\int_0^1 [\bar{d}(x) - \bar{u}(x)] dx$
E866	$0.015 < x < 0.35$	$0.118 \pm 0.012$
NMC	$0.004 < x < 0.80$	$0.148 \pm 0.039$
HERMES	$0.020 < x < 0.30$	$0.16 \pm 0.03$

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

# Sea Flavor Asymmetry

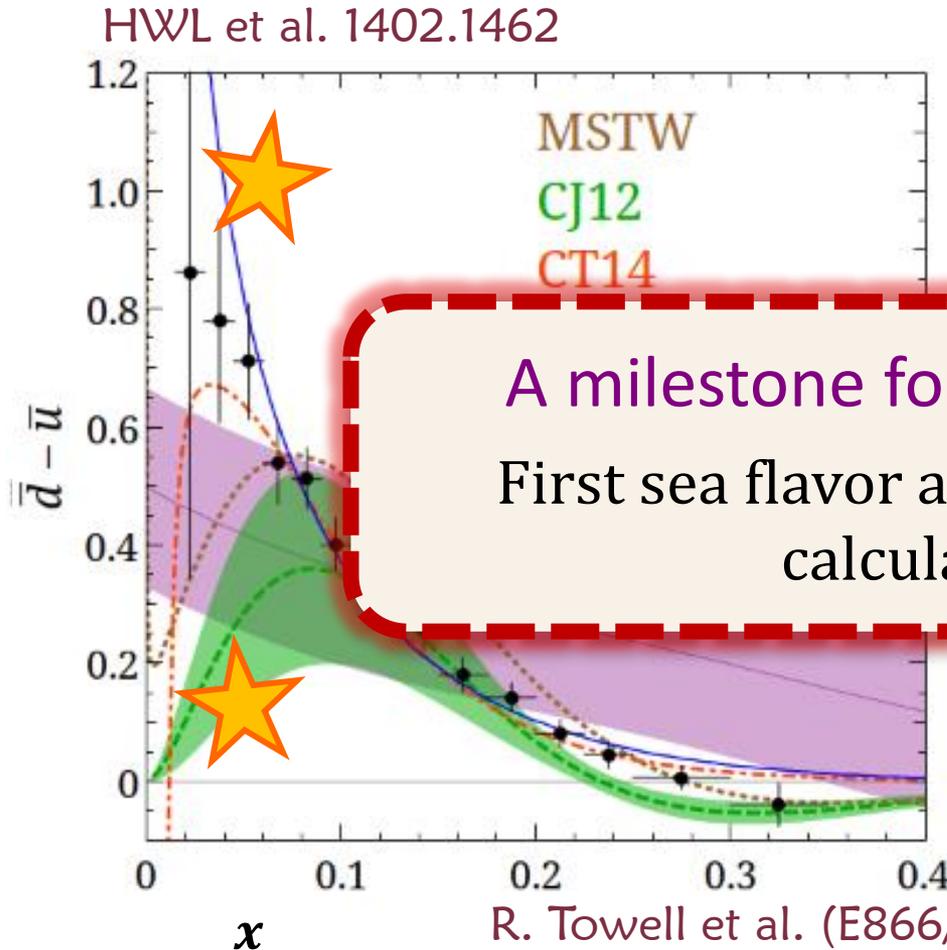
§ First time in LQCD history to study antiquark distribution!

$$\approx M_\pi \approx 310 \text{ MeV}$$



$$\bar{q}(x) = -q(-x)$$

Lost resolution in small-x region



A milestone for lattice QCD!  
First sea flavor asymmetry ever calculated!

moment:  
volume

$$\approx -0.16(7)$$

Experiment	$x$ range	$\int_0^{1/2} [\bar{d} - \bar{u}(x)] dx$
E866	$0.015 < x < 0.35$	$0.118 \pm 0.012$
NMC	$0.004 < x < 0.80$	$0.148 \pm 0.039$
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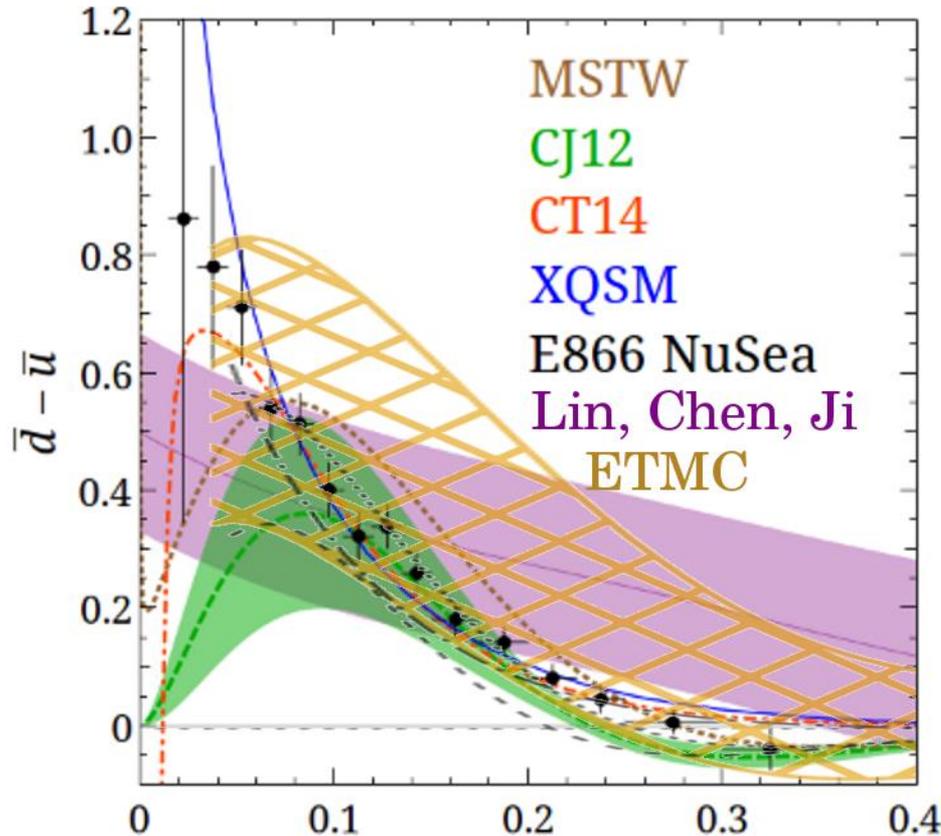
R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

# Sea Flavor Asymmetry

## § Lattice exploratory study

$$\approx M_\pi \approx 310 \text{ MeV}$$

HWL et al 1402.1462



Compared with E866

Too good to be true?

Lost resolution in  
small-x region

Similar results repeated  
by ETMC,  
at  $M_\pi \approx 373 \text{ MeV}$

ETMC, 1504.07455

(7)

Experiment	$x$ range	$\int_0^1 [\bar{d}(x) - \bar{u}(x)] dx$
E866	$0.015 < x < 0.35$	$0.118 \pm 0.012$
NMC	$0.004 < x < 0.80$	$0.148 \pm 0.039$
HERMES	$0.020 < x < 0.30$	$0.16 \pm 0.03$

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

# *Missing Ingredient: Renormalization*

Recent progress:

1705.00246, 1705.11193, 1706.00265, 1706.01295, ...



# Renormalization

## § Long-link operator

$$O_{\Gamma}(z) = \bar{\psi}(z)\Gamma W_z(z, 0)\psi(0)$$

## § Vector operator mixing with scalar ones T. Ishikawa, this conference

$$\begin{pmatrix} O_{\gamma_z}^R(z) \\ O_{\mathbb{I}}^R(z) \end{pmatrix} = \begin{pmatrix} Z_{VV}(z) & Z_{VS}(z) \\ Z_{SV}(z) & Z_{SS}(z) \end{pmatrix} \begin{pmatrix} O_{\gamma_z}(z) \\ O_{\mathbb{I}}(z) \end{pmatrix}$$

## § RI/MOM renormalization scheme 1706.01295 (LP3)

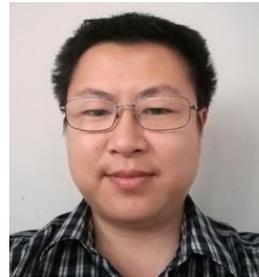
$$\mathfrak{Z} Z^{-1} =$$

$$\frac{1}{12e^{-ip_z z}} \begin{pmatrix} \text{Tr}[\tilde{\Gamma}\Lambda(p, z, \gamma_z)] & \text{Tr}[\tilde{\Gamma}\Lambda(p, z, \mathbb{I})] \\ \text{Tr}[\Lambda(p, z, \gamma_z)] & \text{Tr}[\Lambda(p, z, \mathbb{I})] \end{pmatrix}_{p^2=\mu_R^2, p_z=P_z}$$

$$\Lambda(p, z, \Gamma) = S(p)^{-1} \left\langle \sum_w S^\dagger(p, w + zn)\Gamma W_z(w + zn)S(p, w) \right\rangle S(p)^{-1}$$

projected with  $\tilde{\Gamma} = \not{p}/p_z$

$\mathfrak{Z}$  Test case:  $a \approx 0.12$  fm,  $M_\pi \approx 310$  MeV, clover/HISQ



Yi-Bo Yang  
(MSU)

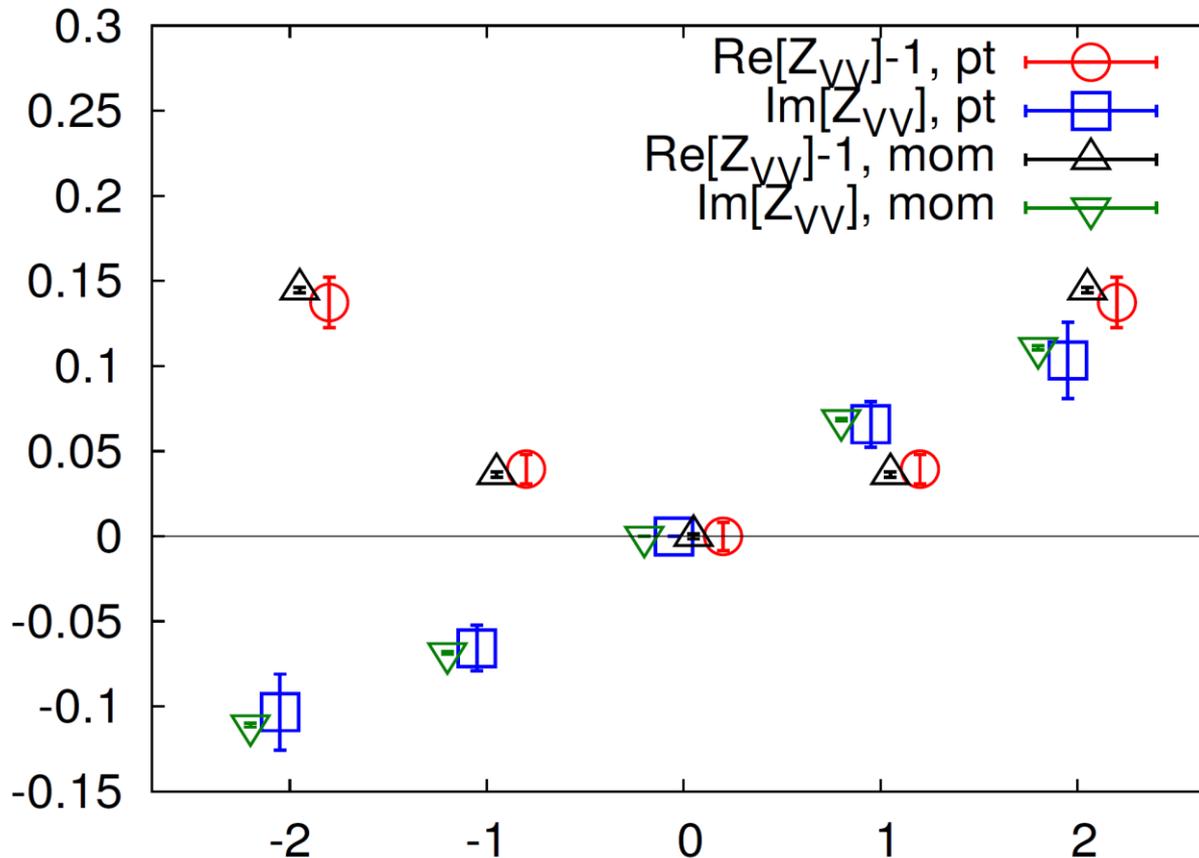


Yong Zhao  
(MIT)

# Renormalization

## § RI/MOM renormalization scheme

Momentum source vs point source for  $|z| \leq 2$

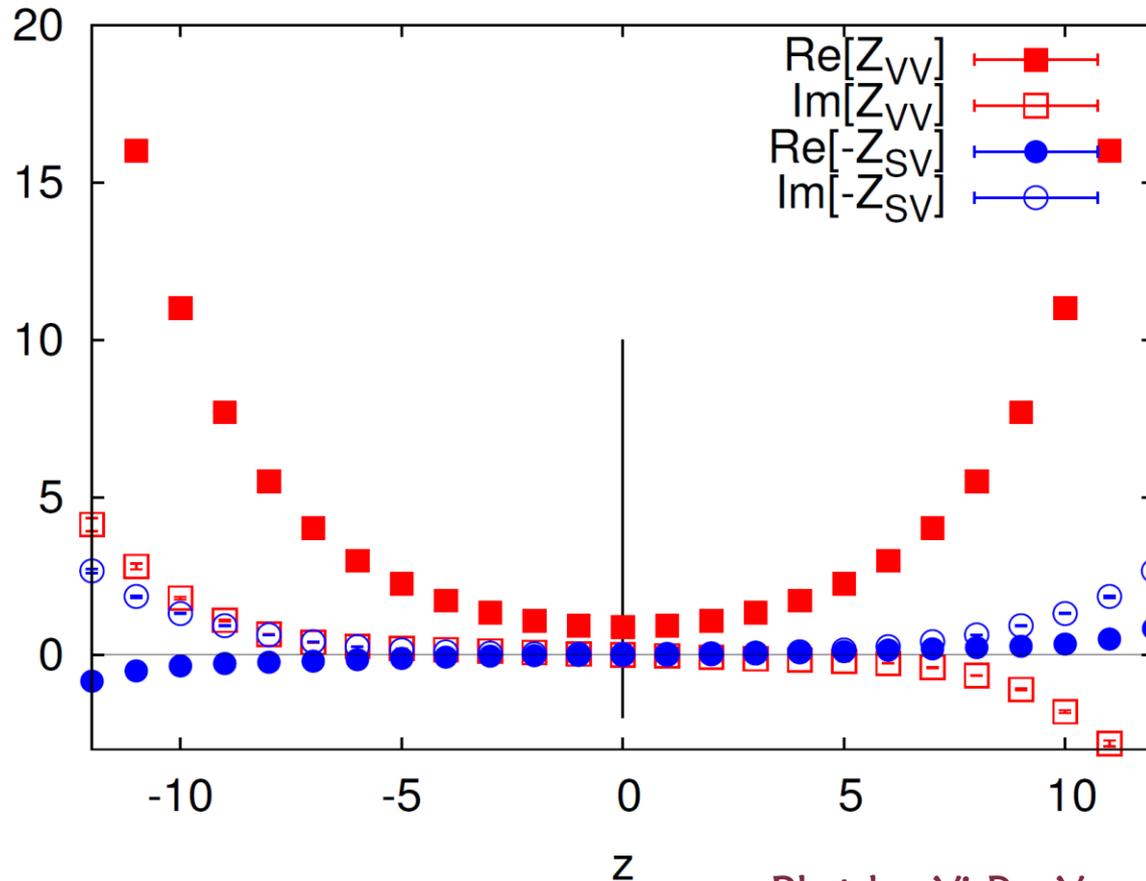


$$p_z = 6\pi/L_s$$
$$\mu_R^2 = p^2$$
$$= 5.74 \text{ GeV}^2$$

Plot by Yi-Bo Yang; 1706.01295 (LP3)

# Renormalization

## § RI/MOM renormalization scheme



$$p_z = 6\pi/L_s$$
$$\mu_R^2 = p^2$$
$$= 5.74 \text{ GeV}^2$$

$$h_R = Z_{VV}h_{\gamma_Z} + Z_{SV}h_{\text{II}}$$

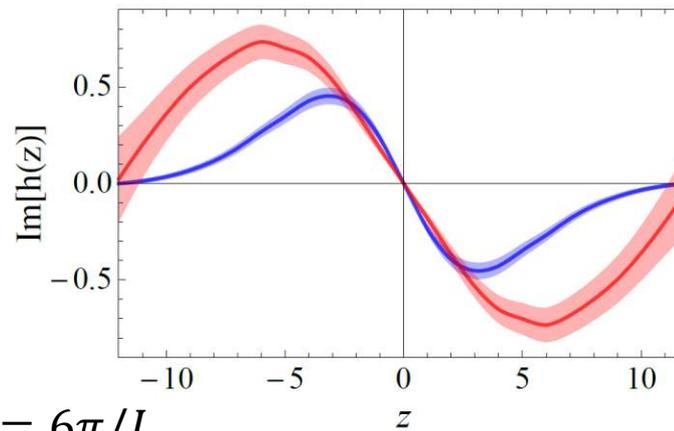
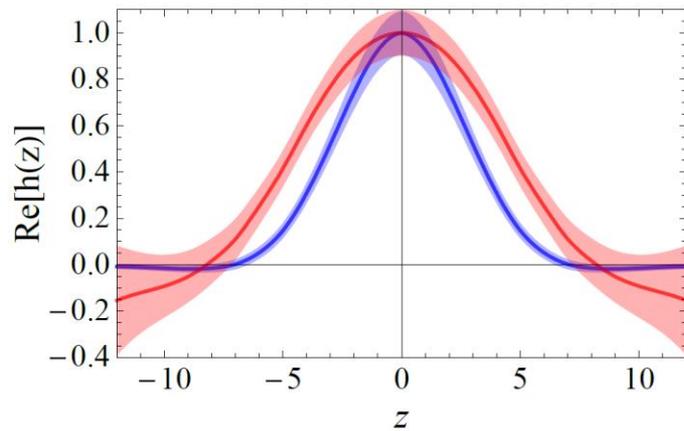
Plot by Yi-Bo Yang; 1706.01295 (LP3)

# Renormalization

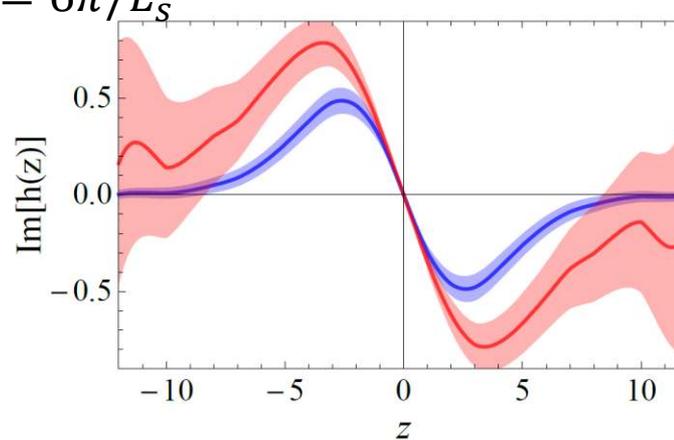
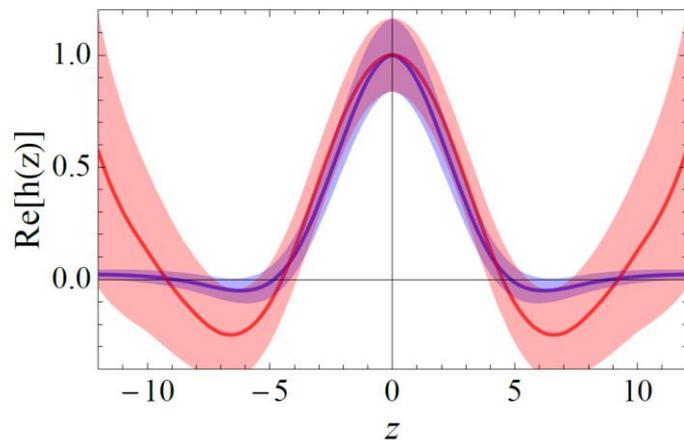
§ Effect on nucleon matrix elements as function of  $z$

$$\approx h_R \approx Z_{VV} h_{\gamma Z}$$

$$p_z = 4\pi/L_s$$



$$p_z = 6\pi/L_s$$



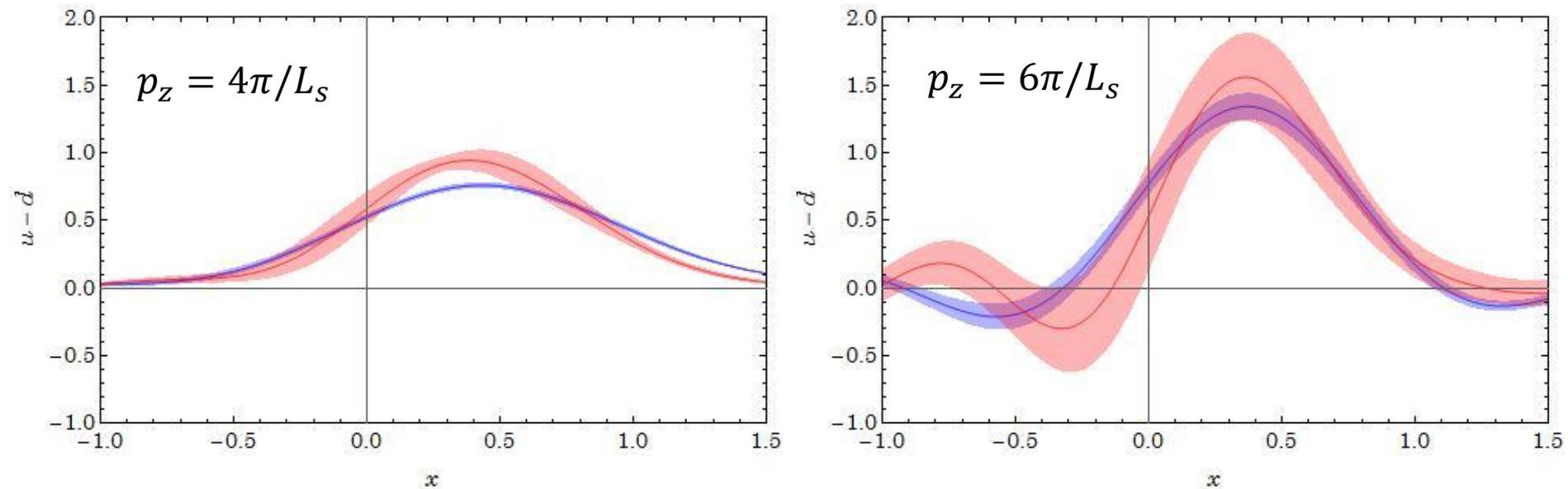
Jian-Hui Zhang  
(Regensburg)

Plot by Jianhui Zhang; 1706.01295 (LP3)

# Renormalization

## § Effect on quasi-PDFs

$$\tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP_z z} \tilde{h}_R(z, P_z, \mu_R)$$



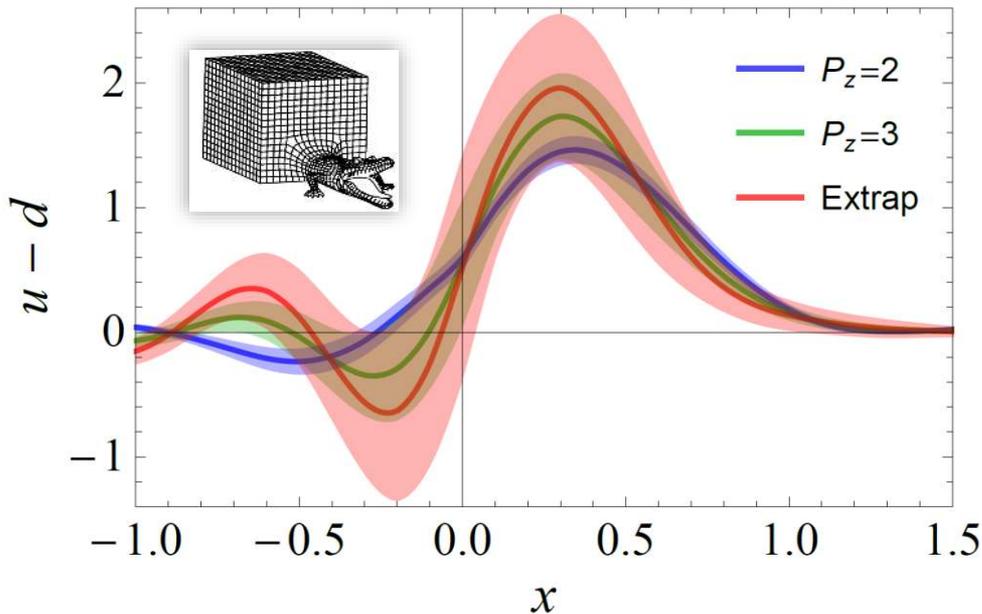
Plot by Jianhui Zhang

# Renormalization

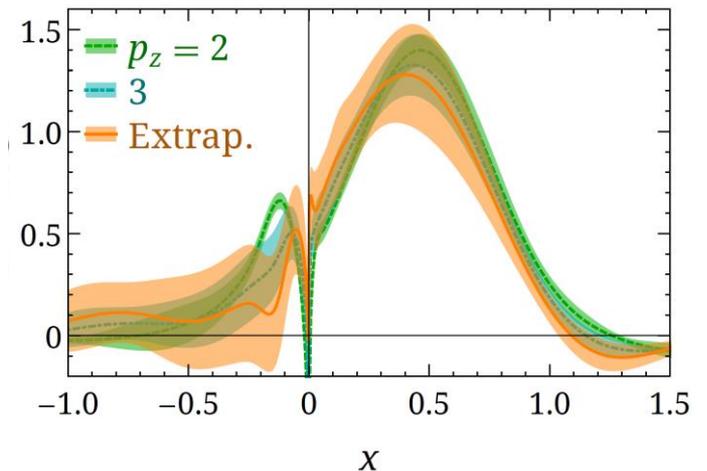
## § Effect on quasi-PDFs

$$\tilde{q}_R(x, P_z, \mu_R) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{ixP_z z} \tilde{h}_R(z, P_z, \mu_R) + O(M_N^n/P_z^n) + O(\alpha_s) \text{ error} + \text{RI/MOM to } \overline{\text{MS}} \text{ matching (Zhao)}$$

$$a \approx 0.12 \text{ fm}, M_\pi \approx 310 \text{ MeV}$$



Earlier work (ignore renormalization)



1603.06664 (LP3)

Plot by Jianhui Zhang; 1706.01295 (LP3)

# A NEW HOPE

*It is a period of war and economic uncertainty.*

*Turmoil has engulfed the galactic republics.*

*Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.*

*A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion-Cygnus arm of the galaxy.*

*The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.*

*They carry secret plans to build the most powerful*

# Summary & Outlook

Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full  $x$ -distribution

- ↪ Most importantly, this can be done with today's computer
- ↪ First lattice approach to study sea asymmetry
- ↪ First look into PDA 1702.00008

§ Moving on to remove the systematic in earlier study

- ↪ Working on **renormalization**, statistics (all-mode averaging?), larger momentum boost, finer lattice-spacing ensembles, ...
- ↪ Larger  $P_z$  with smaller  $a$  may reduce issues associated w/ larger  $z$

§ Explore new operators (no mixing) for the future

