Inclusive B decay calculations with analytic continuation

based on arXiv:1703.01881 [hep-lat];
PTEP 2017 (5): 053B03
plus developments since then

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@ Lattice 2017, Granada,

June 21, 2017

計算基礎科学連携拠点
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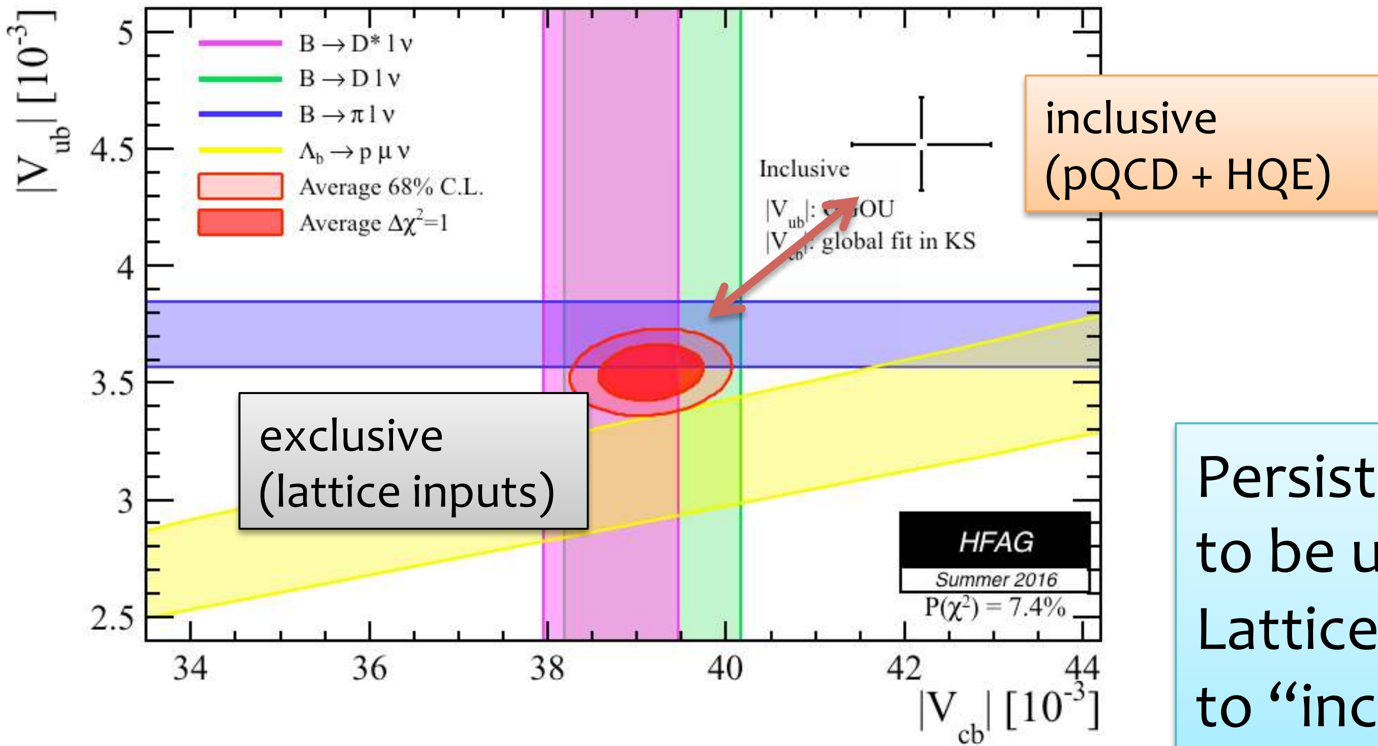
Inclusive semileptonic B decays

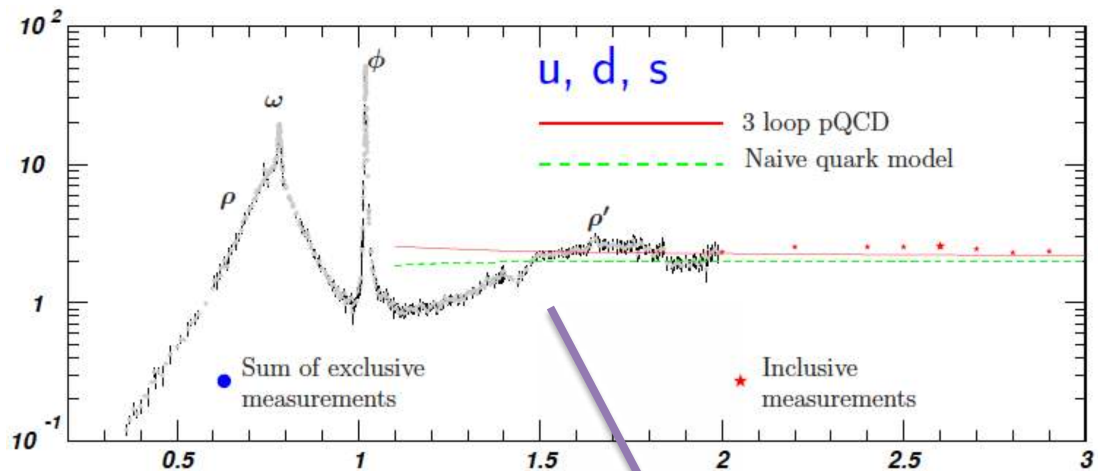
- Determination of $|V_{cb}|$, $|V_{ub}|$



Persistent tension to be understood. Lattice contribution to "inclusive"?

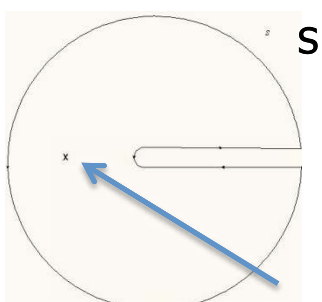
"inclusive" measurements

measure all possible final states.



e.g.) The R-ratio

may be related to VPF by analyticity



$$\Pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{(s-q^2)}$$

Euclidean
(lattice calculation is possible)

Not limited to Euclidean

- Analytic continuation

  Starting from the lattice data in the Euclidean space, do the "Fourier transform" to reach

  - space-like: q^2 (= $-Q^2$) = $-q_0^2 - \mathbf{q}^2$

$$\Pi(q^2) = \int dt\, e^{iq_0 t} \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \langle J^E(\mathbf{x},t) J^E(\mathbf{0},0)\rangle$$

  - time-like: $q^2 = \omega^2 - \mathbf{q}^2$

$$\Pi(q^2) = \int dt\, e^{-\omega t} \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \langle J^E(\mathbf{x},t) J^E(\mathbf{0},0)\rangle$$

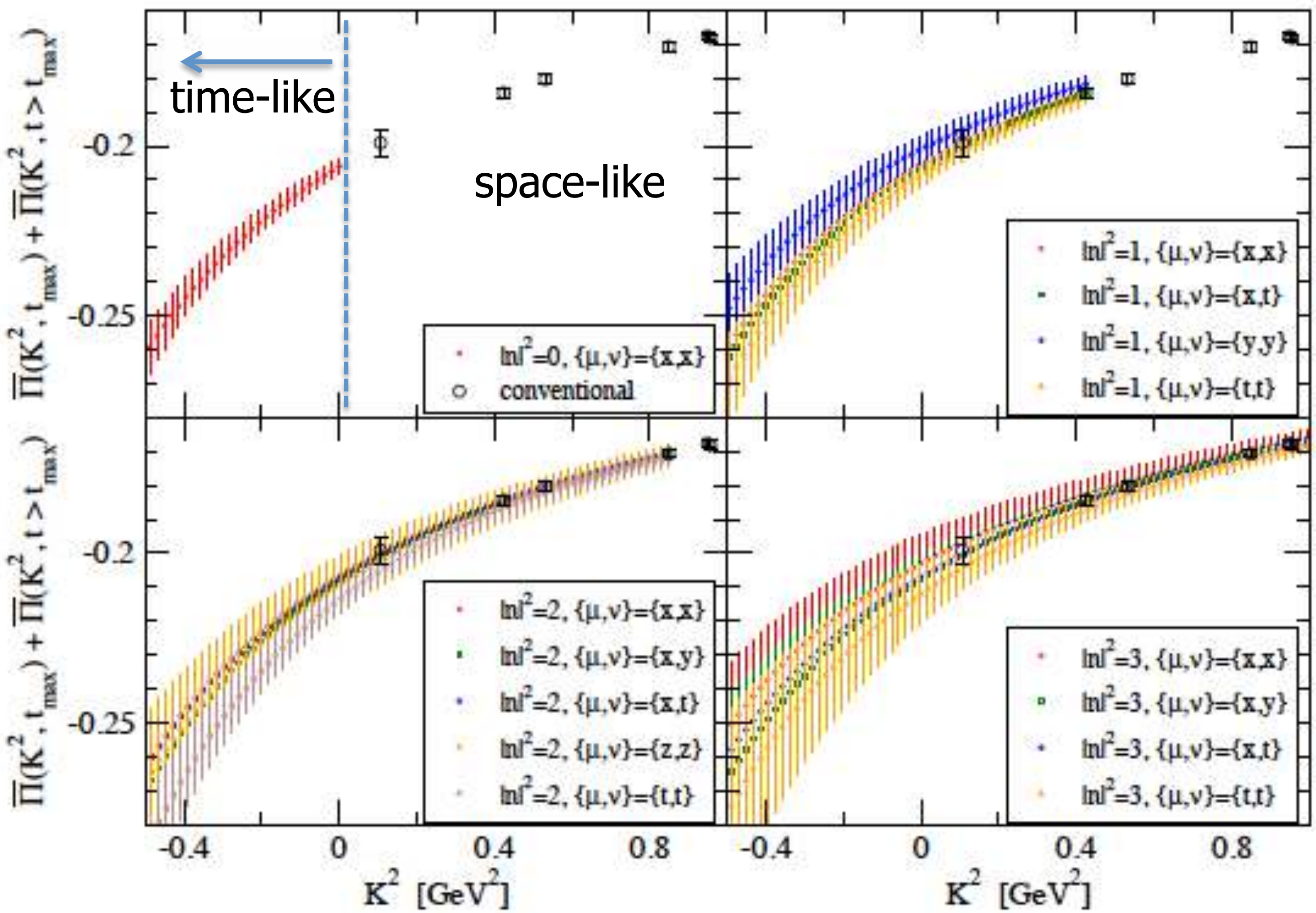
    with an arbitrary (real) ω

  - Justified, until one hits any singularity, i.e. the threshold to produce real particles, i.e. $\omega < m_V$.

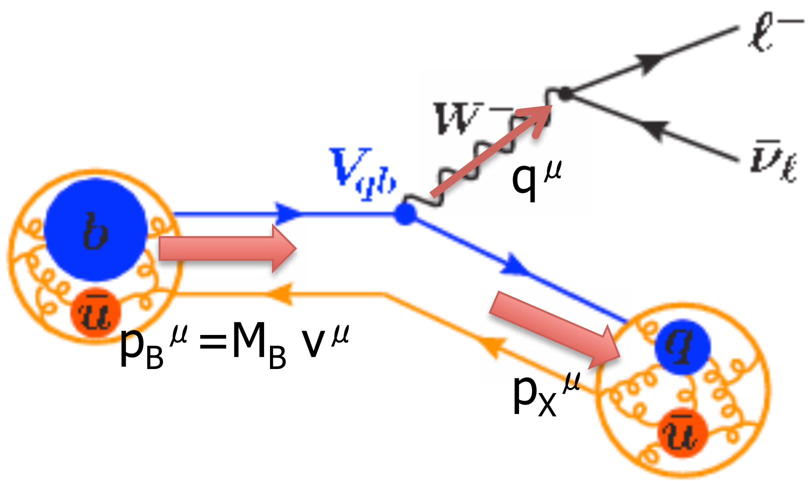
See, e.g., Feng et al. (2013)

Vacuum polarization function

Feng et al, PRD88, 034505 (2013)



Semi-leptonic B decays



Inclusive decays:

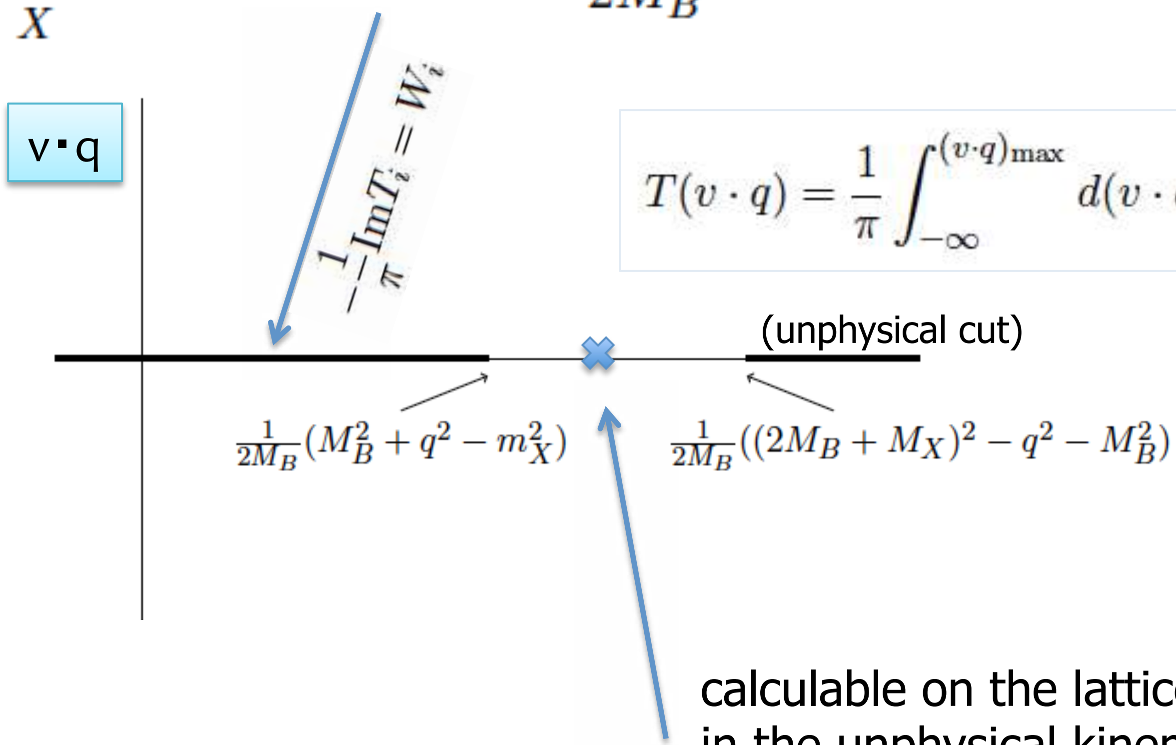
$p_X{}^2 = m_X{}^2$ arbitrary

One may specify the four-momentum:
$p_X{}^\mu = (\omega, \mathbf{p}_X)$.
Analytic continuation is possible as far as $\omega < m_D$

Decay amplitude: $|\mathcal{M}|^2 = |V_{qQ}|^2 G_F^2 M_B l^{\mu\nu} W_{\mu\nu}$ (function of v·q and q^2)

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$



$$T(v\cdot q) = \frac{1}{\pi}\int_{-\infty}^{(v\cdot q)_{\max}} d(v\cdot q') \frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q}$$

calculable on the lattice
in the unphysical kinematical regime

Matrix element:

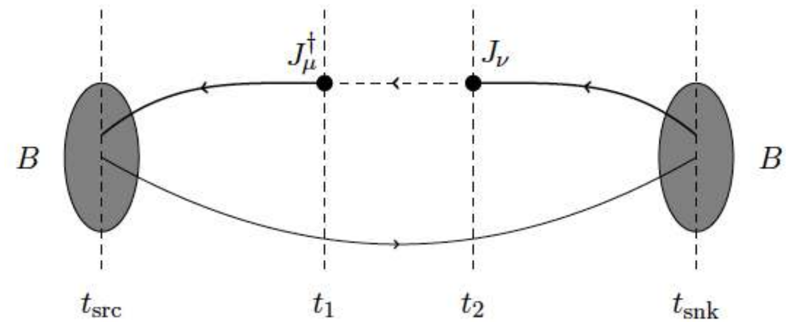
$$T_{\mu\nu} = i\int d^4x\, e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\}B\rangle$$

Strategy

- Matrix element: $$T_{\mu\nu} = i\int d^4x\, e^{-iqx}\frac{1}{2M_B}\langle B|T\{J_\mu^\dagger(x)J_\nu(0)\}B\rangle$$
  - Lattice:
    - Calculate at unphysical kinematics.
  - pQCD + HQE:
    - Perturbation theory should work away from the cut → Direct comparison is possible
  - Experiment:
    - Need a reanalysis to perform the Cauchy integral.

Lattice calculation: recipe

1. Four-point function:



2. after taking appropriate ratios to cancel the external B meson source, we construct

$$C_{\mu\nu}^{JJ}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J_\mu^\dagger(\mathbf{x},t)J_\nu(0)|B(\mathbf{0})\rangle$$

3. do the "Fourier transform" in the time direction

$$T_{\mu\nu}^{JJ}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t;\mathbf{q})$$

– Corresponds to $T_{\mu\nu}(v\cdot q, q^2)$ at $p_X=(\omega,-\boldsymbol{q})$, $q=(m_B-\omega,\boldsymbol{q})$

Ensembles from JLQCD

- With Mobius domain-wall fermion (2012~)
  - 2+1 flavor (uds)
  - Mobius domain-wall fermion [with stout link]
    - residual mass < O(1 MeV)
  - lattice spacing : 1/a = 2.4, 3.6, 4.5 GeV
  - volume : L = 2.7 fm（32^3, 48^3, 64^3 lattices）
  - light quark mass : m_π = 230, 300, 400, 500 MeV
  - statistics : 50-200 measurements
- Valence sector
  - heavy (MDW) + strange (MDW)
  - tuned charm + (unphysical) bottom $m_b = (1.25)^2\, m_c, (1.25)^4\, m_c$
  - on Oakforest-PACS with IROIRO++

(up to 3.4 GeV B_s meson)

Related talks: T. Kaneko (D→π), B. Colquhoun (B→π)

β = 4.17, 1/a ~ 2.4 GeV, 32^3x64 (x12)

m_{ud}	m_π [MeV]	MD time
m_s = 0.030		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
m_s = 0.040		
0.0035	230	10,000
0.0035 (48^3x96)	230	10,000
0.007	320	10,000 ←
0.012	410	10,000
0.019	510	10,000

β = 4.35, 1/a ~ 3.6 GeV, 48^3x96 (x8)

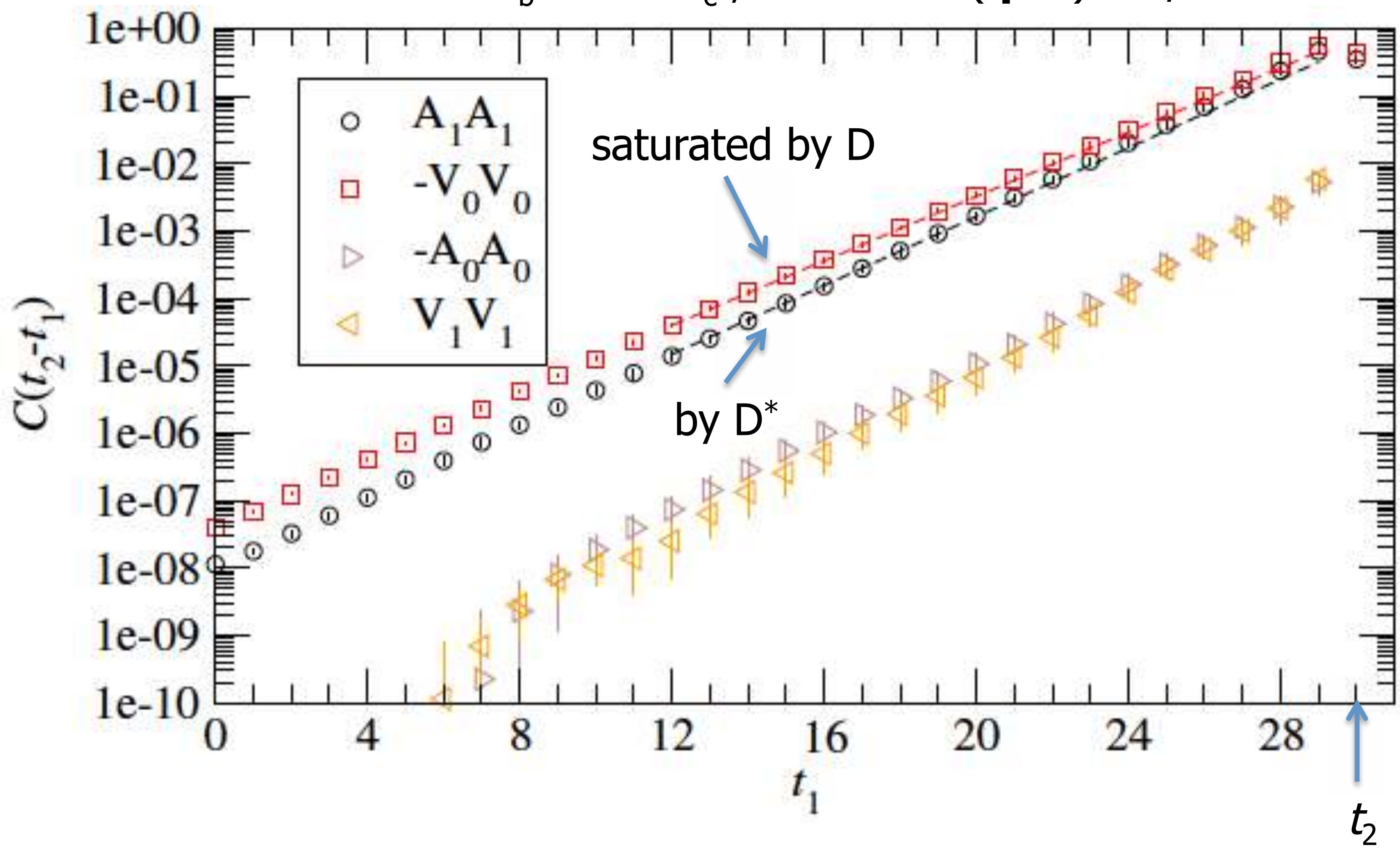
m_{ud}	m_π [MeV]	MD time
m_s = 0.018		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
m_s = 0.025		
0.0042	300	10,000 ←
0.080	410	10,000
0.0120	510	10,000

β = 4.47, 1/a ~ 4.6 GeV, 64^3x128 (x8)

0.0030	~ 300	10,000 ←

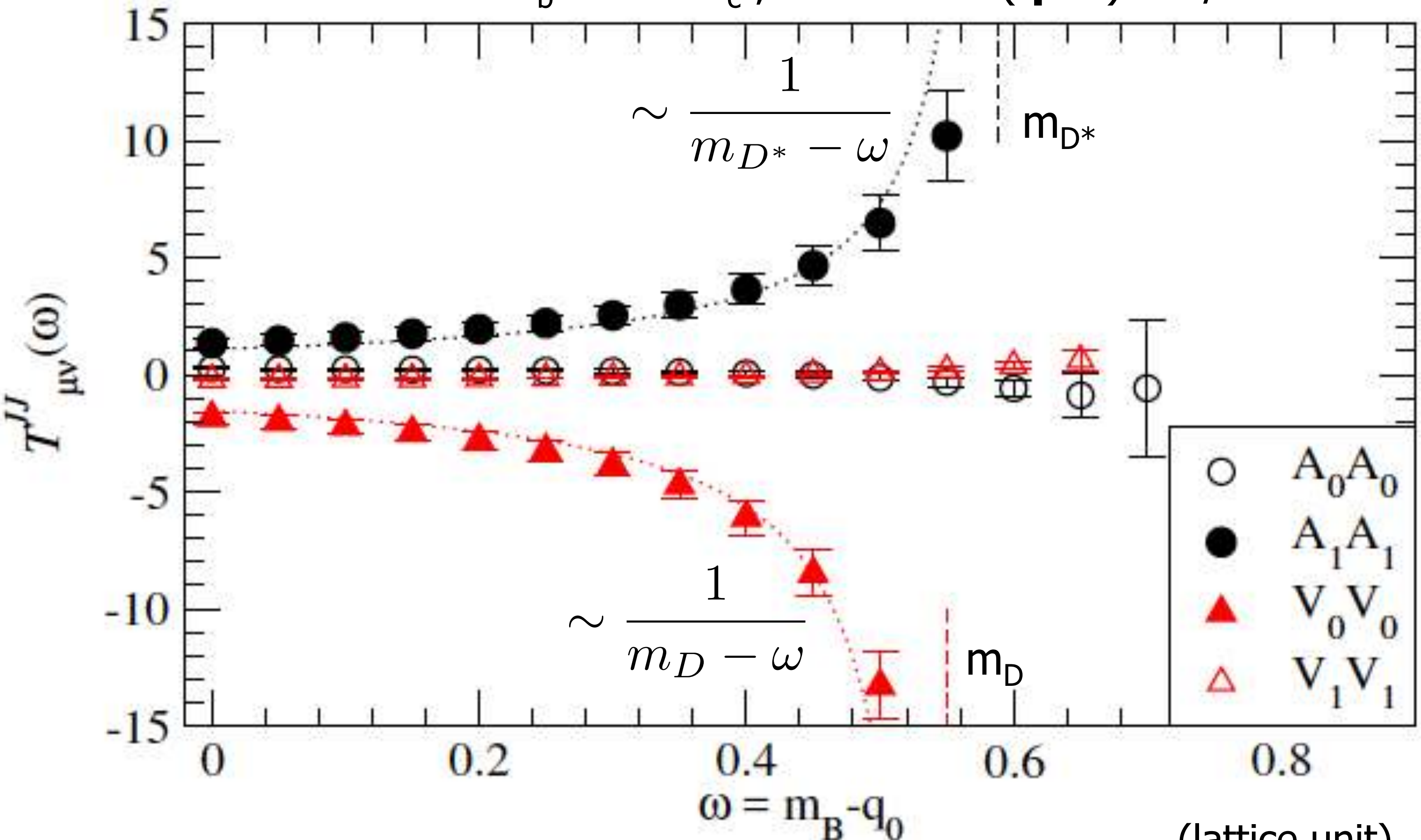
$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t)J_\nu(0)|B(\mathbf{0})\rangle$$

$m_b=1.25^4 m_c$, zero recoil (**q**=**0**)   1/a = 3.6 GeV



$$T^{JJ}_{\mu\nu}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathbf{q})$$

$m_b = 1.25^4 m_c$, zero recoil ($\mathbf{q}=\mathbf{0}$) $1/a$ = 3.6 GeV



(lattice unit)

Saturation by D$^{(*)}$?

Four-point function:

$$C_{\mu\nu}^{JJ}(t;0) = \frac{1}{2M_B}\sum_X \langle B(0)|J_\mu^\dagger|X(0)\rangle \frac{e^{-E_X t}}{2E_X}\langle X(0)|J_\nu|B(0)\rangle$$

Ground-state contribution:

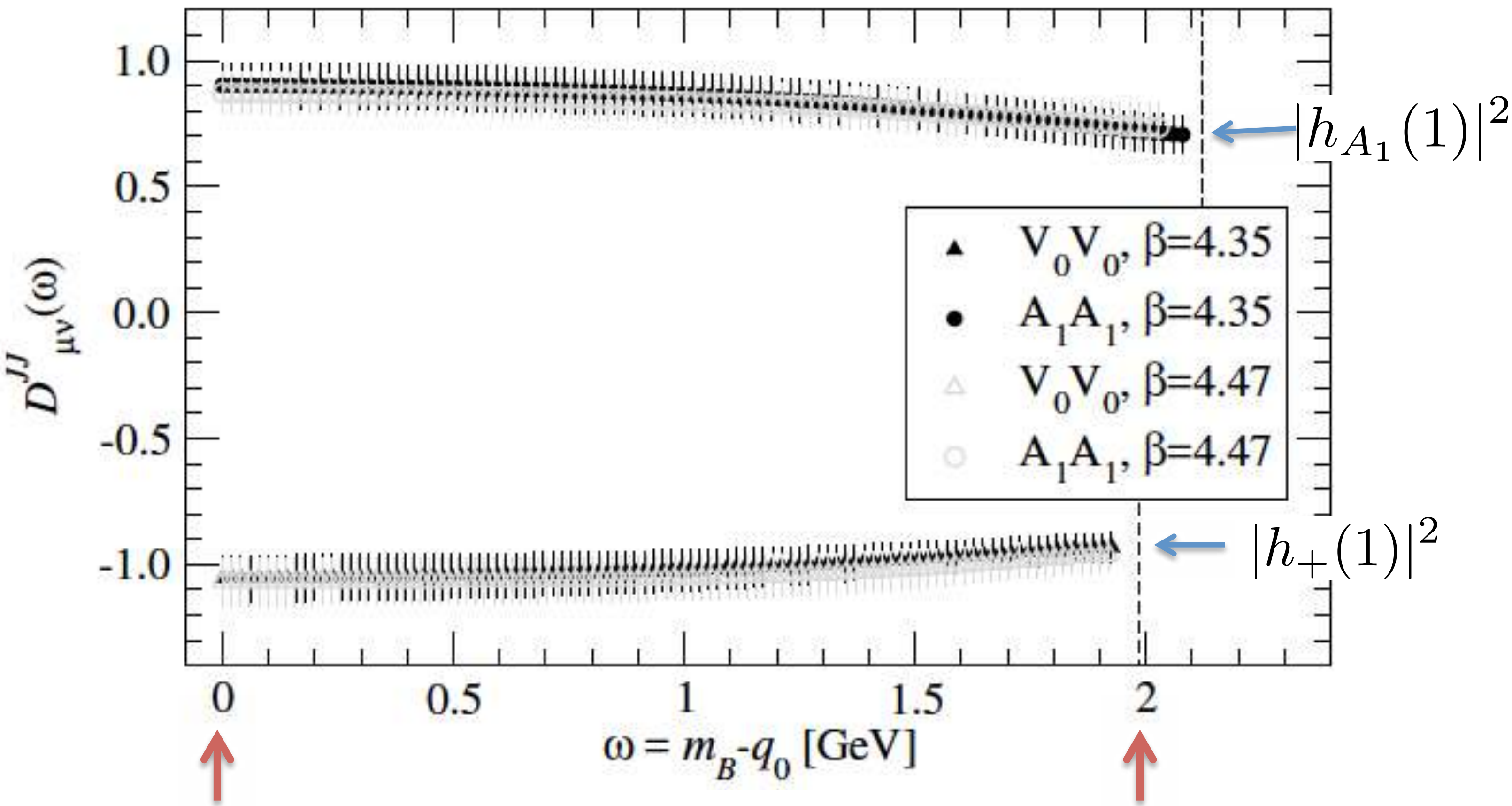
$$\begin{aligned}\langle D(0)|V^0|B(0)\rangle &= 2\sqrt{M_B M_D}\, h_+(1),\\ \langle D^*(0)|A^k|B(0)\rangle &= 2\sqrt{M_B M_{D^*}}\, h_{A_1}(1)\,\varepsilon^{*k}.\end{aligned}$$

$$\begin{aligned}T_{00}^{VV}(\omega,0) &= \frac{|h_+(1)|^2}{M_D-\omega},\\ T_{kk}^{AA}(\omega,0) &= \frac{|h_{A_1}(1)|^2}{M_{D^*}-\omega}.\end{aligned}$$

zero-recoil form factors h(1)
~ Isgur-Wise function ξ(1)=1

$$D^{JJ}_{\mu\nu}(\omega) \equiv (m_{D_s^{(*)}} - \omega)^2 \frac{d}{d\omega} T^{JJ}_{\mu\nu}(\omega) \sim |h(1)|^2$$

$m_b = 1.25^4 m_c$, zero recoil (**q**=**0**); 1/a = 3.6, 4.6 GeV



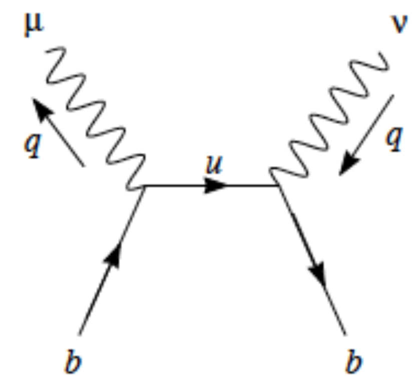
perturbative regime

reproduce the exclusive modes in the limit

Comparison to HQE

Tree-level formulae:

Blok, Koyrakh, Shifman, Vainshtein, PRD49, 3356 (1994).
Manohar, Wise, PRD49, 1310 (1993).
Falk, Ligeti, Neubert, Nir, PLB326, 145 (1994)
Balk, Korner, Pirjol, Schilcher, ZP C64, 37 (1994).

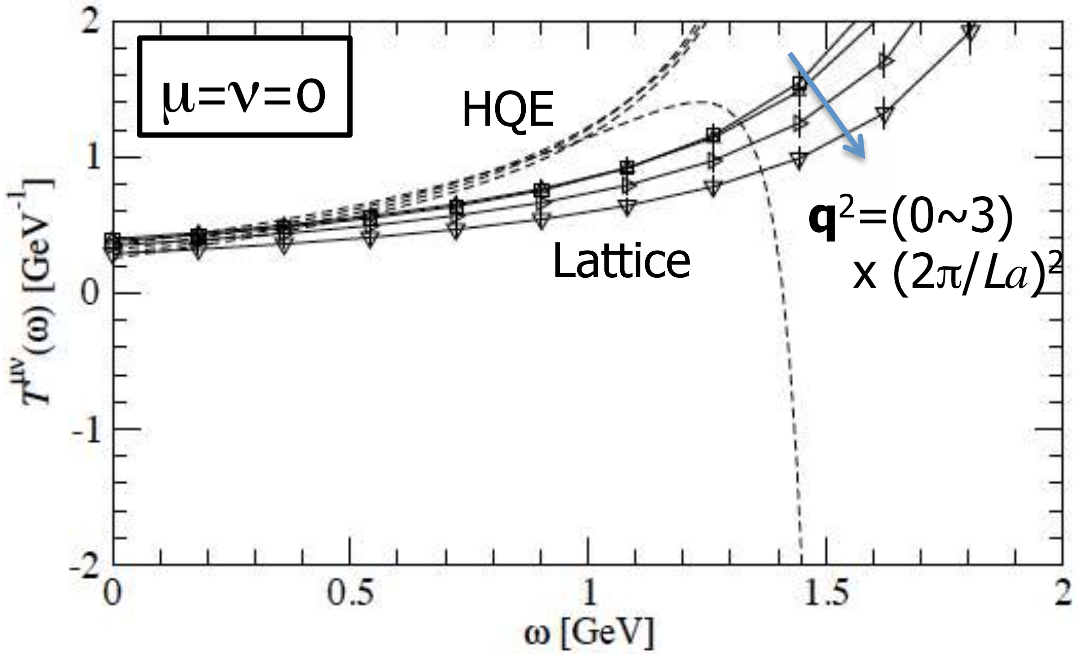


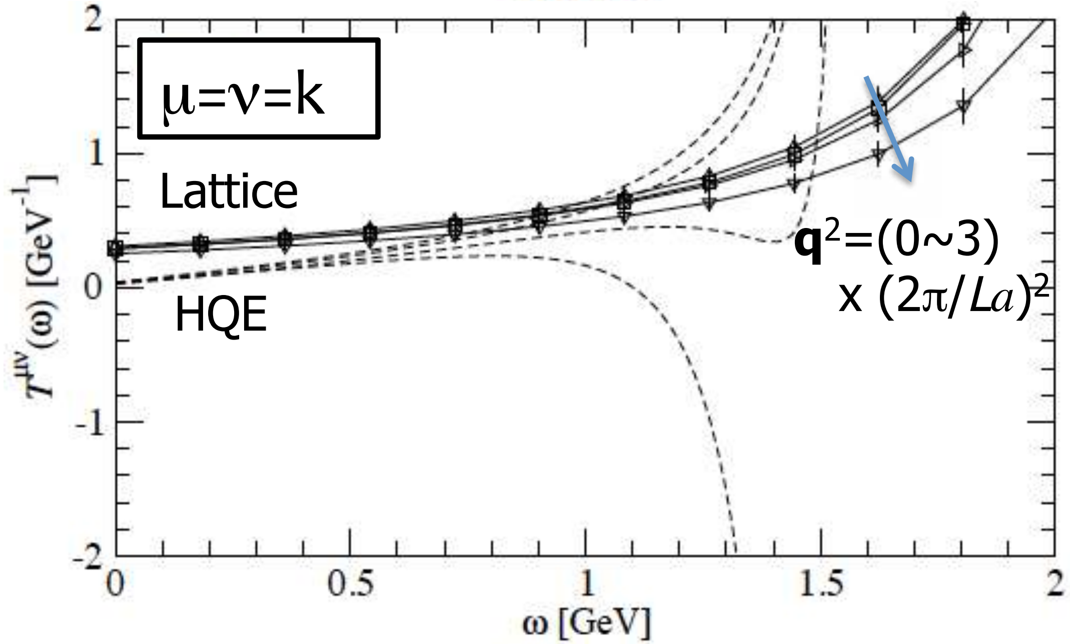
Expand $\dfrac{1}{m_b \not{v} - \not{q} + \not{k} - m_c}$ in small *k*.

Involves matrix elements:

$$\mu_\pi^2 = \frac{1}{2M_B}\left\langle B \left| \bar{b}(i\vec{D})^2 b \right| B \right\rangle \qquad \sim 0.5\ \mathrm{GeV}^2,$$

$$\mu_G^2 = \frac{1}{2M_B}\left\langle B \left| \bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu} b \right| B \right\rangle = 0.37\ \mathrm{GeV}^2$$





- VV+AA corresponding to the LL current insertion.
- HQE evaluated with rough (arbitrary) choices of parameters:
  - m_c = 1.6 GeV
  - μ_π^2 $(=-\lambda_1)$ ~ 1.6 GeV^2
  - μ_G^2 $(=3\lambda_2)$= 0.42 GeV^2

  Details are not important at this order.

Marginal agreement with Heavy quark expansion at 1/M, but at $O(\alpha_s{}^0)$

Need more studies including $O(\alpha_s)$.

Summary and Outlook (1)

- Inclusive structure functions calculable on the lattice, but at unphysical kinematics; Use Cauchy integral to match Exp
- Numerical test of the b→c transition on 2+1 flavor configs with DW heavy. The b quark mass is lighter than physical: m_b~2.5 m_c. Physical other than that.
- The region of ground state saturation (D or D*) is consistent with expectation (from form factors).
- The inclusive region compared to HQE. Need more study to check the consistency.

Summary and Outlook (2)

- Range of applications:
  - B physics
    - b→c semileptonic and $|V_{cb}|$
    - Test of b→c zero-recoil sum-rule
    - Test of Uraltsev sum rules
    - b→u semileptonic and $|V_{ub}|$
  - Nucleon structure
    - Direct calculation of structure functions without recourse to PDF
    - Not-so-deep inelastic scattering
  - Crucial to analyse experimental data accordingly!