Probing σ-model and dilaton signatures of the emergent light scalar

with the Lattice Higgs Collaboration (L_{at}HC)

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story to follow…

1. basic goal: how to describe the light scalar and make it consistent with χPT when the CW is approached influenced by recent work from Golterman-Shamir, Yamawaki et al., and Appelquist et al.

2. testing the β-function in χSB phase ultimately requires to solve this problem Holland talk

3. the linear σ-model regime not reached in simulations with $m_\pi \gtrsim m_\sigma$

4. should the linear σ-model be replaced by dilaton physics from the interplay of spontaneously broken scale and chiral symmetries?

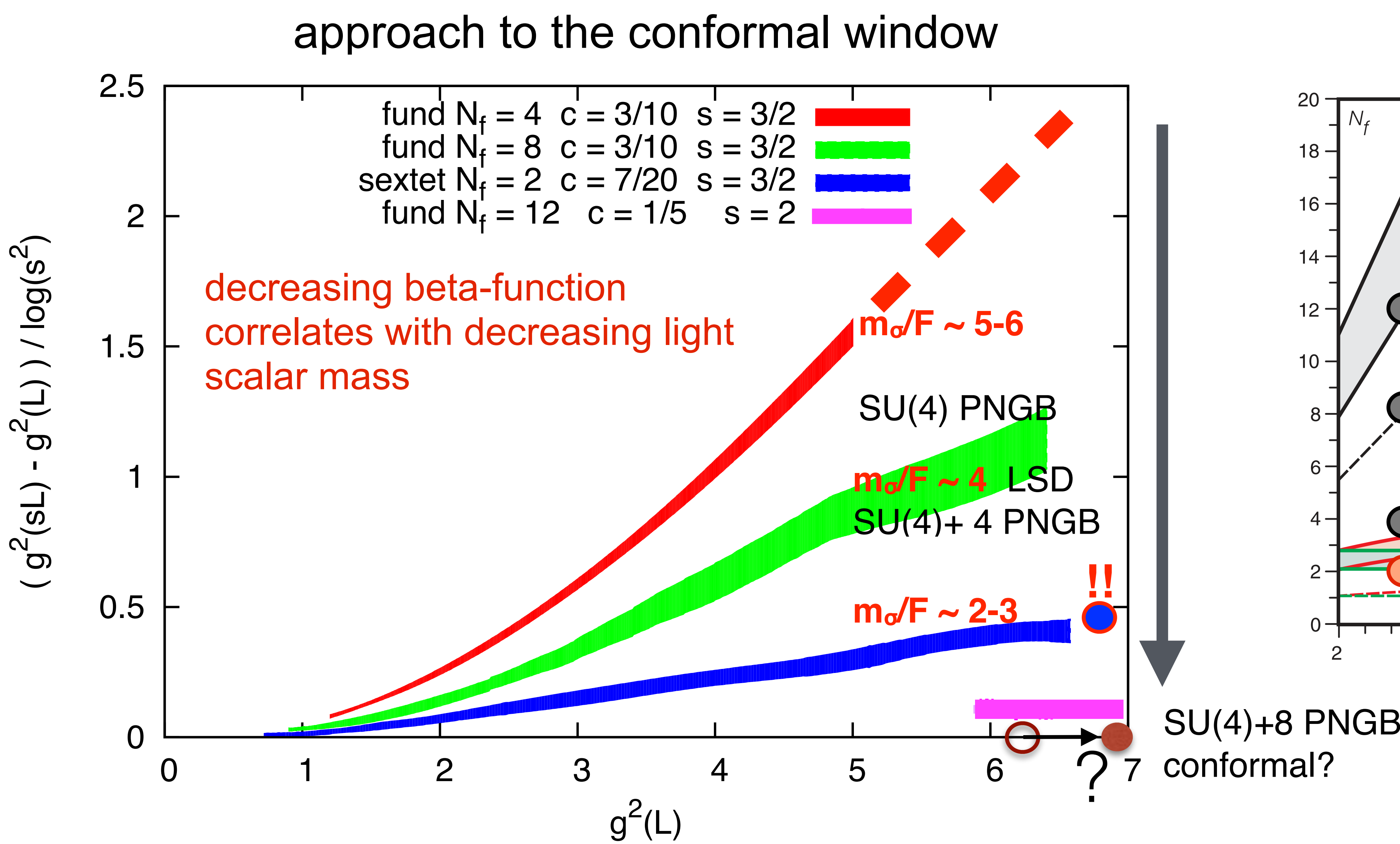
5. EFT for the coupled dynamics of the dilaton and the goldstone pions

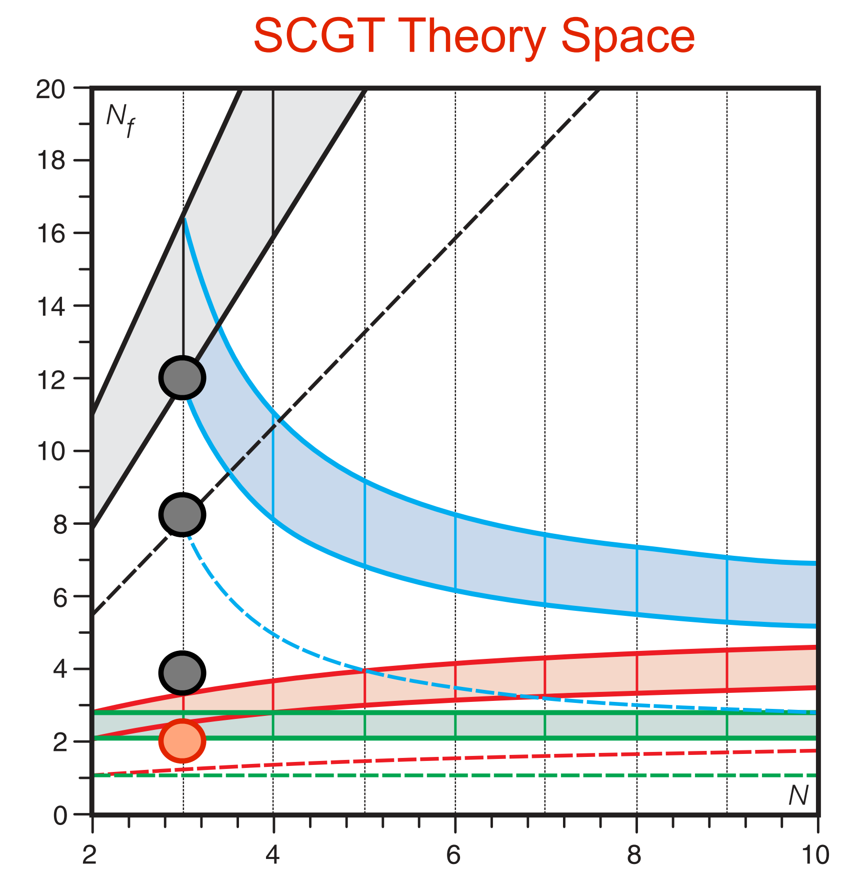
6. probing dilaton signatures in the sextet model

7. how to Monte Carlo the effective potential of the light scalar coupled to fermion operators

8. decoupling the dilaton in the ε-regime and the δ-regime

the running coupling and the β-function as the conformal window is approached:

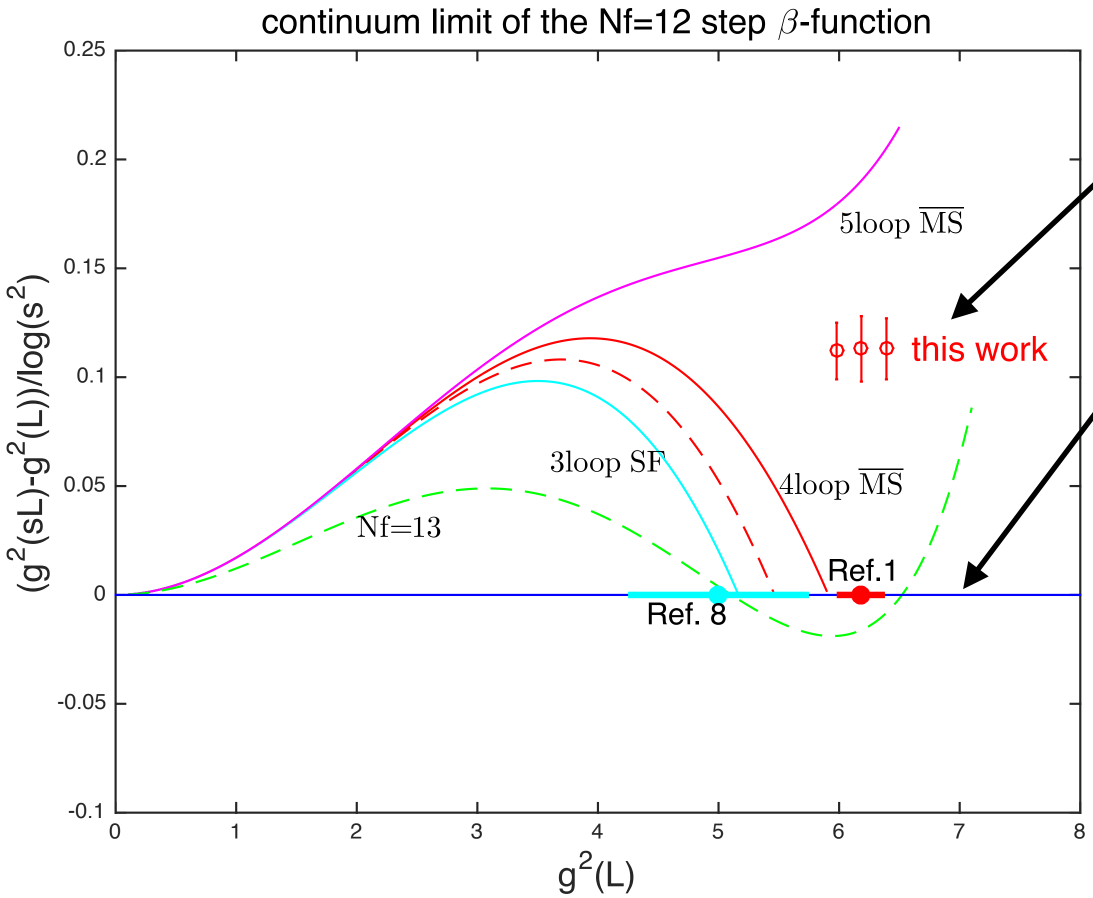




- models of interest: sextet Nf=2, fund Nf=8, fund Nf=12(?) are they near-conformal
- linear σ-model or dilaton signatures? both are legit with light σ-particle
- sextet β-function is reached from the chiral symmetry breaking phase (for first time!)
- is Nf=12 conformal or near-conformal? new results on fate of the IRFP: LatHC poster #8

could the Nf=12 model hide a near-conformal dilaton ?

only if bridge to χSB phase is established in large volumes

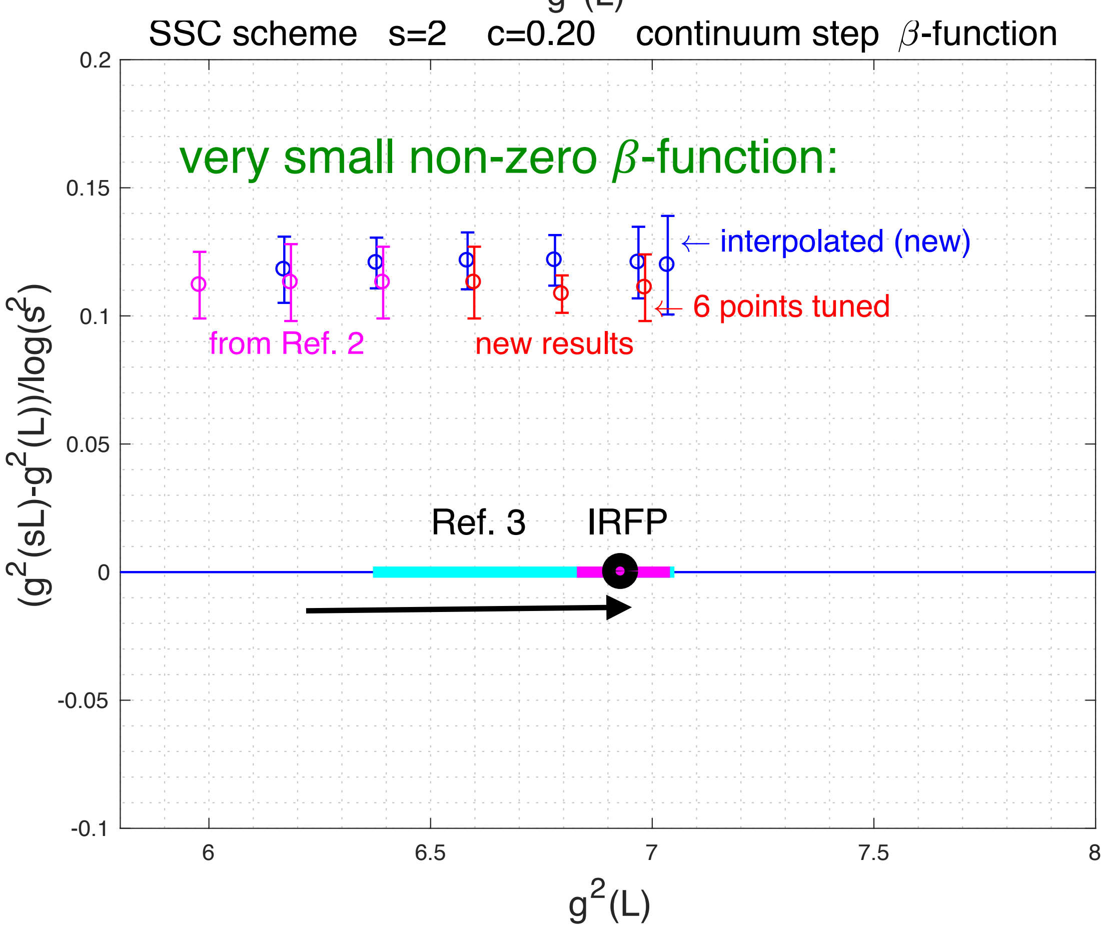


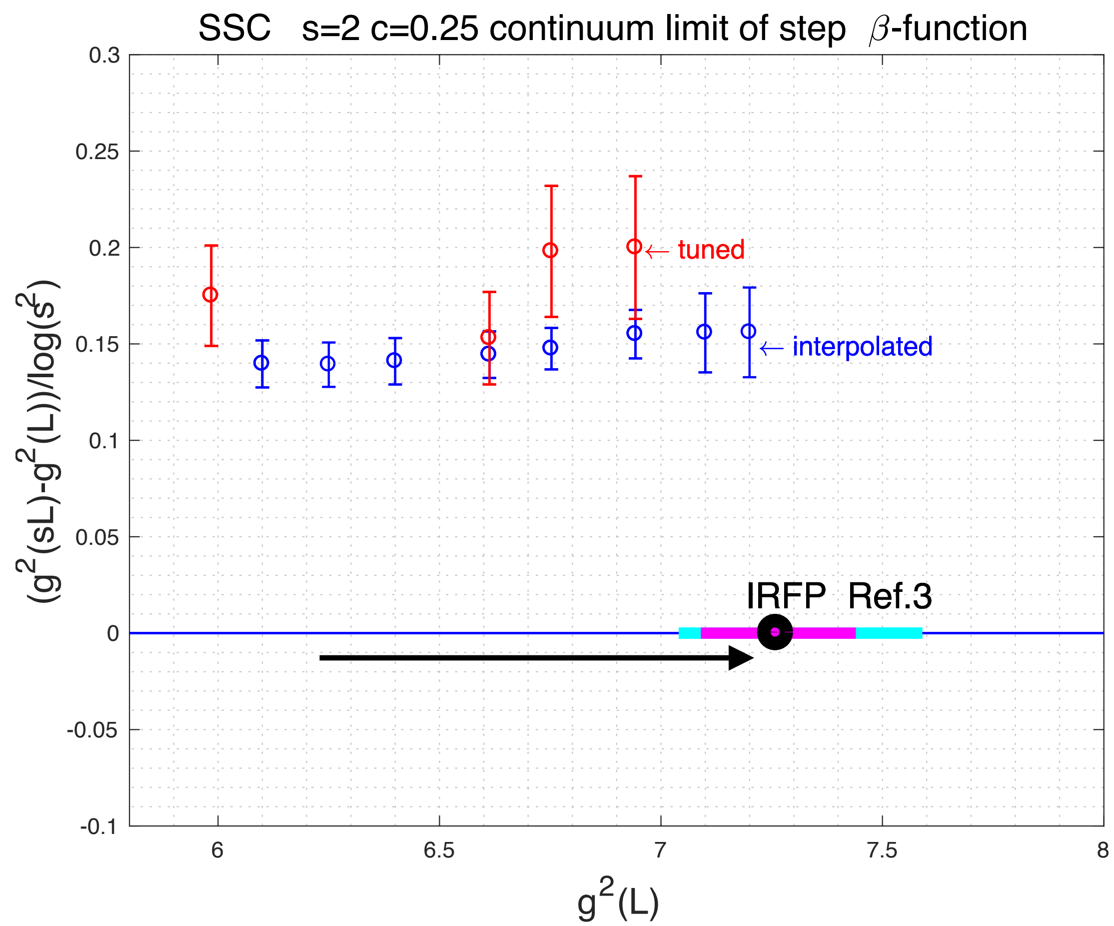
The plot from [2], referred as "this work" in the plot, summarizes the fate of the IRFP reported in [1]. Ref. 8 from [2] in the plot is the original publication of the IRFP by the LSD collaboration:

We are reporting new work which extends the three points of this plot into the $g^2 \sim 7$ region where the new IRFP location moved in [3]. see our #8 poster

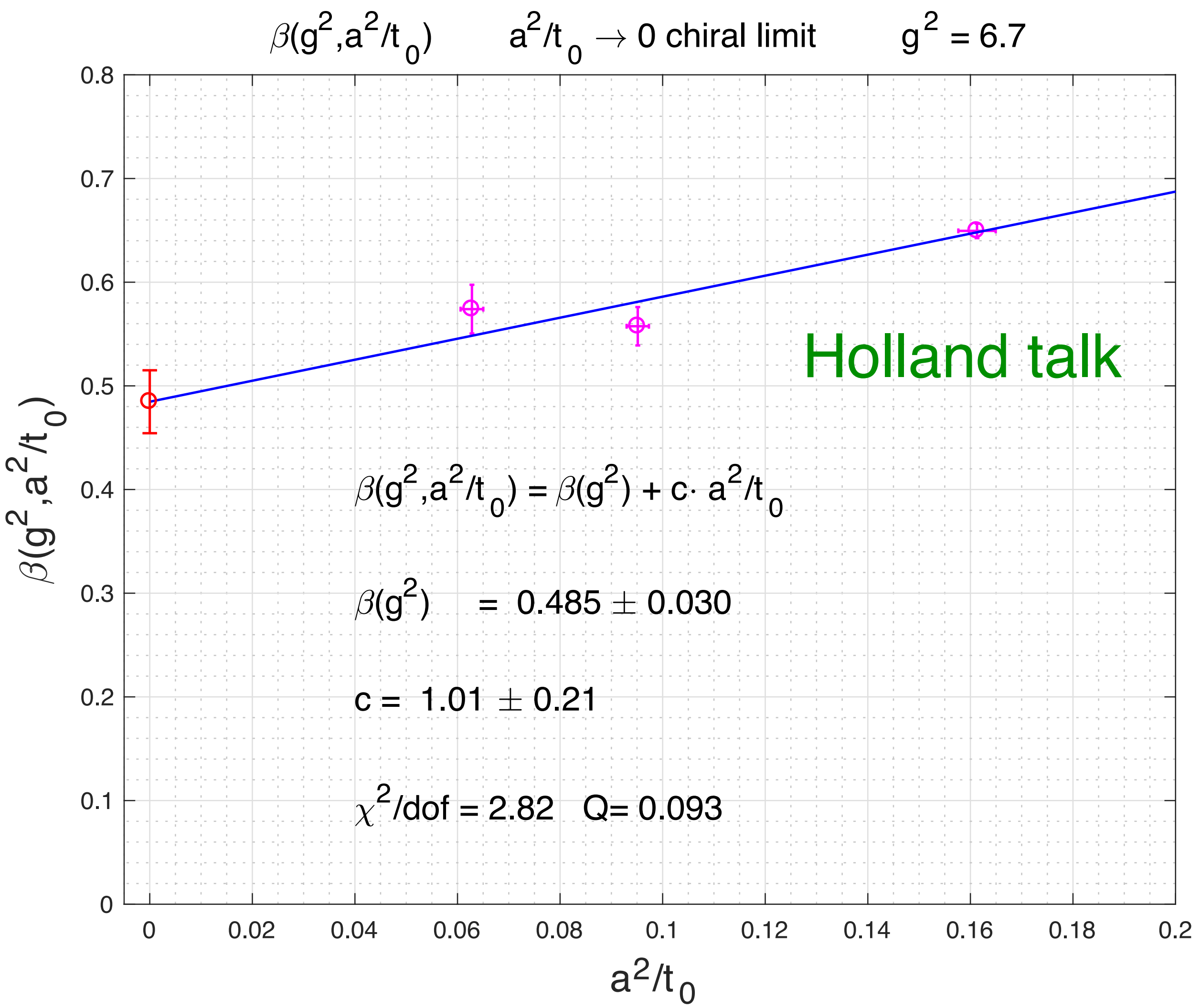
Recent results from the 5-loop β-function calculations do not show IRFP at Nf=12 which only appears at NF=13 for the 5-loop β-function. 3-loop and 4-loop calculations indicated IRFP at Nf=12 before.

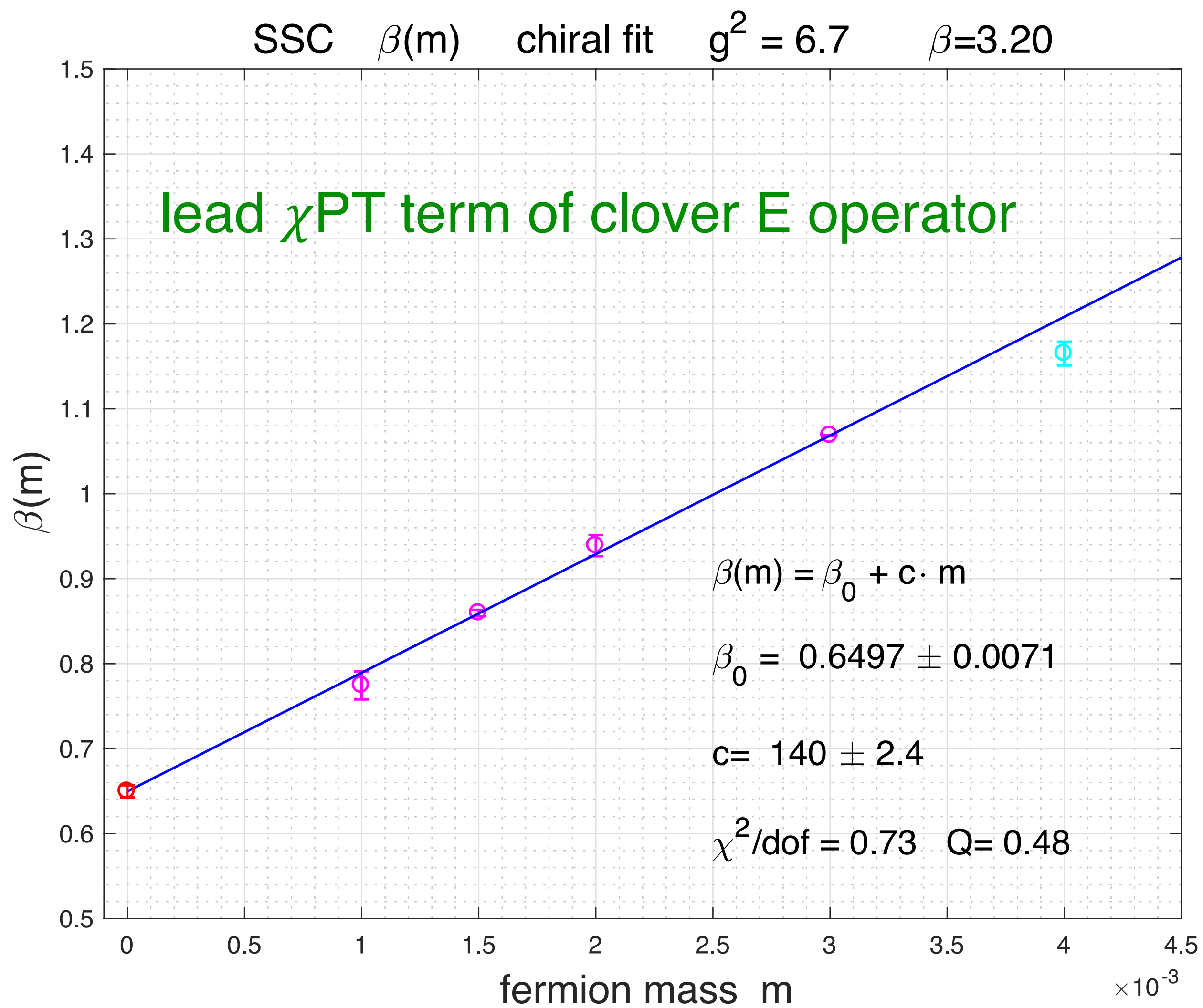
our results (poster!) demonstrate that large volumes are absolutely necessary to probe signals for Nf=12 IRFP ⇒ we found nothing!





bridge to χSB phase in sextet model:

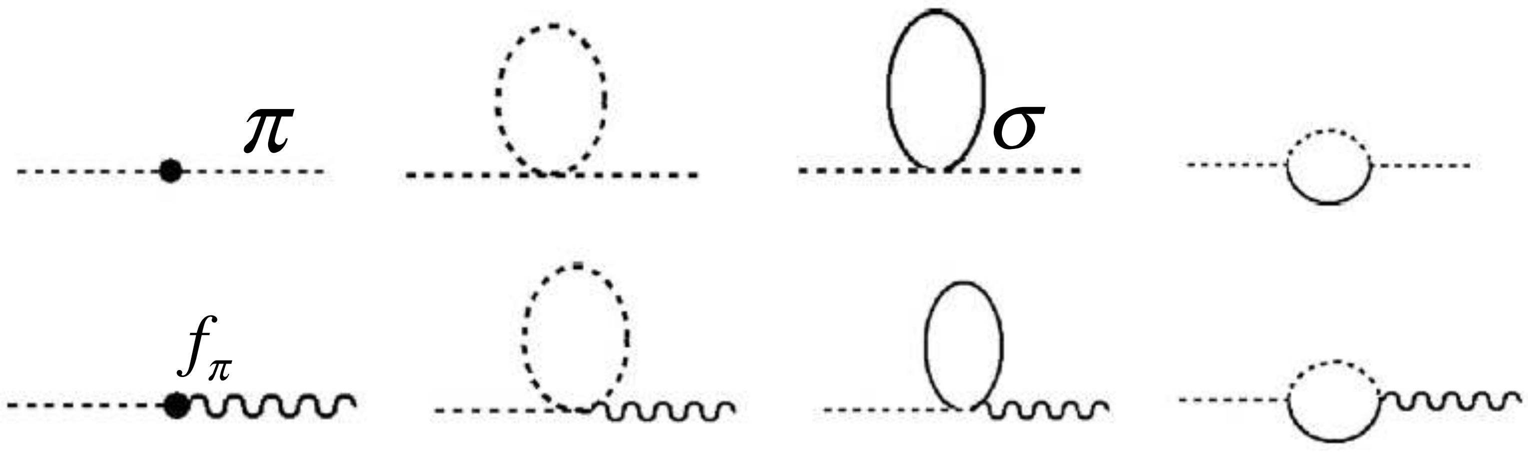




- matching the β-function from chiPT in the sextet model
- chiPT works for clover operator

- role of light scalar entangled with pion dynamics?
- motivates σ-model and dilaton model studies
- we will focus on sextet model (Nf=8 model: LSD data studied by Appelquist et al.)
- could Nf=12 hide a very interesting near-conformal dilaton?

extended EFT of σ-π entanglement in the BSM Higgs sector:



$$L = \frac{1}{2}\partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4}\left(D_\mu \Sigma^\dagger D_\mu \Sigma\right)\cdot\left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + b_3\frac{h^3}{v^3} + \ldots\right)$$

$\Sigma = e^{i\pi^a \tau^a / v}$ with τ^a Pauli matrices

$$V(h) = \frac{1}{2}m_h^2 \cdot h^2 + d_3\left(\frac{m_h^2}{2v}\right)\cdot h^3 + d_4\left(\frac{m_h^2}{8v^2}\right)\cdot h^4 + \ldots$$

σ-model limit (SM): $a = b = d_3 = d_4 = 1$ χSB of the underlying gauge theory

dilaton model limit: $a = b^2,\ b_3 = 0$ χSB below broken scale symmetry scale f_d

M_π, F_π, M_σ are calculated to 1-loop: extended SU(2) flavor chiral dynamics

We have been analyzing the small pion mass region in the M_π= 0.07- 0.015 range of the p-regime, also targeting the ε-regime and δ-regime because

linear sigma model limit in χPT p-regime simulations requires very small pion masses

$m_\pi \ll m_\sigma$

linear σ-model is not tested in simulations with $m_\pi \geq m_\sigma$

$SU(2)\otimes SU(2) \sim O(4)$

$$L=\frac{1}{2}\left(\partial_\mu \vec{\pi}\right)^2+\frac{1}{2}\left(\partial_\mu \sigma\right)^2-\frac{1}{2}\mu^2\left(\sigma^2+\vec{\pi}^2\right)+\frac{1}{4}g\left(\sigma^2+\vec{\pi}^2\right)^2-\varepsilon\sigma$$

$m_\sigma^2 \geq 3m_\pi^2$ tree level relation

$$L=\frac{F^2}{4}tr\left(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\right)-\frac{F^2M^2}{4}tr\left(\Sigma+\Sigma^\dagger\right)$$

pion field $\Sigma=e^{i\pi_a\tau_a/F}$ with τ_a Pauli matrices,

tree level pion mass $M^2=2Bm$

$$M_\pi^2=M^2\left\{1-\frac{1}{2}\frac{M^2}{16\pi^2F^2}\bar{l}_3+O(M^4)\right\} \qquad \bar{l}_3=\frac{16\pi^2}{g_R}-\ln\frac{M_\pi^2}{M_R^2}-\frac{14}{3}$$

$$F_\pi=F\left\{1+\frac{M^2}{16\pi^2F^2}\bar{l}_4+O(M^4)\right\} \qquad \bar{l}_4=\frac{8\pi^2}{g_R}-\ln\frac{M_\pi^2}{M_R^2}-\frac{1}{3}$$

$m_\sigma^2 \geq 2m_\pi^2$ 1-loop relation

$$\frac{1}{2}\frac{M_\pi^2}{16\pi^2F^2}\bar{l}_3<0.5 \Rightarrow \frac{M_\sigma}{M}>\sqrt{2} \text{ with the condition } \frac{M_\sigma}{F}=2$$

similar condition from $\bar{l}_4=\frac{8\pi^2}{g_R}-\ln\frac{M_\pi^2}{M_R^2}-\frac{1}{3}$

Low energy effective theory of the $\sigma(x)$ dilaton field and the $\pi^a(x)$ Goldstone bosons separated from the higher resonance states with SU(2) flavor in sextet model:

$$L = \frac{1}{2}\partial_\mu\chi\,\partial_\mu\chi - V(\chi) + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right] - \frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

Golterman and Shamir
Appelquist et al.
Matsuzaki and Yamawaki
LatKMI …

$y = 3-\gamma$ where γ is the mass anomalous dimension

$\chi(x) = f_d\, e^{\sigma(x)/f_d}$ describes the dilaton field $\sigma(x)$

pion field $\Sigma = e^{i\pi^a\tau^a/f_\pi}$ with τ^a Pauli matrices, tree level pion mass $m_\pi^2 = 2Bm$

we adapt Appelquist et al. notation for comparison

$$V_1 = \frac{m_d^2}{2f_d^2}\left(\frac{\chi^2}{2} - \frac{f_d^2}{2}\right)^2$$ relevant deformation of IRFP theory

$$V_2 = \frac{m_d^2}{16 f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$ nearly marginal deformation

Golterman and Shamir
Appelquist et al.

two different dilaton potentials illustrate scope of the analysis

Appelquist et al. Nf=8 fundamental rep

we use V_1 and V_2 in sextet model comparative analysis

dictionary for the effective dilaton theory coupled to Goldstone pions:

f_π	Goldstone decay constant	chiral limit $\Rightarrow$	F_π	with fermion mass deformations m
$m_\pi = 2mB$	Goldstone pions		M_π	
f_d	dilaton decay constant		F_d	
m_d	dilaton mass		M_d	

How do we test this theory? General scaling laws can be derived:

Appelquist et al.
Golterman and Shamir

F_d minimum of dilaton potential after fermion mass m is turned on:

for V_1 potential: $$\left(\frac{F_d^2}{f_d^2}\right)^{2-y/2}\left(1-\frac{F_d^2}{f_d^2}\right)=\frac{2yF^2}{f_d^2}\left(\frac{m_\pi^2}{m_d^2}\right)$$

for V_2 potential: $$\left(\frac{F_d^2}{f_d^2}\right)^{2-y/2}\ln\left(\frac{F_d^2}{f_d^2}\right)=\frac{2yF^2}{f_d^2}\left(\frac{m_\pi^2}{m_d^2}\right)$$

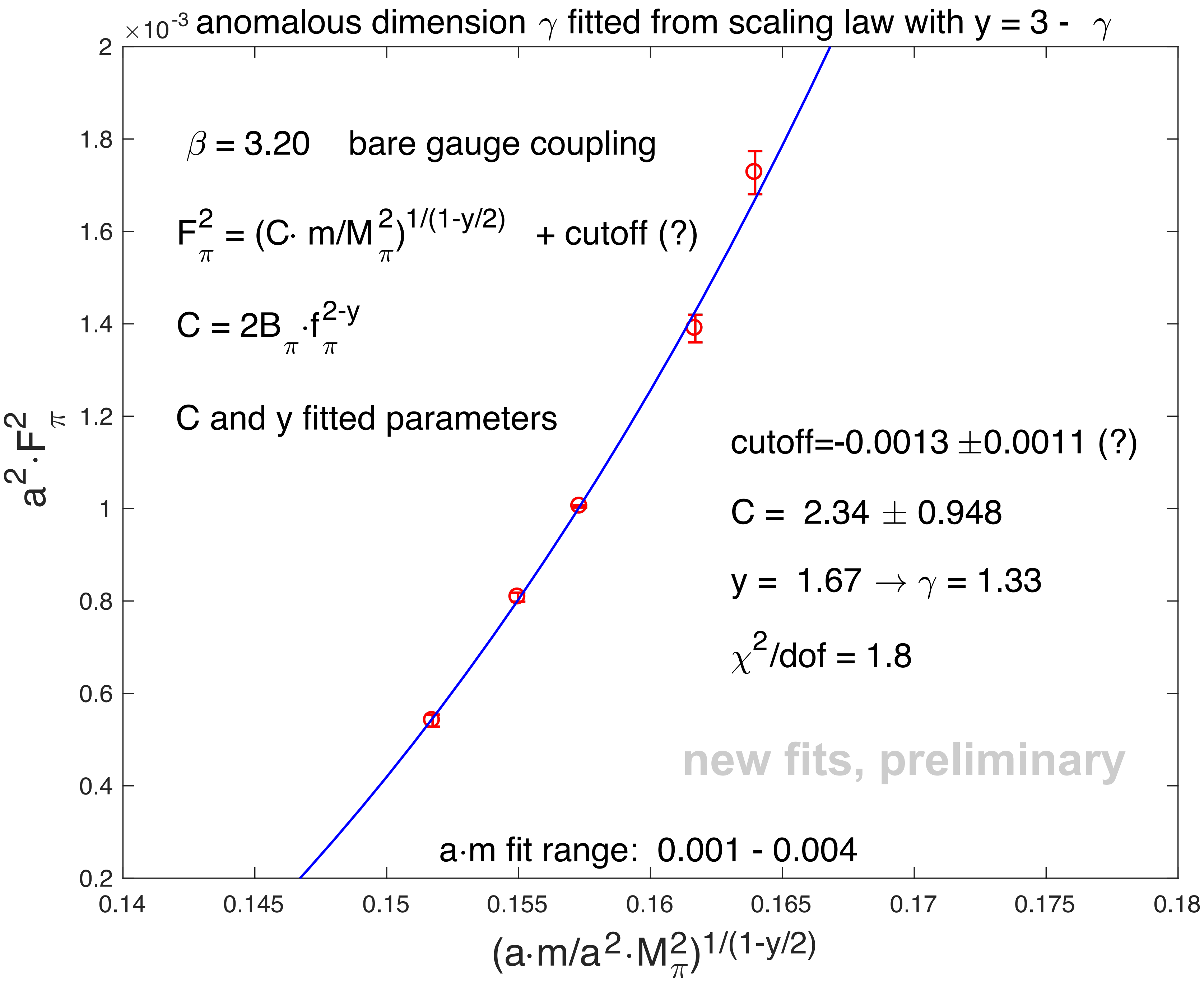
F_π^2 and M_π^2 at finite fermion mass m :

$$\left.\begin{array}{l} \dfrac{F_\pi^2}{f_\pi^2}=\dfrac{F_d^2}{f_d^2} \\[2ex] \dfrac{M_\pi^2}{m_\pi^2}=\left(\dfrac{F_d^2}{f_d^2}\right)^{y/2-1} \end{array}\right\} \Rightarrow \quad M_\pi^2\left(F_\pi^2\right)^{1-y/2} = \underbrace{2B_\pi\left(f_\pi^2\right)^{1-y/2}}_{\substack{C \text{ fitted} \\ y \text{ fitted}}}\cdot m$$

scaling of M_π^2 and F_π^2

now we turn to tests in the sextet model

$$M_\pi^2\left(F_\pi^2\right)^{1-y/2} = \underbrace{2B_\pi\left(f_\pi^2\right)^{1-y/2}}_{\substack{C \text{ fitted} \\ y \text{ fitted}}} \cdot m$$ sextet model test (no loop correction):



large volumes with FSS at each m

dilaton scaling law works well

does not depend on shape of V(χ)

shape of F_π in m is different from χPT

consistent mass anomalous dimension γ

f_π in chiral limit is approximately what was expected

tests at other lattice spacings are in the work

loop analysis is is being worked out

can a conformal theory look like this?

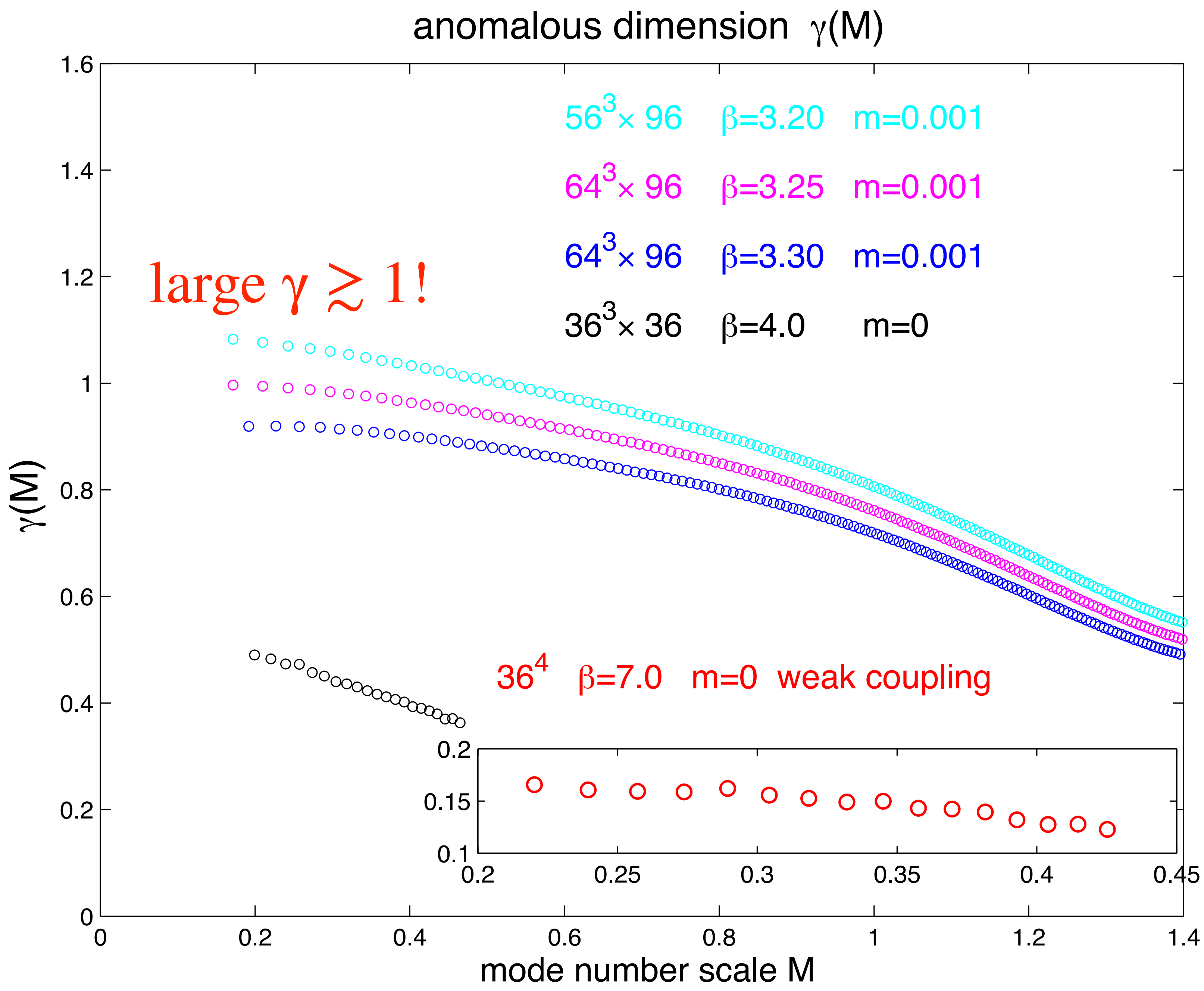
mass anomalous dimension γ from sextet Dirac spectrum:

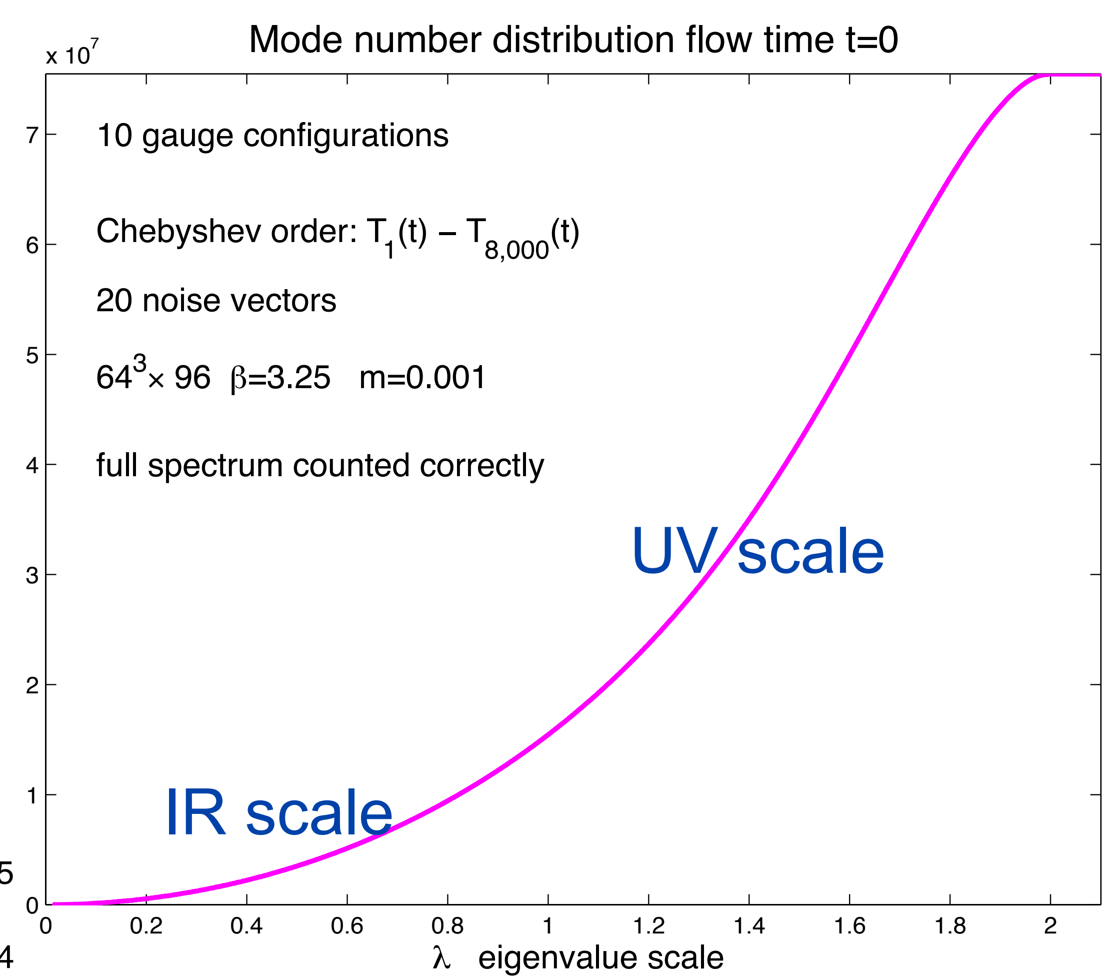
Del Debbio et al., Patella, Hasenfratz et al.

$$\nu_R(M_R, m_R) = \nu(M,m) \approx const \cdot M^{\frac{4}{1+\gamma_m(M)}},$$

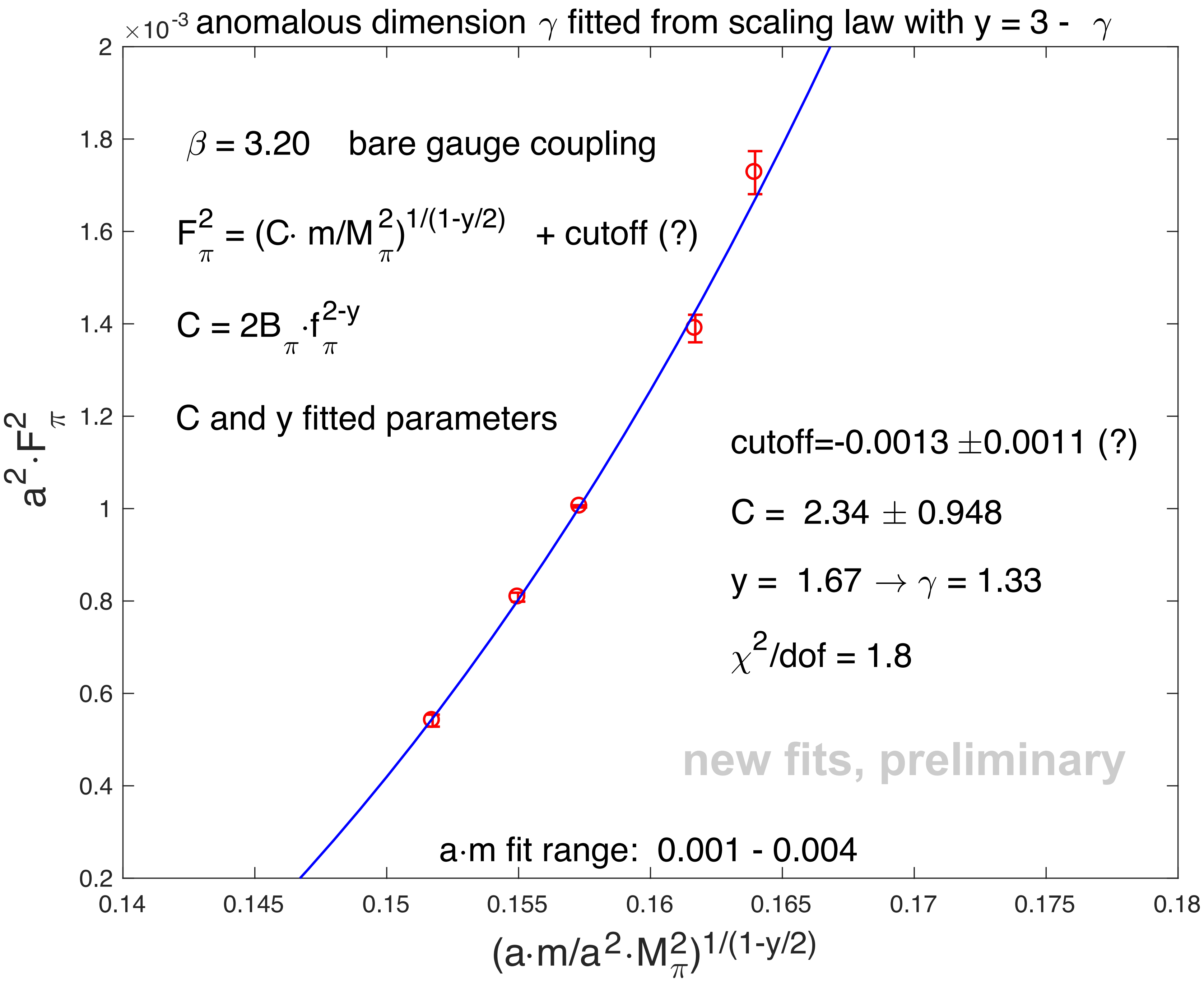
$$M \leftrightarrow \lambda \quad \nu(M,m) \approx const \cdot \lambda^{\frac{4}{1+\gamma_m(\lambda)}}, \text{ with } \gamma_m(\lambda) \text{ fitted}$$

- also from pseudo-scalar correlator
- How to match λ scale and g^2 ?





$$M_\pi^2 \left(F_\pi^2\right)^{1-y/2} = \underbrace{2B_\pi \left(f_\pi^2\right)^{1-y/2}}_{\substack{C \text{ fitted} \\ y \text{ fitted}}} \cdot m$$ sextet model test (no loop correction):



large volumes with FSS at each m

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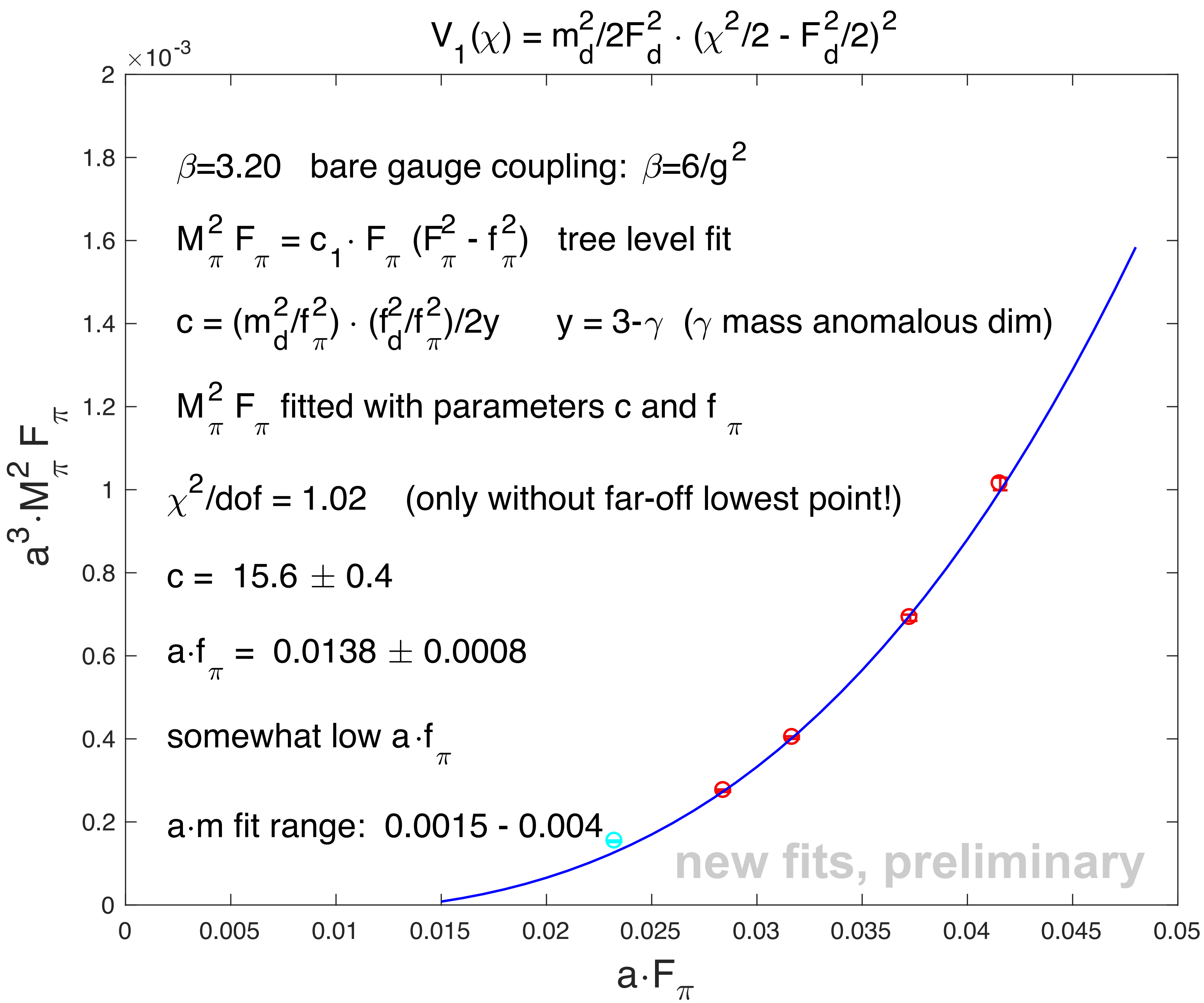
tests at other lattice spacings are in the work

loop analysis is is being worked out

can a conformal theory look like this?

how to get to the shape of the dilation potential?

$$V_1 = \frac{m_d^2}{2 f_d^2}\left(\frac{\chi^2}{2} - \frac{f_d^2}{2}\right)^2$$ sextet model V_1 test (tree level):



large volumes with FSS at each m

dilaton scaling works except

lowest point is problematic (1-loop?)

test does depend on shape of $V(\chi)$

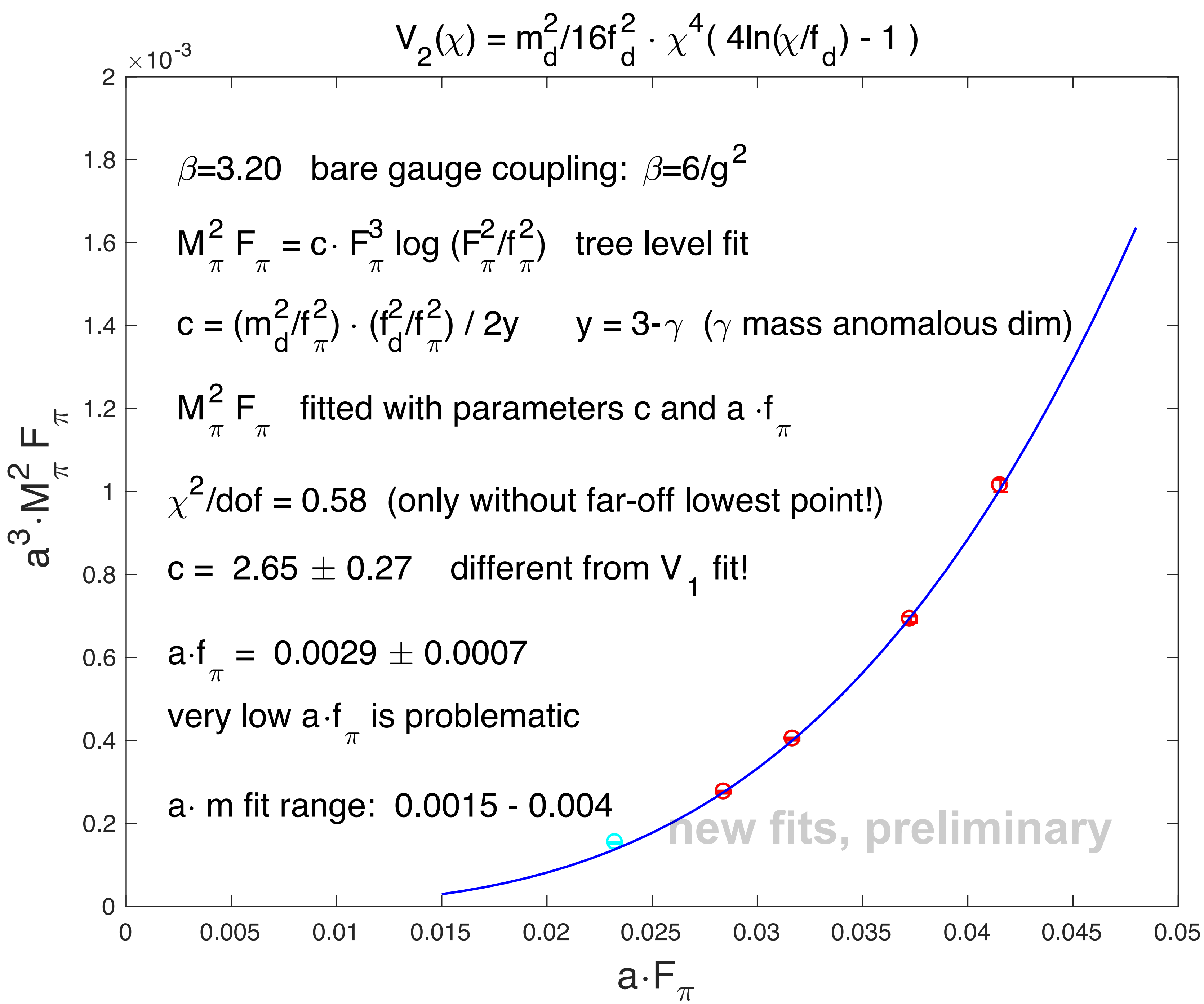
f_π in chiral limit is on the low side

tests at other lattice spacings are in the work

can a conformal theory look like this?

and the V_2 potential?

$$V_2 = \frac{m_d^2}{16 f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$$ sextet model V_2 test (tree level):



large volumes with FSS at each m

dilaton scaling works except

lowest point is problematic (1-loop?)

test does depend on shape of $V(\chi)$

f_π in chiral limit is very different from V_1

can a conformal theory look like this?

can we get to the dilation or other EFT potential in MC simulation?

constraint effective potential

$$\exp(-\Omega U_\Omega(\Phi)) = \prod_x \int d\phi(x)\delta\left(\Phi - \frac{1}{\Omega}\sum_x \phi(x)\right)\exp(-S[\phi])$$

(pseudo)scalar field ϕ(x) elementary or source of composite operator

$$P(\Phi) = \frac{1}{Z}\exp(-\Omega U_\Omega(\Phi)), \quad Z = \int d\Phi' \exp(-\Omega U_\Omega(\Phi'))$$

probability distribution of order parameter in finite volume Ω

implementation with fermion fields: jk, Lee Lin, Pietro Rossi, Yue Shen, **NPB Proc. Suppl. 9 (1989) 99-104**

$$\frac{dU_{\text{eff}}}{d\Phi} = m^2\Phi + \frac{1}{6}\lambda\langle\phi^3\rangle_\Phi - N_F y\langle\bar{\psi}\psi\rangle_\Phi, \quad \langle\bar{\psi}\psi\rangle_\Phi = \langle \mathrm{Tr}(D[\phi]^{-1})\rangle_\Phi$$

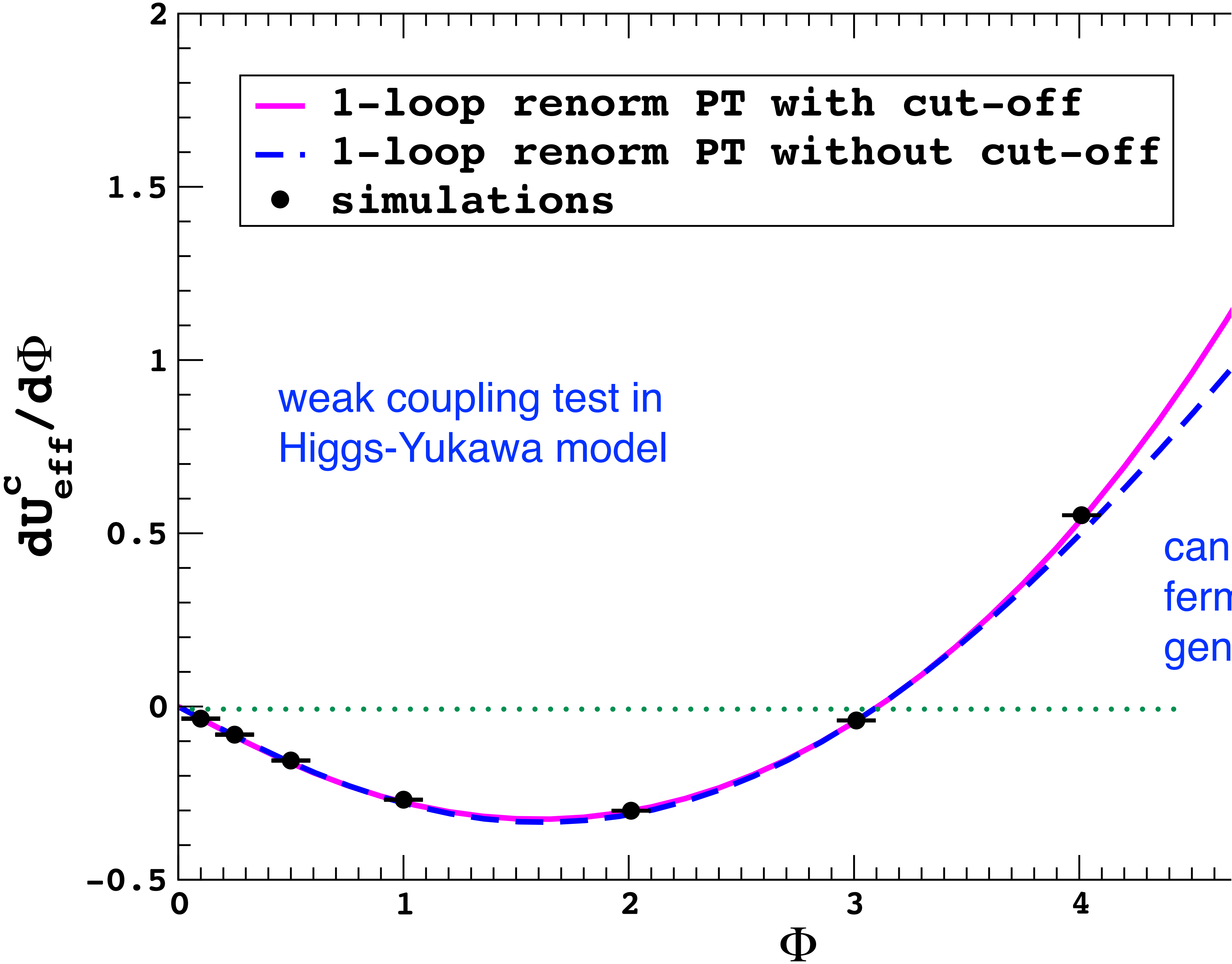
tested in Higgs-Yukawa model:

$$\dot{\phi}(x,t) = \pi(x,t)\ ,$$

$$\dot{\pi}(x,t) = -\left[\frac{\partial S_{\text{eff}}}{\partial\phi(x,t)} - \frac{1}{\Omega}\sum_y \frac{\partial S_{\text{eff}}}{\partial\phi(y,t)}\right]$$

$$\frac{1}{\Omega}\sum_y \phi(y,t) = \Phi, \quad \sum_y \pi(y,t) = 0$$

constraint effective potential



weak coupling test in Higgs-Yukawa model

jk, Kieran Holland
LatHC

can be extended to four-fermion operator and more general BSM models

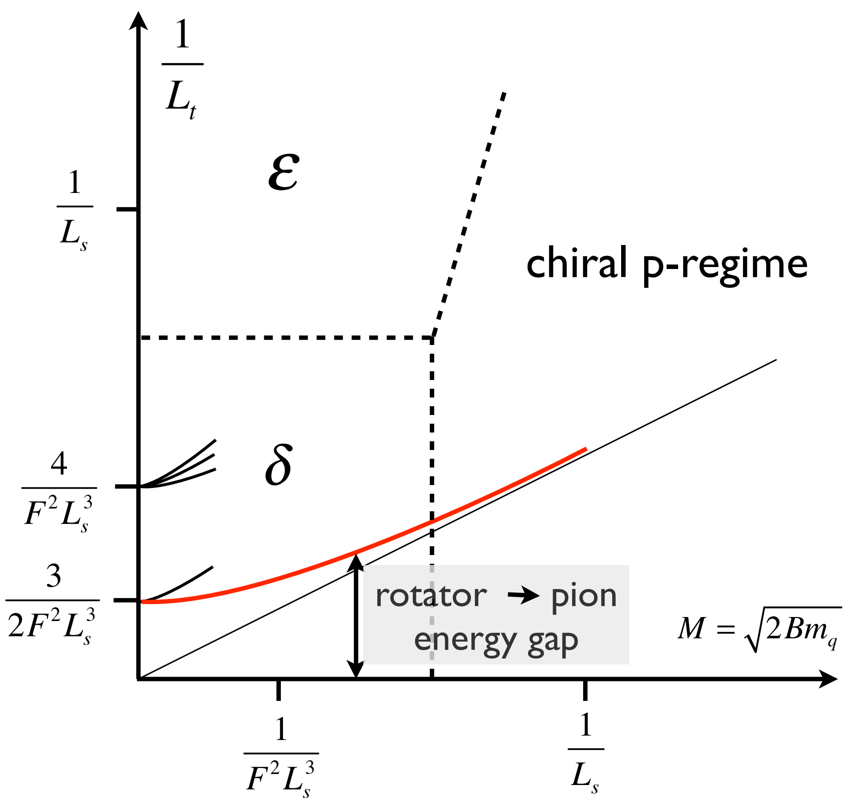
can we extend the analysis from p-regime to δ/ε - regimes?

dilaton in δ-regime and ε-regime

$$L=\frac{1}{2}\partial_\mu\chi\partial_\mu\chi - V(\chi)+\frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 tr\left[\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\right]-\frac{f_\pi^2 m_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y tr\left(\Sigma+\Sigma^\dagger\right)$$

only time derivatives survive in δ-regime

even the time derivatives are missing in ε-regime (RMT)

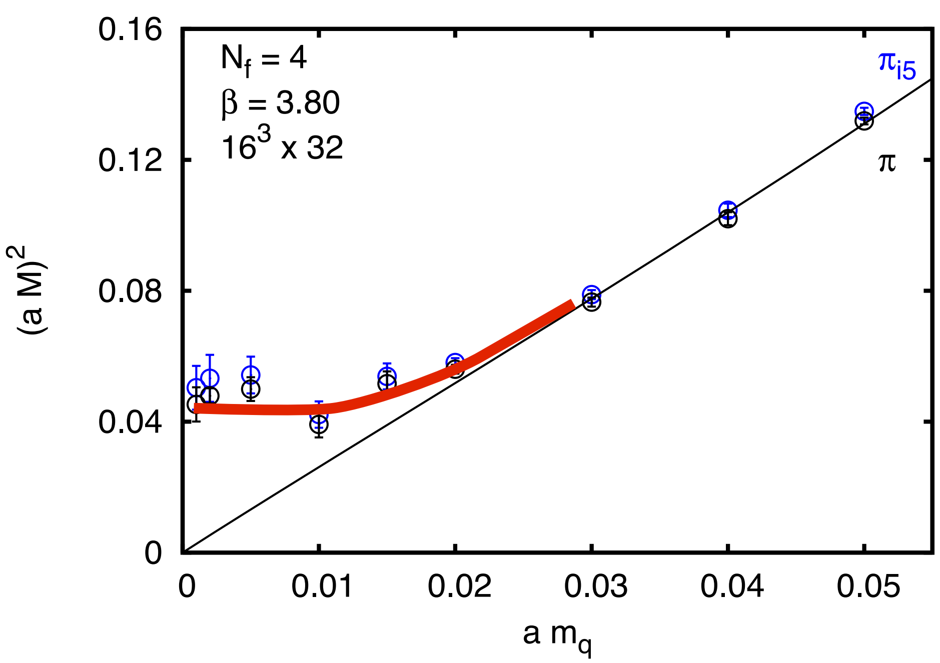


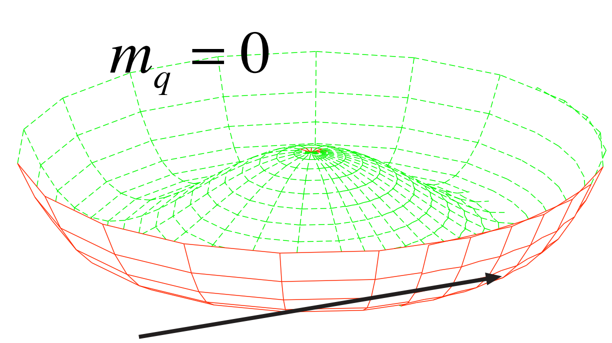
in rotator/RMT dynamics

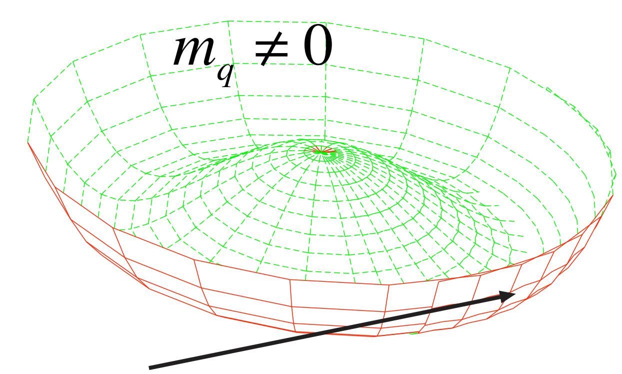
$m=0$ limit: dilaton is decoupled from δ/ε regimes

$m\neq 0$ limit: dilaton remains coupled in δ/ε regimes

access to F_d and M_d under new conditions







Veff: chiral condensate in flavor space
arbitrary orientation of condensate

tilted condensate

Summary: fingerprints of near-conformal sextet scalar in goldstone dynamics may show dilaton or some more general EFT signatures:

- sextet model is consistent with χSB from all angles we looked at
- dilaton signatures or more general EFT approach may completely change the current χPT analysis
- reaching the linear σ-model regime is difficult
- large γ(λ) is consistent with dilaton signatures
- never use data in p-regime without FSS for EFT analysis in this new ball game
- the ε-regime (RMT) and δ-regime are new opportunities for dilaton and other EFT signatures
- Nf=12 model is perhaps hiding a dilation (as interesting field theory)