Electromagnetic pion form factor with strange quark mass reweighting in N_f=2+1 lattice QCD

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(PACS Collaboration)

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Motivation

- Electromagnetic form factor

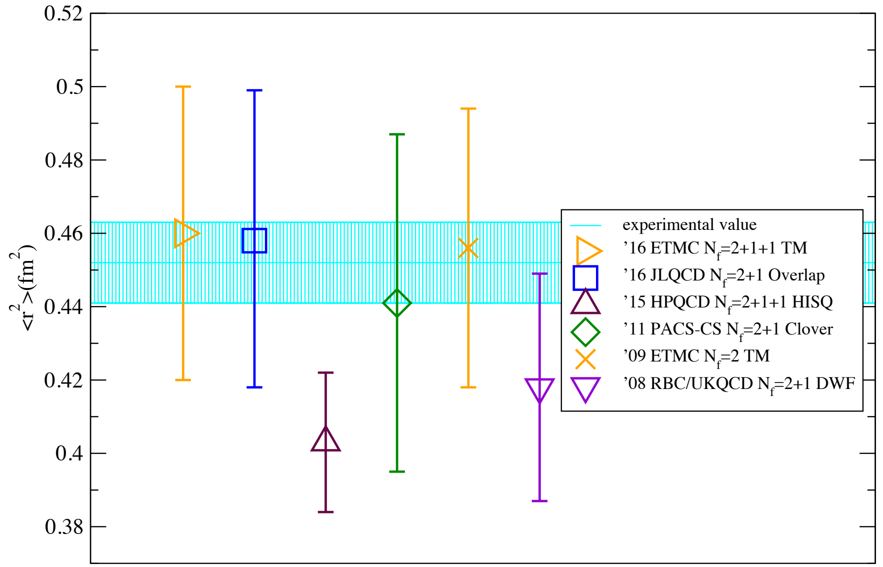
  =deviation from charged point particle

  →Hadron’s structure (cf. charge radius)

However, errors of charge radii in lattice calculation are <u>larger than experimental one</u>

- chiral extrapolation → error increases

- small momentum transfer and suppressing finite size effect → need large box size



previous results of radii in lattice calculation at physical point and experimental value

Purpose

calculation near physical point on the large volume

+extrapolation to physical point (updated from lattice 2016)

Electromagnetic form factor $f_{\pi\pi}(q^2)$

Definition $\langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle = (p_f + p_i)_\mu f_{\pi\pi}(q^2)$

electromagnetic current $V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$ $\psi_f = u, d, s$ Q_f :charge of flavor

$f_{\pi\pi}(0) = 1$ (normalization condition relating to pion's charge)

$q^2 = -(p_f - p_i)^2 \geq 0$ (space-like momentum transfer →calculating directly)

calculating <u>3-pt function</u> and <u>2-pt function</u> by Lattice QCD

$$
\begin{aligned}
C_{\pi V \pi} &= Z_V \left\langle 0 \middle| O_\pi(t_f, \vec{p}_f = \vec{0}) V_4(t, \vec{p} = \vec{p}_f - \vec{p}_i) O_\pi^\dagger(0, \vec{p}_i) \middle| 0 \right\rangle \\
&= Z_V \frac{Z_\pi(0) Z_\pi(p)}{4E_\pi(p) m_\pi} \langle \pi(0) | V_4(0,p) | \pi(p) \rangle e^{-E_\pi(p)t} e^{-m_\pi(t_f - t)} \\
&= Z_V \frac{Z_\pi(0) Z_\pi(p)}{4E_\pi(p) m_\pi} (E_\pi(p) + m_\pi) f_{\pi\pi}(q^2) e^{-E_\pi(p)t} e^{-m_\pi(t_f - t)}
\end{aligned}
$$

$$|Z_\pi(p)|^2 (e^{-E_\pi(p)t} + e^{-E_\pi(p)(T-t)})$$

$$Z_\pi(p) = \langle 0 | O_\pi(0,0) | \pi(p) \rangle$$

$$\rightarrow \quad R(t,p) = \frac{2m_\pi Z_\pi(0)}{(E_\pi(p) + m_\pi) Z_\pi(p)} \frac{C_{\pi V \pi}(t, t_f; p)}{C_{\pi V \pi}(t, t_f, 0)} e^{(E_\pi(p) - m_\pi)t}$$

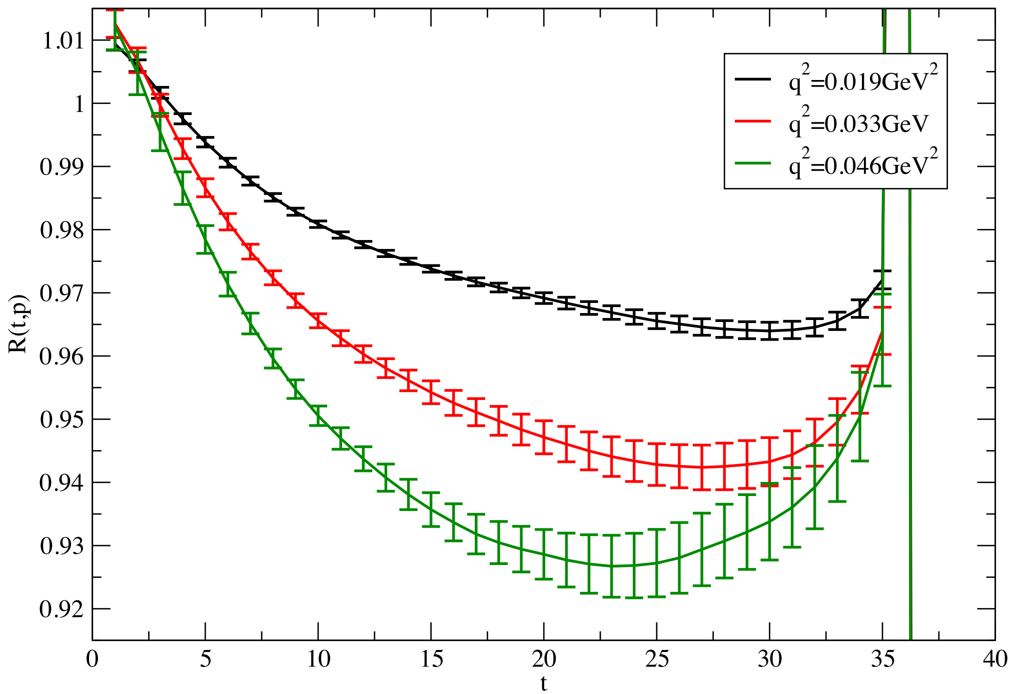
in $0 \ll t \ll t_f$ we can extract form factor $f_{\pi\pi}(q^2)$ from R

Extraction of form factor

light pion mass+ periodic boundary condition in temporal direction

→<u>current time dependence appears in R</u>

we should consider wrapping around effect



pion mass is 0.145 GeV

$$t_f - t_i = 36$$

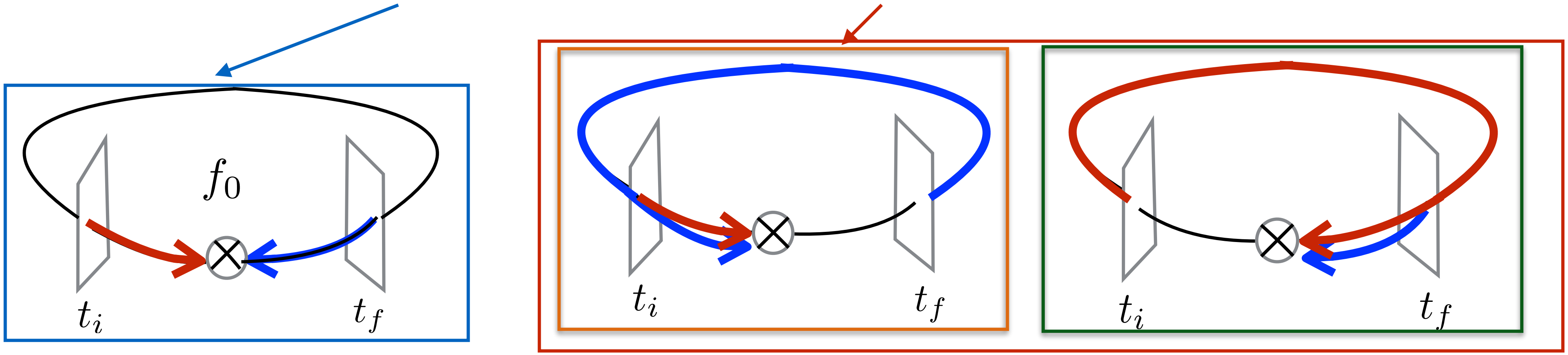
Extraction of form factor

light pion mass+ periodic boundary condition in temporal direction

→<u>current time dependence appears in R</u>

we should consider wrapping around effect

form factor+wrapping around effect



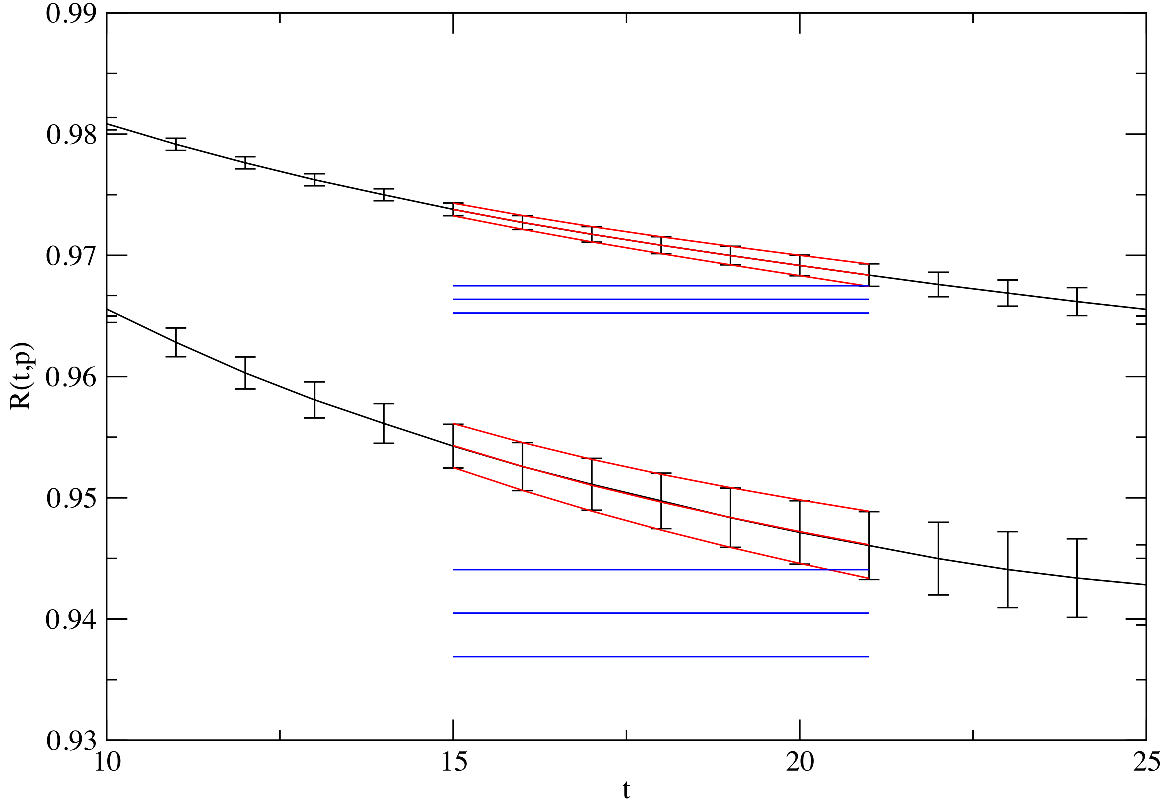
$$f_0 + f_1 \times \frac{e^{-E_{2\pi}\times t} e^{-E_{\pi_f}\times(T-t_f)} - e^{-E_{2\pi}\times(t_f-t)} e^{-E_{\pi_i}\times(T-t_f)}}{e^{-E_{\pi_i}\times t} e^{-E_{\pi_f}\times(t_f-t)}}$$

f_1 is related to $\langle 0|V_\mu|\pi\pi\rangle$

$E_{2\pi} = E_i + E_f$:sum of single pion energy we fit $R(t,p) = \frac{2m_\pi Z_\pi(0)}{(E_\pi(p)+m_\pi)Z_\pi(p)} \frac{C_{\pi V\pi}(t,t_f;p)}{C_{\pi V\pi}(t,t_f,0)} e^{(E_\pi(p)-m_\pi)t}$

fit form $f_0 + f_1 \times \frac{e^{-E_{2\pi}\times t} e^{-E_{\pi_f}\times(T-t_f)} - e^{-E_{2\pi}\times(t_f-t)} e^{-E_{\pi_i}\times(T-t_f)}}{e^{-E_{\pi_i}\times t} e^{-E_{\pi_f}\times(t_f-t)}}$

the extraction form factor of the smallest and second smallest momentum transfer



red : fit including wrapping around effect

blue : form factor

fits including wrapping around effect work well in our data

Simulation details

All the results are preliminary

gauge configuration (HPCI Strategic Program Field 5)

$m_\pi = 0.145(\mathrm{GeV}), L = 8.1(\mathrm{fm})$ $m_\pi L \approx 6$ $(\kappa_{ud}, \kappa_s) = (0.126117, 0.124790)$

strange mass reweighting factors of $\kappa_s = 0.124812$, $\kappa_s = 0.124768$

$L^3 \times T = 96^3 \times 96$ $a^{-1} = 2.333(\mathrm{GeV}) \rightarrow a = 0.084(\mathrm{fm})$ $m_K \approx 0.525\mathrm{GeV}$

$N_f = 2+1$ Iwasaki gauge+stout smeared link Wilson clover action

$\beta = 1.82, n_{stout} = 6, \rho = 0.1, c_{sw} = 1.11$

measurement parameter

100 configurations (every 20 traj.)

4 sources × 4 directions (x, y, z, t) ×2 random sources = 32 meas. per config.

periodic boundary condition for all directions

$t_f - t_i = 36$ $q^2 = 2m_\pi(E_\pi - m_\pi)$

bin size:100 traj.

$$\vec{p} = \frac{2\pi}{L}\vec{n}$$

directions of source momenta $\vec{n}$

$(1,0,0)$	$(0,1,0)$	$(0,0,1)$
$(1,1,0)$	$(1,0,1)$	$(1,1,0)$
$(1,1,-1)$	$(1,-1,1)$	$(1,1,-1)$
$(2,0,0)$	$(0,2,0)$	$(0,0,2)$
$(2,1,0)$	$(0,2,1)$	$(1,0,2)$
$(1,1,2)$	$(1,2,1)$	$(1,1,2)$

sink momentum : 0

resources
PRIMRGY cx400 (tatara), RIIT, Kyusyu University and HAPACS, CCS, University of Tsukuba

preliminary result: NLO SU(2) ChPT fit of generation point

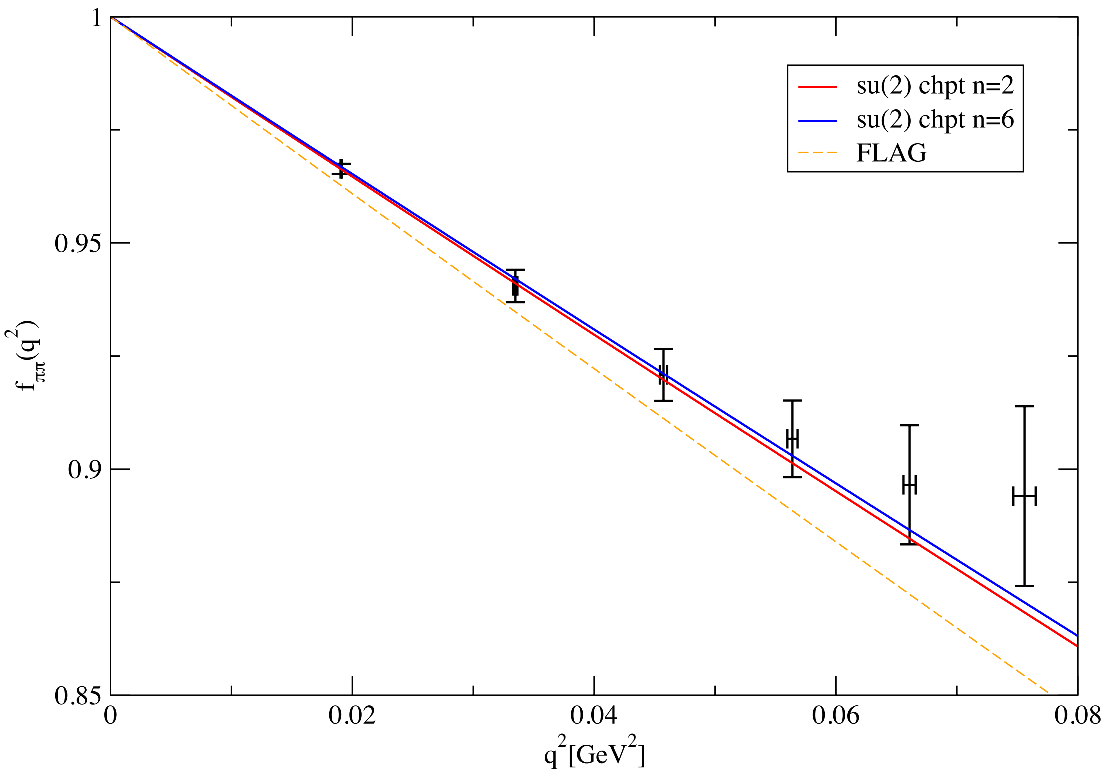
$$f_{\pi\pi}(q^2) = 1 + \frac{1}{f^2}\left[2l_6 q^2 + \frac{m^2 H(x)}{8\pi^2} + \frac{q^2}{48\pi^2}\log\left(\frac{m^2}{\mu^2}\right)\right]$$

$$H(m^2,q^2,\mu^2) = \frac{m^2}{32\pi^2}\left(-\frac{4}{3} + \frac{5}{18}x - \frac{x-4}{6}\sqrt{\frac{x-4}{x}}\log\left(\frac{\sqrt{\frac{x-4}{x}}+1}{\sqrt{\frac{x-4}{x}}-1}\right)\right) - \frac{q^2}{192\pi^2}\log\frac{m^2}{\mu^2} \quad (x = -\frac{q^2}{m^2}).$$

$$\langle r^2 \rangle = \frac{-12 l_6}{f^2} - \frac{1}{8\pi^2 f^2}\left(\log\left(\frac{m_\pi^2}{\mu^2}\right) + 1\right)$$

$$\mu = m_\rho = 0.77(\mathrm{GeV})$$

$$f = 0.12925(\mathrm{GeV})\ (m_u, m_d \to 0)$$



there is one fit parameter

l_6: Low Energy Constant

	l_6	$\chi^2/d.o.f$
n=1	-0.01236(51)	
n=2	-0.01240(49)	0.08
n=3	-0.01238(53)	0.17
n=6	-0.01216(57)	0.55
FLAG	-0.01233(127)	

arXiv:1607.00299 [hep-lat]

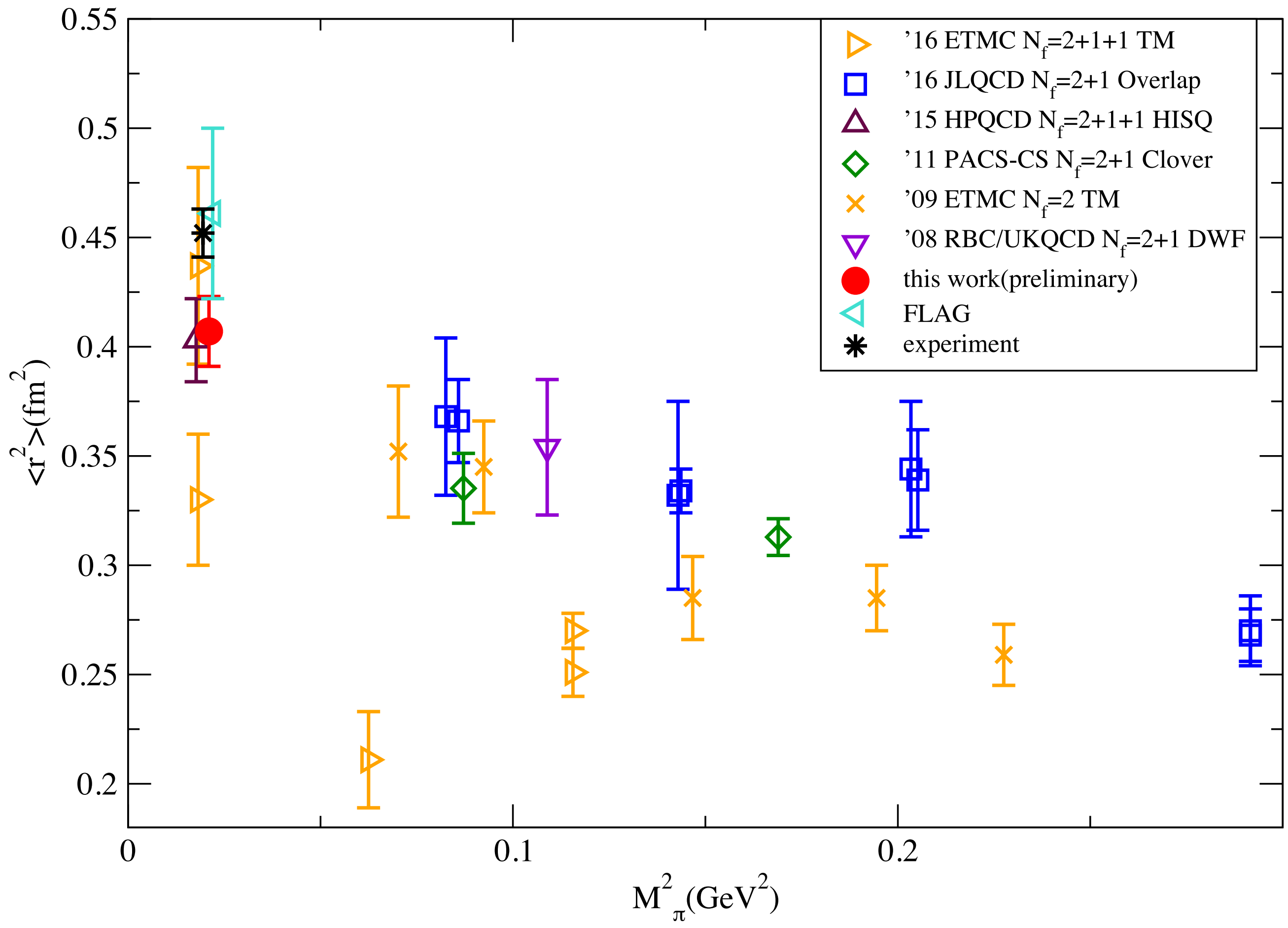
n: the number of q used in the fit
in ascending order from smallest q

In FLAG's line

$$f = 0.122553(\mathrm{GeV}) \quad (m_u, m_d \to 0)$$

NLO ChPT fit works well → almost physical pion mass, small enough q^2

preliminary result: pion mass dependance



the result of n=6
NLO SU(2) ChPT fit

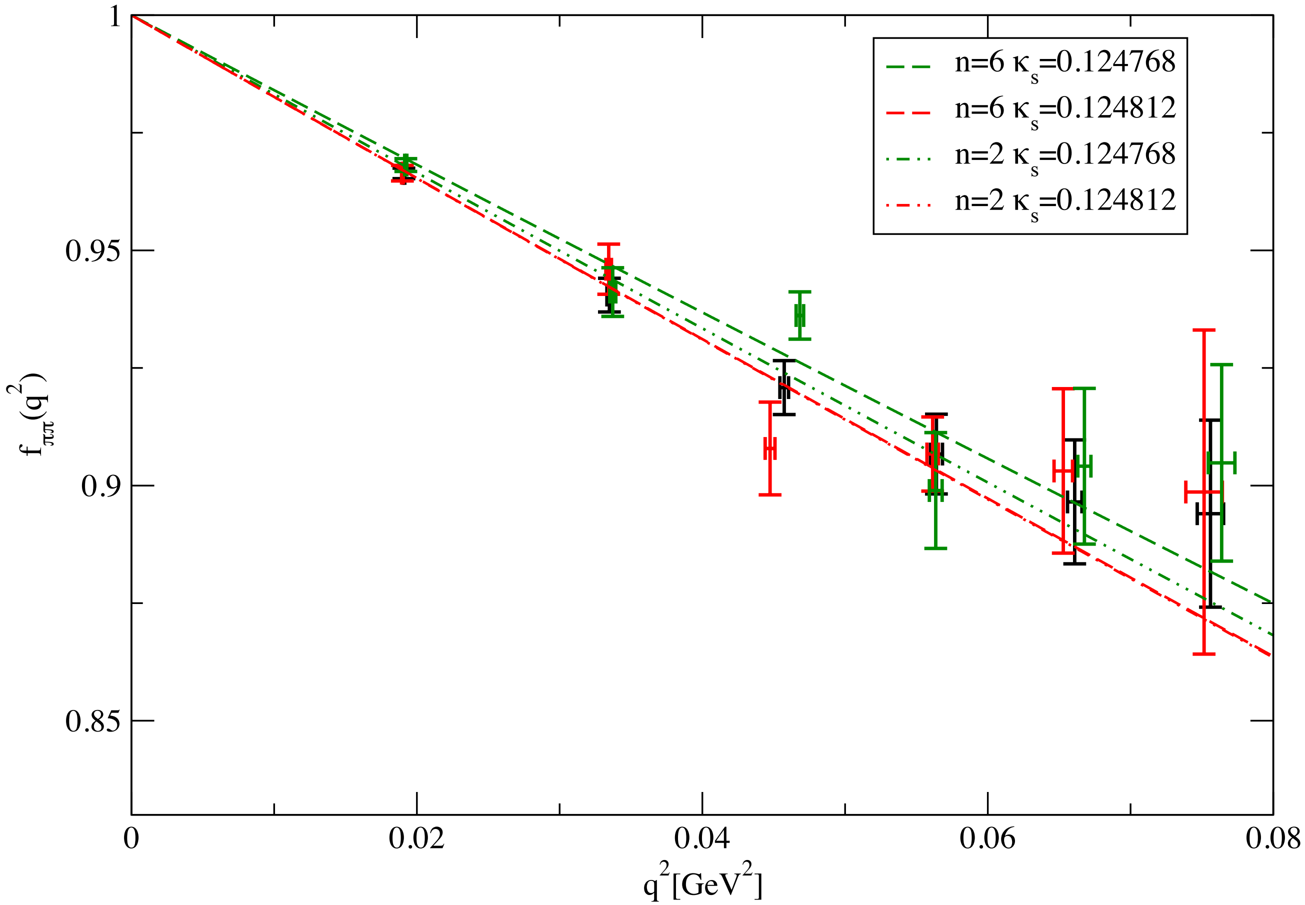
$$l_6 = -0.01216(57)$$
$$\rightarrow \langle r^2 \rangle = 0.407(16)\mathrm{fm}^2$$

$$\langle r^2 \rangle = \frac{-12 l_6}{f^2} - \frac{1}{8\pi^2 f^2}\left(\log\left(\frac{m_\pi^2}{\mu^2}\right) + 1\right)$$

by NLO SU(2)
ChPT formula

NO physical strange mass
extrapolation

preliminary result: NLO SU(2) ChPT fit with m_s reweighted $f_{\pi\pi}(q^2)$



$\kappa_s = 0.124812$

$m_K = 0.523(\mathrm{GeV})$

$f = 0.12875(\mathrm{GeV}) \quad (m_u, m_d \to 0)$

	l_6	$\chi^2/d.o.f$
n=1	-0.01227(72)	
n=2	-0.01199(64)	0.7
n=3	-0.01229(55)	1.4
n=6	-0.01197(59)	0.95
FLAG	-0.01233(127)	

$\kappa_s = 0.124768$

$m_K = 0.526(\mathrm{GeV})$

$f = 0.12956(\mathrm{GeV}) \quad (m_u, m_d \to 0)$

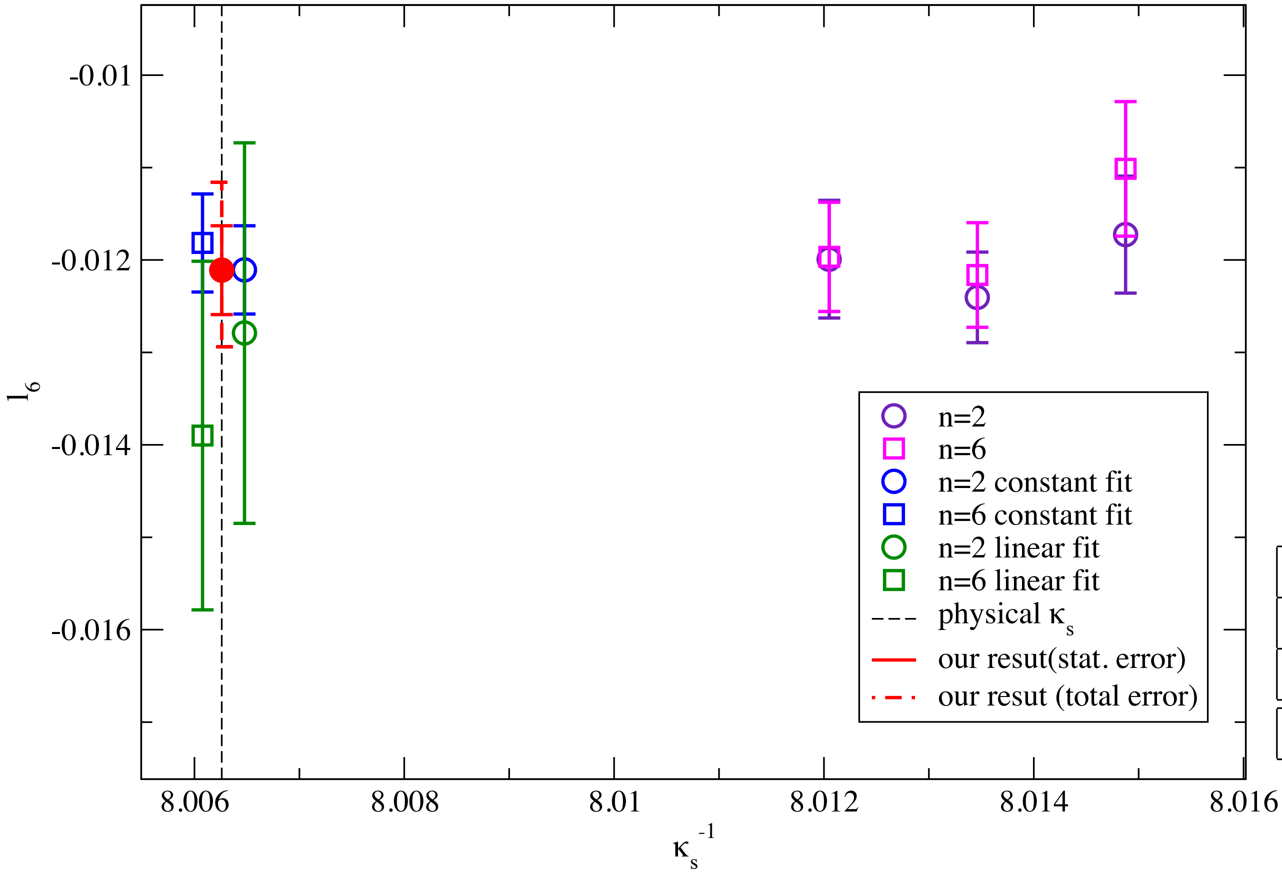
	l_6	$\chi^2/d.o.f$
n=1	-0.01158(62)	
n=2	-0.01173(63)	0.35
n=3	-0.01106(65)	3.0
n=6	-0.01101(73)	1.8
FLAG	-0.01233(127)	

Including third smallest momentum transfer,
$\chi^2/d.o.f$ of these fits increase

→ we decide the result of n=2 as central value
in our analysis of strange mass extrapolation.

preliminary result: physical strange mass extrapolation

$\kappa_s = 0.124902$



In all κ_s

constant fit(blue)

linear fit(green)

$$l_6(\kappa_s^{-1}) = c_1\kappa_s^{-1} + c_0$$

	l_6 (const.)	l_6 (linear)
n=2	-0.01211(48)	-0.01279(206)
n=6	-0.01182(53)	-0.01390(189)
FLAG	-0.01233(127)	

our result with systematic error

$$l_6 = -0.01211(48)\binom{+83}{0}\binom{0}{-68}$$

1st error : statistical

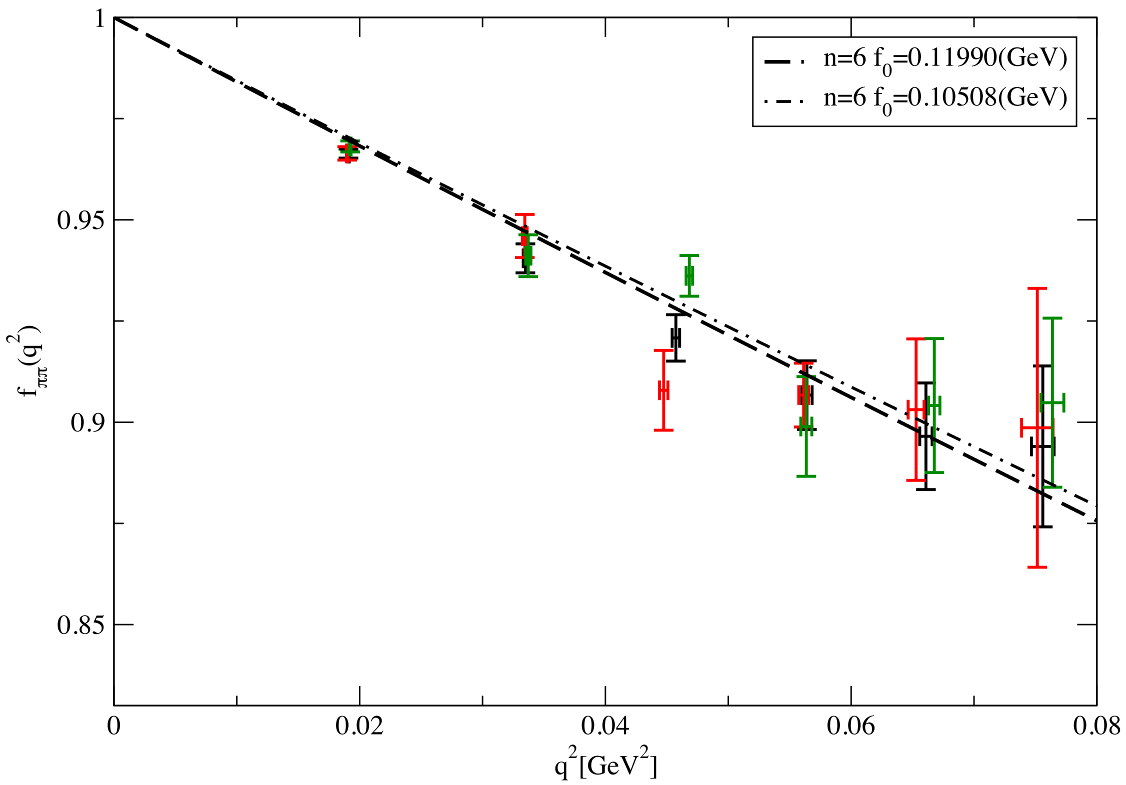
2nd : combined error by choice 2pt fit range & choice of q fit range of NLO SU(2) ChPT

3rd : model dependance of strange mass extrapolation

preliminary result: NLO SU(3) ChPT fit

$$f_{\pi\pi}^{SU(3)}(q^2) = 1 + \frac{1}{f_0^2}\left[-4L_9(\mu)q^2 + 4H(m_\pi^2, q^2, \mu^2) + 2H(m_K^2, q^2, \mu^2)\right],$$

$$\langle r^2 \rangle = -6\frac{df(q^2)}{dq^2}\bigg|_{q^2=0} = \frac{24L_9(\mu)}{f_0^2} - \frac{1}{8\pi^2 f_0^2}\left(\log\left(\frac{m_\pi^2}{\mu^2}\right)+1\right) - \frac{1}{16\pi^2 f_0^2}\left(\log\left(\frac{m_K^2}{\mu^2}\right)+1\right).$$



NLO SU(3) ChPT fit works in our data

our result is consistent with other calculations.

but there is the large uncertainty of the decay constant

previous SU(3) ChPT studies

$$1.078 \le \frac{f}{f_0} \le 1.229$$

$$0.10508 \le f_0 \le 0.11990(\text{GeV})$$

$$(m_u, m_d, m_s \to 0)$$

$f_0 = 0.10508$(GeV)	$1000L_9$
$3m_s$(n=6)	$3.603(0.656)(^{+1.496}_{0})$
JLQCD(15A)	$4.6(1.1)(^{+0.1}_{-0.5})(0.4)$
JLQCD(14)	2.4(0.8)(1.0)
RBC/UKQCD(08A)	3.08(23)(51)

$$\chi^2/d.o.f \approx 0.7$$

there is one fit parameter

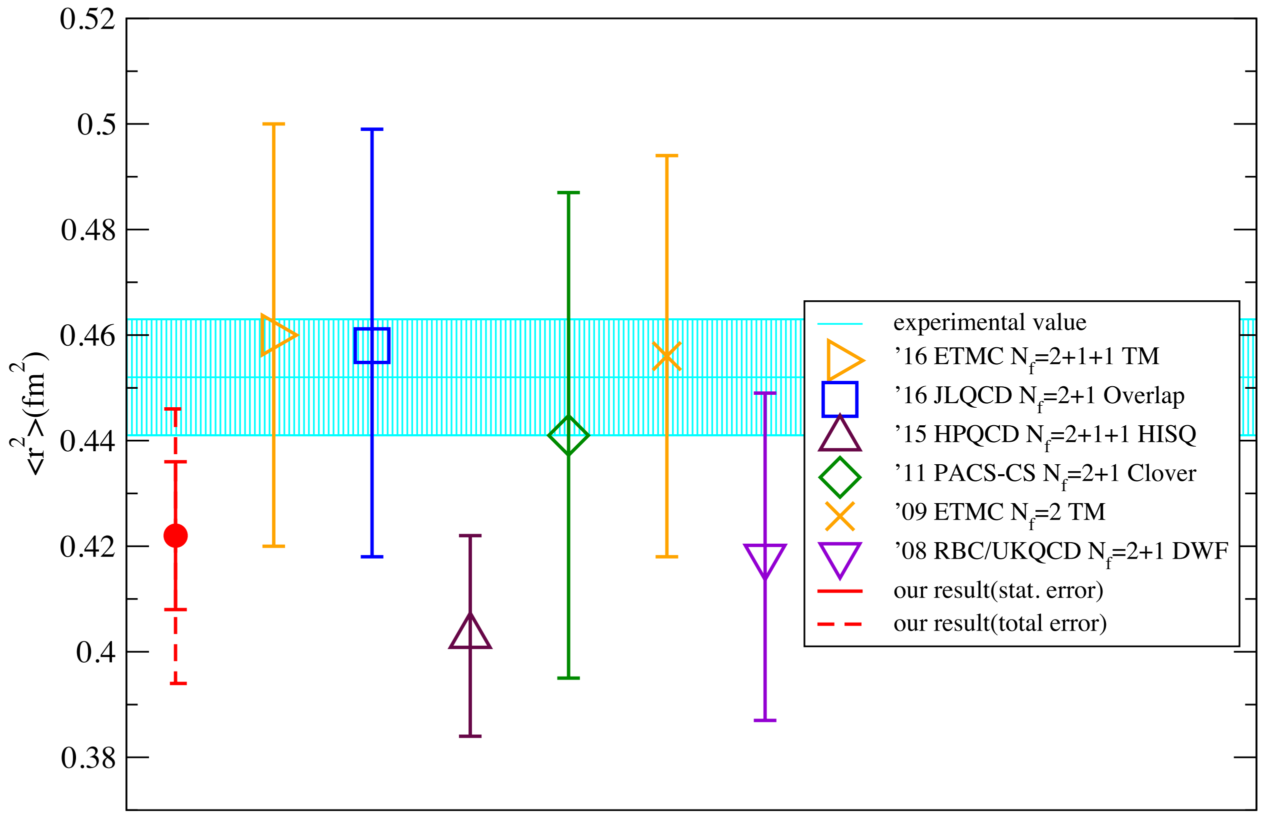
L_9: Low Energy Constant

$$\langle r^2 \rangle = 0.410(71)(^{+118}_{0})(\text{fm})$$

extrapolation with NLO SU(3) ChPT formula to the physical π, K mass

comparison with other calculation at physical point

$m_\pi = 0.13957(\mathrm{GeV})$



our result
with systematic error

$$\langle r^2 \rangle = 0.422(14)\binom{0}{-24}\binom{+20}{0}(\mathrm{fm}^2)$$

extrapolation with
NLO SU(2) ChPT formula

$$\langle r^2 \rangle = \frac{-12 l_6}{f^2} - \frac{1}{8\pi^2 f^2}\left(\log\left(\frac{m_\pi^2}{\mu^2}\right) + 1\right)$$

+extrapolation to the
physical strange mass

consistent with previous results and
consistent with experiment

summary

We calculated pion electromagnetic form factor
in N=2+1 Lattice QCD

- $m_\pi = 0.145(\mathrm{GeV}), L = 8.1(\mathrm{fm})$
- fits including wrapping around effect to extract form factor
- strange quark mass reweighting technique
- NLO SU(2) ChPT

preliminary result: $\langle r^2 \rangle = 0.422(14)(^{0}_{-24})(^{+20}_{0})(\mathrm{fm}^2)$

(NLO SU(2) ChPT fit at physical pion mass and physical strange quark mass)
consistent with experiment and consistent previous results

- NLO SU(3) ChPT fit works in our data

future works

- curvature by NNLO SU(2) ChPT fit
- excited state contribution
- estimate error from finite lattice spacing
- higher precision calculation

(using finer and larger lattice, on the physical point quark masses, including QED)

- back up

Charge radius

In non-relativistic limit , ff is regarded as <u>3D Fourier transformation of charge density</u>

$$f_{\pi\pi}(q^2) = \int d^3x e^{i\vec{q}\cdot\vec{x}} \rho_\pi(\vec{x})$$

Assuming spherical symmetry of density and $\int d^3x \rho_\pi(x) = 1$

Expand ff by $\vec{q}\cdot\vec{x} \ll 1$

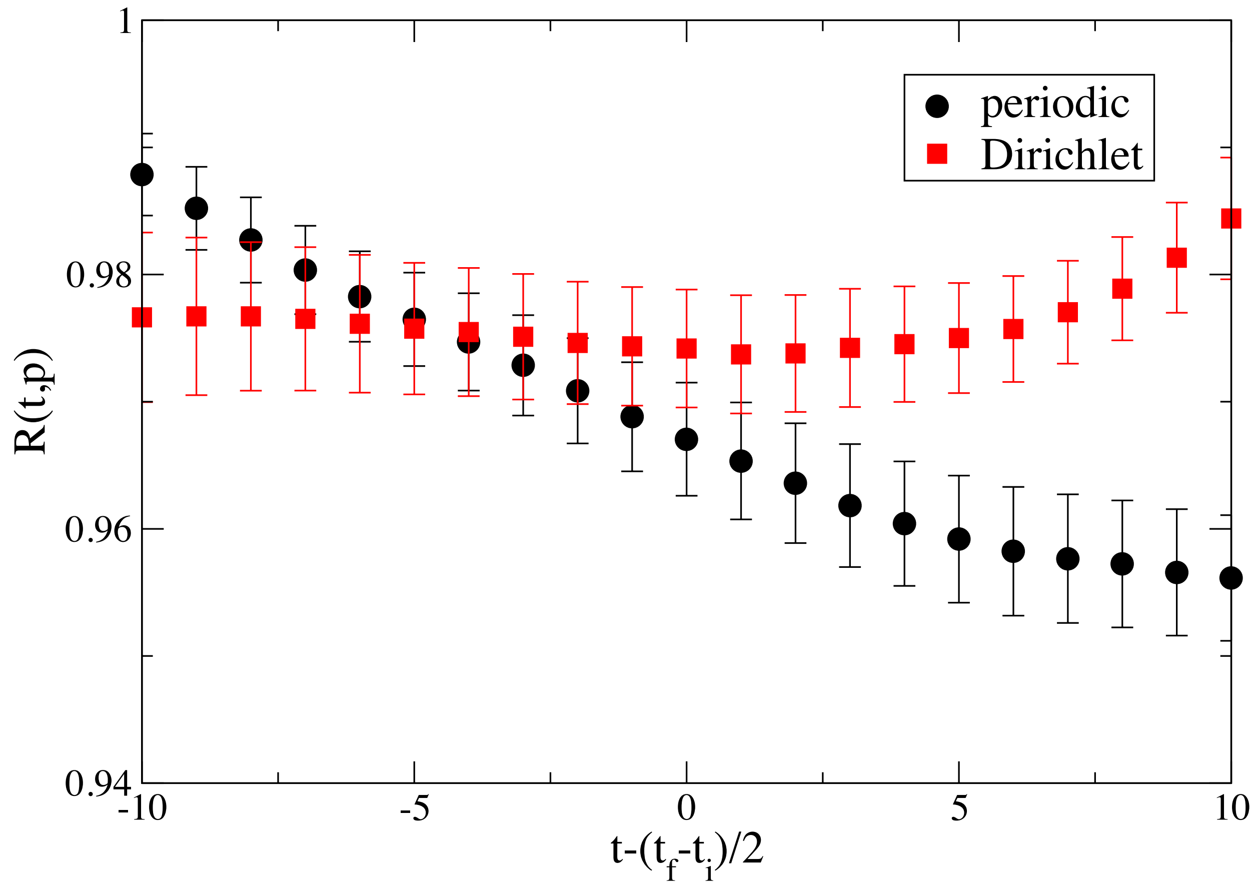
$$\begin{aligned} f_{\pi\pi}(q^2) &= \int_0^\infty r^2 dr \int_{-1}^{1} d(cos\theta) \int_0^{2\pi} d\phi (1 - \frac{1}{2} q^2 \cdot r^2 cos^2\theta + \ldots) \rho_\pi(r) \\ &= 1 - \frac{1}{6} q^2 \int_0^\infty 4\pi r^2 dr r^2 \rho_\pi(r) + \ldots \end{aligned}$$

$$f_{\pi\pi}(q^2) = 1 - \frac{\langle r_\pi^2 \rangle}{6} q^2 + \ldots$$

We can consider mean square of charge radius as <u>1st differential coefficient of ff</u> → $\langle r_\pi^2 \rangle = -6 \frac{d}{dq^2} f_{\pi\pi}(q^2) \Big|_{q^2=0}$

(so we need large box size for small momentum)

R in Dirichlet BC



Dirichlet

$11conf, q^2 \approx 0.019(\mathrm{GeV}^2)$

$|t_f - t_i| = 25$

periodic

$19conf, q^2 \approx 0.019(\mathrm{GeV}^2)$

$|t_f - t_i| = 36$

there is no t dependence in Dirichlet BC

Extraction of form factor with heavy pion mass

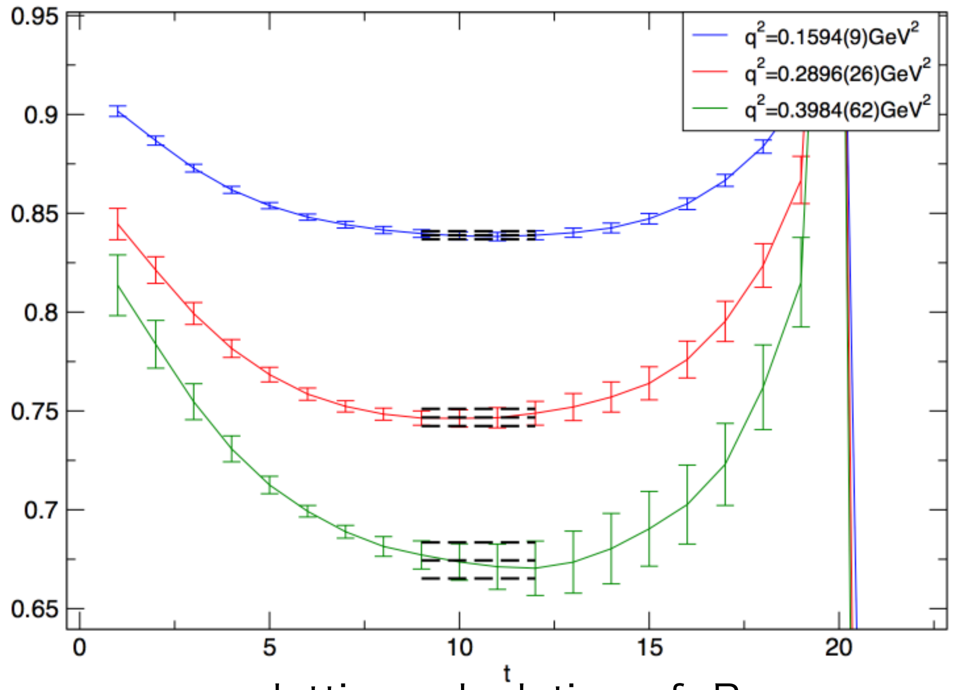
heavy pion mass

$m_\pi = 0.51(\mathrm{GeV}), L = 2.9(\mathrm{fm})$

when $0 \ll t \ll t_f$

ff is extracted by fitting

plateau of R as constant

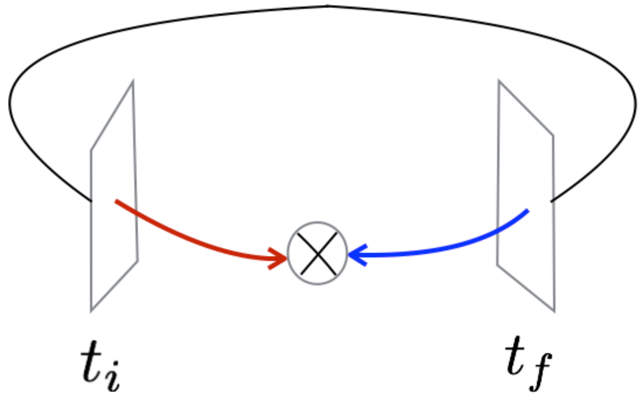


lattice calculation of R

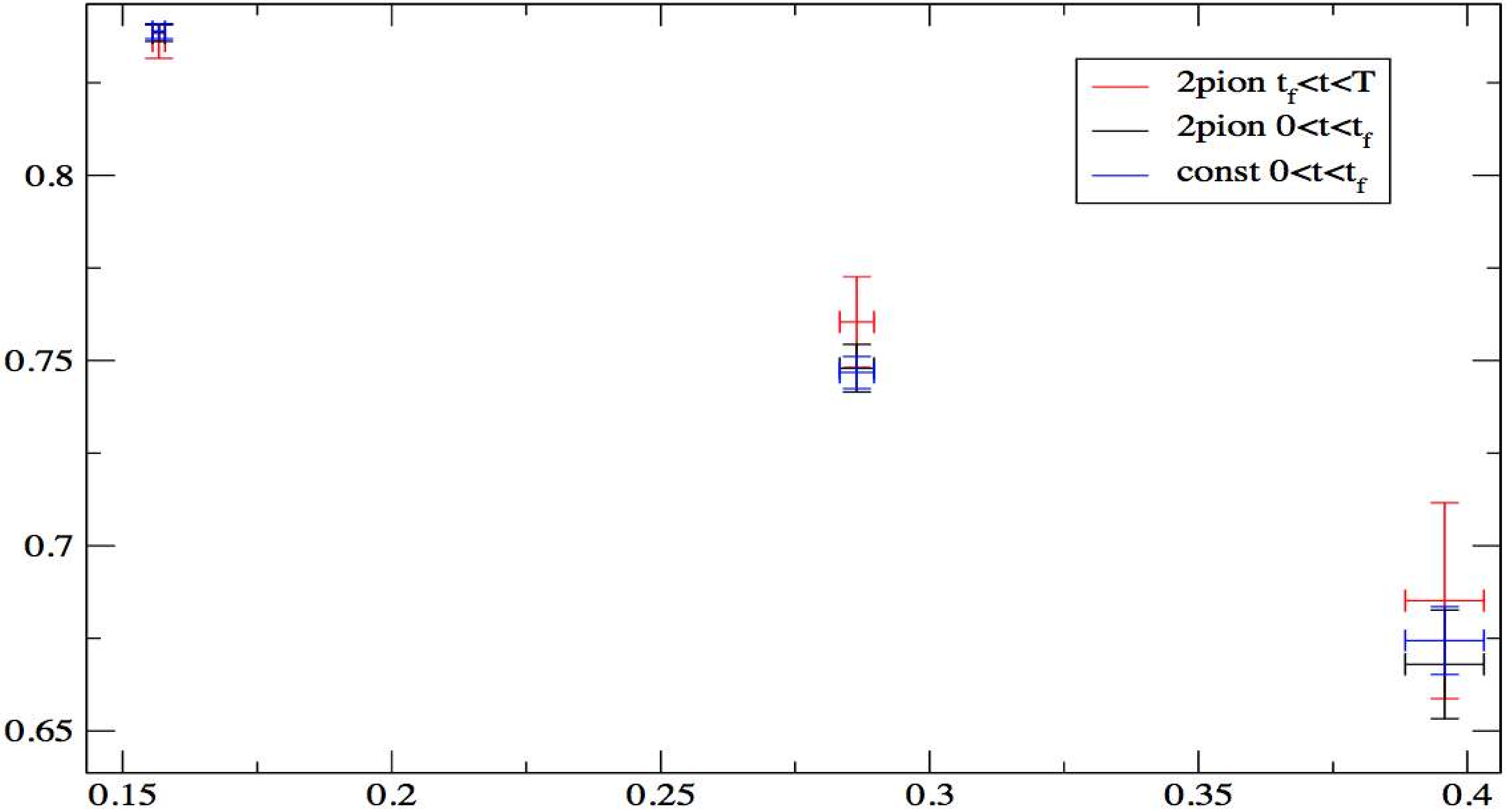
Diagrammatically,

one pion from source

the other pion from sink



(using periodic boundary condition in temporal direction)



	f_0	$f_1 \times 10^6$
q_1^2	0.8362(47)	1.350(172)
q_2^2	0.7603(122)	1.122(215)
q_3^2	0.6851(264)	1.0342(366)

	f_0	f_1
q_1^2	0.8383(22)	0.4714 (4104)
q_2^2	0.7479 (64)	-0.7725(1.7470)
q_3^2	0.6679(147)	3.9180(3.6210)

	f_0
q_1^2	0.8388(20)
q_2^2	0.7467(43)
q_3^2	0.6744(91)

Calculation 3-point function

- connected 3-point function is consist of 3 quark propagators

$$C_{\pi V \pi} = Z_V \left\langle 0 \middle| O_\pi(t_f, \vec{p}_f) V_4(t, \vec{q}) O_\pi^\dagger(t_i = 0, \vec{p}_i) \middle| 0 \right\rangle$$

- 1 random wall source

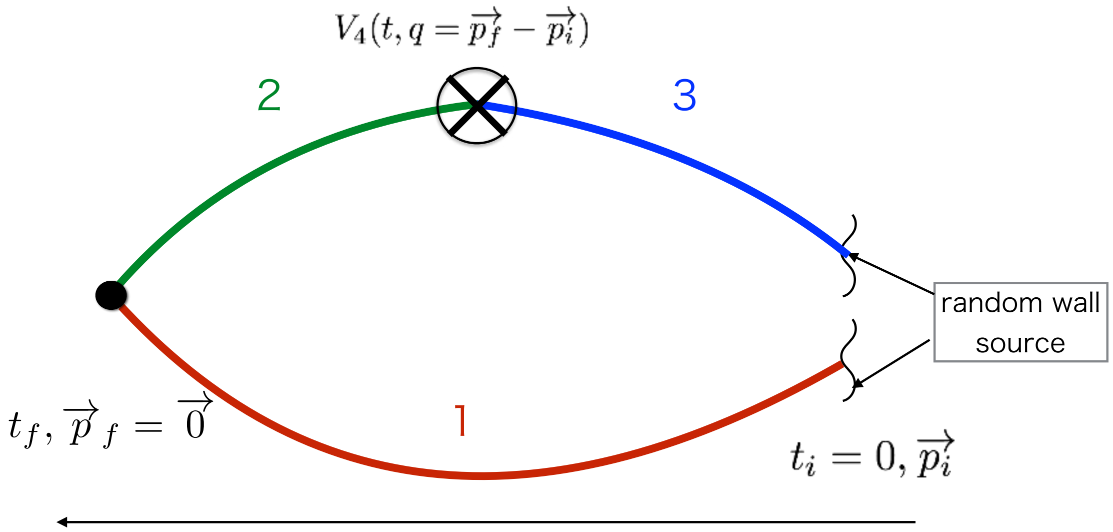
$$\vec{p}_i = \vec{0}$$

- 2 sequential source

$$\vec{p}_f = \vec{0}$$

- 3 random wall source

$$\vec{p}_i \neq \vec{0}$$



random wall source (A,B:color&spinor index)

$$\begin{aligned}\eta_B(\vec{y}, t_i) &= \left\{ \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}} \right\} \\ &\in \mathbb{Z}(2) \otimes \mathbb{Z}(2)\end{aligned}$$

$$\lim_{N \to \infty} \frac{1}{N} \sum_{j=0}^{N} \eta_A^j(\vec{x}, t_i) \eta_B^{\dagger j}(\vec{y}, t_i) = \delta(\vec{x} - \vec{y}) \delta_{AB}$$

calculation cost is reduced by random wall source

RBC&UKQCD:JHEP(0807 (2008)112)

disconnected term is vanished by charge symmetry

disconnect diagram of 3-point function

DRAPER and WOLOSHYN,Nucl. Phys., B318 (1989), p. 319-336

$$C^{disc}_{\pi J_\mu \pi}(U) = \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U) + \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U^*)$$

from charge conjugation

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, CUC^{-1} = U^* = (U^\dagger)^T$$

$$C^{disc}_{\pi J_\mu \pi}(U) = -C^{disc}_{\pi J_\mu \pi}(U^*)$$

$$C^{disc}_{\pi J_\mu \pi}(U) = \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U) - \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U) = 0$$

disconnected term is vanished by charge symmetry