Decoupling of charm beyond leading order

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based on 1706.04982

Lattice 2017, Granada, Spain, 21 Jun 2017

Motivation

Do we need to simulate a dynamical charm quark?

- Simulations of QCD_3 with $N_f = 2+1$ (up, down, strange) quarks are simpler than QCD_4 with $N_f = 2+1+1$ (+ charm) quarks
- At energies $E \ll M_{\rm charm} \equiv M_c$ the charm quark decouples. QCD_4 can be described by an effective theory
- The gauge couplings of QCD_4 and QCD_3 can be matched in perturbation theory. This is used in the determination of α_s from QCD_3 simulations [Bruno et al., 1706.03821; see T. Korzec's plenary talk]
- Power corrections are neglected. Is M_c large enough? What is the precision needed to see them?

Motivation

Model study

- To avoid a multi-scale problem and control the continuum limit we study a model, QCD_2 with $N_f = 2$ degenerate quarks of mass M
- Effective theory for $E \ll M$ is a Yang–Mills theory ($N_f = 0$, $M = \infty$) at leading order
- Beyond leading order there are power corrections starting at $\propto M^{-2}$
- In a previous work [Bruno et al., 1410.8374] we simulated masses up to $M \approx M_c/2$ and estimated the power corrections to be at the permille level
- However we could not see a behaviour $\propto M^{-2}$

Effective field theory

Expansion in $(E/M)^n$: EFT for $E \ll M$ [Weinberg, Phys. Lett. B91 (1980)]

- Only virtual effects of quark with mass M
  No states with explicit heavy quark (the HQET part)
- Effective Lagrangian (here for $N_\mathrm{f} = 2 \to N_\mathrm{f} = 0$)

$$\begin{aligned} \mathcal{L}_\mathrm{dec} &= \mathcal{L}_\mathrm{YM} + \frac{1}{M^2}\mathscr{L}_6 + \mathrm{O}\left(\frac{\Lambda^4}{M^4}\right) \\ \mathscr{L}_6 &= \omega_1 \mathrm{tr}\left\{D_\mu F_{\nu\rho} D_\mu F_{\nu\rho}\right\} + \omega_2 \mathrm{tr}\left\{D_\mu F_{\mu\rho} D_\nu F_{\nu\rho}\right\} \end{aligned}$$

  Due to gauge invariance no dimension 5 operator
- $\mathcal{L}_\mathrm{YM}$ has one free parameter, the gauge coupling
- Matching means specifying a value of the coupling at some scale or equivalently the Λ parameter

Effective field theory

Matching

- Relation between the Λ parameters

$$\Lambda_{\rm YM}(M,\Lambda) \;=\; P(M/\Lambda)\,\Lambda$$

where $\Lambda \equiv \Lambda^{(N_{\rm f}=2)}$. We use the $\overline{\rm MS}$ scheme

- Consider a low energy observable $m^{\rm had}$. After matching

$$m^{\rm had}(M) \;=\; m^{\rm had}_{\rm YM} + {\rm O}(\Lambda^2/M^2)$$

$m^{\rm had}$ can be a hadronic scale such as $1/\sqrt{t_0}$ or $1/r_0$

- Note: the value $m^{\rm had}_{\rm YM}$ in the Yang–Mills theory depends on M through the matching since $c = m^{\rm had}_{\rm YM}/\Lambda_{\rm YM}$ is a pure number

Effective field theory

Ratios

Consider two hadronic scales, $m^{\rm had,1}(M)$ and $m^{\rm had,2}(M)$, whose values in the Yang–Mills theory are $m^{\rm had,1}_{\rm YM}$ and $m^{\rm had,2}_{\rm YM}$ respectively. A consequence of matching is

$$R(M) \;=\; \frac{m^{\rm had,1}(M)}{m^{\rm had,2}(M)} = \frac{m^{\rm had,1}_{\rm YM}}{m^{\rm had,2}_{\rm YM}} + {\rm O}(\Lambda^2/M^2)$$

Note that

$$\frac{m^{\rm had,1}_{\rm YM}}{m^{\rm had,2}_{\rm YM}} = \frac{m^{\rm had,1}_{\rm YM}/\Lambda_{\rm YM}}{m^{\rm had,2}_{\rm YM}/\Lambda_{\rm YM}} = \frac{c_1}{c_2}$$

is a pure number, matching of the couplings is irrelevant

$$R(M) \;=\; R(\infty) + k\Lambda^2/M^2$$

Hadronic scales

Scales t_0, t_c and w_0 from the Wilson flow

Scale t_0 [Lüscher, 1006.4518]

$$\mathcal{E}(t_0) = 0.3\,, \quad \mathcal{E}(t) = t^2 \left\langle E(x,t) \right\rangle\,, \quad E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

Similarly

$$\mathcal{E}(t_c) = 0.2$$

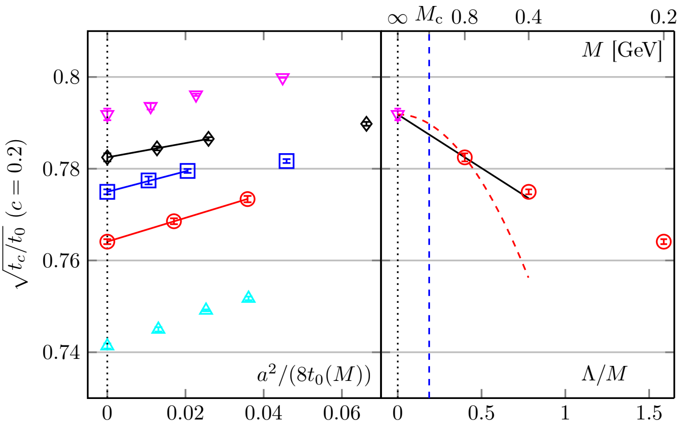
Scale w_0 [Borsanyi et al., 1203.4469]

$$w_0^2 \mathcal{E}'(w_0^2) = 0.3\,, \quad \mathcal{E}'(t) = \frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}(t)$$

Scale r_0 from the static force [Sommer, hep-lat/9310022]

$$r^2 F(r)\big|_{r=r_0} = 1.65\,, \quad F(r) = V'(r)$$

$R = \sqrt{t_c/t_0}$ from [Bruno et al., 1410.8374]



Masses up to $M_{\text{charm}}/2$, looks like linear behavior $R(M) - R(\infty) \propto \Lambda/M$. Masses are not large enough to see M^{-2} (dashed line) expected from EFT.

Simulations

The data of [Bruno et al., 1410.8374] were produced from simulations of $N_\mathrm{f} = 2$ O(a) improved Wilson quarks with plaquette gauge action

New ensembles

- We use $N_\mathrm{f} = 2$ twisted mass Wilson quarks at maximal twist with clover term and plaquette gauge action. We simulated new masses $M_\mathrm{c}/\Lambda = 4.8700$ (charm) and $M/\Lambda = 5.7781$ and also quenched ensembles ($M = \infty$)
- We use open boundary conditions and the openQCD simulation package
- We measure

$$R = \sqrt{t_c/t_0}\,,\ \sqrt{t_0}/w_0\,,\ r_0/\sqrt{t_0}$$

Ensembles at $M = M_c$, $1.2M_c$ and quenched ($M = \infty$)

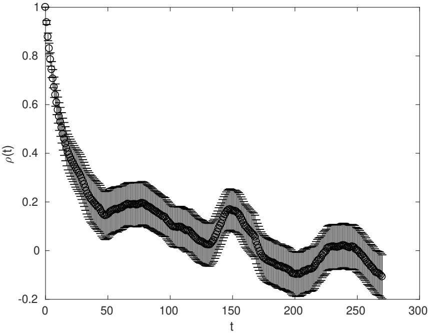
β	a [fm]	A	BC	$T \times L^3$	$M/\Lambda_{\overline{MS}}$	t_0/a^2	kMDU	τ_{exp} [kMDU]
5.6	≈ 0.042	tm	o	192×48^3	4.8700	6.609(15)	2.0	0.08
				192×48^3	5.7781	6.181(11)	2.1	0.08
5.7	≈ 0.036	tm	o	120×32^3	4.8703	9.104(36)	17.2	0.14
				192×48^3	5.7781	8.565(31)	2.7	0.12
5.88	≈ 0.028	tm	o	192×48^3	4.8700	14.622(62)	23.1	0.24
				120×32^3	5.7781	14.916(93)	59.9	0.23
6.0	≈ 0.023	tm	o	192×48^3	4.8700	22.39(12)	22.4	0.36
6.100	0.0778	–	o	120×32^3	∞	4.4329(32)	64.0	0.05
6.340	0.0545	–	o	120×32^3	∞	9.034(29)	20.1	0.13
$*$				120×24^3	∞	9.002(31)	60.9	0.13
6.672	0.0350	–	o	192×48^3	∞	21.924(81)	73.9	0.35
6.900	0.0261	–	o	192×64^3	∞	39.41(15)	160.2	0.65

Lattice spacing for $N_f = 2$ is determined from the scale L_1 [Blossier et al., 1203.6516, Fritzsch et al., 1205.5380]; for $N_f = 0$ from r_0

$Lm_{PS} \gg 4$; * finite volume study

Autocorrelation function of t_0

Aurocorrelation function of t_0 (units of 16 MDU) for $N_\mathrm{f} = 2$, $\beta = 6.0$, $M = M_\mathrm{c}$



Fit $A\exp(-t/\tau_\mathrm{exp})$ of the tail gives $\tau_\mathrm{exp} = 357$ MDU
On our ensembles we find $\tau_\mathrm{exp} = -32(23) + 17.4(2.8)\,t_0/a^2$
as expected with open boundary conditions

Continuum extrapolations

Fits

- From Symanzik's theory we expect O(a^2) cut-off effects. We fit our data to

$$R(a, M/\Lambda, \mathrm{A}) = R^{\mathrm{cont}}(M/\Lambda) + \frac{a^2}{t_0} c(M/\Lambda, \mathrm{A})$$

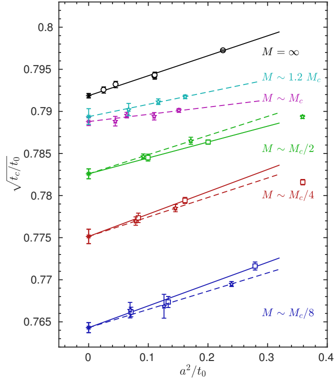
  where A is the action, "W" for Wilson, "tm" for twisted mass and "q" for quenched

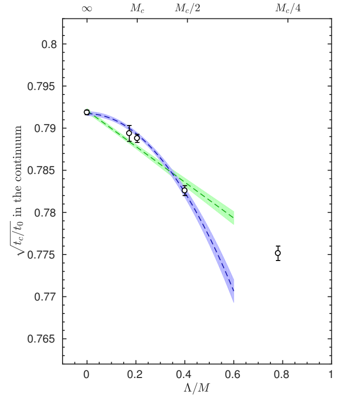
- We apply a cut, $a^2/t_0(M) < 0.32$, to the fits
- For a check we also do a global fit

$$R(a, M/\Lambda, \mathrm{A}) = R^{\mathrm{cont}}(M/\Lambda) + \frac{a^2}{8t_0}\left[c(\mathrm{A}) + \alpha(\mathrm{A})\frac{M}{\Lambda} + \beta(\mathrm{A})\frac{M^2}{\Lambda^2}\right]$$

  with M-dependent slopes

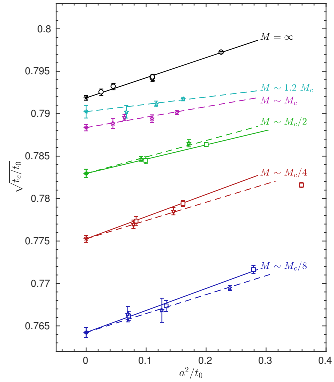
$R = \sqrt{t_c/t_0}$

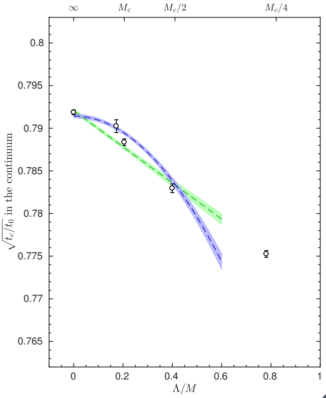




The dashed line in the blue band represents the EFT prediction $R(M) = R(\infty) + k\Lambda^2/M^2$ fitted through $M/\Lambda \geq 2.50$ data points. It has a $\chi^2/\mathrm{dof} = 1.75/2$. A linear fit in M^{-1} (green band) has a far worse $\chi^2/\mathrm{dof} = 9.55/2$

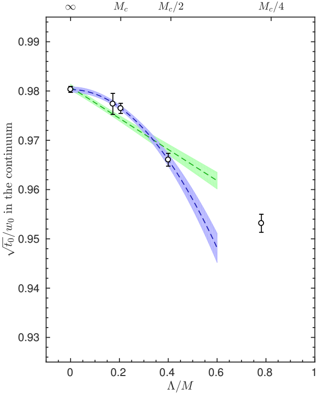
$R = \sqrt{t_c/t_0}$ (global fit)

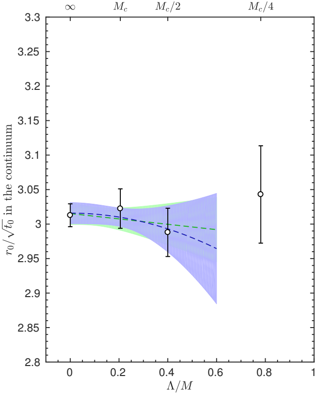




The global fit yields consistent continuum values and favors M^{-2} behavior

$R = \sqrt{t_0}/w_0\,,\ r_0/\sqrt{t_0}$





The ratio $\sqrt{t_0}/w_0$ also strongly favors the M^{-2} behavior

The precision of the ratio $r_0/\sqrt{t_0}$ is not enough to resolve the power corrections

Conclusions

- We studied the decoupling of a charm quark non-perturbatively in QCD with two heavy quarks of mass M
- By comparing ratios of hadronic flow scales to Yang–Mills we are able to measure effects of a dynamical charm quark which are of 2 permille size
- Our data can be very well fitted by the EFT prediction for the power corrections $\propto M^{-2}$ down to masses $M_{\rm c}/2$

Outlook

Compute effects of dynamical charm in other observables such as strong coupling from the static force, charmonium, f_{D_s}, ...