HQET form factors for $B_s \to K\ell\nu$ decays beyond leading order

Mateusz Koreń

NIC, DESY Zeuthen

In collaboration with D. Banerjee, H. Simma, R. Sommer

Lattice 2017, Granada, 19-06-2017

Table of contents

B physics without Perturbation Theory

- The goal: Calculate the form factor of the $\mathrm{B_s} \to \mathrm{K}\ell\nu$ decay.
- The QCD matrix element in the rest-frame of the $\mathrm{B_s}$:

$$(2m_{\mathrm{B_s}})^{-1/2}\langle \mathrm{K}(p_\mathrm{K})|V^0(0)|\mathrm{B_s}(0)\rangle = h_{\parallel}(E_\mathrm{K})$$
$$(2m_{\mathrm{B_s}})^{-1/2}\langle \mathrm{K}(p_\mathrm{K})|V^k(0)|\mathrm{B_s}(0)\rangle = p_\mathrm{K}^k h_{\perp}(E_\mathrm{K})$$

- And our way to obtain it:
  1. Use *HQET* for the b quark.
  2. Obtain the bare ground-state matrix elements $\langle \mathrm{K}|V^k|\mathrm{B_s}\rangle$.
  3. *Non-perturbatively* renormalize the currents in EFT and relate to QCD ("matching").
  4. Take their *continuum limit*.
  5. Extrapolate to physical quark masses in Nature.
  6. Map out the q^2 dependence.

Order by order in $1/m_h$ (steps 2–4)

HQET expansion

- $\mathcal{O}(1/m_h)$ terms in the heavy-quark expansion treated as operator insertions in static correlation functions:

$$\langle O\rangle = \langle O\rangle_{\rm stat} + \omega_{\rm kin}\sum_x \langle O\mathcal{O}_{\rm kin}(x)\rangle_{\rm stat} + \omega_{\rm spin}\sum_x \langle O\mathcal{O}_{\rm spin}(x)\rangle_{\rm stat}$$

$$\mathcal{O}_{\rm kin}(x) = \bar\psi_h(x)\mathbf{D}^2\psi_h(x), \quad \mathcal{O}_{\rm spin}(x) = \bar\psi_h(x)\boldsymbol{\sigma}\cdot\mathbf{B}\psi_h(x)$$

- Additional terms at $\mathcal{O}(1/m_h)$ to vector current:

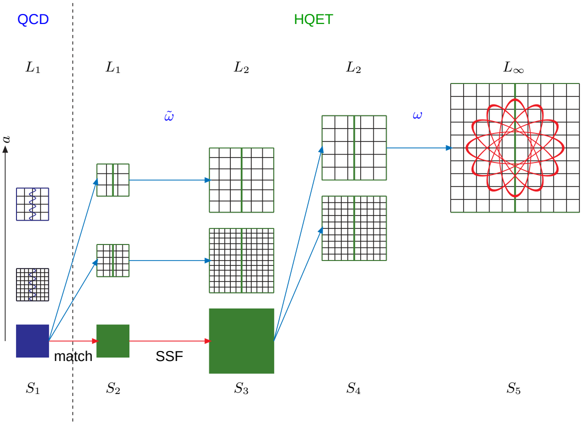
$$V_0^{\rm HQET}(x) = Z_{V_0}^{\rm HQET}\left(V_0^{\rm stat}(x) + \sum_{l=1}^{2}\omega_{0,l}V_{0,l}(x)\right),$$
$$V_k^{\rm HQET}(x) = Z_{V_k}^{\rm HQET}\left(V_k^{\rm stat}(x) + \sum_{l=1}^{4}\omega_{k,l}V_{k,l}(x)\right)$$

where e.g. $V_{k,1}(x) = \bar\psi_\ell(x)\frac{1}{2}(\nabla_i^S - \overleftarrow{\nabla}_i^S)\gamma_i\gamma_k\psi_h(x)$.

- For the final result: *large-volume matrix elements* and *matching coefficients* needed

Non-perturbative matching to QCD

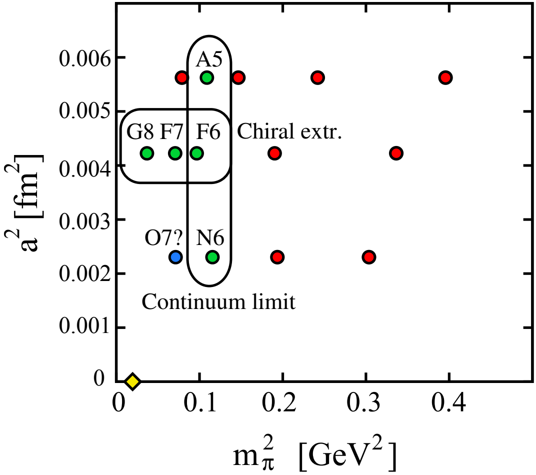
A finite-volume, recursive strategy:



Matching volume $L_1 \approx 0.5 fm \Rightarrow$ relativistic b-quark feasible

[Della Morte et al., PoS (LATTICE2016) 199 and references therein]

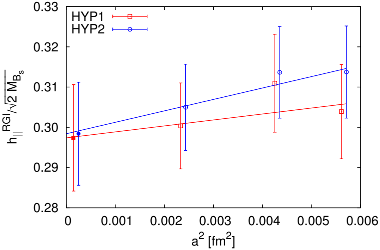
Ensembles: CLS $N_f = 2$

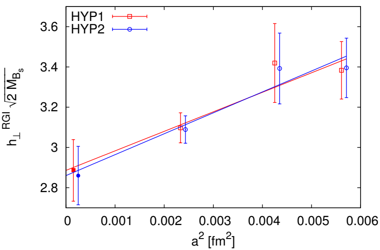


- Twisted b.c. to stay at fixed q^2, $p_K = (1,0,0) \times 0.535\text{GeV}$

Table of contents

Static results: reminder





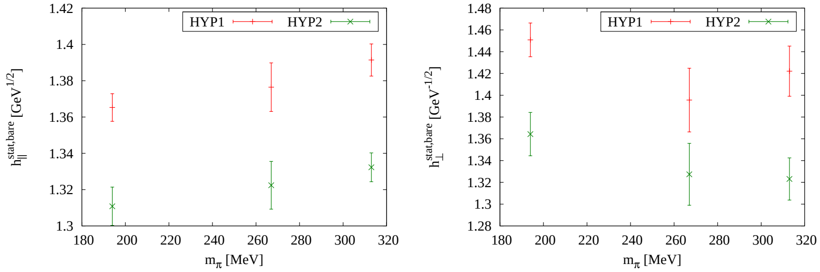
Continuum limit with renormalized static form factors [Bahr, MK et al. 2016]:

$$h_{\|}(E_K) = C_{V_0}(M_b/\Lambda_{\bar{MS}})h_{\|}^{stat,RGI}(E_K)\left[1+\mathcal{O}(1/m_b)\right]$$

$$h_{\perp}(E_K) = C_{V_k}(M_b/\Lambda_{\bar{MS}})h_{\perp}^{stat,RGI}(E_K)\left[1+\mathcal{O}(1/m_b)\right]$$

with C_x being perturbative conversion functions *(to be removed when full matching available)*

Static results: Light quark mass dependence

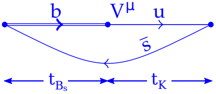


- Chiral trajectory taken so that $m_K/f_K = \text{Const.}$
- No signs of strong m_{light} dependence in the static case

Table of contents

Ratios at order $1/m_h$

- For simplicity only analyze $t_{\mathrm{K}} = t_{\mathrm{B_s}} = t$:



$$\mathcal{R}_{\mathrm{I}}^{\mu}(t) = \mathcal{Z} \frac{\mathcal{C}_{\mu}^{3\mathrm{pt}}(t,t)}{\left[\mathcal{C}^{\mathrm{K}}(2t)\mathcal{C}^{\mathrm{B_s}}(2t)\right]^{1/2}}$$

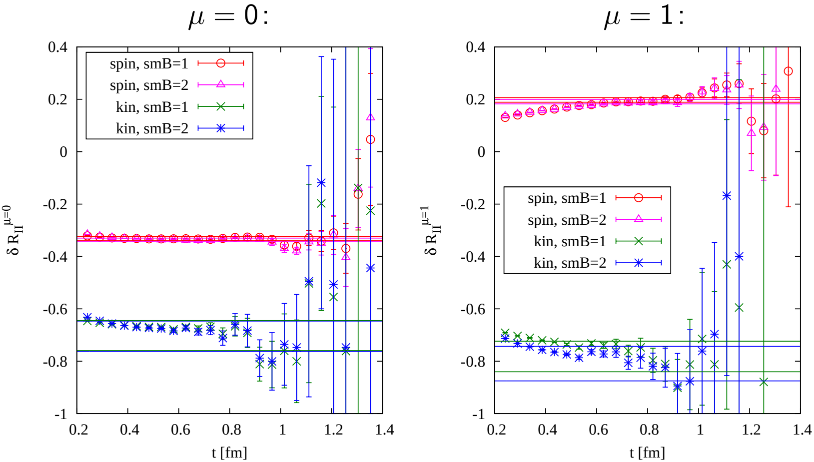
- HQET expansion:

$$\begin{aligned}
\mathcal{R}_{\mathrm{I}}^{\mu}(t) &= \mathcal{Z} \frac{\mathcal{C}_{\mathrm{stat}}^{3\mathrm{pt},\mu}(t,t) + \omega_{\varsigma}\mathcal{C}_{\varsigma}^{3\mathrm{pt},\mu}(t,t)}{\left[\mathcal{C}^{\mathrm{K}}(2t)[\mathcal{C}_{\mathrm{stat}}^{\mathrm{B_s}}(2t) + \omega_{\varsigma}\mathcal{C}_{\varsigma}^{\mathrm{B_s}}(2t)]\right]^{1/2}} \\
&= \mathcal{Z}\mathcal{R}_{\mathrm{I}}^{\mathrm{stat},\mu}(t)\left[1 + \omega_{\varsigma}\left(\frac{\mathcal{C}_{\varsigma}^{3\mathrm{pt},\mu}(t,t)}{\mathcal{C}_{\mathrm{stat}}^{3\mathrm{pt},\mu}(t,t)} - \frac{1}{2}\frac{\mathcal{C}_{\varsigma}^{\mathrm{B_s}}(2t)}{\mathcal{C}_{\mathrm{stat}}^{\mathrm{B_s}}(2t)}\right)\right] \\
&= \mathcal{Z}\mathcal{R}_{\mathrm{I}}^{\mathrm{stat},\mu}(t)\left[1 + \omega_{\varsigma}\delta^{\varsigma}\mathcal{R}_{\mathrm{I}}^{\mu}(t)\right]
\end{aligned}$$

- In practice, better signal obtained with

$$\mathcal{R}_{\mathrm{II}}^{\mu}(t) = \mathcal{Z} \frac{\mathcal{C}_{\mu}^{3\mathrm{pt}}(t,t)}{\left[\mathcal{C}^{\mathrm{K}}(t)\mathcal{C}^{\mathrm{B_s}}(t)\right]^{1/2}} e^{(E_{\mathrm{B_s}}^{\mathrm{eff}} + E_{\mathrm{K}}^{\mathrm{eff}})\frac{t}{2}}$$

$1/m_h$ ratios, fixed $a \approx 0.048$ fm: kin and spin insertions



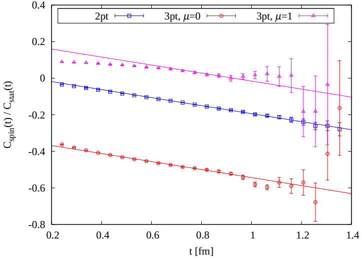
Kin and spin insertions for two B smearings. Fit bands explained on the next slide.

Kin and spin insertions: fit procedure

- Neglecting excited states one has:

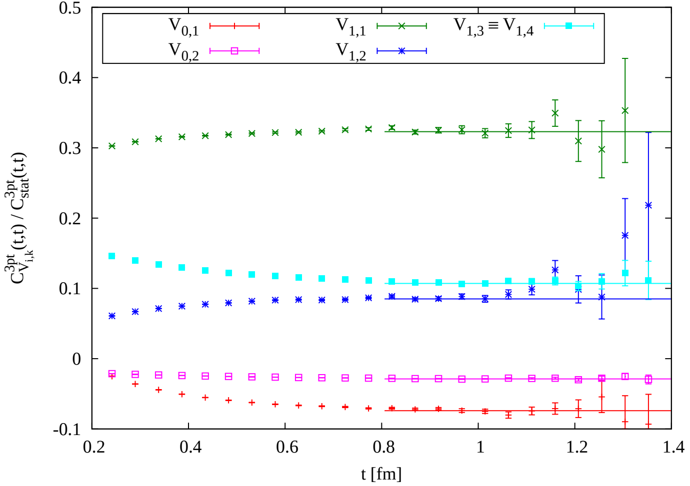
$$\frac{\mathcal{C}_{\varsigma}^{B_s}(t)}{\mathcal{C}_{\mathrm{stat}}^{B_s}(t)} = \rho_{\varsigma}^{\mathrm{2pt}} - E_{\varsigma} t + \cancel{\mathcal{O}(te^{-\Delta E t})}$$

$$\frac{\mathcal{C}_{\varsigma}^{\mathrm{3pt},\mu}(t,t)}{\mathcal{C}_{\mathrm{stat,j}}^{\mathrm{3pt},\mu}(t,t)} = \rho_{\varsigma}^{\mathrm{3pt},\mu} - E_{\varsigma} t + \cancel{\mathcal{O}(te^{-\Delta E t})}$$



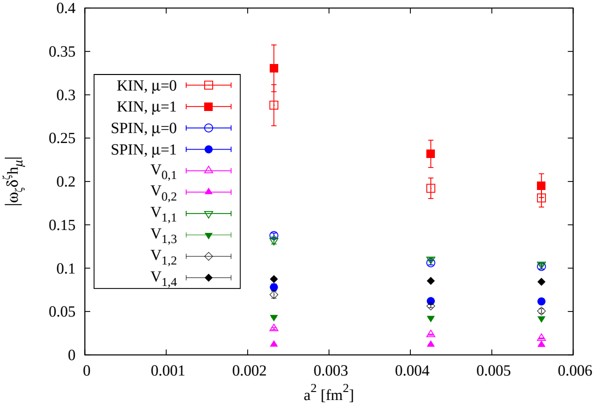
- For increased precision: parameters are obtained by fitting the equations simultaneously for both $\mu = 0, 1$ in the range [0.8 fm, 1.4 fm].
- Final result is then given by: $\delta^{\varsigma}\mathcal{R}_{\mathrm{II}}^{\mu} = \rho_{\varsigma}^{\mathrm{3pt},\mu} - \frac{1}{2}\rho_{\varsigma}^{\mathrm{2pt}}$

$1/m_h$ ratios, fixed $a \approx 0.048$ fm: current insertions



Current corrections have different size but all can be determined rather precisely.

Ratios, lattice spacing dependence



Absolute values of $1/m_h$ ratios with tree-level matching coefficients.

Divergence in a will be removed with the non-perturbative matching results:

$$\log h_\mu = \log \mathcal{Z}_V^{\mathrm{HQET}} + \log h_\mu^{\mathrm{stat}} + \omega_\varsigma \delta^\varsigma h_\mu$$

Table of contents

Summary & Outlook

- $N_f = 2$ large-volume matrix elements now available including $1/m_h$ terms
- Precision limited by signal to noise problem
- Most challenging: $\mathcal{O}_{kin}$ insertions, contributing 2.5% error at $a \approx 0.05$ fm

TODO:

- Finish matching
- Then: continuum limit + physical mass extrapolation
- Outlook on future improvements:
  - Better smearing [Bali et al., 2016]
  - Multilevel [Ce, Giusti, Schaefer 2016]

Thank you for your attention!