Temperature-dependence of the QCD topological susceptibility

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work with

Borsanyi, Fodor, Guenther, Kampert, Katz, Kawanai, Mages, Pasztor, Pittler, Redondo, Ringwald, Szabo

Based on

Calculation of the axion mass based on high-temperature QCD
Nature **539** 69 (2016).

Lattice 2017, June 22, 2017

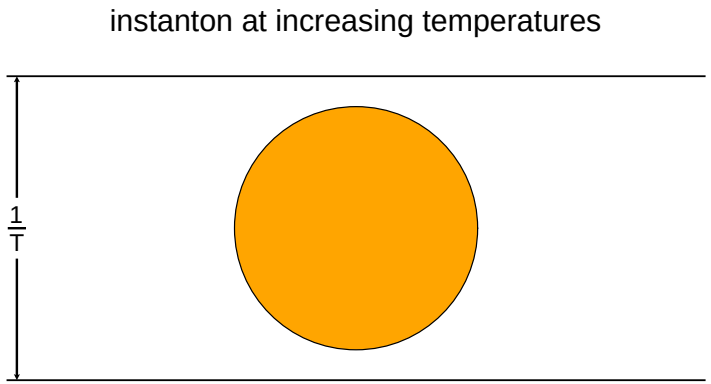
Main goal

Compute QCD topological susceptibility $\chi(T)$

- Temperature range: $0 < T < 2\text{GeV}$
- Physical quark masses (m_u, m_d, m_s, m_c)
- Continuum limit
- Using
  - $N_f = 2+1+1$, with isospin splitting correction
  - staggered and overlap quarks
  - Lattices with $N_t = 8, 10, 12, 16, 20$

Physics to be captured

- Typical instanton size $1/T$ [Callan, Dashen, Gross '76]



- Dilute gas of small ($r \approx 1/T$) instantons remain
- Zero modes in the light quark det suppress topology
- $\Rightarrow$ $\chi(T)$ falls sharply above T_c

Challenges for the simulation

1) Unusually large cut-off effects

- Small-instanton zero modes badly captured by lattice Dirac operator
- Higher Q sectors not properly suppressed
- Cut-off effects much larger at higher T
- Solution: identify would-be zero eigenvalues and shift them to zero $\rightarrow$ reweighting

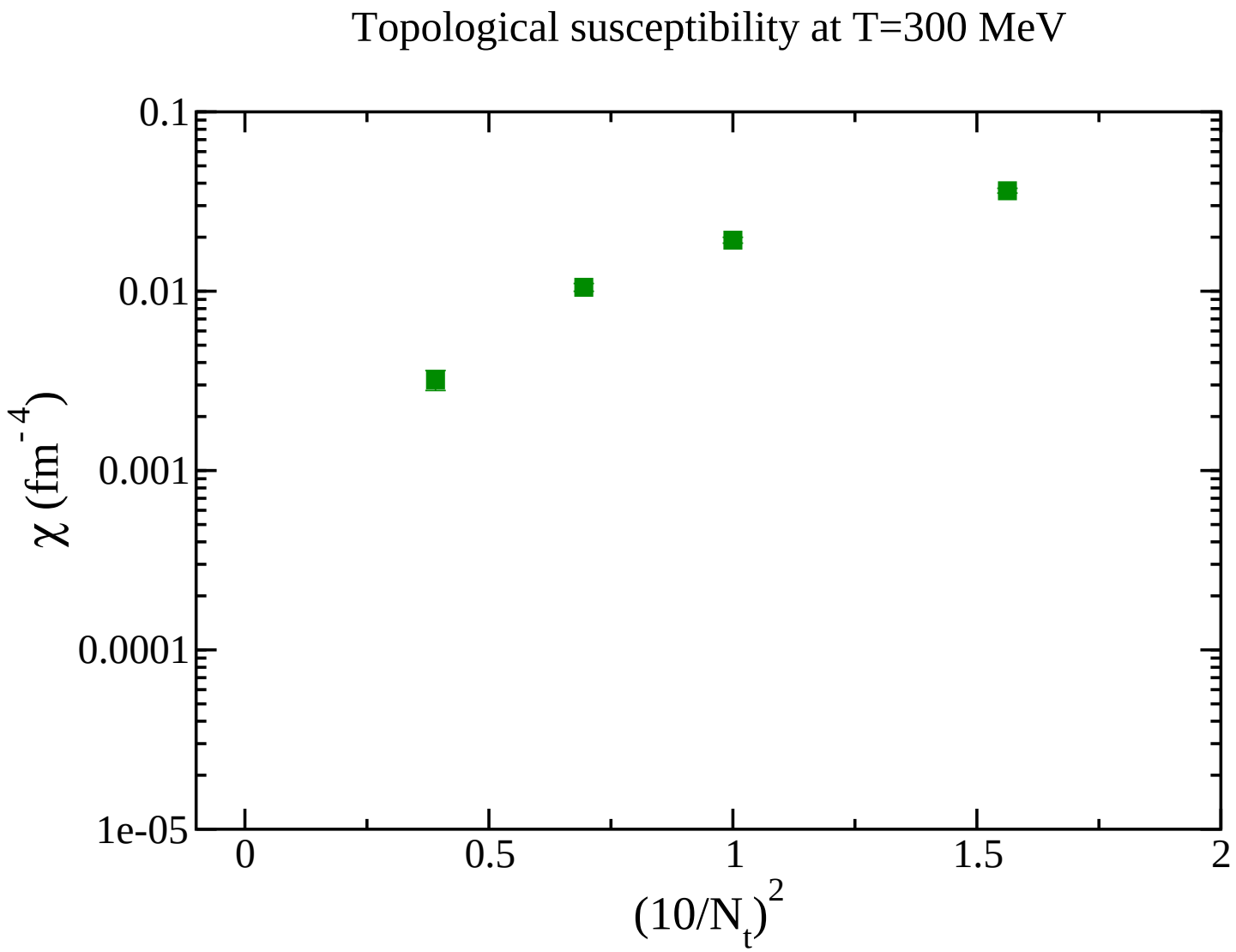
Challenges for the simulation

2) Tiny χ hard to measure

- No statistics for $Q \neq 0$ sectors (dictated by physics)
- Topology change slow on fine lattices (algorithmic)
- Solution:
  Derivative of $\chi(T)$ much easier to measure than χ
- Measure $\chi(T_0)$ at low enough T_0
- Using $d\chi/dT$ integrate up to T $\rightarrow$ integral method
  Also suggested for the quenched case by [Frison et al '16]

Unusually large cut-off effects, bad scaling

$N_f = 2+1+1$ staggered quarks on 4-stout gauge fields



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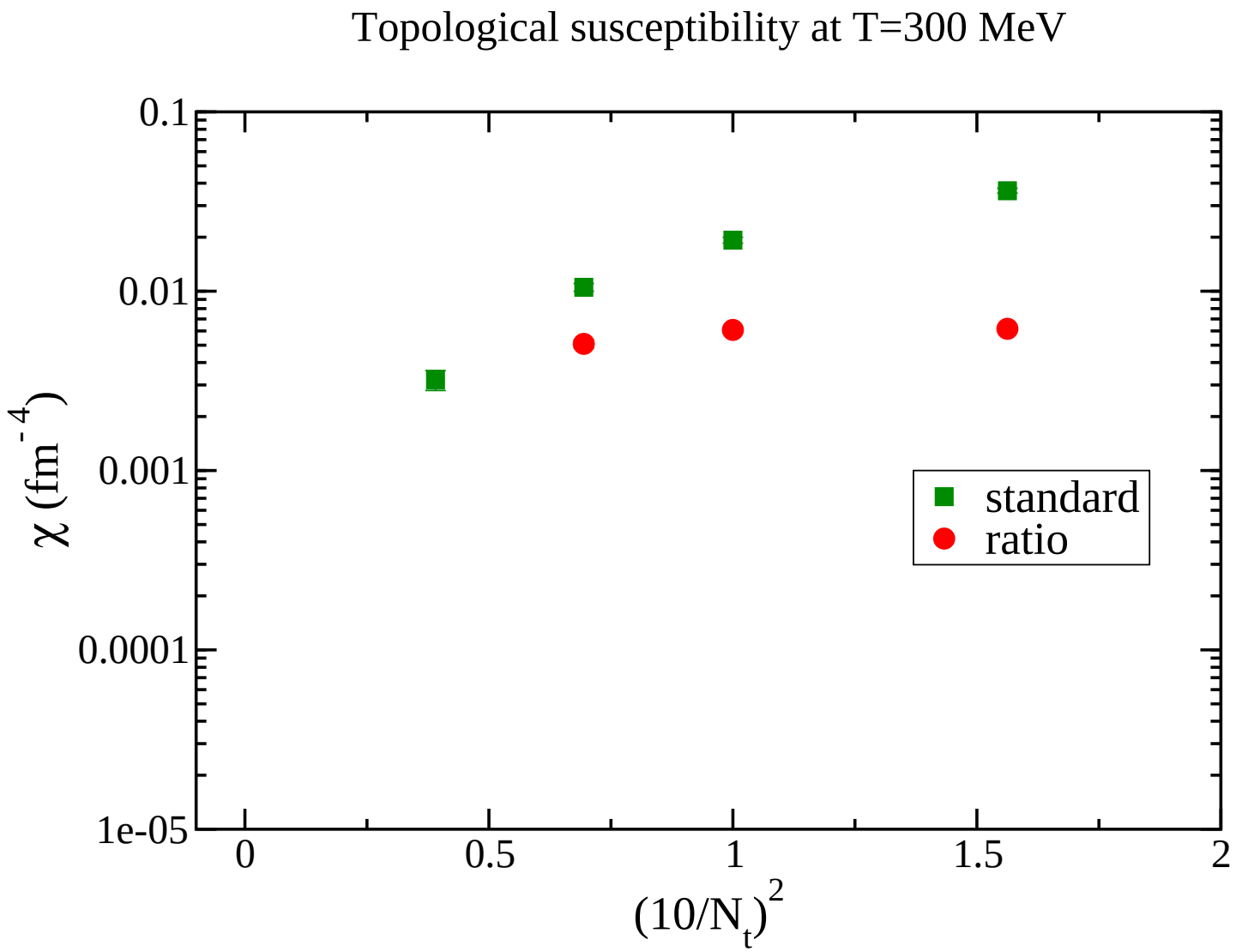

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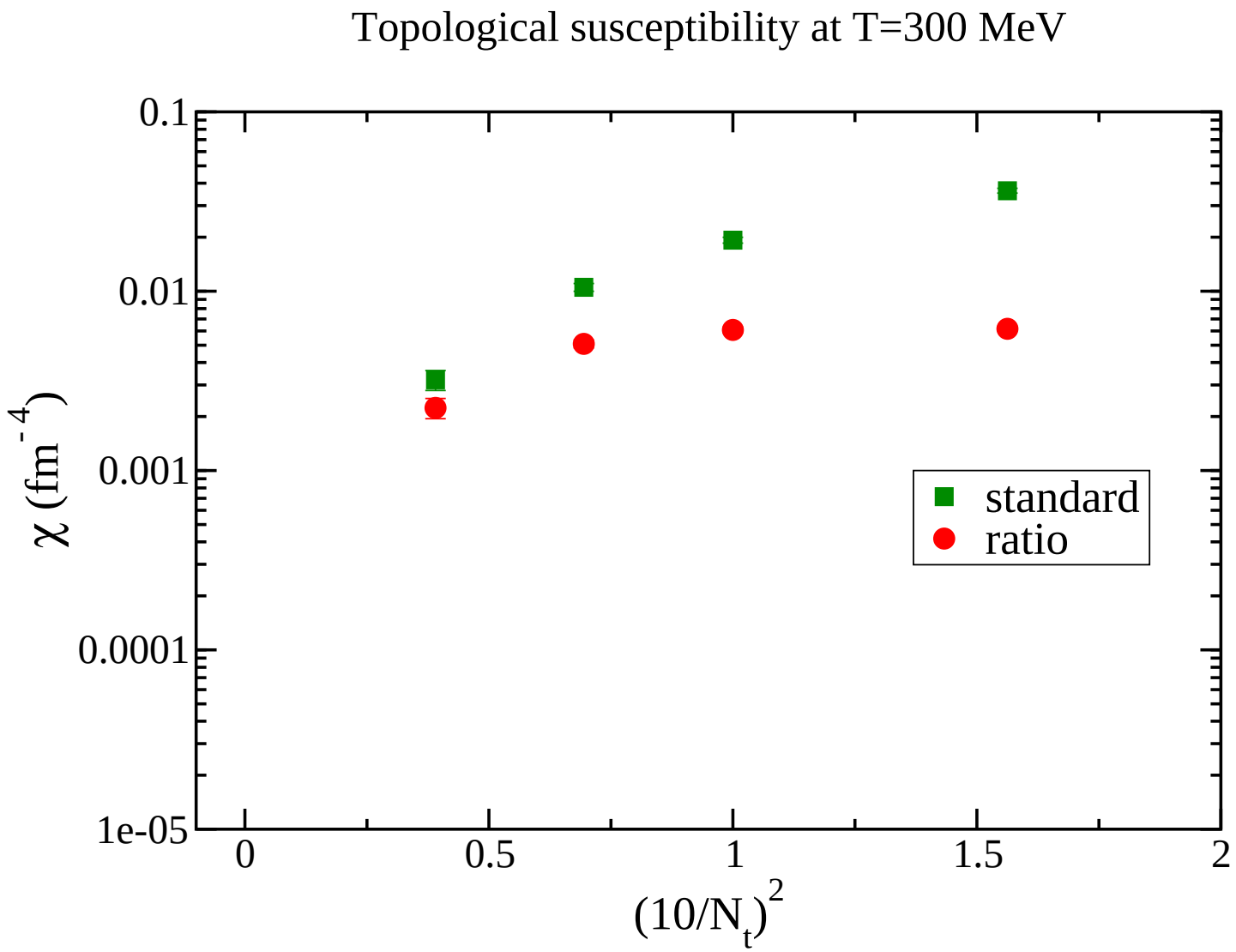

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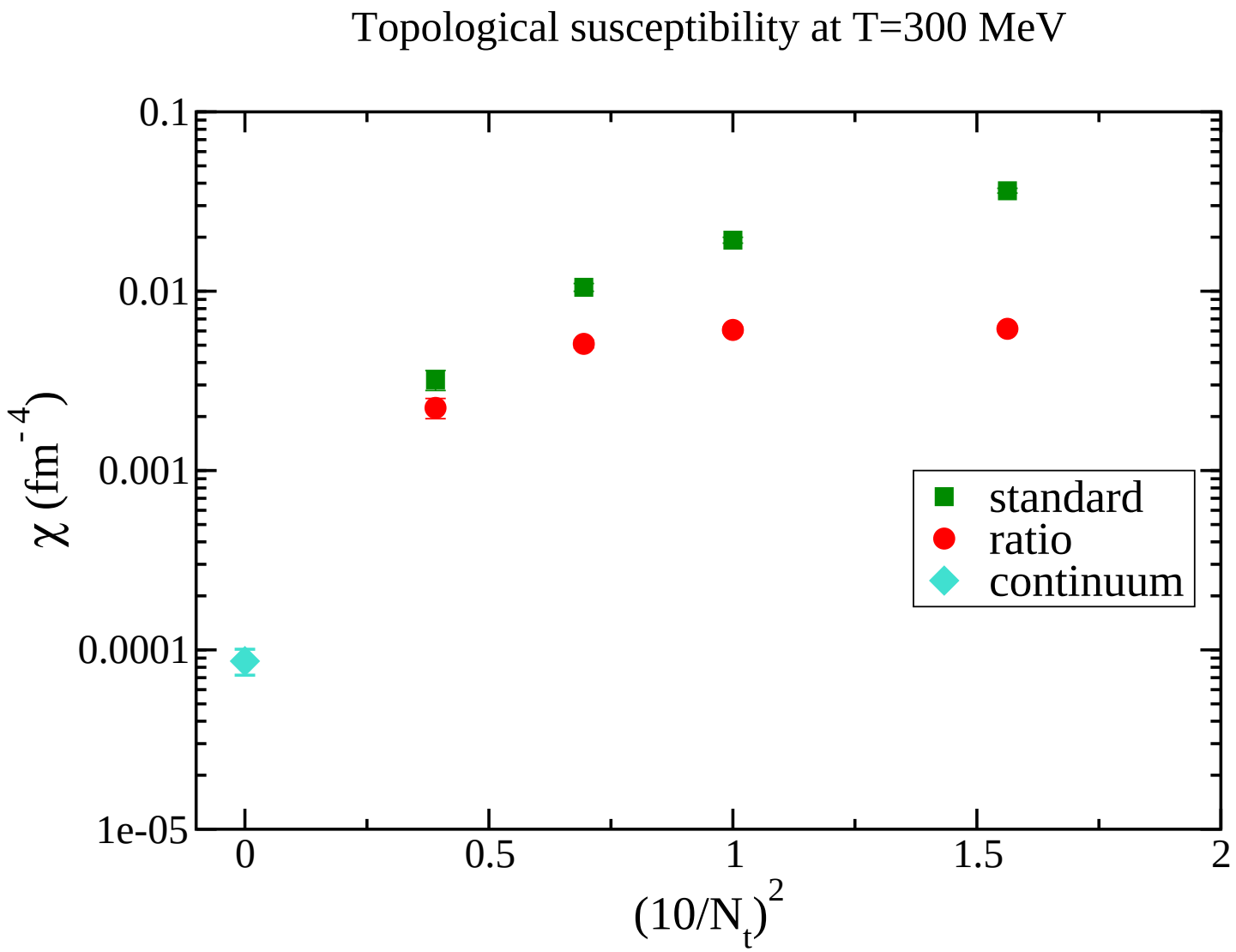
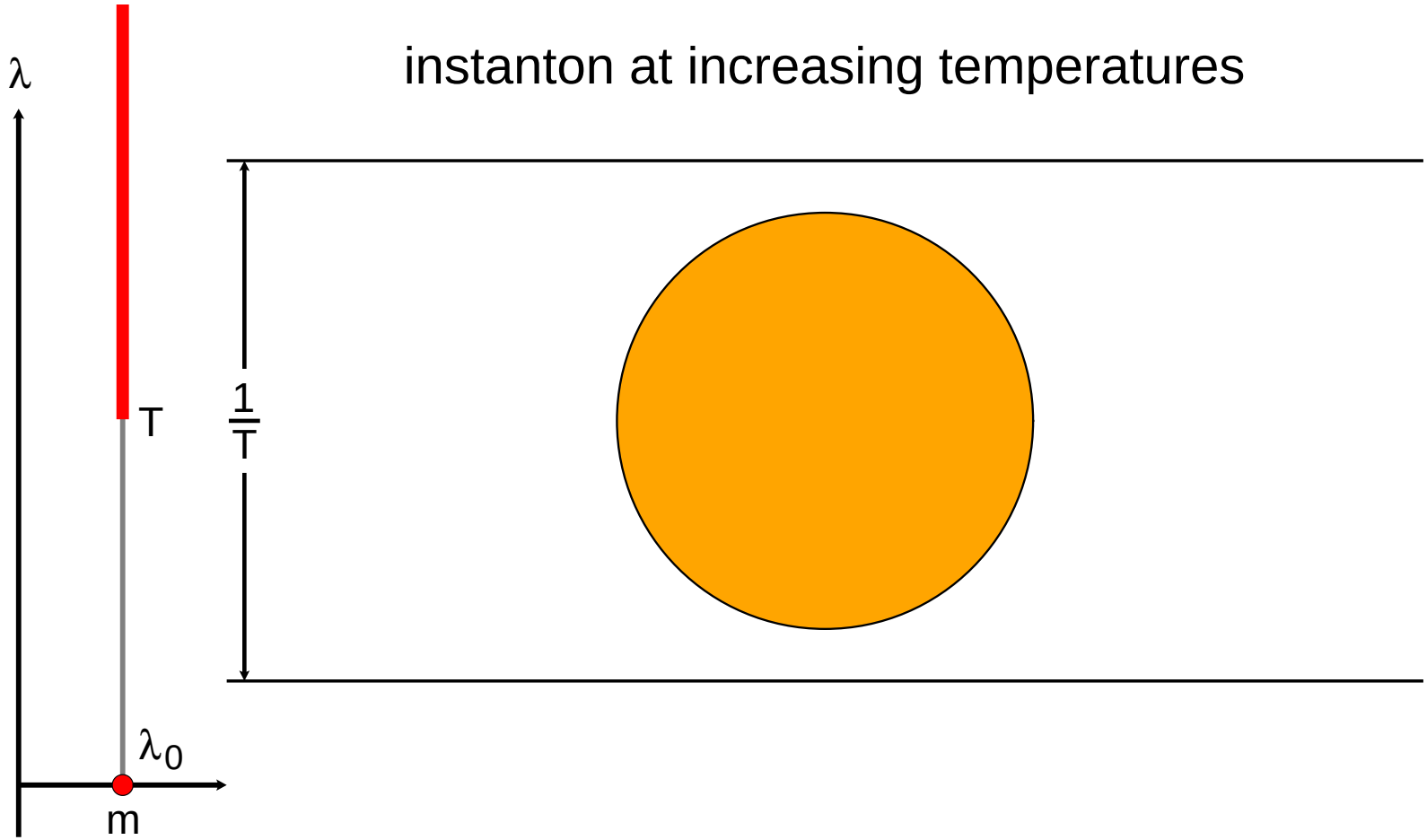


Continuum instanton and zero mode



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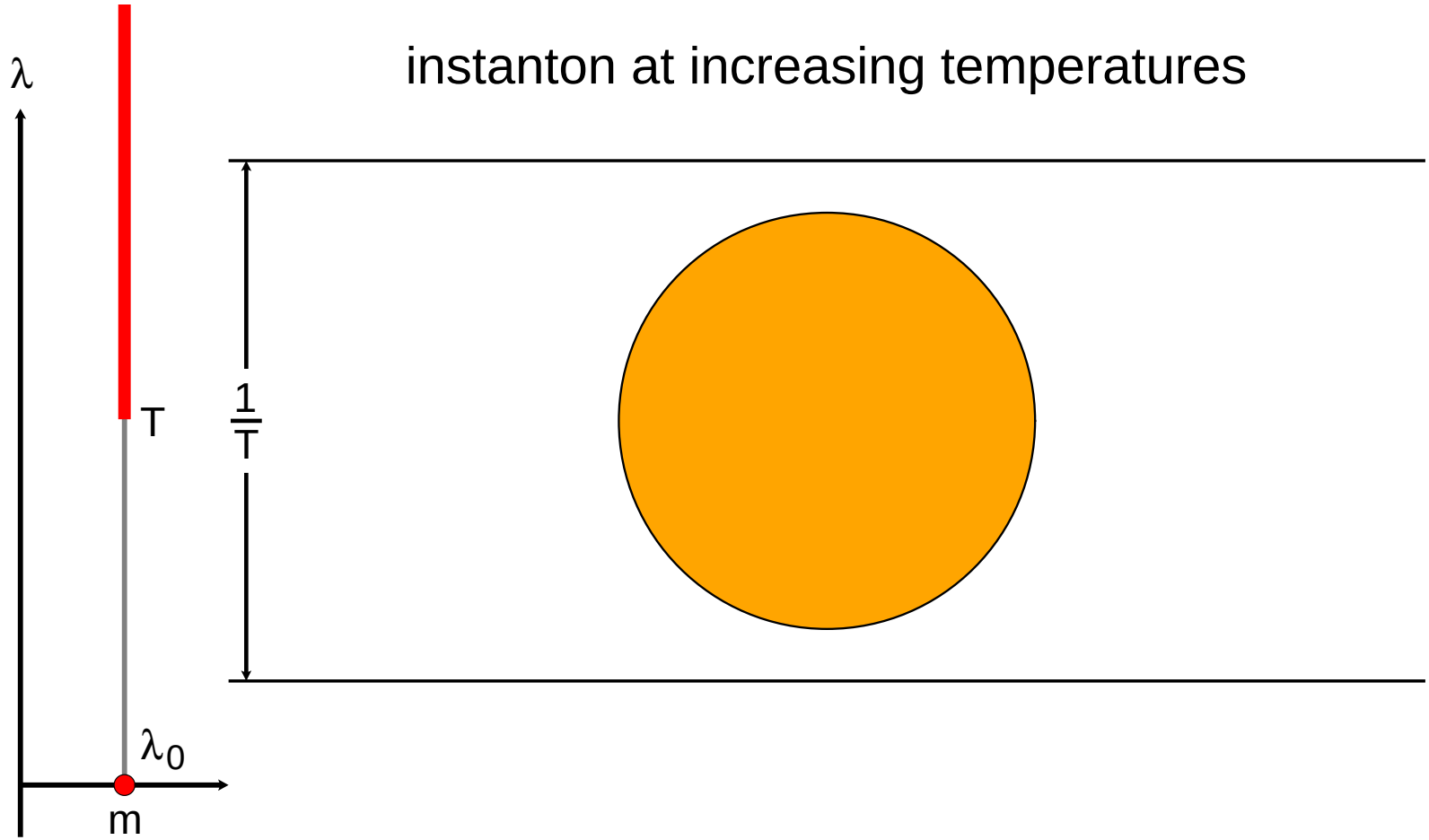
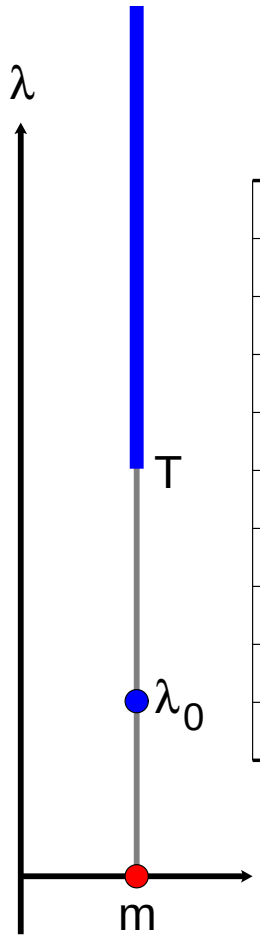
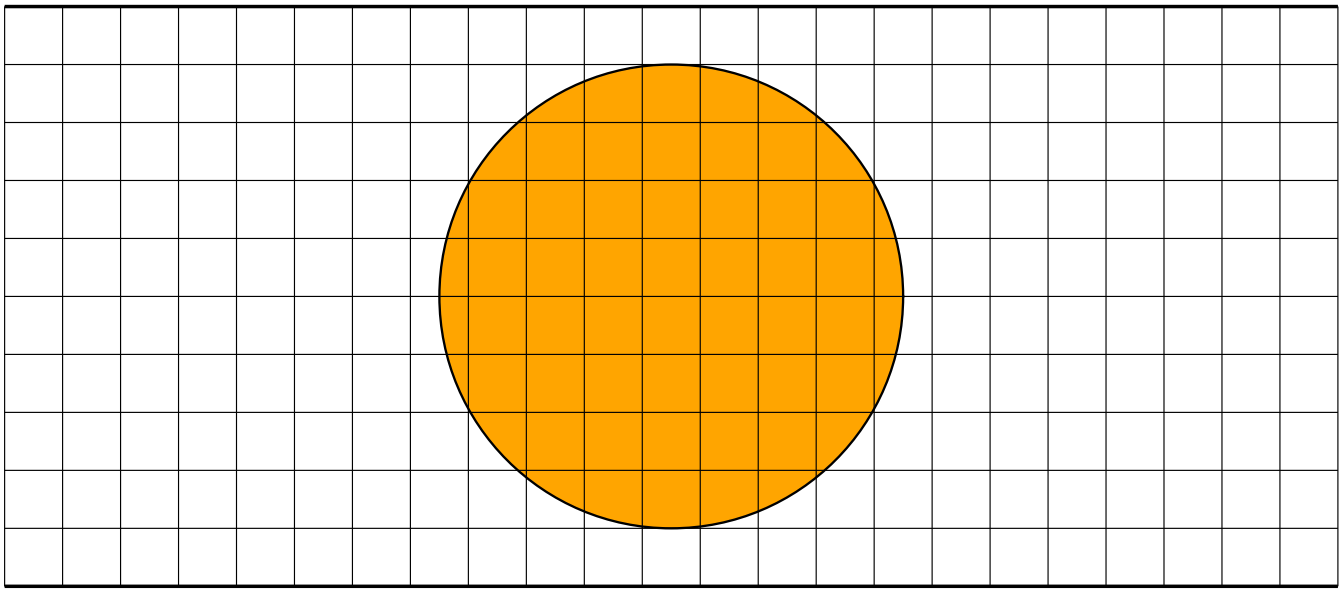


Lattice instanton and zero mode

N_t fixed, go up in T by increasing β



instanton at increasing temperatures
with staggered fermions



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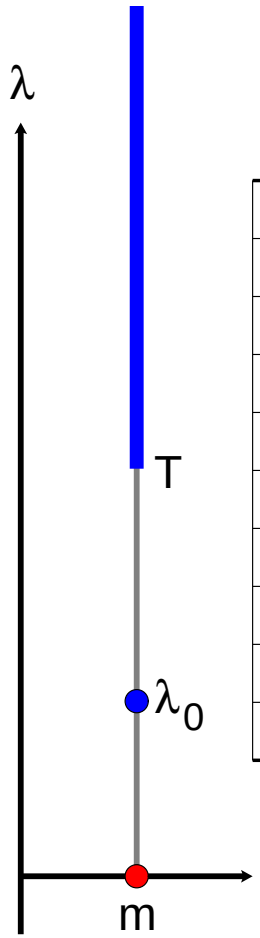

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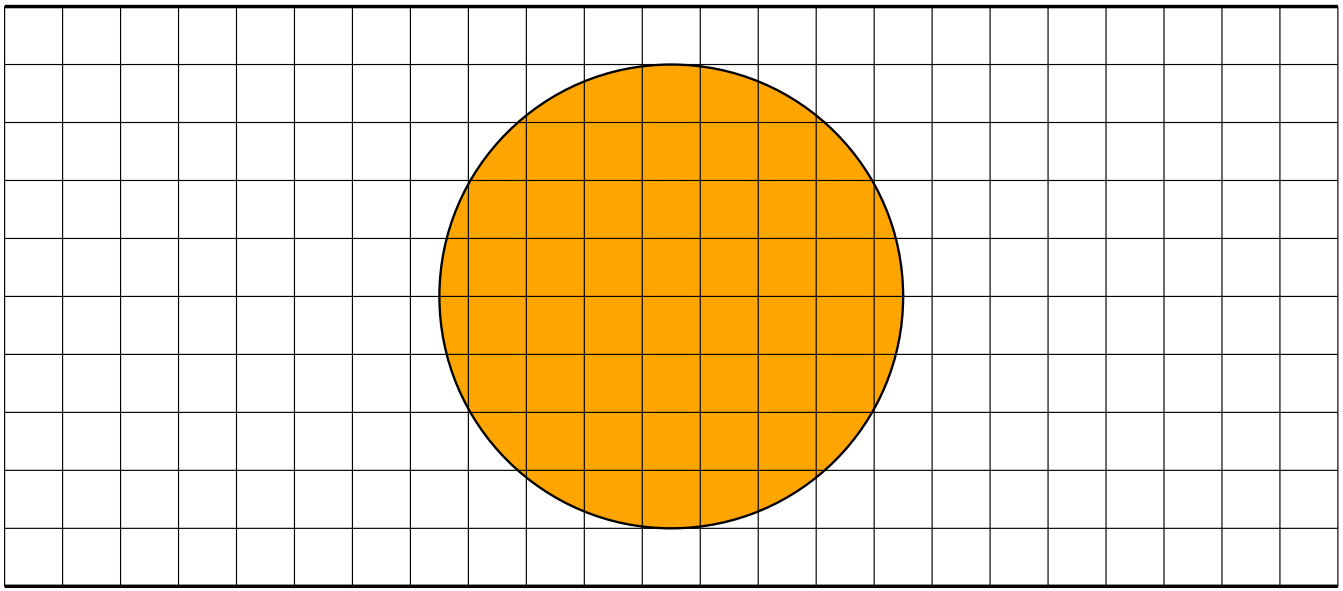


Reason for bad scaling

- Would-be zero eigenvalues too big
- Weight in det is

  Lattice: $\lambda_0 + m_f$

  Instead of continuum: m_f

- Even if $a \propto 1/T$ (fix N_t, increase β)

  λ_0/m increases with T

Reweighting

- Weight in det is $\lambda_0 + m_f$ instead of m_f

Solution: identify would-be zero modes and shift them to 0

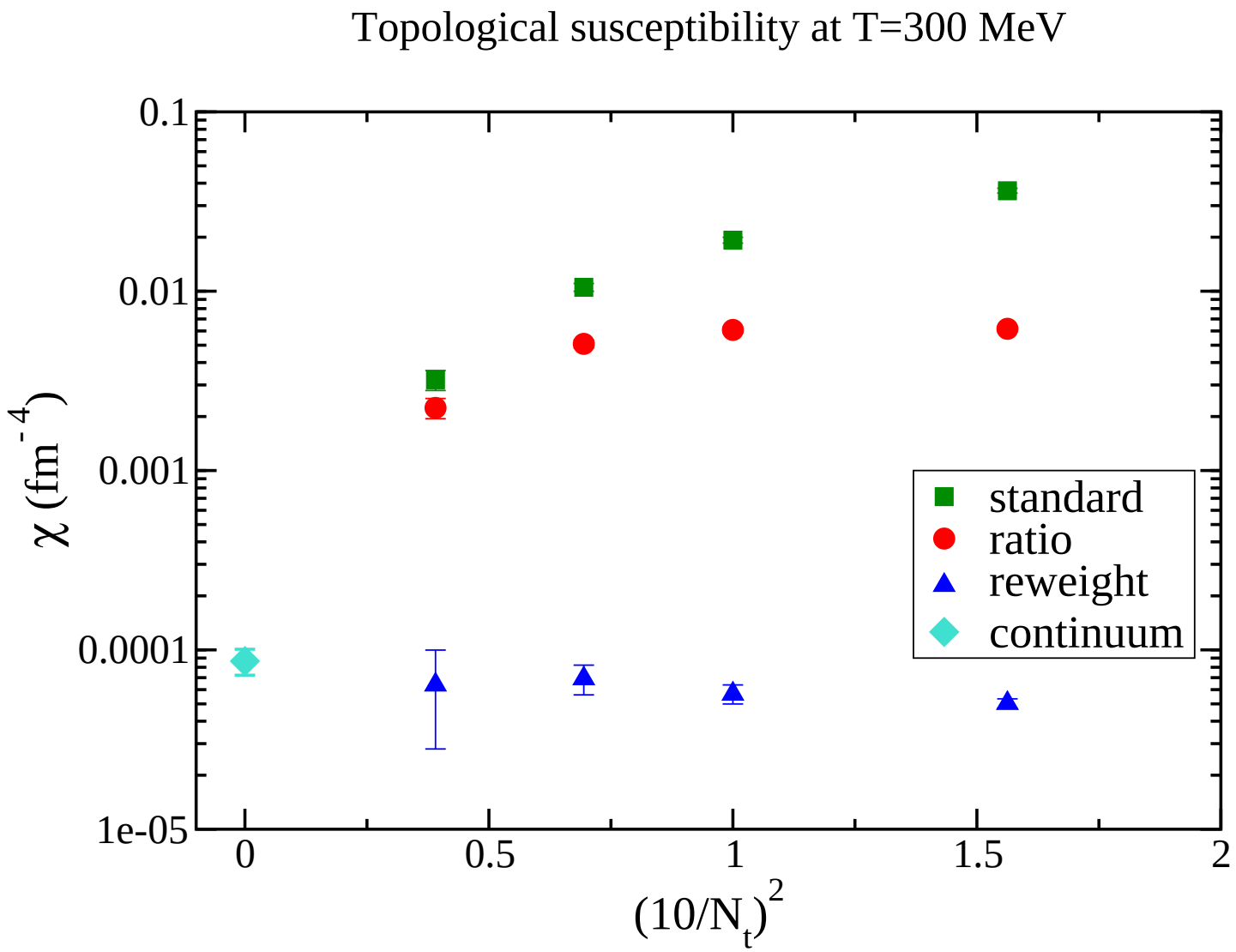
- Compute topological charge Q with Wilson flow [Lüscher '10]
- Identify $4|Q|$ would-be zero eigenvalues $\lambda_1, \lambda_2 \ldots \lambda_{4|Q|}$
- Modify quark determinant by reweighting with factor

$$w[U] = \prod_f \prod_{n=1}^{4|Q|} \left(\frac{m_f}{\lambda_n[U] + m_f} \right)^{1/4}$$

- Approaching the continuum it is getting better $w[U] \to 1$

Susceptibility after reweighting

$T = 300$ MeV



Integral method

- Direct measurement of χ up to 2 GeV not feasible due to bad sampling of $Q \neq 0$ sectors at high T (10 orders of magnitude beyond available resources)

- Calculate derivative

$$\frac{d \log Z_Q/Z_0}{d \log T} = \frac{d\beta}{d \log a} \langle S_g \rangle_{Q-0} + \sum_f \frac{d \log m_f}{d \log a} \langle \bar{\psi}\psi \rangle_{Q-0},$$

  where $\langle . \rangle_{Q-0}$ is the difference of expectations between sectors Q and 0.

- Compute Z_Q/Z_0 at temperature T_0

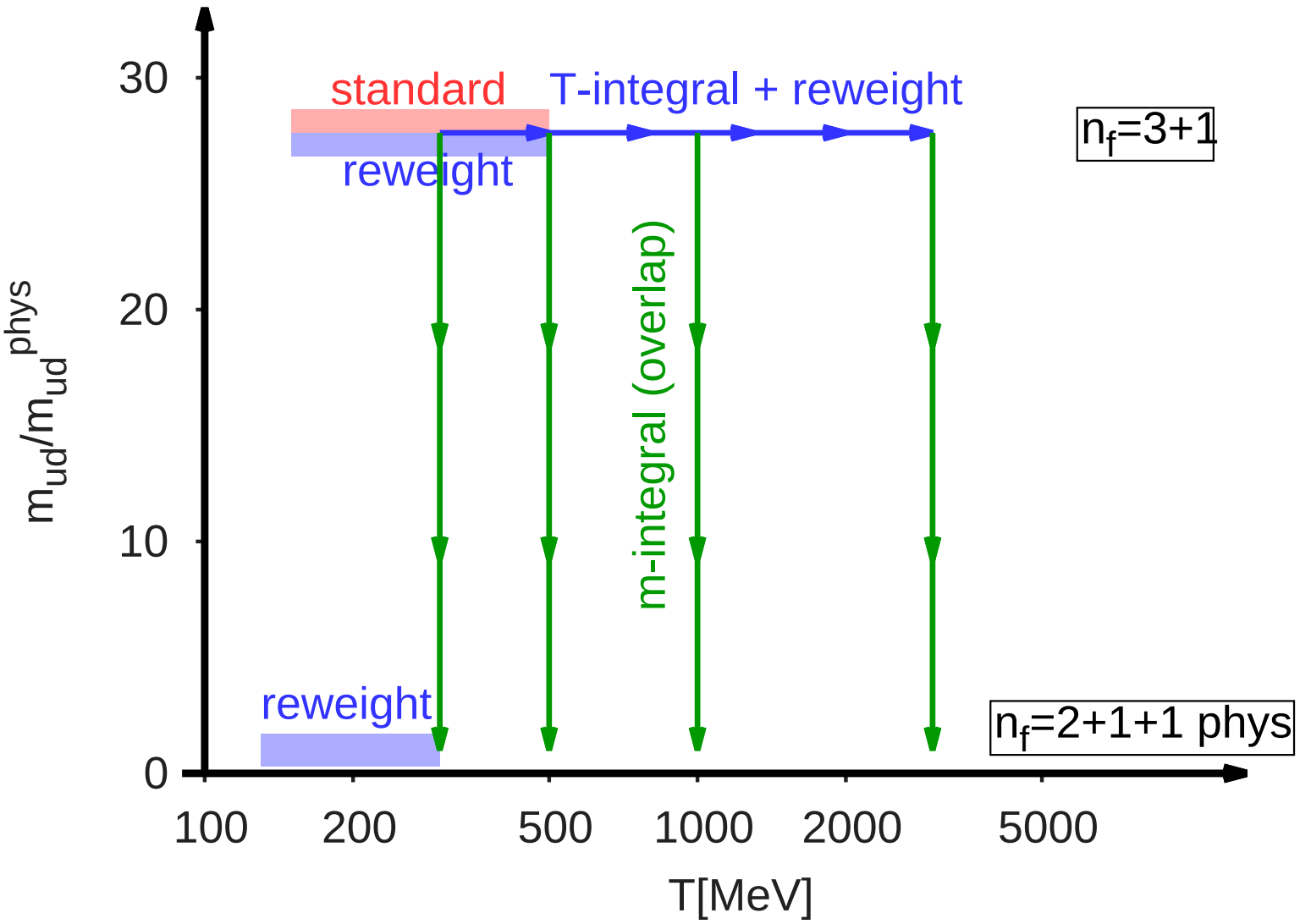
- Integrate starting from temperature T_0 to T

- Combine Z_Q/Z_0's to get χ

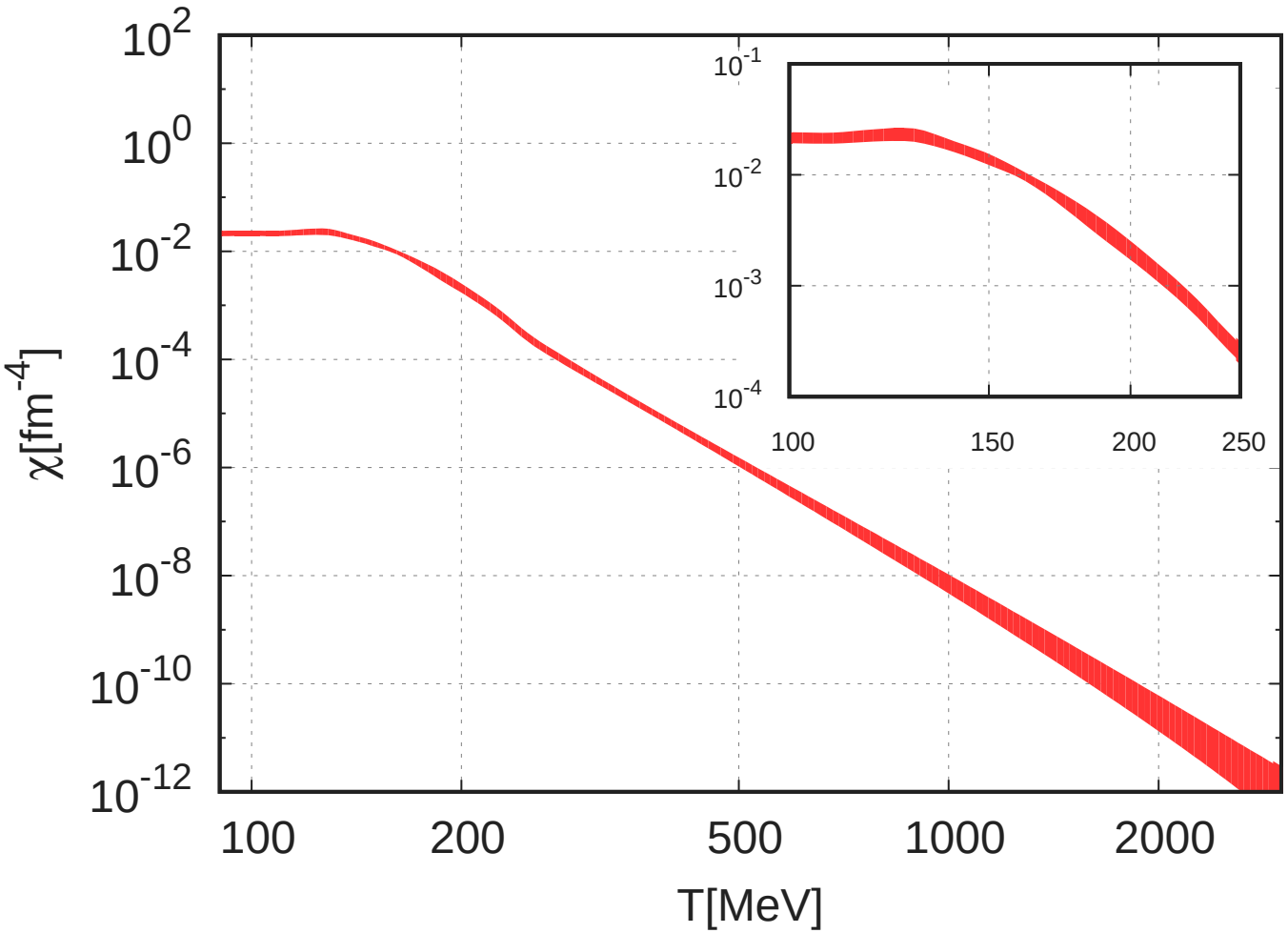
Integral method cont'd

- At high T dilute $\rightarrow$ non-interacting instanton gas
  $\rightarrow$ Z_1/Z_0 determines χ
- Can integrate also in light quark mass (cheaper than in T)
- Do T integral in $N_f = 3 + 1$ ($m_{ud} = m_s$)
- For each T integrate down in m_{ud} to physical quark mass
- Have to combine integral method with reweighting

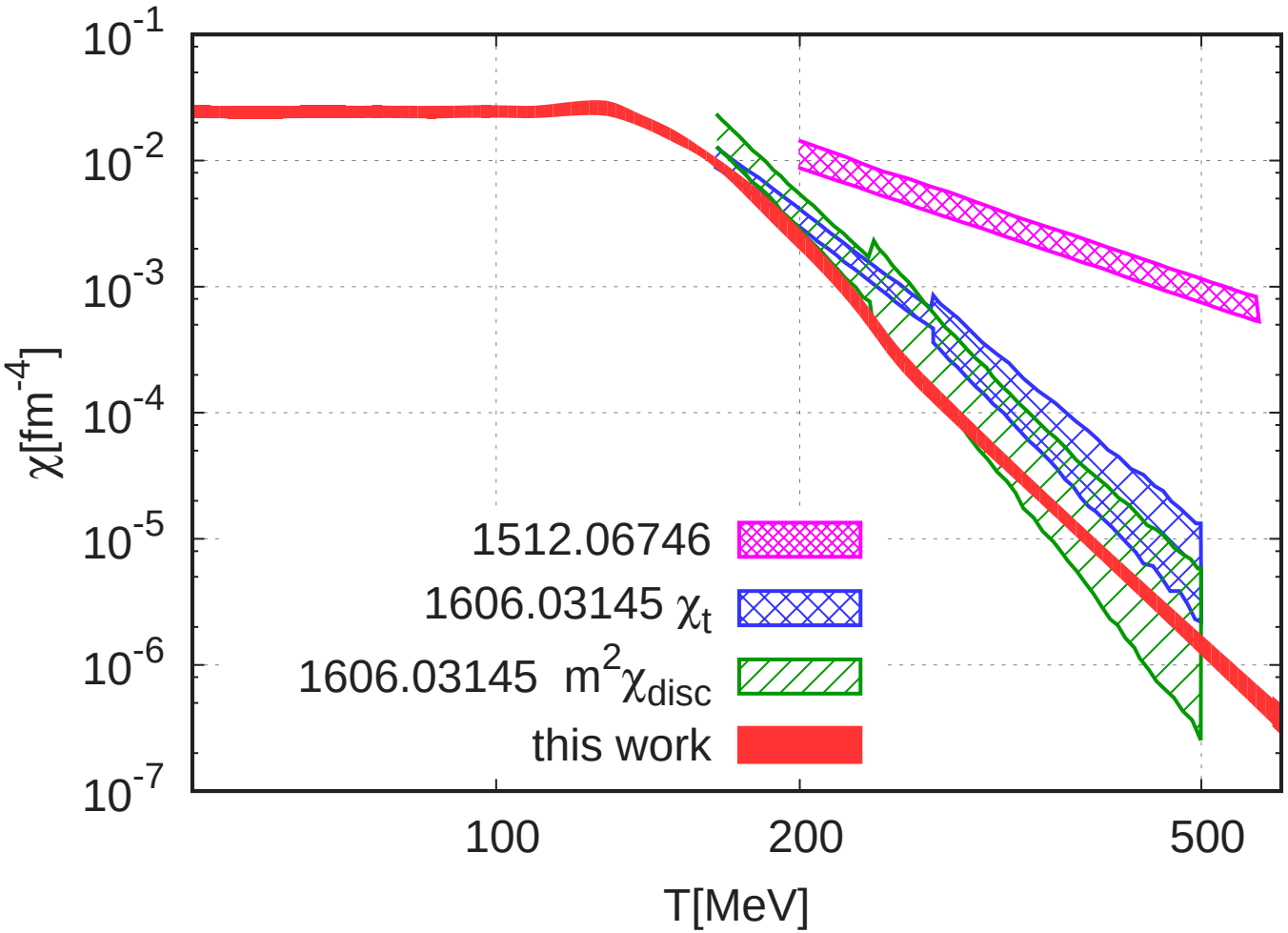
Map of simulations



Final result for the susceptibility



Comparison to other works



1512.06746 [Bonati et al '15] 1606.03145 [Petreckzy et al '16]

Conclusions

Calculated T-dependence of the QCD topological susceptibility

- Temperature range: $0 \leq T \leq 2$ GeV
  (follow change of χ over 10 orders of magnitude)
- Physical quark masses
- Continuum limit

Main lesson: keep in mind the physics of the problem

- Large cut-off effects due to instanton zero-modes
- At high T : tiny $\chi \quad \rightarrow \quad$ ideal instanton gas