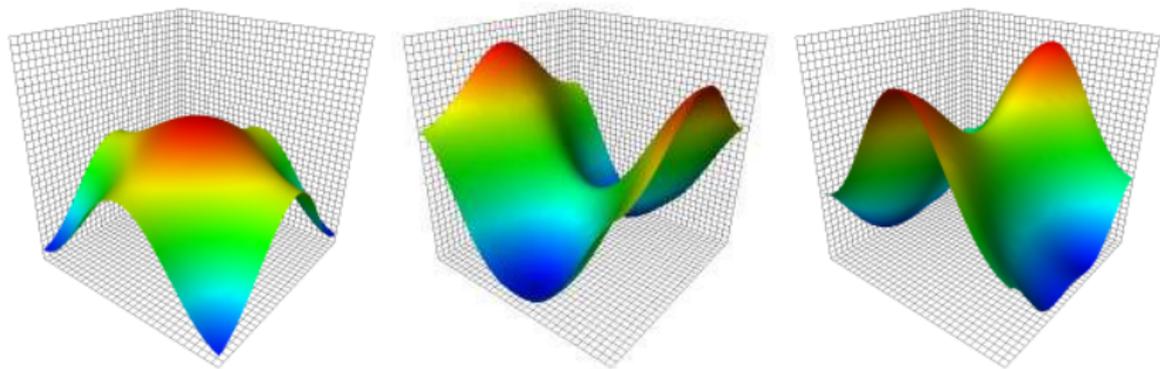


Background field Landau mode operators for the nucleon

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Collaborators

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Background Field Method

- Uniform magnetic field (in the z direction) on the lattice.

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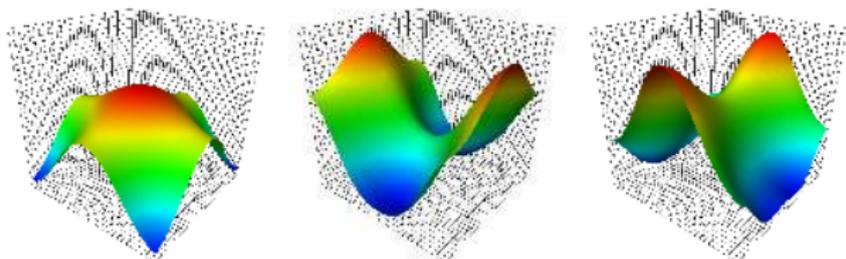
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- 3D spatial symmetry is broken by the magnetic field.

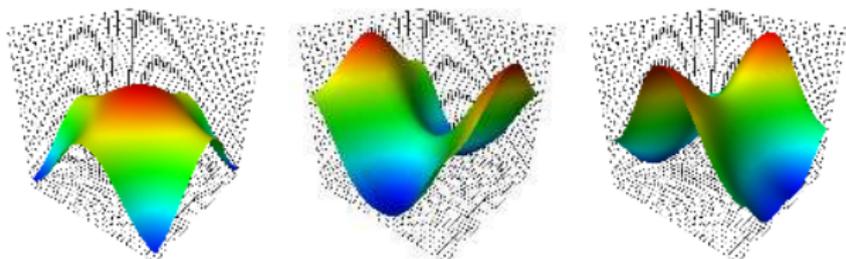
QED Eigenmodes



Lowest lying eigenmode probability densities of lattice 2D
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- Origin is centre of the x - y plane illustrated by bottom surface of the grid.

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- Origin is centre of the x - y plane illustrated by bottom surface of the grid.
- Project to these modes, i.e. $\sum_{i=1}^n \langle x | \nu_i$

QCD+QED Eigenmodes

- Define QED eigenmode projection operator (after Landau gauge fixing)

$$P_{QED}^n(x, y) = \sum_{i=1}^{n=|3q_f k_d|} \langle x | \nu_i \rangle \langle \nu_i | y \rangle$$

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- QCD+QED mixes lowest Landau modes.

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 - Truncation of $QCD+QED$ completeness relation is equivalent to smearing.

Configurations

- PACS-CS configs via ILDG: S. Aoki, et al., Phys. Rev. D79 (2009) 034503.
 - Lattice Volume: $32^3 \times 64$
 - Non-perturbative $\mathcal{O}(a)$ -improved Wilson quark action
 - Iwasaki gauge action
 - 2 + 1 flavour dynamical-fermion QCD
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- **Electro-quenched:**
 - Dynamical QCD configurations only - 'sea' quarks experience no B field.

Ratio Construction – Polarisability

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- Construct ratios of different spin and field direction different 2pt correlation functions.

$$R(B, t) = \left(\frac{G_{\downarrow}(B+, t) + G_{\uparrow}(B-, t)}{G_{\downarrow}(0, t) + G_{\uparrow}(0, t)} \right) \left(\frac{G_{\downarrow}(B-, t) + G_{\uparrow}(B+, t)}{G_{\downarrow}(0, t) + G_{\uparrow}(0, t)} \right)$$

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- Then extract an effective energy in the standard manner.

$$\delta E(B) = \frac{1}{\delta t} \log \left(\frac{R(B, t)}{R(B, t + \delta t)} \right) = \frac{1}{2} \left(\frac{|qeB|}{2M} - \frac{4\pi}{2} \beta B^2 \right)$$

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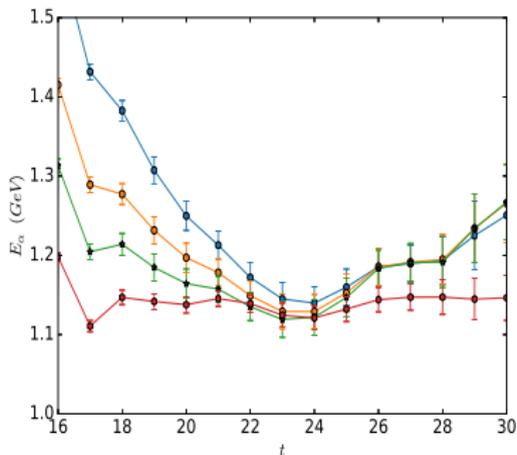
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Optimised Smeared Sources

- Avoid large amounts of sink smearing as it reduces signal.
- Use optimised smeared source \rightarrow QCD ground state is reached quickly.



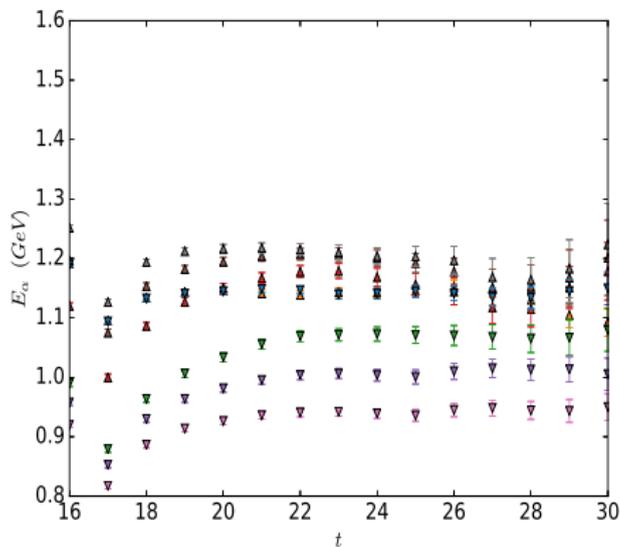
Smeared source to point sink effective masses.
Smearing increases as mass at $t = 16$ decreases.

Fit window selection

$$\delta E(B) = \left(\frac{|qeB|}{2M} - \frac{4\pi}{2} \beta B^2 \right)$$

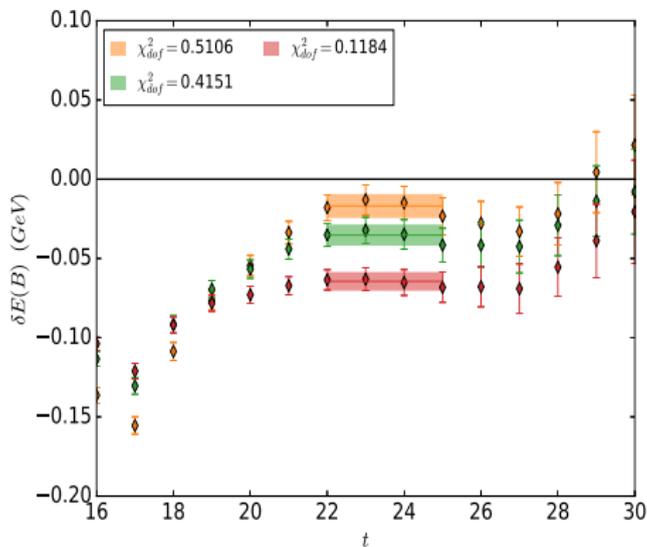
- To choose where to fit and obtain polarisability values, a number of factors are considered.
 - The constant fits to the energy shifts as function of time.
 - The fits to energy shifts as function of field strength.
 - We only consider the same fit window across all field strengths.
- The χ^2_{dof} of each of these fits must be in an acceptable range.

Neutron Energy Shifts for polarisability



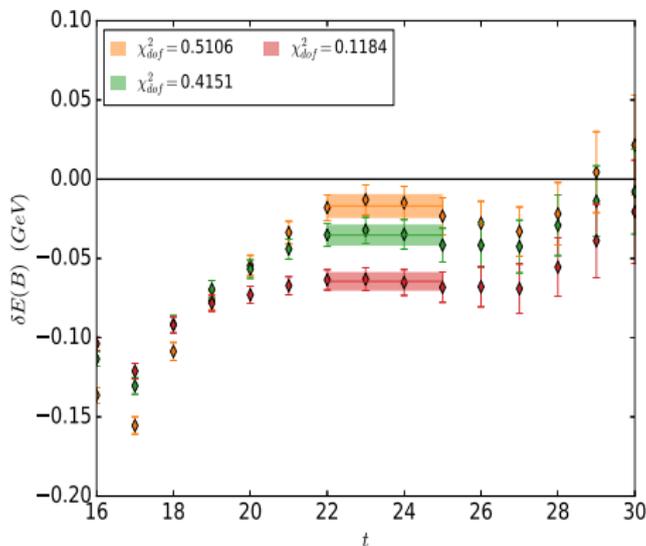
Smearred Source to QED eigenmode projected sink neutron energy shift

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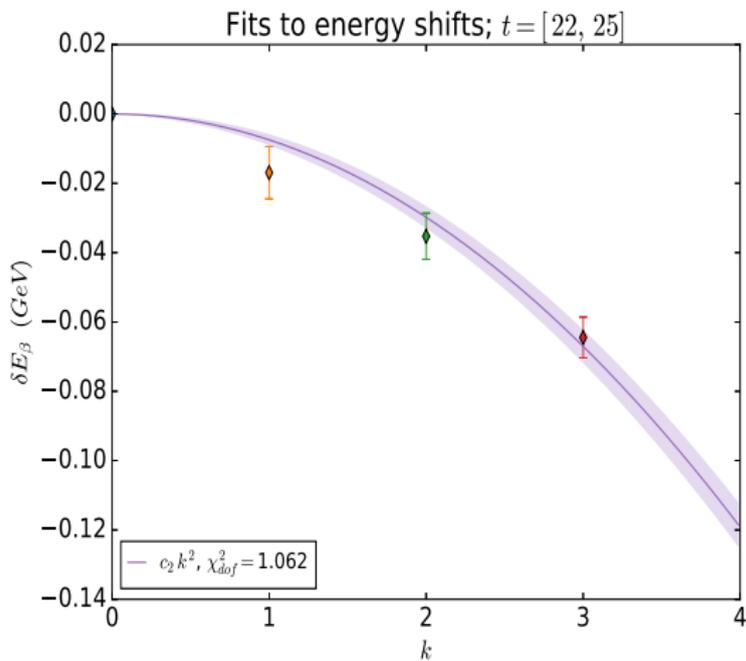
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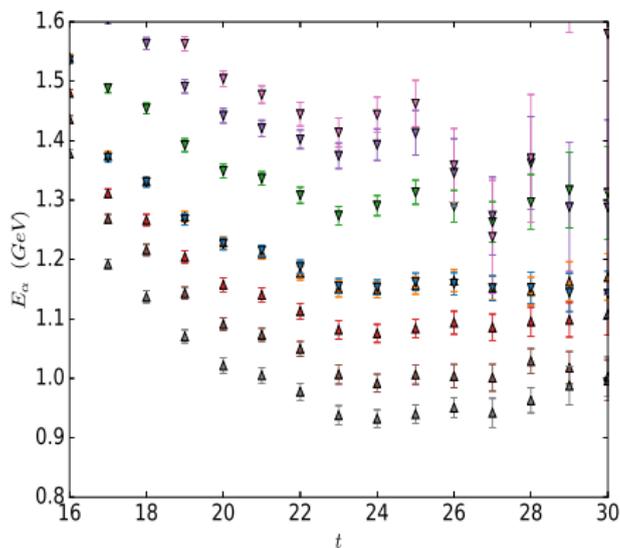
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- It is now possible to get plateaus in the energy shifts by using the projection methods.

Neutron

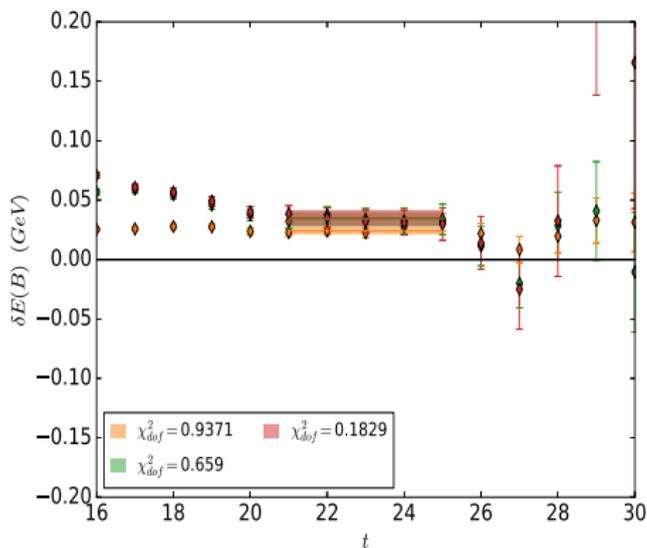


Proton Energy Shifts for polarisability



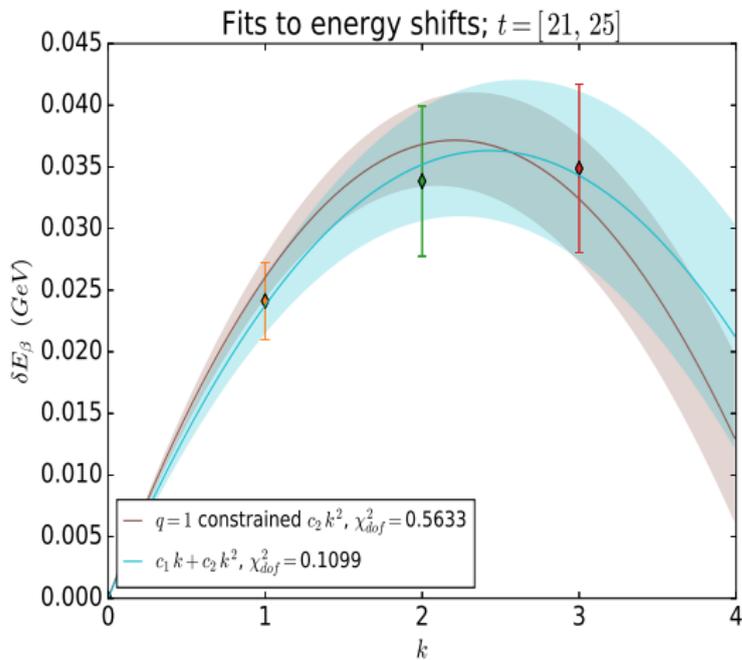
Smearred Source to QCD+QED eigenmode projected sink proton energy shift

Proton Energy Shifts for polarisability



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Proton



Nucleon Polarizability

- From the quadratic term extract the polarizability

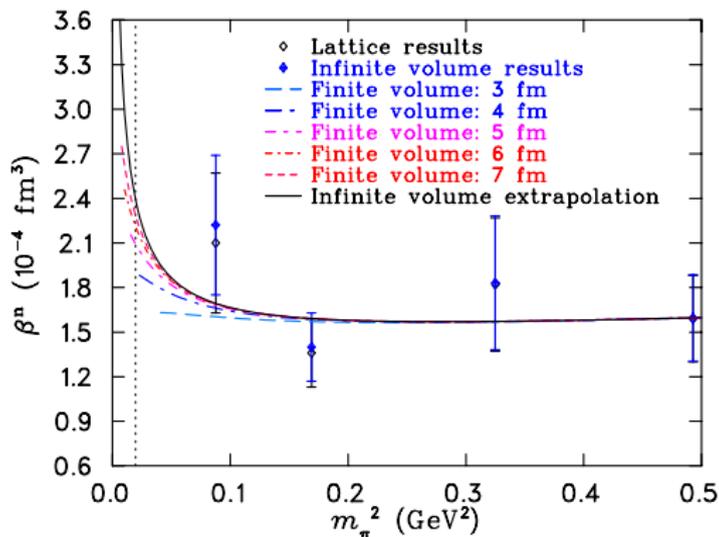
$$\delta E(B) \propto -\frac{4\pi}{2} \beta B^2$$

- Nucleon polarisabilities are found to be

	Experiment ($m_\pi = 138$ MeV)	This Work ($m_\pi = 413$ MeV)
proton	$2.5(4) \times 10^{-4} \text{ fm}^3$	$1.15(24) \times 10^{-4} \text{ fm}^3$
neutron	$3.7(12) \times 10^{-4} \text{ fm}^3$	$1.39(15) \times 10^{-4} \text{ fm}^3$

- Potential exists to make interesting predictions from lattice QCD.

Chiral Extrapolations



Extrapolation of the magnetic polarisability of the neutron for several volumes, correcting for partial quenching effects.

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Future Work

- Use QCD + QED eigenmode projection at more quark masses.
- Investigate the proton.

Magnetic Moment

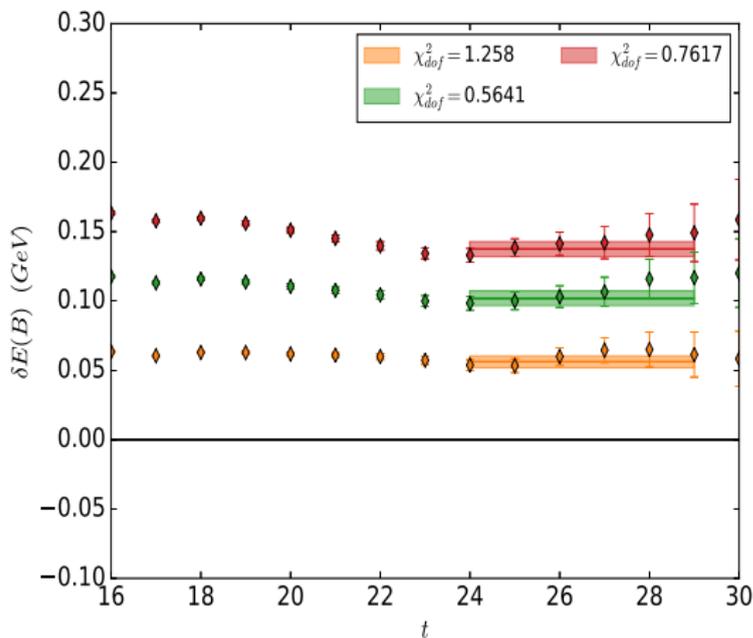
- Considerably easier than magnetic polarisability
- Take a different ratio

$$R(B, t) = \left(\frac{G_{\downarrow}(B-, t) + G_{\uparrow}(B+, t)}{G_{\downarrow}(B+, t) + G_{\uparrow}(B-, t)} \right)$$

- Get an energy shift of

$$\delta E_{\mu}(B) = -\mu B + \mathcal{O}(B^3)$$

Magnetic Moment



Energy shift for magnetic moment of the neutron.

Magnetic Moment

- Extract magnetic moment from linear term
- Background field results are preliminary only

	3PT ($m_\pi = 413$ MeV)	BFM ($m_\pi = 413$ MeV)
proton	$2.184(22) \mu_N$	$2.244(61) \mu_N$
neutron	$-1.371(14) \mu_N$	$-1.36(10) \mu_N$