QCD in a moving frame: an exploratory study

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Introduction

The equation of state (EoS) of QCD is instrumental for our understanding of a plethora of phenomena involving the strong interactions, ranging from the dynamics inside a nucleon star to the evolution of the early Universe.

First principles determinations of the EoS of QCD are very challenging, as one needs non-perturbative control of QCD over a wide range of temperatures. Lattice QCD is therefore, the only known framework that allows us to tackle this problem in a systematic way.

Results for the EoS of QCD with $N_\mathrm{f} = 2+1$ quark-flavours have been recently obtained using well-established lattice techniques: the so-called integral methods [1, 2]. These techniques, however, suffer from several numerical limitations that severely limit the accessible range of temperatures ($T \lesssim 500\,\mathrm{MeV}$). Hence, in the last years a lot of effort has been invested into devising new methods to address that problem and overcome those limits (see e.g [3, 4, 5, 6]).

In this contribution, we present first results from *QCD in a moving frame.* In particular, we investigate the feasibility of a determination of the EoS of $N_\mathrm{f} = 2+1$ QCD using this set-up, and show how precise results can be obtained in a wide range of temperatures i.e up to $T \sim 45\,\mathrm{GeV}$.

QCD in a moving frame

In a relativistic thermal field theory one can relate the entropy to the total momentum of the system as measured by an observer in a moving reference frame. Thus, if one was able to measure the total momentum in a moving frame, one could directly compute the entropy, and from it any other thermodynamic potential.

Remarkably, the Euclidean path integral formulation of a thermal field theory in a moving frame is rather simple: it amounts to imposing *shifted boundary conditions* (SBC) to the fields in the compact (time) direction [3]. In the lattice theory, that means

$$U_\mu(L_0,\boldsymbol{x}) = U_\mu(0,\tilde{\boldsymbol{x}}), \quad \tilde{\boldsymbol{x}} = \boldsymbol{x} - L_0\boldsymbol{\xi},$$
$$\psi(L_0,\boldsymbol{x}) = -\psi(0,\tilde{\boldsymbol{x}}), \quad \overline{\psi}(L_0,\boldsymbol{x}) = -\overline{\psi}(0,\tilde{\boldsymbol{x}}),$$

where L_0 is the length of the compact direction and $\boldsymbol{\xi}$ is the shift parameter or (imaginary) velocity of the system; periodic boundary conditions are assumed in the spatial directions of length L.

Given these definitions, the entropy density $s(T)$ at the temperature T can be computed as [3]

$$\frac{s(T)}{T^3} = -\frac{L_0^4(1+\boldsymbol{\xi}^2)^3}{\xi_k}\langle T^R_{0k}\rangle_\xi,\;\; T^{-1} = L_0\sqrt{1+\boldsymbol{\xi}^2} \qquad (1)$$

where $\langle\cdots\rangle_\xi$ denotes the path-integral expectation value in presence of SBC, while T^R_{0k} is the momentum k-component of the renormalized *energy-momentum tensor*, i.e.,

$$T^R_{0k} = Z(g_0)\big[T^G_{0k} + z(g_0)T^F_{0k}\big].$$

In the above definition $Z(g_0)$ and $z(g_0)$ are finite renormalization constants, which depend on the exact discretization of the QCD action, and of the bare fields T^G_{0k} and T^F_{0k} [7]. We choose

$$T^G_{0k} = \frac{1}{g_0^2}F^a_{0\alpha}F^a_{k\alpha},$$

where g_0 is the bare gauge coupling and $F_{\mu\nu}$ is the clover discretization of the field strength tensor [7, 4]. For the fermionic part, we take [7]

$$T^F_{0k} = \frac{1}{8}\{\overline{\psi}[\overleftrightarrow{\nabla}_0 + \overleftrightarrow{\nabla}^*_0]\gamma_k\psi + \overline{\psi}[\overleftrightarrow{\nabla}_k + \overleftrightarrow{\nabla}^*_k]\gamma_0\psi\},$$

being $\overleftrightarrow{\nabla}_\mu = [\nabla_\mu - \overleftarrow{\nabla}_\mu]$, with ∇_μ the usual covariant lattice derivative. Note that in this exploratory investigation we neglect any $\mathrm{O}(a)$ operator counterterm.

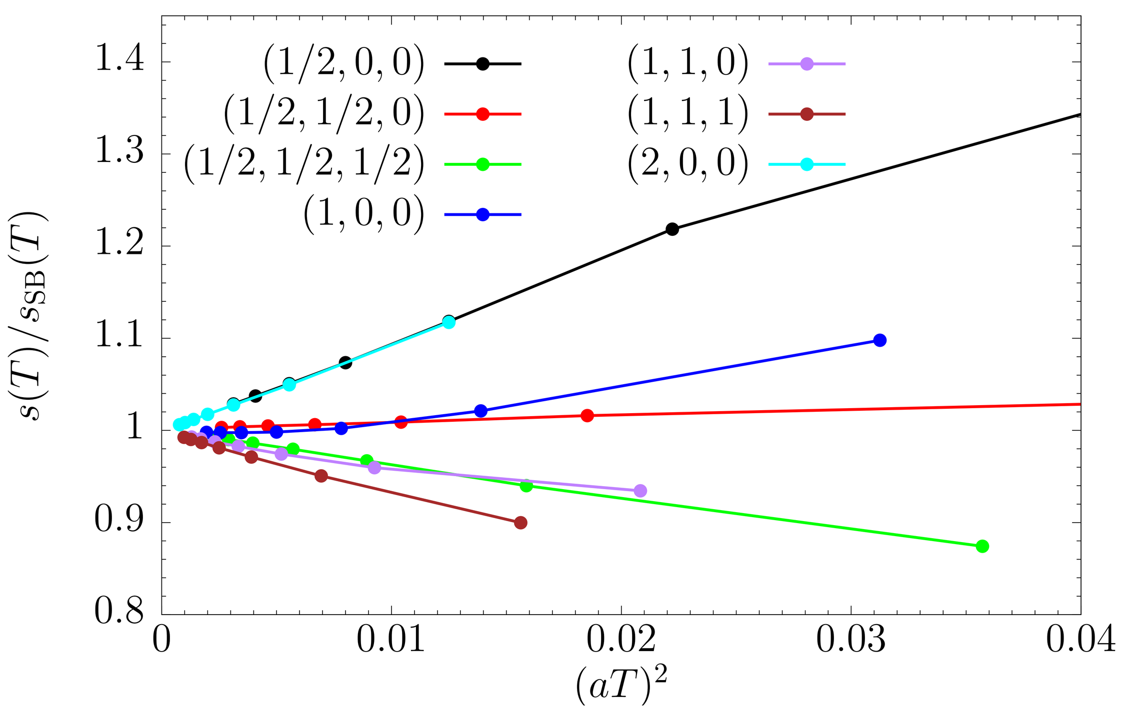
For the lattice action we consider *3-flavours of O(a)-improved Wilson quarks*, and, if not stated otherwise, the Wilson gauge-action.

The entropy in free-field theory

To gain some insight on which set of kinematic parameters might lead to smaller discretization errors in non-perturbative investigations, it is useful to first study eq. (1) in free-field theory i.e. for $g_0 = 0$. As this information is expected to be more trustworthy at high-temperatures, we consider massless quarks. In the figure we show the deviation of eq. (1) from its continuum value

$$\frac{s_\mathrm{SB}(T)}{T^3} = \frac{\pi^2}{45}(32 + 21\times N_\mathrm{f}),$$

for $N_\mathrm{f} = 3$, $L/L_0 = 32$, and different choices of shift $\boldsymbol{\xi}$. The cases $\boldsymbol{\xi} = (1,0,0)$ and $(1/2,1/2,0)$ appear to have remarkably small lattice artifacts for $aT \lesssim 0.14$. We also note that for $g_0 = 0$ the only $\mathrm{O}(a)$ effects are of $\mathrm{O}(am_0)$, and thus absent for massless quarks.



Simulation results

To access the feasibility of the determination of the EoS in 3-flavour QCD with SBC, we addressed the crucial issue of the computational effort necessary to obtain a given precision for the basic quantities $\langle T^{G,F}_{0k}\rangle_\xi$. Particularly important is how this depends on the temperature. To this end, we considered 3 well-separated temperatures. For all simulations we chose: $\boldsymbol{\xi} = (1,0,0)$, $L_0 = 6$, $L = 96$, thus having $TL > 11$. Based on the results in pure Yang-Mills theory [4] and perturbative estimates in QCD, we expect finite-volume effects to be well-below our target precision (see below). Considering larger spatial volumes is anyway not an issue in our set-up [4], and will also help with topology freezing.

The results we obtained are collected in the table, for the 3 chosen temperatures:

1. $T \sim 0.36\,\mathrm{GeV}$: the lattice bulk-action and action bare parameters match ensemble N203 of CLS [8]; the Lüscher-Weisz gauge action is hence employed. These correspond to having $a \sim 0.064\,\mathrm{fm}$, and quark-masses such that $m_\pi \sim 340\,\mathrm{MeV}$ and $m_K \sim 440\,\mathrm{MeV}$ at $T = 0$.
2. $T \sim 4\,\mathrm{GeV}$: the bare parameters are specified by fixing the Schrödinger functional coupling $\bar{g}^2_\mathrm{SF}(T) \sim 2.0$, and by setting the quark masses (roughly) to zero [9]. This is feasible as the Dirac operator has a spectral gap $\sim \pi T$.
3. $T \sim 45\,\mathrm{GeV}$: similarly we considered bare parameters such that $\bar{g}^2_\mathrm{SF}(T) \sim 1.26$, and the quark masses are close to vanishing [9].

T (GeV)	$\langle T^G_{0k}\rangle \times 10^4$	$\langle T^F_{0k}\rangle \times 10^4$	N_ms	CPU Time
~ 0.36	$-2.545(45)$	$-9.445(77)$	704	1.3 Mch
~ 4	$-4.700(80)$	$-11.426(65)$	548	0.2 Mch
~ 45	$-5.200(88)$	$-11.994(57)$	654	0.2 Mch

† In these exploratory runs we measured every 10 MDU. Measuring every 2 MDU, however, may reduce of $3-5$ times the CPU time at fixed precision.

Conclusions

The results presented show that a precision of $0.5-1\%$ on the bare quantities is reachable with modest computational effort in a wide range of temperatures. The next question is thus, whether the necessary renormalization can be obtained with a similar level of precision. This issue, together with the $\mathrm{O}(a)$ improvement of $T_{\mu\nu}$, is currently under investigation. In this respect, it is interesting to note that in general chiral symmetry "restoration" at high-temperatures may have interesting consequences on the $\mathrm{O}(a)$ effects of the theory, as a partial $\mathrm{O}(a)$ improvement may "automatically" be obtained.

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