Pion decay in magnetic fields

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in collaboration with
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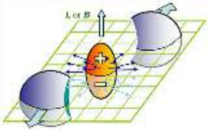
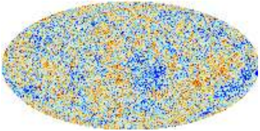
Lattice '17, 23. June 2017

Introduction

- pions in magnetic fields
  - off-central heavy-ion collisions [Kharzeev et al '07]
  - inner core of magnetized neutron stars [Duncan, Thompson '92]
  - evolution of the early universe [Vachaspati '91]





- typical field strength $eB = (100 \text{ MeV})^2 \ldots 1 \text{ GeV}^2$
  comparable or bigger than $M_\pi^2 = (135 \text{ MeV})^2$
- most important properties
  - B-dependence of mass
  - B-dependence of decay rate

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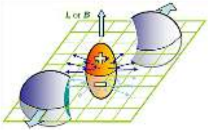

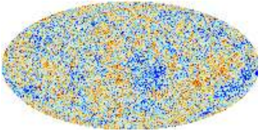

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Outline

- introduction: pion decay rate
  - electroweak factor
  - QCD factors
  - effect of the magnetic field
- perturbative calculation
- nonperturbative simulation
- conclusions

Pion decay: introduction

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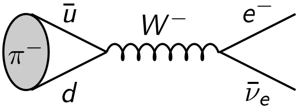
- leptonic decay of charged pion

$$\pi^- \to e^- \bar{\nu}_e$$

- through weak interactions (violates isospin and parity)



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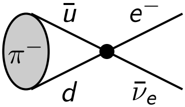

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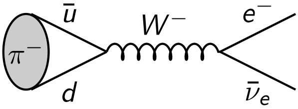

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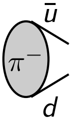
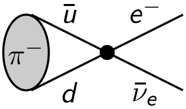


$\propto f_\pi$



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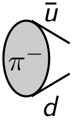
Pion decay: impact of magnetic field

- coupling of B to electric charges



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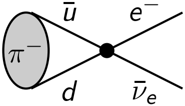

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- W-boson: effectively $G(B)$
- electron

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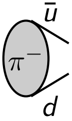

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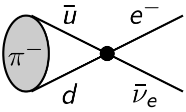

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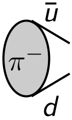

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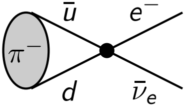

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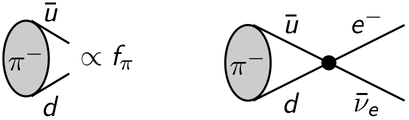

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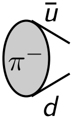

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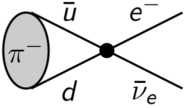

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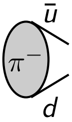

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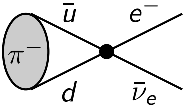

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- separation of scales $M_e^2 \ll eB \ll M_W^2$
- here: perturbation theory for e^- on lowest Landau level
  quenched Wilson quarks for π^- (nonperturbative)

Pion decay: more details

Pion decay: decay rate

- Fermi's golden rule

$$\Gamma = \int \mathrm{d}\Phi \sum_{\{s\}} |\mathcal{M}|^2$$

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probability = squared amplitude

Pion decay rate

- textbook calculation at $B = 0$
  [Huang: Quarks leptons and gauge fields]

$$\Gamma_{B=0} = \frac{G^2}{8\pi} \cos^2\theta_c \, f_1^2 \, (M_\pi^2 - M_e^2)^2 \, \frac{M_e^2}{M_\pi^3} \, ,$$

notation: $f_1 = f_\pi = 131$ MeV

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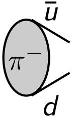
- decay rate proportional to B

Pion decay constants

- QCD matrix element



$$= \langle 0|\bar{d}\gamma_\mu(1+\gamma^5)u|\pi^-(p)\rangle$$

- parameterize using Lorentz invariance

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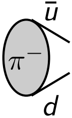

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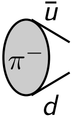

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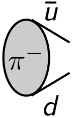

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- for a magnetic field $F_{12} = B$ and pion at rest $p_0 = M_\pi$

$$f_1 M_\pi \delta_{\mu 0} + f_2 eBM_\pi \delta_{\mu 3}$$

Decay rate

- take ratio of decay rates

$$\frac{\Gamma}{\Gamma_{B=0}} = 4\left(\frac{f_1(B)+f_2(B)eB}{f_1(0)}\right)^2\left(1-\frac{M_e^2}{M_\pi(0)^2}\right)^{-2}\frac{eB}{M_\pi(0)^2}\frac{M_\pi(0)}{M_\pi(B)}\,.$$

- remains to calculate nonperturbatively:

$$M_\pi(B),\ f_1(B),\ f_2(B)$$

Lattice simulations

Lattice setup

- quenched ensembles $12^3 \times 36 \ldots 24^3 \times 72$
- lattice spacing $a = 0.12 \ldots 0.06$ fm
- valence Wilson quarks $M_\pi = 415$ MeV $\ldots 770$ MeV
- physical size $M_\pi L \gtrsim 3$
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- magnetic field $\mathbf{B} \parallel \hat{\mathbf{z}}$, strength $eB = 0 \ldots 4$ GeV^2
- improvement: renormalized quark mass kept fixed
  (requires B-dependent tuning of bare quark mass parameter)
  [Bali, Brandt, Endrődi, Gläßle '15]

Matrix elements on the lattice

- remember we need

$$\langle 0|\bar{d}\gamma_0\gamma^5 u|\pi^-(\mathbf{p}=0)\rangle + \langle 0|\bar{d}\gamma_3 u|\pi^-(\mathbf{p}=0)\rangle = f_1 M_\pi + f_2 eBM_\pi$$

- strategy for f_1:

$$C_{PP} = \sum_{\mathbf{x}} \left\langle P(t,\mathbf{x}) P^\dagger(0) \right\rangle, \qquad P = \bar{d}\gamma^5 u$$
$$C_{AP} = \sum_{\mathbf{x}} \left\langle A_0(t,\mathbf{x}) P^\dagger(0) \right\rangle, \quad A_0 = \bar{d}\gamma_0\gamma^5 u$$

- large-time behavior

$$C_{AP} = \frac{1}{2M_\pi}\langle 0|\bar{d}\gamma_0\gamma^5 u|\pi^-\rangle\langle\pi^-|\bar{u}\gamma^5 d|0\rangle e^{-M_\pi t} + \ldots$$
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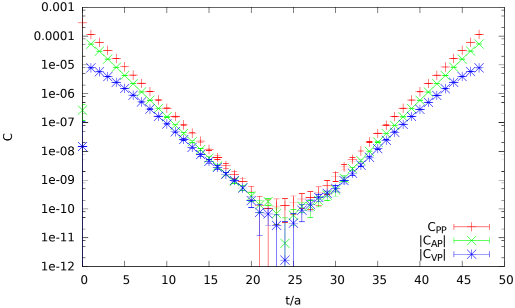
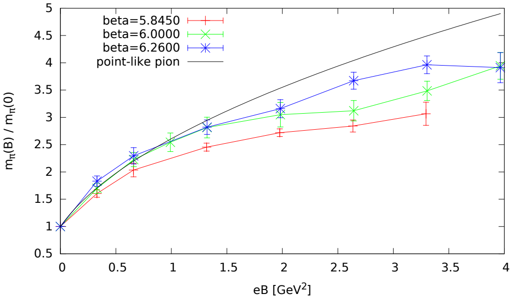
Correlators

- f_1 and f_2 are related to the amplitude of the exponential decay of C_{AP} and of C_{VP}
- M_π is the slope of decay



Results

Pion mass



$a = 0.062$ fm

$a = 0.093$ fm

$a = 0.124$ fm

- for a point-like scalar particle that only interacts with B

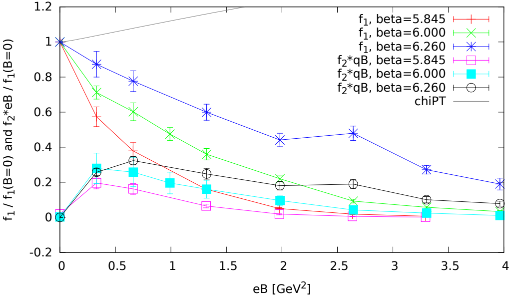
$$M(B) = \sqrt{M(0) + eB}$$

[Bali et al '11, Hidaka et al '12, Luschevskaya et al '16]

Pion decay constants

- normalization

$$\frac{f_1(B)}{f_1(0)} \quad \text{and} \quad \frac{f_2(B)eB}{f_1(0)}$$

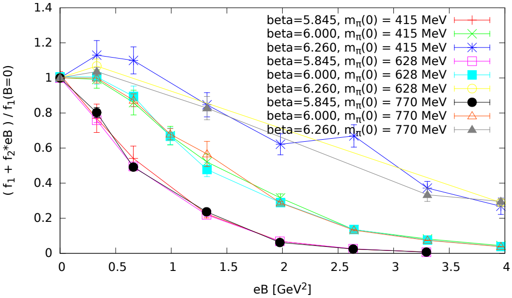


- comparison to chiral perturbation theory [Andersen '12] for $f_1(B)$
- $f_2 eB$ indeed starts linearly with $f_2/f_1 \approx 0.9\ \mathrm{GeV}^{-2}$

Pion decay constants

- the combination we need for $\Gamma/\Gamma_{B=0}$:

$$\left[\frac{f_1 + f_2 eB}{f_1(0)}\right]^2$$



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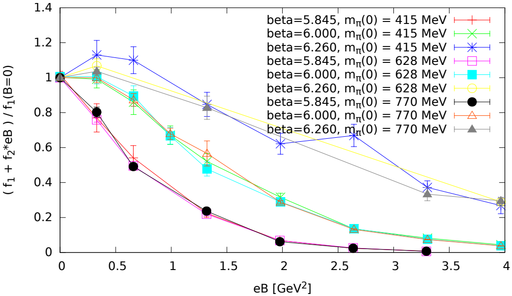


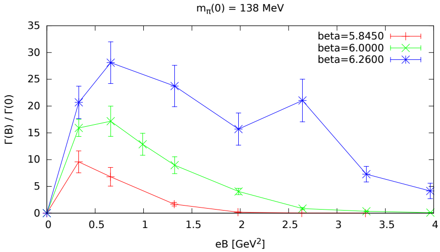
- almost no mass-dependence is visible between $M_\pi(B=0) = 410\text{ MeV}\ldots 770\text{ MeV}$
- assume mass-independence holds down to physical point

Full decay rate

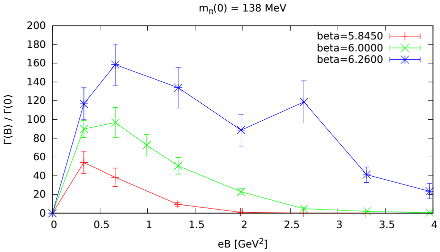
- remember decay rate

$$\frac{\Gamma}{\Gamma_{B=0}} = 4\left(\frac{f_1(B) + f_2(B)eB}{f_1(0)}\right)^2 \left(1 - \frac{M_e^2}{M_\pi(0)^2}\right)^{-2} \frac{eB}{M_\pi(0)^2}\frac{M_\pi(0)}{M_\pi(B)}$$

- extrapolating the QCD factor to the physical point $M_\pi(0) = 138$ MeV



electrons

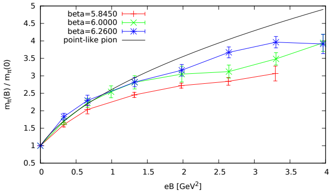


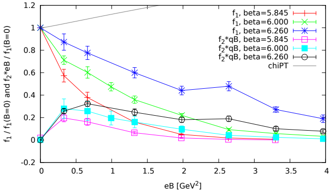
muons

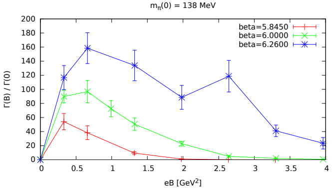
Conclusions

Summary

- charged pion mass increases with B as if π were point-like and free
- tree-level electroweak calculation reveals a new decay channel for $B > 0$
- two decay constants f_1, f_2 behave differently
- full decay rate is enhanced drastically $\rightarrow$ pion lifetime becomes very short







Backup

Interpretation

- for a typical magnetic field $eB \approx 0.3\ \text{GeV}^2$
- pion decay to $e\bar{\nu}_e$ enhanced by ~ 20
- pion decay to $\mu\bar{\nu}_\mu$ enhanced by ~ 100
- remember, the result is valid for $eB \gg m^2_{\mathrm{lepton}}$
  (lowest Landau Level approximation for the lepton)
- ratio of decay rates is independent of B

$$eB \gg m^2_{\mathrm{lepton}}: \qquad \frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)} = 2.27 \cdot 10^{-5}$$

- compare to $B = 0$ ratio:

$$B = 0: \qquad \frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)} = 1.28 \cdot 10^{-4}$$

Electroweak part: spin sums

- sum over outgoing spins

$$\sum_{s_\nu, s_e} |\mathcal{M}|^2 = \ldots \cdot \sum_{s_\nu, s_e} \bar{\nu}\gamma^\alpha(1+\gamma^5)e\bar{e}\gamma_\alpha(1+\gamma^5)\nu$$

  - $B = 0$: e is a plane wave solution of the free Dirac equation

$$\sum_s e_s(k)\bar{e}_s(k) = \not{k} + M_e$$

  - $eB \gg M_e^2$: e is a Landau-level solution of the $B > 0$ Dirac eq.

$$\sum_{\text{multiplicity}} e_-(k)\bar{e}_-(k) = (\not{k}_\parallel + M_e) \cdot \frac{1-\sigma^{12}}{2} \cdot \frac{eB \cdot L^2}{2\pi}$$

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momentum $\parallel B$

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momentum $\parallel B$ → $\not{k}_\parallel$ spin $\parallel -B$ → $\frac{1-\sigma^{12}}{2}$

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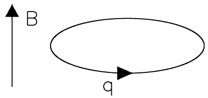
momentum $\parallel B$ spin $\parallel -B$ degeneracy

Magnetized spin sums

- solutions of the Dirac equation for $B > 0$: Landau levels



multiplicity $\propto \Phi = eB \cdot L^2$

- energy levels: $E_n(k) = \pm\sqrt{2neB + k_z^2 + M_e^2}$
- if B is large, only $n = 0$ is important!
- lowest Landau level: only spin-down states $\mathbf{s} \parallel e\mathbf{B}$