Dynamical Simulation of QCD + Axion: First Results

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Outline

QCD Axion

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QCD Axion

Classical action

$$\mathcal{S}_\Sigma = \mathcal{S}_{\rm QCD} + \sum_x \left\{ (\partial_\mu a(x))^2 + i\left(\theta + \frac{a(x)}{f_a}\right) q(x) \right\} \qquad q(x) = \frac{1}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x)$$

$$= \mathcal{S}_{\rm QCD} + \sum_x \left\{ (\partial_\mu a(x))^2 - i\left(\theta + \frac{a(x)}{f_a}\right) \frac{\hat{m}}{N_f}\sum_q \bar{q}(x)\gamma_5 q(x) \right\}$$

The action is invariant under a global shift of $a(x)$. Replacing $a(x)$ by $a(x) - \theta f_a$ cancels the CP violating θ term in the action, at the expense of introducing a physical axion field with the action

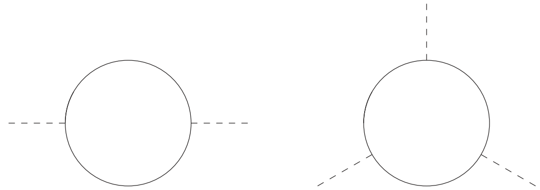
$$\mathcal{S}_\Sigma = \mathcal{S}_{\rm QCD} + \sum_x \left\{ (\partial_\mu a(x))^2 - i\,\frac{a(x)}{f_a}\frac{\hat{m}}{N_f}\sum_q \bar{q}(x)\gamma_5 q(x) \right\}$$

It is expected that QCD effects induce a potential for $a(x)$, whose minimum is at $a(x) = 0$, thus restoring CP symmetry. The resulting axion mass is given by

$$m_a^2\, f_a^2 \approx \frac{2}{9}\chi_t \qquad \chi_t = \frac{Q^2}{V} \approx (190\,\text{MeV})^4$$

Peccei & Quinn

Quantum mechanically there is the possibility of generating double and triple axion couplings in the Wilson effective action. This would invalidate the shift symmetry and the Peccei-Quinn mechanism, and question the use of axions



For example: ϕ^3 theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_0)^2 + \frac{1}{2} m_0^2\, \phi_0^2 + \frac{1}{6} g_0\, \phi_0^3 + y_0\, \phi_0$$

Bare Lagrangian

$$\mathcal{L} = \frac{1}{2} Z_\phi (\partial_\mu \phi)^2 + \frac{1}{2} Z_m m^2 \phi^2 + \frac{1}{6} Z_g\, g\, \phi^3 + y\, \phi$$

Effective (renormalized) Lagrangian

The result would be, that the action $\mathcal{S}_\Sigma$ has to be augmented with possibly all local operators (composed of $a(x)$) of dimension four or less, which are not forbidden by symmetry, in order that the theory is renormalizable

In this presentation we will simulate $\mathcal{S}_\Sigma$, as it stands, on the lattice, to subject the model to a first nonperturbative test

Lattice Simulation

The QCD + axion action $\mathcal{S}_\Sigma$ lends itself to numerical simulations for imaginary values of the axion decay constant $\bar{f}_a = i f_a$

$$\mathcal{S}_\Sigma = \mathcal{S}_{\rm QCD} + \sum_x \left\{ (\partial_\mu a(x))^2 + \frac{a(x)}{\bar{f}_a} \frac{\hat{m}}{N_f} \sum_q \bar{q}(x)\gamma_5 q(x) \right\}$$

We expect the results to be analytic in $1/f_a^2$ for small values $\left|1/f_a^2\right| \ll \pi^2$. At the end of the calculation the results are rotated back to real f_a

$$N_f = 3$$

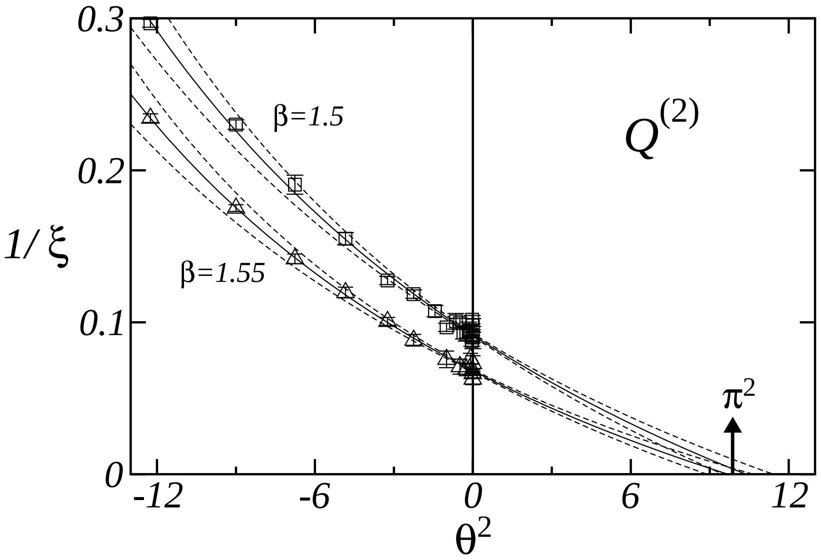
$$(\partial_\mu a(x))^2 \to \sum_\mu \left(\frac{a(x+\alpha\hat{\mu}) - a(x)}{\alpha} \right)^2$$

$$\frac{1}{\hat{m}} = \frac{1}{3} \left(\frac{1}{m_u} + \frac{1}{m_d} + \frac{1}{m_s} \right)$$

α: lattice spacing

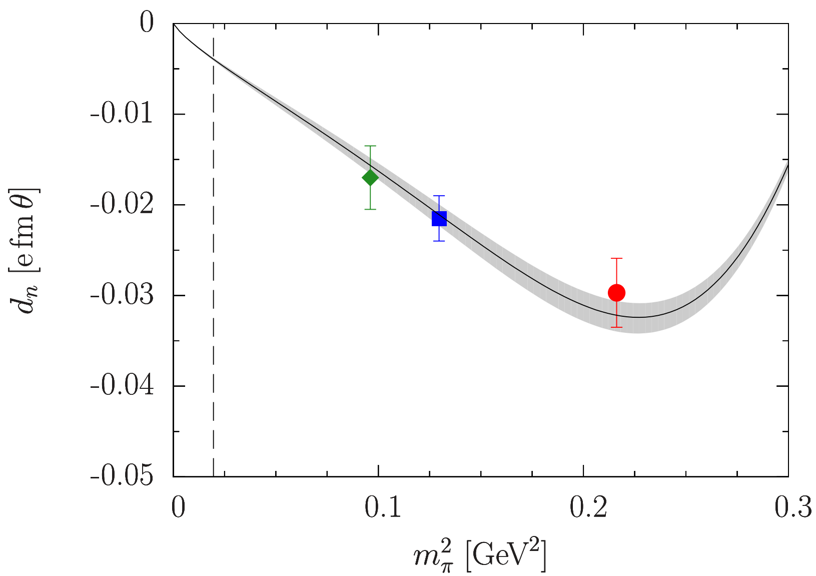
Useful approach ?

$O(3)$ sigma model



Alles & Papa

EDM



QCDSF

Lattices

The simulations so far are done at the SU(3) flavor symmetric point, $m_u = m_d = m_s$, corresponding to a pion mass of $m_\pi \equiv m_K = 410\,\text{MeV}$ using the SLiNC action

QCDSF

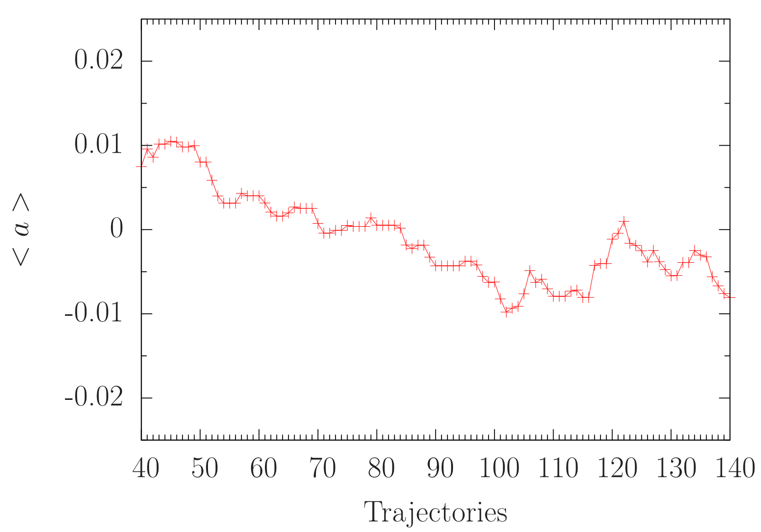
#	V	$\bar{f}_a$
1	$12^3 \times 24$	0.055
2	$12^3 \times 24$	0.550
3	$12^3 \times 24$	5.500
4	$24^3 \times 48$	0.055
5	$24^3 \times 48$	0.550
6	$24^3 \times 48$	5.500

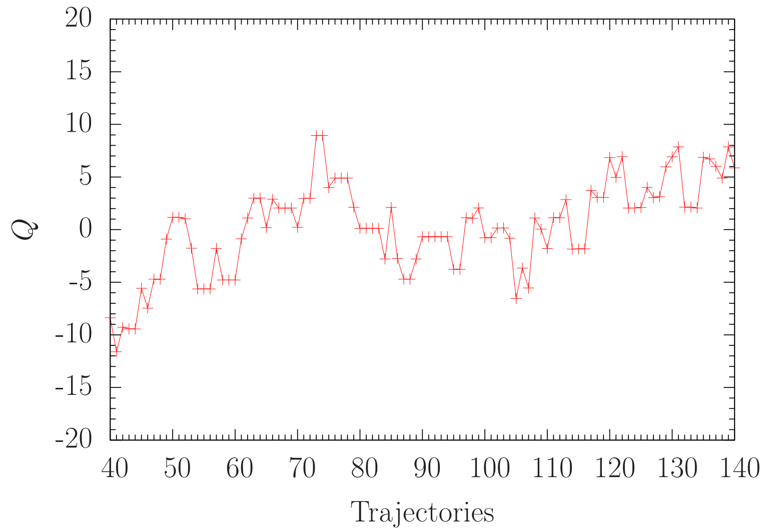
$$\beta = 5.50\,,\ \alpha = 0.074\,\text{fm}$$

First Results

Interaction ?

$$\bar{f}_a = 5.50$$





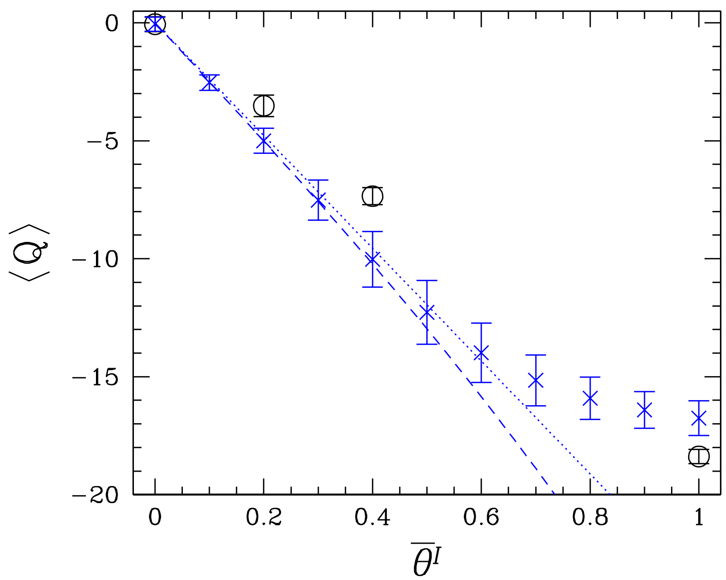
$$r = \frac{(\sum a\, Q) - (\sum a)(\sum Q)/N}{\sqrt{(\sum a^2) - (\sum a)^2/N}\sqrt{(\sum Q^2) - (\sum Q)^2/N}} = -0.4(2)$$ Strongly anticorrelated

In comparison

$$\bar{f}_a \, \frac{\partial \, Q}{\partial <a>} \approx -2500$$

$$\updownarrow \qquad V(\theta) = V(<a>/f_a)$$

$$\frac{\partial \, Q}{\partial \, \bar{\theta}} = -V \, \chi_t \, \cosh \bar{\theta} \approx -20$$

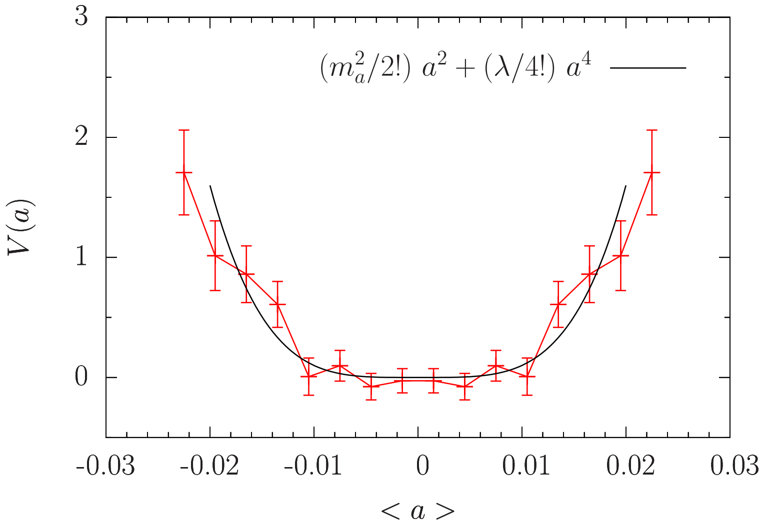


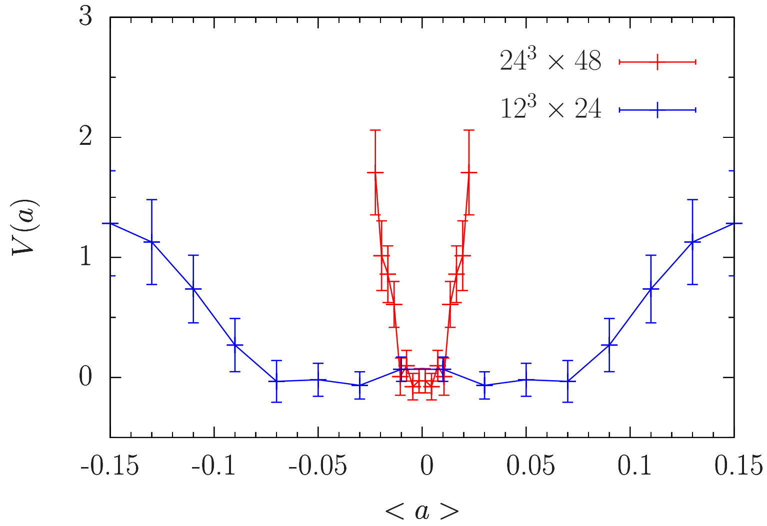
arXiv:0808.1428

$$V(\theta) = \chi_t \, (1 - \cos \theta)$$

$$m_a^2 f_a^2 = \frac{\partial^2}{\partial \, a^2} V(a/f_a)\big|_{a=0} = \chi_t$$

Effective potential





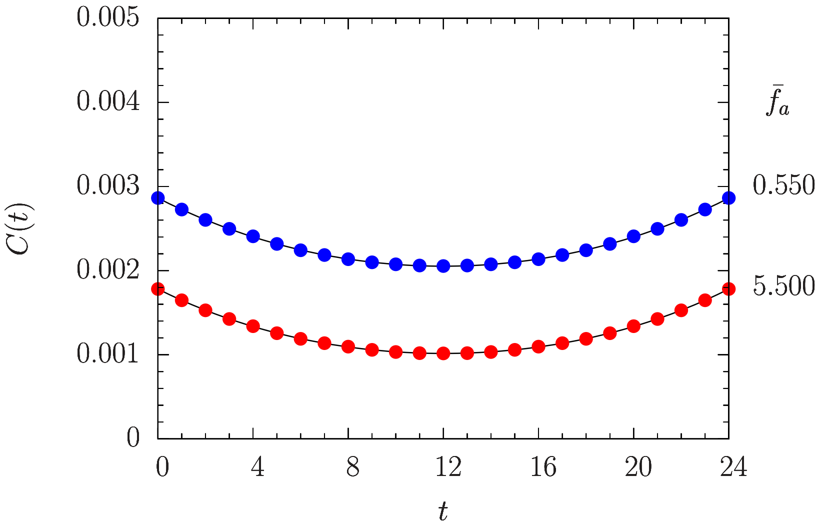
Shows higher order polynomials of a in the effective potential

12^3: $\quad \chi_t = 0.05\ 10^{-4}$

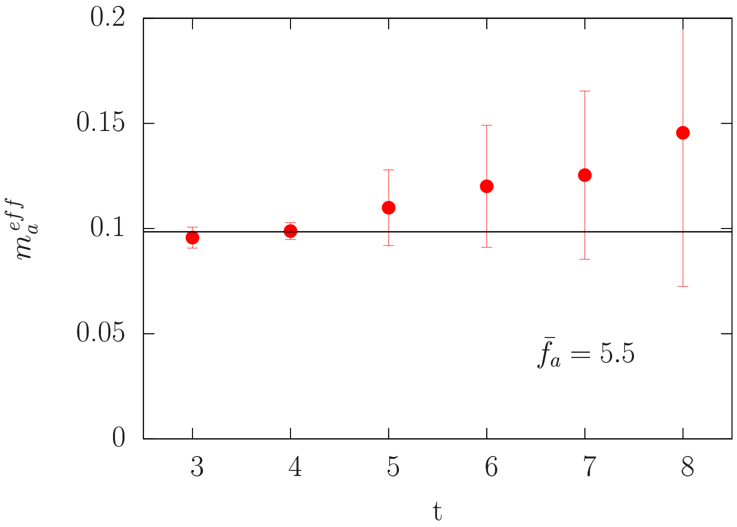
24^3: $\quad \chi_t = 0.28\ 10^{-4} \mathrel{\widehat{=}} (195\,\text{MeV})^4$

Axion mass

$12^3 \times 24$

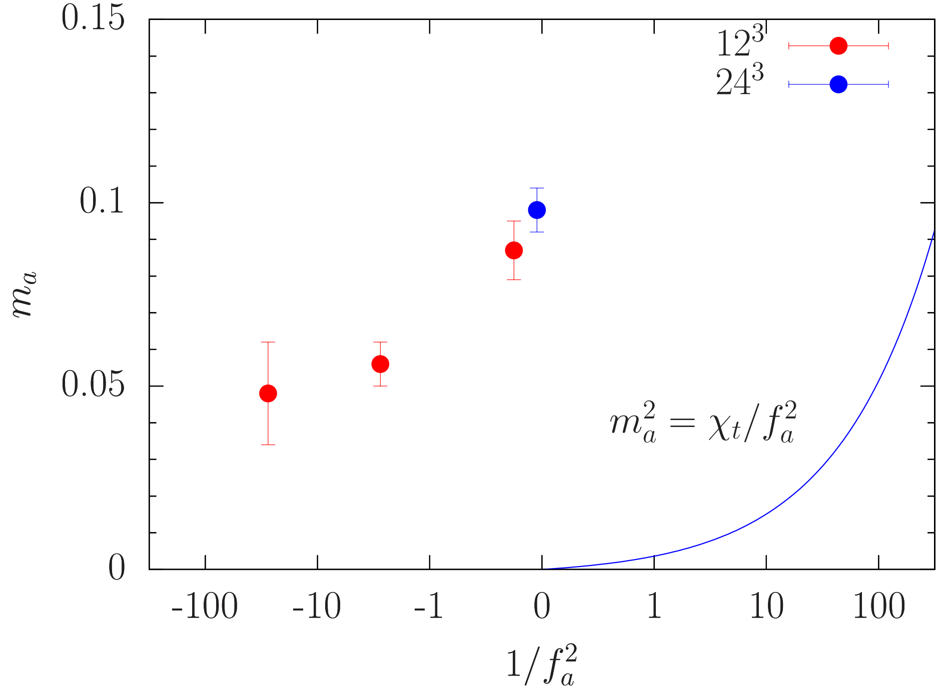


$24^3 \times 48$



$$\langle a(0)a(t)\rangle - \langle a\rangle^2 = A\cosh(m_a t - T/2)$$

Effective Mass



In physical units:

$m_a = O(300)$ MeV

Conclusions

- Axion field $a(x)$ and topological charge density $q(x)$ are strongly correlated/interacting, even at small values of $1/f_a^2$, which requires a dynamical treatment

- Effective potential $V(a)$ receives contributions from higher polynomials/operators of a, far larger than $\chi_t \left(1 - \cosh(<a> / \bar{f}_a)\right)$

- Axion mass turns out to be unexpectedly large - approximately half of the η' mass (at the SU(3) symmetric point). Possibly due to dynamical generation of mass term

- Cannot exclude a very small mass, $m_0 \ll$ 100 MeV, underlying the correlation function $C(t)$, at present