

# SU(3) breaking and the pseudoscalar spectrum in multi-taste QCD

Michael Creutz  
BNL

Are rooted staggered fermions QCD?

- **NO** despite 10 years of controversy

Issue: symmetries of rooted staggered fermions

- incompatible with the chiral anomaly

Concentrate on two ideosyncracies in the pseudoscalar spectrum

- 1. Different octet states have different taste degeneracies
- 2. Spurious states appear at unphysical masses

## Review: Pseudoscalar octet masses in 3 flavor QCD

$$\begin{array}{ccccc} & & K_0 & & K_+ \\ & & & & \\ \pi_- & & & \pi_0, \eta & & \pi_+ \\ & & K_- & & \bar{K}_0 \end{array}$$

Lowest order meson mass spectrum in the sigma model

Ignore the 9th pseudoscalar  $\eta'$

acquires large mass from the anomaly

Effective theory for fluctuations about  $\langle \bar{\psi}\psi \rangle = v$

- model fluctuations  $\bar{\psi}_L^j \psi_R^k \sim v \Sigma^{jk}$  ( $j$  and  $k$  label flavors)
- pion fields encoded in  $\Sigma = \exp(i\pi_\alpha \lambda_\alpha / f_\pi) \in SU(3)$
- Gell-Mann matrices  $\lambda_\alpha$  generate  $SU(3)$  ( $\text{Tr} \lambda_\alpha \lambda_\beta = 2\delta_{\alpha\beta}$ )
- $f_\pi$  phenomenological constant, about 93 MeV.

## Effective theory continued

- Kinetic term

$$L_0 = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)$$

- Expanding to second order in the pion fields

$$L_0 = \text{const} + \frac{1}{2} \partial_\mu \pi_\alpha \partial_\mu \pi_\alpha + \dots$$

Chiral symmetry: without mass, two independent  $SU(3)$  symmetries

$$\begin{aligned}\psi_L &\rightarrow \psi_L g_L \\ \psi_R &\rightarrow \psi_R g_R\end{aligned}\quad g_L, g_R \in SU(3)$$

in the effective theory

$$\Sigma \rightarrow g_L^\dagger \Sigma g_R.$$

Spontaneous breaking gives usual octet of 8 pseudoscalars

## Masses break SU(3) and chiral symmetries

- for the effective theory take

$$L = L_0 - \frac{v f_\pi^2}{4} \text{Re Tr}(m \Sigma).$$

- diagonalize the mass matrix (use  $m \rightarrow g_R^\dagger m g_L$ )

$$m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

- expand  $L$  to quadratic order in pion fields

$$L = \text{const} + \frac{1}{2} \partial_\mu \pi_\alpha \partial_\mu \pi_\alpha + \frac{1}{2} \pi_\alpha M_{\alpha\beta} \pi_\beta$$

where

$$M_{\alpha\beta} = \text{Re Tr } \lambda_\alpha m \lambda_\beta$$

- diagonalize  $M_{\alpha\beta}$  for the lowest order meson masses

And the answer is

$$M_{\pi_+}^2 = M_{\pi_-}^2 \propto \frac{m_u + m_d}{2}$$

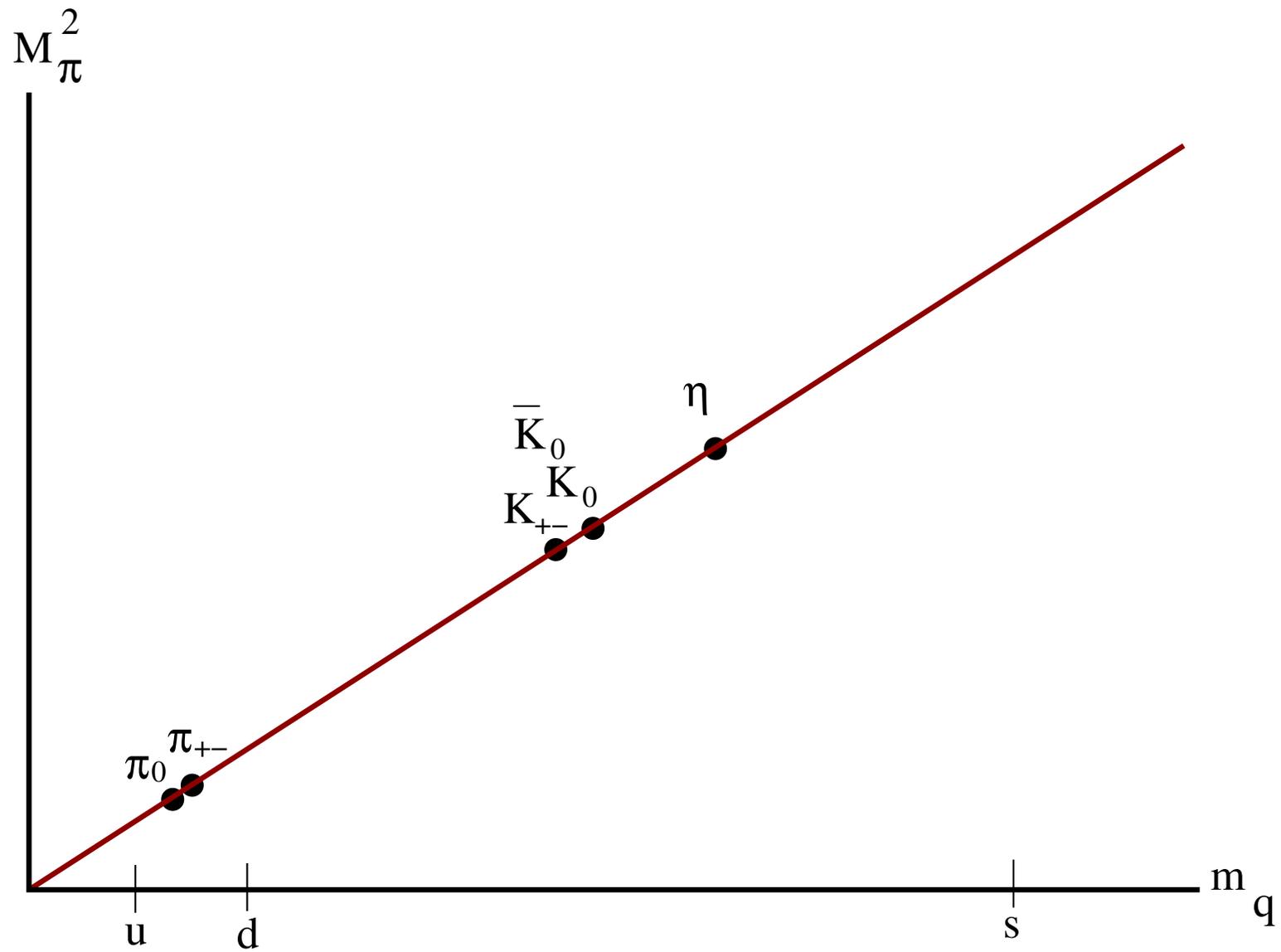
$$M_{K_+}^2 = M_{K_-}^2 \propto \frac{m_u + m_s}{2}$$

$$M_{K_0}^2 = M_{\bar{K}_0}^2 \propto \frac{m_d + m_s}{2}$$

$$M_{\pi_0}^2 \propto \frac{1}{3} \left( m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s} \right)$$

$$M_{\eta}^2 \propto \frac{1}{3} \left( m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s} \right)$$

( $\pi_0$   $\eta$  mixing since  $M_{38} \neq 0$ )



Turn on a  $N_t = 4$  factor for each flavor

Motivated by 4 “tastes” of staggered fermions

12 distinct quark species

Assume exact taste symmetry

Before flavor breaking:  $SU(12)$  symmetry

- expect  $143 = 12^2 - 1$  pseudo-goldstone bosons
- 8 by 8 meson mass matrix  $\longrightarrow$  143 by 143

Actually three distinct  $SU(4)$  taste groups

- one for each flavor  $u, d, s$
- classify states by representations in each
- relevant  $SU(4)$  representations:  $1, 4, \bar{4}, 15$

analog of  $SU(3)$ :  $1, 3, \bar{3}, 8$

## Kaons and charged pions

- each involves two distinct flavors
- appear in a  $(4_q, \bar{4}_{q'})$  representation

16 equivalent taste combinations

- meson masses average constituents

$$M^2 \propto \frac{1}{2}(m_q + m_{q'})$$

- this accounts for  $16 \times 6 = 96$  of 143 expected pseudo-Goldstones

Neutral mesons:  $q = q' \longrightarrow 4 \otimes \bar{4} \rightarrow 1 \oplus 15$

- for each flavor  $\bar{q}q \rightarrow$  a taste 15 plus a taste singlet

The taste 15 combinations **cannot mix**: (independent taste groups)

a taste 15 of  $i\bar{u}\gamma_5 u$  states  $M^2 \propto m_u$

a taste 15 of  $i\bar{d}\gamma_5 d$  states  $M^2 \propto m_d$

a taste 15 of  $i\bar{s}\gamma_5 s$  states  $M^2 \propto m_s$

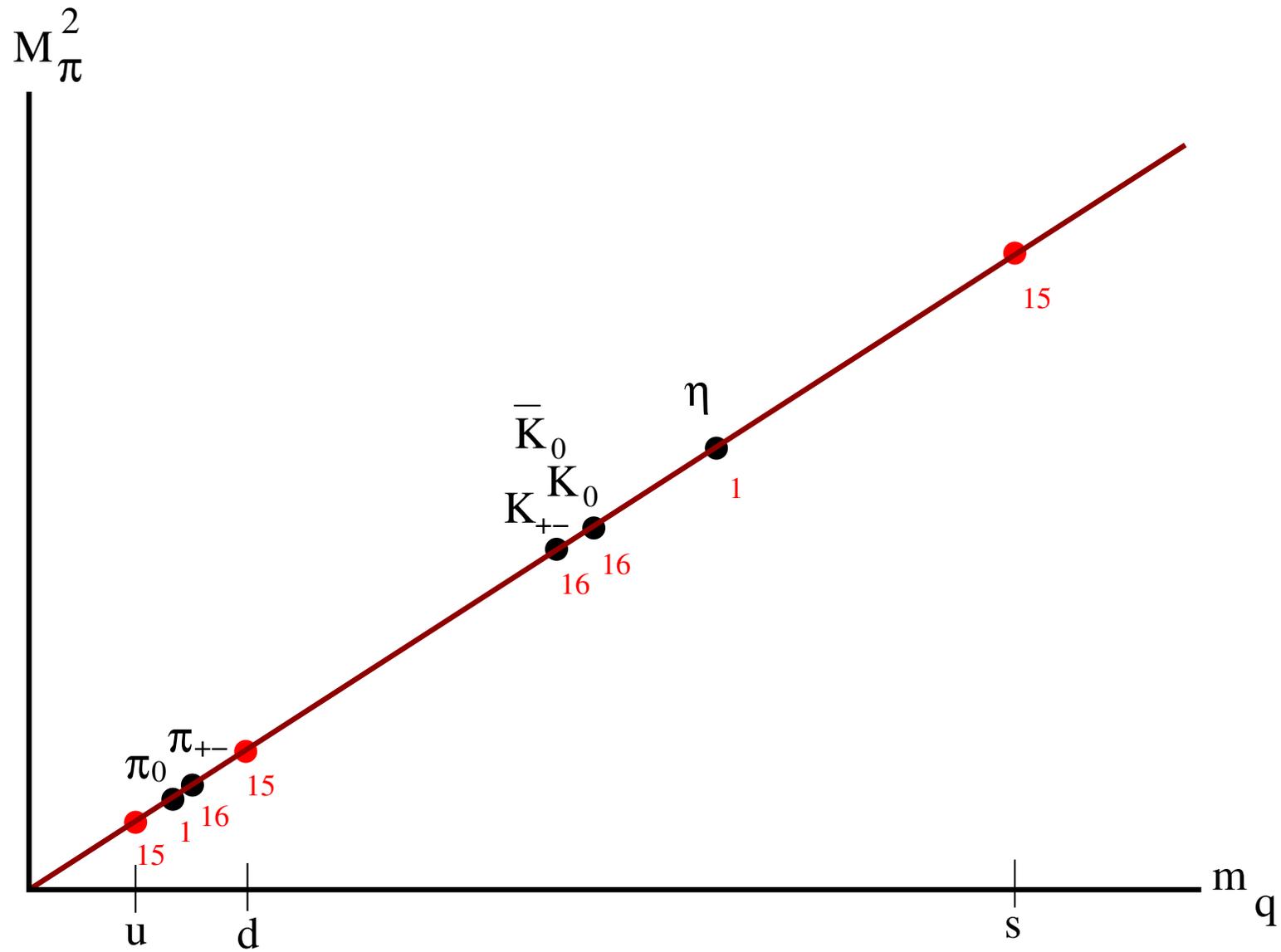
Remaining: three taste singlet combinations  $\bar{u}u, \bar{d}d, \bar{s}s$

- $\eta'$ : taste and flavor singlet, heavy and ignored here
- flavor non-singlet combinations do mix  
same mixing matrix as the single taste theory

$$M_{\pi_0}^2 \propto \frac{1}{3} \left( m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s} \right)$$

$$M_{\eta}^2 \propto \frac{1}{3} \left( m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s} \right)$$

Aubin and Bernard



$$143 = 16 \times 6 + 15 \times 3 + 2$$

## Rooting replicas

Replace  $|D| \longrightarrow |D^4|^{1/4}$

With  $N_t$  copies of a valid fermion formulation; i.e. Wilson fermions

- rooting is a mathematical identity (if  $|D| > 0$ )
- propagator always has only one pole
- $16 \rightarrow N_t^2$  goes to unity as  $N_t \rightarrow 1$
- $15 \rightarrow N_t^2 - 1$  goes to zero as  $N_t \rightarrow 1$

# Staggered fermions

NOT replicas: doublers are chiral partners

- propagator always has four poles, even with rooting
- the taste **15** multiplets remain

One exact chiral symmetry for each flavor

- a member of each **15** must survive as a Goldstone boson

# The questions

1. How can the SU(3) octet be recovered?

- Different flavors have different taste degeneracies

$\pi_0$  and  $\eta$ : a single taste combination

kaons and charged pions: 16 equivalent taste combinations

2. What happens to the 3 unphysical 15's?

- masses incompatible with any physical particles

$$M^2 \propto m_u, m_d, m_s$$

