TECHNISCHE UNIVERSITÄT WIEN

Preliminary QCD phase transition line with 695 MeV dynamical staggered fermions from effective Polyakov line actions

Roman Höllwieser[a], Jeff Greensite[b]

[a]Institute of Atomic and Subatomic Physics, Nuclear Physics Dept., Vienna University of Technology, Operngasse 9, 1040 Vienna, Austria

[b]Physics and Astronomy Dept., San Francisco State University, San Francisco, CA 94132, USA

Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model

Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model
- fix Polyakov line holonomies $U_0(\vec{x}, 0) = P_x$ (temporal gauge) and integrate out all other d.o.f.

Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model
- fix Polyakov line holonomies $U_0(\vec{x},0) = P_x$ (temporal gauge) and integrate out all other d.o.f.

$$e^{S_P(P_x)} = \int DU_0(\vec{x},0) DU_k D\psi \prod_x \delta[P_x - U_0(\vec{x},0)] e^{S_L}$$

Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model
- fix Polyakov line holonomies $U_0(\vec{x},0) = P_x$ (temporal gauge) and integrate out all other d.o.f.

$$e^{S_P(P_x)} = \int DU_0(\vec{x},0) DU_k D\psi \prod_x \delta[P_x - U_0(\vec{x},0)] e^{S_L}$$

- derive S_P at $\mu = 0$, for $\mu > 0$ we have (true to all orders of strong coupling/hopping parameter expansion)

Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model
- fix Polyakov line holonomies $U_0(\vec{x},0) = P_x$ (temporal gauge) and integrate out all other d.o.f.

$$e^{S_P(P_x)} = \int DU_0(\vec{x},0) DU_k D\psi \prod_x \delta[P_x - U_0(\vec{x},0)] e^{S_L}$$

- derive S_P at $\mu = 0$, for $\mu > 0$ we have (true to all orders of strong coupling/hopping parameter expansion)

$$S_P^{\mu}(P_x, P_x^{\dagger}) = S_P^{\mu=0}[e^{N_t\mu} P_x, e^{-N_t\mu} P_x^{\dagger}]$$

Effective Polyakov Line Action

- map LGT to Polyakov line action (SU(3) spin) model
- fix Polyakov line holonomies $U_0(\vec{x},0) = P_x$ (temporal gauge) and integrate out all other d.o.f.

$$e^{S_P(P_x)} = \int DU_0(\vec{x},0) DU_k D\psi \prod_x \delta[P_x - U_0(\vec{x},0)] e^{S_L}$$

- derive S_P at $\mu = 0$, for $\mu > 0$ we have (true to all orders of strong coupling/hopping parameter expansion)

$$S_P^{\mu}(P_x, P_x^{\dagger}) = S_P^{\mu=0}[e^{N_t\mu} P_x, e^{-N_t\mu} P_x^{\dagger}]$$

- hard to compute $\exp[S_P(P_x)]$ directly, but action ratios are easily computed as expectation values $\rightarrow$ relative weights via derivatives of S_P w.r.t. Fourier components a_k of P_x

- setting a particular $a_k = 0$, we construct from the resulting configuration $\tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3$

- setting a particular $a_k = 0$, we construct from the resulting configuration $\tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3$

$P'_x = (\alpha + \Delta\alpha/2) e^{ikx} + f \tilde{P}_x$ and $P''_x = (\alpha - \Delta\alpha/2) e^{ikx} + f \tilde{P}_x$

- setting a particular $a_k = 0$, we construct from the resulting configuration $\tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3$

$$P'_x = (\alpha + \Delta\alpha/2) e^{ikx} + f \tilde{P}_x \text{ and } P''_x = (\alpha - \Delta\alpha/2) e^{ikx} + f \tilde{P}_x$$

- effective Polyakov line action motivated by heavy-dense action, where h is some inverse power of hopping parameter and satisfies the Pauli exclusion principle

- setting a particular $a_k = 0$, we construct from the resulting configuration $\tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3$

$$P'_x = (\alpha + \Delta\alpha/2) e^{ikx} + f \tilde{P}_x \text{ and } P''_x = (\alpha - \Delta\alpha/2) e^{ikx} + f \tilde{P}_x$$

- effective Polyakov line action motivated by heavy-dense action, where h is some inverse power of hopping parameter and satisfies the Pauli exclusion principle

$$\begin{aligned} S_{eff}[P_x] &= \sum_{x,y} P_x K(x-y) P_y \\ &+ p \sum_x \log(1 + h e^{\mu/T} Tr[P_x] + h^2 e^{2\mu/T} Tr[P_x^\dagger] + h^3 e^{3\mu/T}) \\ &\qquad \log(1 + h e^{-\mu/T} Tr[P_x] + h^2 e^{-2\mu/T} Tr[P_x^\dagger] + h^3 e^{-3\mu/T}) \end{aligned}$$

- setting a particular $a_k = 0$, we construct from the resulting configuration $\tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3$

$$P'_x = (\alpha + \Delta\alpha/2)e^{ikx} + f\tilde{P}_x \text{ and } P''_x = (\alpha - \Delta\alpha/2)e^{ikx} + f\tilde{P}_x$$

- effective Polyakov line action motivated by heavy-dense action, where h is some inverse power of hopping parameter and satisfies the Pauli exclusion principle

$$\begin{aligned} S_{eff}[P_x] = &\sum_{x,y} P_x K(x-y) P_y \\ &+ p \sum_x \log(1 + he^{\mu/T} Tr[P_x] + h^2 e^{2\mu/T} Tr[P_x^\dagger] + h^3 e^{3\mu/T}) \\ &\qquad \log(1 + he^{-\mu/T} Tr[P_x] + h^2 e^{-2\mu/T} Tr[P_x^\dagger] + h^3 e^{-3\mu/T}) \end{aligned}$$

- determine $K(x-y)$ and h from fitting to lattice data

- setting a particular $a_k = 0$, we construct from the resulting configuration $\tilde{P}_x = P_x - e^{ikx} \sum_y P_y e^{-iky} / L^3$

$$P'_x = (\alpha + \Delta\alpha/2)e^{ikx} + f\tilde{P}_x \text{ and } P''_x = (\alpha - \Delta\alpha/2)e^{ikx} + f\tilde{P}_x$$

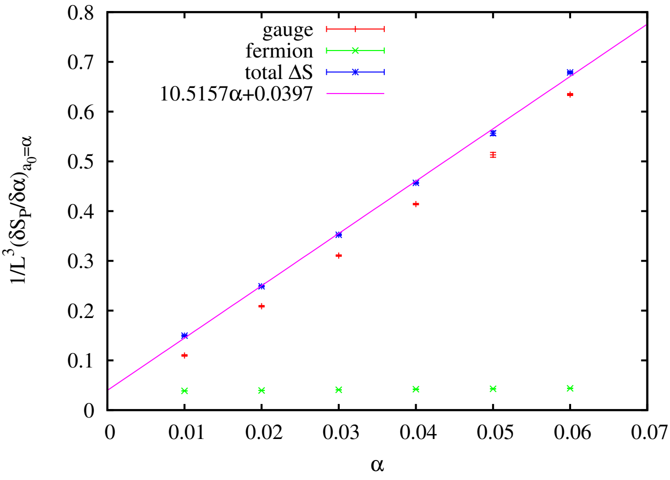
- effective Polyakov line action motivated by heavy-dense action, where h is some inverse power of hopping parameter and satisfies the Pauli exclusion principle

$$\begin{aligned} S_{eff}[P_x] = & \sum_{x,y} P_x K(x-y) P_y \\ & + p \sum_x \log(1 + he^{\mu/T} Tr[P_x] + h^2 e^{2\mu/T} Tr[P_x^\dagger] + h^3 e^{3\mu/T}) \\ & \log(1 + he^{-\mu/T} Tr[P_x] + h^2 e^{-2\mu/T} Tr[P_x^\dagger] + h^3 e^{-3\mu/T}) \end{aligned}$$

- determine $K(x-y)$ and h from fitting to lattice data

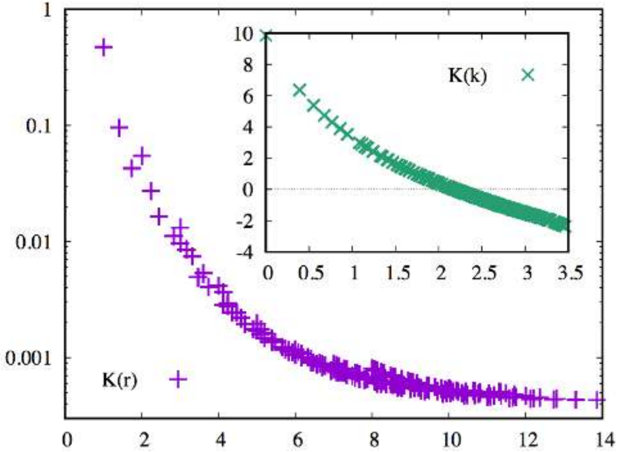
$$\frac{1}{L^3}\left(\frac{\partial S_P}{\partial a_k}\right)_{a_k=\alpha} = 2K(k)\alpha + \frac{p}{L^3} \sum_x (3he^{ikx} + 3h^2 e^{-ikx} + c.c.)$$

Fitting to lattice data

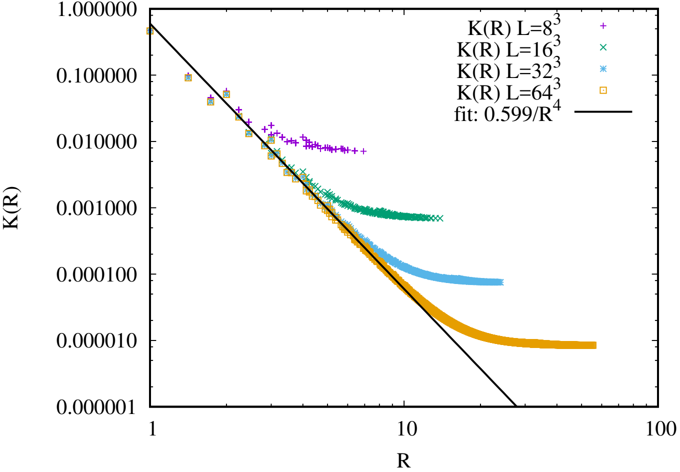


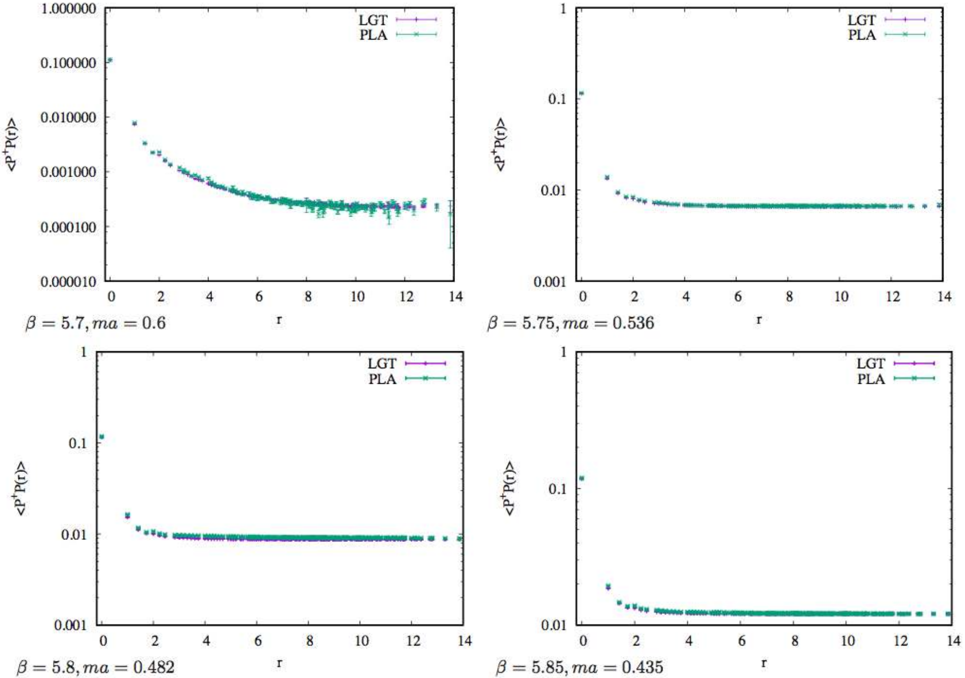
k=0

Fourier transform $K(k)$ to $K(r)$



Finite size cutoff R_{cut} for $K(r)$





$\beta = 5.7, ma = 0.6$ | $\beta = 5.75, ma = 0.536$

$\beta = 5.8, ma = 0.482$ | $\beta = 5.85, ma = 0.435$

Solve sign problem for the effective action

- remaining sign problem can be solved by mean field theory (see also *Splittorff and Greensite, 2012*)

Solve sign problem for the effective action

- remaining sign problem can be solved by mean field theory (see also *Splittorff and Greensite, 2012*)
- treatment of $SU(3)$ spin models at finite μ is a minor variation of standard mean field theory at zero μ

Solve sign problem for the effective action

- remaining sign problem can be solved by mean field theory (see also *Splittorff and Greensite, 2012*)
- treatment of $SU(3)$ spin models at finite μ is a minor variation of standard mean field theory at zero μ
- basic idea is that each spin is effectively coupled to the average spin on the lattice, not just nearest neighbors

Solve sign problem for the effective action

- remaining sign problem can be solved by mean field theory (see also *Splittorff and Greensite, 2012*)
- treatment of $SU(3)$ spin models at finite μ is a minor variation of standard mean field theory at zero μ
- basic idea is that each spin is effectively coupled to the average spin on the lattice, not just nearest neighbors

$$S_P^0 = \frac{1}{9}[\sum_{x,y\neq x} \mathrm{Tr}U_x \mathrm{Tr}U_y^\dagger K(x-y) + \sum_x \mathrm{Tr}U_x \mathrm{Tr}U_x^\dagger K(0)]$$

Solve sign problem for the effective action

- remaining sign problem can be solved by mean field theory (see also *Splittorff and Greensite, 2012*)
- treatment of $SU(3)$ spin models at finite μ is a minor variation of standard mean field theory at zero μ
- basic idea is that each spin is effectively coupled to the average spin on the lattice, not just nearest neighbors

$$S_P^0 = \frac{1}{9}[\sum_{x,y\neq x} \mathrm{Tr} U_x \mathrm{Tr} U_y^\dagger K(x-y) + \sum_x \mathrm{Tr} U_x \mathrm{Tr} U_x^\dagger K(0)]$$

- we introduce two magnetizations u, v for $\mathrm{Tr}U$ and $\mathrm{Tr}U^\dagger$

Solve sign problem for the effective action

- remaining sign problem can be solved by mean field theory (see also *Splittorff and Greensite, 2012*)
- treatment of $SU(3)$ spin models at finite μ is a minor variation of standard mean field theory at zero μ
- basic idea is that each spin is effectively coupled to the average spin on the lattice, not just nearest neighbors

$$S_P^0 = \frac{1}{9}[\sum_{x,y \neq x} \mathrm{Tr} U_x \mathrm{Tr} U_y^\dagger K(x-y) + \sum_x \mathrm{Tr} U_x \mathrm{Tr} U_x^\dagger K(0)]$$

- we introduce two magnetizations u, v for $\mathrm{Tr} U$ and $\mathrm{Tr} U^\dagger$

$$\mathrm{Tr} U_x = (\mathrm{Tr} U_x - u) + u \ , \ \mathrm{Tr} U_x^\dagger = (\mathrm{Tr} U_x^\dagger - v) + v$$

$$
\begin{aligned}
S_P^0 &= \frac{1}{9}\sum_{x\neq 0} K(x)[\sum_x (v\mathrm{Tr}U_x + u\mathrm{Tr}U_x^\dagger) - uvL^3] \\
&+ \frac{1}{9}\sum_x \mathrm{Tr}[U_x]\mathrm{Tr}U_x^\dagger K(0) + E_0
\end{aligned}
$$

with $E_0 = \sum_{x,y\neq x}(\mathrm{Tr}U_x - u)(\mathrm{Tr}U_y^\dagger - v)K(x-y)$

$$
\begin{aligned}
S_P^0 &= \frac{1}{9}\sum_{x\neq 0} K(x)[\sum_x (v\mathrm{Tr}U_x + u\mathrm{Tr}U_x^\dagger) - uvL^3] \\
&+ \frac{1}{9}\sum_x \mathrm{Tr}[U_x]\mathrm{Tr}U_x^\dagger K(0) + E_0
\end{aligned}
$$

with $E_0 = \sum_{x,y\neq x}(\mathrm{Tr}U_x - u)(\mathrm{Tr}U_y^\dagger - v)K(x-y)$

- if we drop E_0 the total action (including $\mu \neq 0$) is local and group integrations can be carried out analytically

$$S_P^0 = \frac{1}{9}\sum_{x\neq 0} K(x)[\sum_x (v\mathrm{Tr}U_x + u\mathrm{Tr}U_x^\dagger) - uvL^3] + \frac{1}{9}\sum_x \mathrm{Tr}[U_x]\mathrm{Tr}U_x^\dagger K(0) + E_0$$

with $E_0 = \sum_{x,y\neq x}(\mathrm{Tr}U_x - u)(\mathrm{Tr}U_y^\dagger - v)K(x-y)$

- if we drop E_0 the total action (including $\mu \neq 0$) is local and group integrations can be carried out analytically
- parameters u and v are chosen such that E_0 can be treated as a perturbation, $\langle E_0\rangle = 0$ when $u = \langle \mathrm{Tr}U_x\rangle, v = \langle \mathrm{Tr}U_x^\dagger\rangle$

$$S_P^0 = \frac{1}{9}\sum_{x\neq 0} K(x)[\sum_x (v\mathrm{Tr}U_x + u\mathrm{Tr}U_x^\dagger) - uvL^3]$$
$$+ \frac{1}{9}\sum_x \mathrm{Tr}[U_x]\mathrm{Tr}U_x^\dagger K(0) + E_0$$

with $E_0 = \sum_{x,y\neq x}(\mathrm{Tr}U_x - u)(\mathrm{Tr}U_y^\dagger - v)K(x-y)$

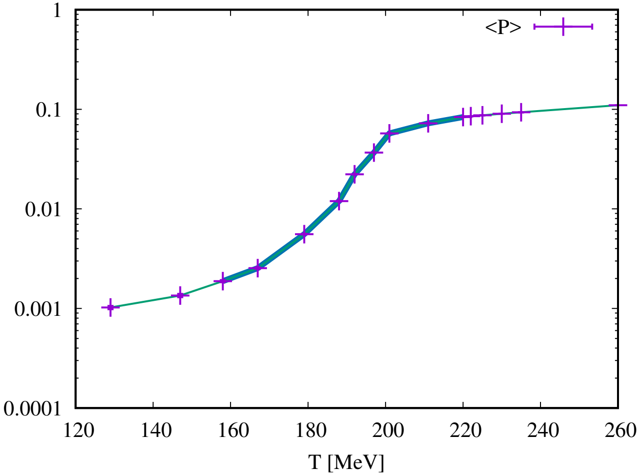
- if we drop E_0 the total action (including $\mu \neq 0$) is local and group integrations can be carried out analytically
- parameters u and v are chosen such that E_0 can be treated as a perturbation, $\langle E_0\rangle = 0$ when $u = \langle \mathrm{Tr}U_x\rangle, v = \langle \mathrm{Tr}U_x^\dagger\rangle$
- equivalent to the stationarity of the mean field free energy with respect to variations in u and v $\rightarrow$ solve numerically

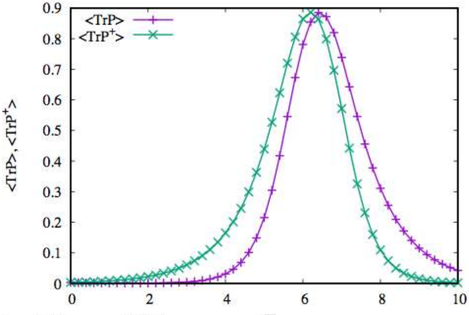
Simulation parameters and mean field results

- for effective Polyakov line actions derived from LGT
- on 16^3x6 lattices with Wilson gauge action and
- dynamical staggered fermions with $m_q = 695$MeV
- note good agreement of Polyakovs and small h!!!

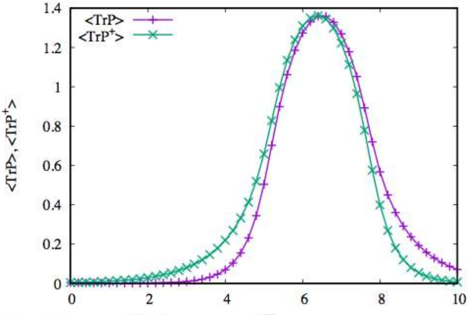
β	T[MeV]	a[fm]	ma	P (LGT)	P (mfd)	R_{cut}	h	μ_1/T	μ_2/T
5.55	129	0.248	0.875	0.00102	0.00101	5.5	0.0018	-	-
5.60	147	0.217	0.767	0.00135	0.00133	5.5	0.0014	-	-
5.63	158	0.201	0.711	0.00188	0.00189	5.5	0.0017	4.4	8.5
5.65	167	0.192	0.677	0.00254	0.00249	5.5	0.0019	4.2	8.3
5.70	188	0.170	0.601	0.01198	0.01195	5.0	0.0069	3.5	6.5
5.73	201	0.159	0.561	0.05734	0.05731	5.0	0.0220	2.9	5.5
5.75	211	0.152	0.536	0.07235	0.07110	5.0	0.0272	2.1	5.3
5.77	220	0.145	0.513	0.08354	0.08347	5.0	0.0387	1.1	5.1
5.775	222	0.144	0.508	0.08522	0.08523	4.0	0.0565	-	-
5.78	225	0.142	0.502	0.08703	0.08679	4.0	0.0610	-	-
5.80	235	0.136	0.482	0.09332	0.09437	4.0	0.0710	-	-
5.85	260	0.123	0.435	0.10992	0.01115	4.0	0.0900	-	-

Finite temperature transition at $\mu = 0$

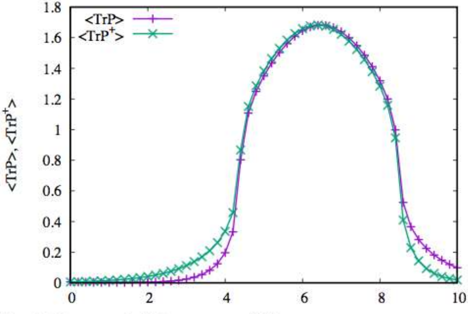




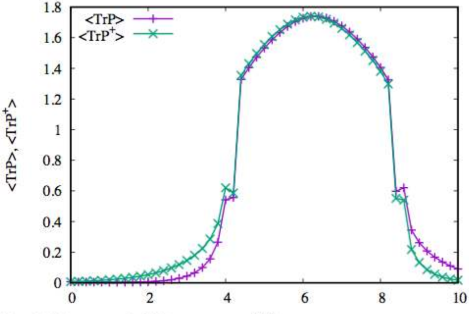
$\beta = 5.55, ma = 0.875$



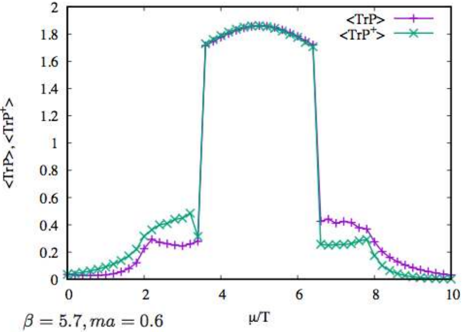
$\beta = 5.6, ma = 0.767$



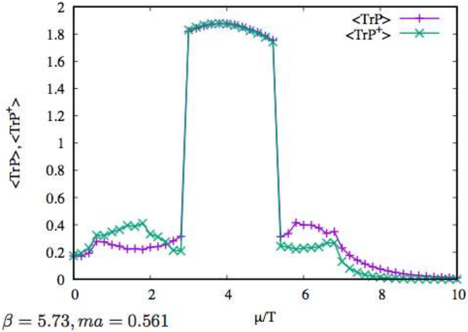
$\beta = 5.63, ma = 0.711$



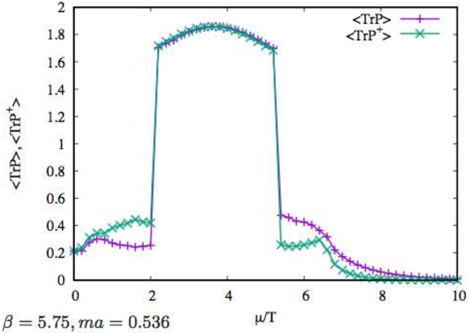
$\beta = 5.65, ma = 0.677$



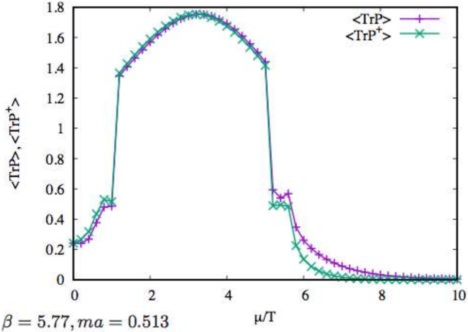
$\beta = 5.7, ma = 0.6$



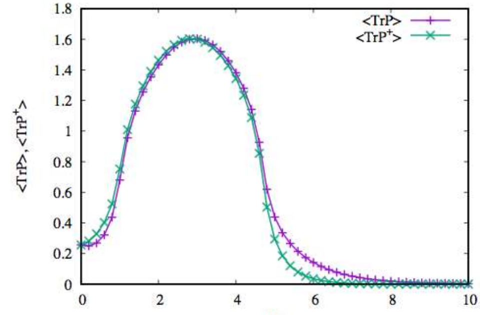
$\beta = 5.73, ma = 0.561$



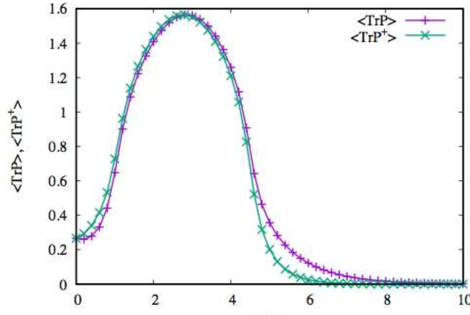
$\beta = 5.75, ma = 0.536$



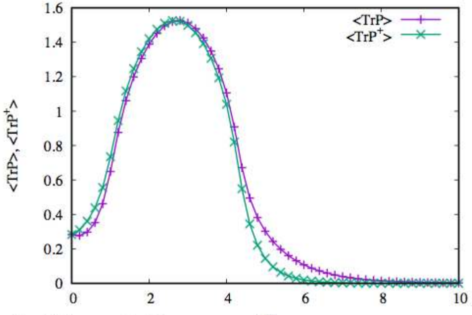
$\beta = 5.77, ma = 0.513$



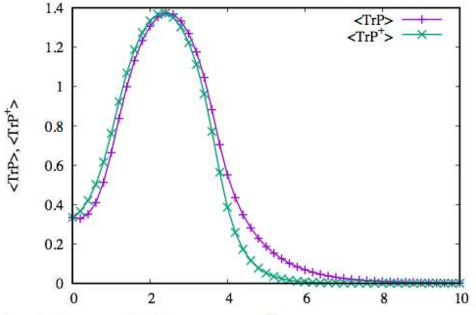
$\beta = 5.775, ma = 0.508$



$\beta = 5.78, ma = 0.502$

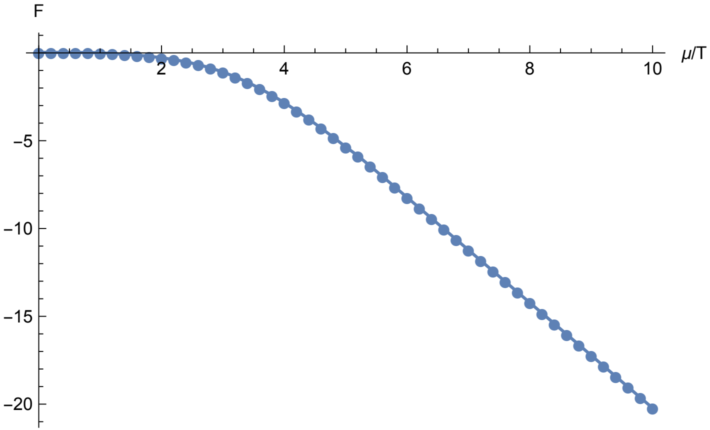


$\beta = 5.8, ma = 0.482$

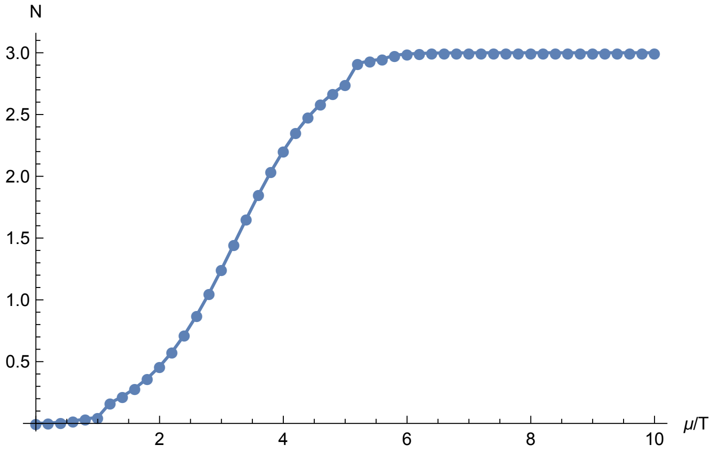


$\beta = 5.85, ma = 0.435$

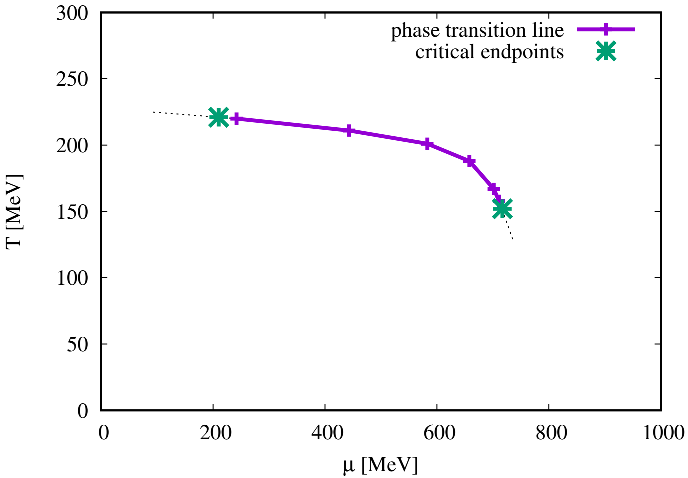
Free energy



Number density



Preliminary Phase Diagram



Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695\text{MeV}$ with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices

Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695\text{MeV}$ with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices
- good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory

Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695\text{MeV}$ with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices
- good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory
- solved sign problem for the effective theory by mean field and find a phase transition line and correct density limit

Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695$MeV with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices
- good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory
- solved sign problem for the effective theory by mean field and find a phase transition line and correct density limit
- second critical endpoint suggests heavy quark regime?

Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695$MeV with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices
- good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory
- solved sign problem for the effective theory by mean field and find a phase transition line and correct density limit
- second critical endpoint suggests heavy quark regime?
- simple PLA ansatz may not hold on finer lattices (diverging interaction range) and for lighter quarks?

Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695$MeV with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices
- good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory
- solved sign problem for the effective theory by mean field and find a phase transition line and correct density limit
- second critical endpoint suggests heavy quark regime?
- simple PLA ansatz may not hold on finer lattices (diverging interaction range) and for lighter quarks?
- higher order (multi-body interactions) and chiral/center symmetry breaking terms suppressed by small h?

Conclusions & Discussion

- determined effective Polyakov line action for staggered fermions with $m_q = 695$MeV with standard Wilson gauge action for a range of gauge couplings on $16^3 \times 6$ lattices
- good agreement for the Polyakov line correlators computed in the effective theory and underlying lattice gauge theory
- solved sign problem for the effective theory by mean field and find a phase transition line and correct density limit
- second critical endpoint suggests heavy quark regime?
- simple PLA ansatz may not hold on finer lattices (diverging interaction range) and for lighter quarks?
- higher order (multi-body interactions) and chiral/center symmetry breaking terms suppressed by small h?
- can the mean-field PLA still locate transition lines and determine critical properties reliably?

Questions?

Thank You &

Tareq Alhalholy, Derar Altarawneh, Michael Engelhardt, Manfried Faber, Martin Gal, Jeff Greensite, Urs M. Heller, James Hettrick, Andrei Ivanov, Thomas Layer, Štefan Olejnik, Luis Oxman, Mario Pitschmann, Jesus Saenz, Thomas Schweigler, Wolfgang Söldner, David Vercauteren, Markus Wellenzohn

Preliminary QCD phase transition line with 695 MeV dynamical staggered fermions from effective Polyakov line actions

Roman Höllwieser[a], hroman@kph.tuwien.ac.at
Jeff Greensite[b], greensit@sfsu.edu

[a]Institute of Atomic and Subatomic Physics, Nuclear Physics Dept., Vienna University of Technology, Operngasse 9, 1040 Vienna, Austria
[b]Physics and Astronomy Dept., San Francisco State University, San Francisco, CA 94132, USA