Determination of the Strong Coupling Constant by the ALPHA Collaboration

M. Bruno, M. Dalla Brida, P. Fritzsch, T. Korzec, A. Ramos, S. Schaefer, H. Simma, S. Sint, R. Sommer

ALPHA Collaboration

Lattice2017

Motivation

- Fix QCD parameters $\alpha_S(\mu), \overline{m}_u(\mu), \overline{m}_d(\mu), \ldots$ from experiments
  $\Rightarrow$ everything else is a prediction

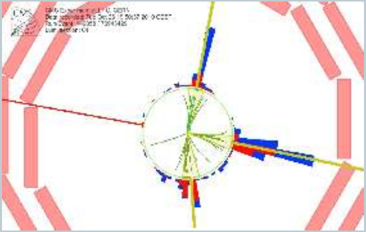
Example

differential inclusive jet production cross section

$$\frac{d\sigma}{dp_T} \stackrel{\text{NLO}}{=} \alpha_S^2(\mu)\, \hat{X}^{(0)}(\mu_F, p_T) \times [1 + \alpha_S(\mu) K1(\mu, \mu_F, p_T)]$$

- Measure $\frac{d\sigma}{dp_T}$ at LHC for transverse jet momenta $p_T = 74 - 2500$ GeV
- Best fit $\rightarrow \alpha_S(\mu)$ with $\mu \approx p_T$
- E.g. $p_T = 1410 - 2500$ GeV $\rightarrow \alpha_S(1508.04\ \text{GeV}) = 0.0822^{+0.0034}_{-0.0031}$

[V. Khachatryan et al. (CMS), JHEP 03, 156 (2017)]



[CMS, CERN]

Motivation

[Particle Data Group, Chin. Phys. C, 40, 100001 (2016)]

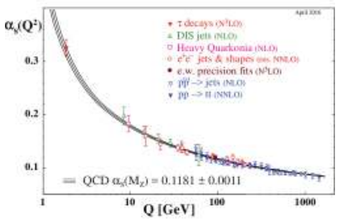
- Various experiments
  $\leftrightarrow$ various high Q
- Use $\beta^{\text{5-loop}}$

  [P.Baikov, K.Chetyrkin, J.Kühn, PRL 118 (2017)]

  [F.Herzog, B.Ruijl, T.Ueda, J.Vermaseren, JHEP 02 (2017)]

  $\alpha_{\overline{\mathrm{MS}}}(Q) \to \alpha_{\overline{\mathrm{MS}}}(M_Z)$
- Experiment at $Q \approx 100$ MeV
  Needs $\beta^{\text{non-perturbative}}$
  $\to$ not with $\overline{\mathrm{MS}}$
- We use:
  finite volume schemes
- replace $\beta \to \beta^{\mathrm{PT}}$ in

  $$\frac{\Lambda}{\mu} = \left[b_0 \bar{g}^2(\mu)\right]^{-b_1/(2b_0{}^2)} e^{-1/(2b_0\bar{g}^2(\mu))} \times \exp\left[-\int_0^{\bar{g}(\mu)} \left(\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0{}^2 x}\right) dx\right]$$



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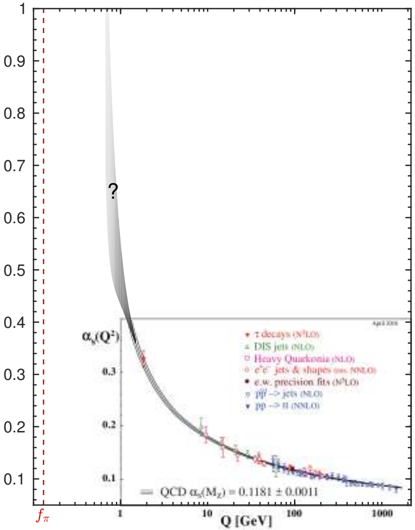

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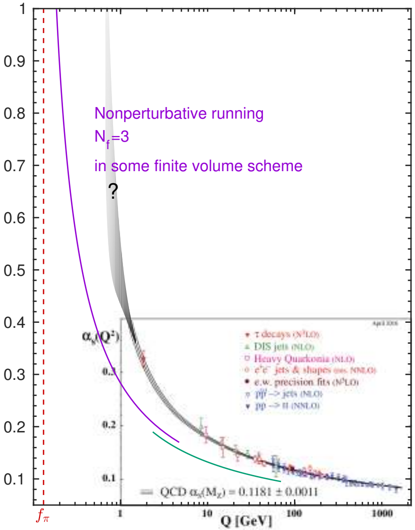

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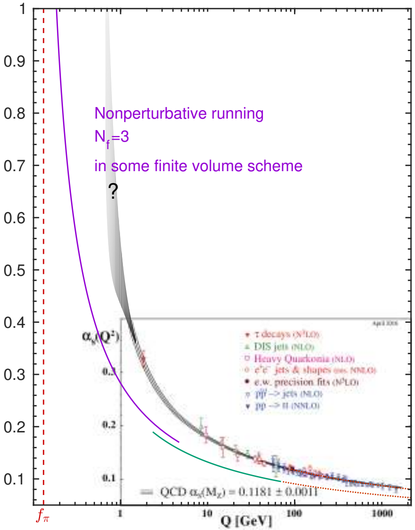
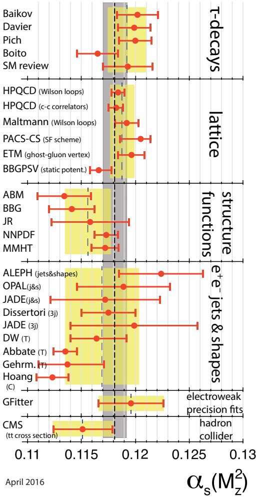


The QCD Coupling Constant: $\alpha_{\overline{MS}}^{N_f=5}(M_Z)$



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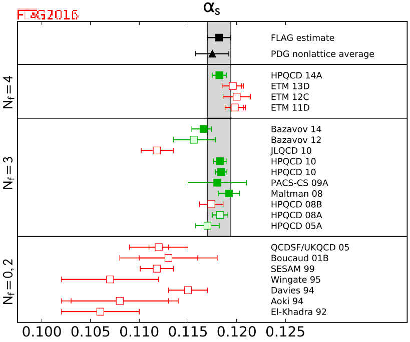
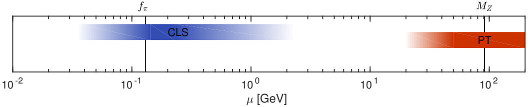


[FLAG Working Group, Phys. J. C (2017)]

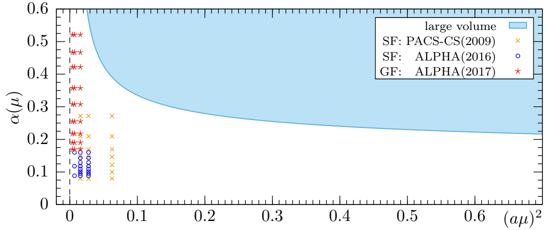
Finite Volume Schemes



- A "single lattice" approach is impossible without compromises

$$L \gg \frac{1}{m_\pi} \quad \text{and} \quad a \ll \frac{1}{\mu} \qquad \Rightarrow \frac{L}{a} \approx O(1000)$$

- Solution: finite-size scaling, $\mu \equiv 1/L$ $\qquad \Rightarrow \frac{L}{a} \approx O(10)$
  But: requires separate sets of simulations for each value of μ



Choosing a Good Finite Volume Scheme

We need

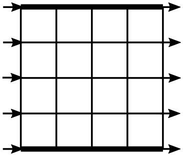
- Non-perturbative definition
- Accessible with Monte-Carlo methods
  often: massless=impossible
- Good statistical precision from low to high energies
- Mild lattice artifacts
- Available perturbation theory
  - Normalization
  - Relation to $\overline{\text{MS}}$-scheme
  - For precision also: $\beta^{\text{3-loop}}$

The Schrödinger Functional

Finite volume needs boundary conditions. We use: Schrödinger Functional

Gauge fields $U_\mu(x)$:

$$\begin{aligned} U_k(x)|_{x_0=0} &= \exp(a\,C_k) \\ U_k(x)|_{x_0=T} &= \exp(a\,C'_k) \\ U_\mu(x+L\hat{k}) &= U_\mu(x) \end{aligned}$$



We choose:

$$C_k = \frac{i}{L}\begin{pmatrix} \eta - \frac{\pi}{3} & & \\ & \eta\nu - \frac{\eta}{2} & \\ & & -\eta\nu - \frac{\eta}{2} + \frac{\pi}{3} \end{pmatrix}, \quad C'_k = \frac{i}{L}\begin{pmatrix} -\eta - \pi & & \\ & \eta\nu + \frac{\eta}{2} + \frac{\pi}{3} & \\ & & \frac{\eta}{2} - \eta\nu + \frac{2\pi}{3} \end{pmatrix}$$

Matter fields $\psi(x)$:

$$\begin{aligned} \frac{1}{2}[\mathbb{1} - \gamma_0]\,\psi(x)|_{x_0=0} &= 0 \\ \frac{1}{2}[\mathbb{1} + \gamma_0]\,\psi(x)|_{x_0=T} &= 0 \\ \psi(x+L\hat{k}) &= e^{i\theta}\psi(x) \end{aligned}$$

Traditional Schrödinger-Functional Scheme

Couplings: "susceptibility" of Γ to a change of the boundaries

$$\bar{g}_\nu^2 \propto \left[\frac{\partial \Gamma}{\partial \eta}\bigg|_{\eta=0} \right]^{-1}, \qquad \bar{g}_{\mathrm{SF}}^2 \equiv \bar{g}_{\nu=0}^2$$

$\Gamma = -\ln\left[\int_{\mathrm{fields}} e^{-S[U,\bar{\psi},\psi]}\right]$, massless theory

- Well tried and extensively studied

  [M.Lüscher, R.Narayanan, P.Weisz, U.Wolff, Nucl.Phys. B384 (1992)]

  [P.Fritzsch, F.Knechtli, B.Leder, M.Marinkovic, S.Schaefer, R.Sommer, F.Virotta, Nucl.Phys. B865 (2012)]

  [S.Aoki et al., JHEP 0910 (2009)]

- With SF boundaries: Dirac operator has a solid spectral gap
- Perturbation theory: $\beta^{\text{3-loop}}$ known , perturbative lattice artifacts known (with plaquette gauge action)
- Very mild $O(a^2)$ lattice artifacts
- Statistical precision: good at large μ, problematic at low μ

⇒ Very good, but we can do better at low energies: gradient-flow couplings

The Gradient Flow

Gradient flow $\sim$ (covariant) diffusion in “flow time” t

[M.F. Atiyah, R. Bott, Phil.Trans.Roy.Soc.Lond. A308 (1982)]

$$\begin{aligned} \partial_t B_\mu(t,x) &= D_\nu G_{\nu\mu}(t,x), \qquad\qquad B_\mu(0,x) = A_\mu(x) \\ D_\mu &= \partial_\mu + [B_\mu, \cdot] \\ G_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu] \end{aligned}$$

- Correlators of B at $t > 0$ need no renormalization

  [M. Lüscher, JHEP 1008 (2010)]

  [M. Lüscher and P. Weisz, JHEP 1102 (2011)]

- Most common use: definition of scales, e.g. t_0

$$\frac{t_0^2}{4} \left\langle G^a_{\mu\nu}(t_0,x) G^a_{\mu\nu}(t_0,x) \right\rangle = 0.3$$

Gradient Flow Couplings

Finite-size-scheme couplings based on the GF

[Z. Fodor, K. Holland, J. Kuti, D. Nogradi, and C. H. Wong, JHEP 1211 (2012)]

[P. Fritzsch, A. Ramos, JHEP 1310 (2013)]

$$\bar{g}^2_{\mathrm{GF}} \propto \frac{t^2}{4} \frac{\left\langle G^a_{\mu\nu}(t,x) G^a_{\mu\nu}(t,x)\, \delta_{Q,0} \right\rangle}{\left\langle \delta_{Q,0} \right\rangle}\Bigg|_{\sqrt{8t}=cL, x_0=T/2}$$

Our coupling:

- SF boundaries
  - $C = C' = 0$
  - $\theta_k = \frac{1}{2}$
  - $L = T$
- Massless scheme $\overline{m}_u = \overline{m}_d = \overline{m}_s = 0$
- $c = 0.3$
- Use only "magnetic" components $\mu, \nu = 1, 2, 3$
- Evaluate the action density at $x_0 = T/2$
- Project to trivial topological sector $Q = \frac{1}{32\pi^2} \int d^4x\, \epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu}(t,x) G^a_{\rho\sigma}(t,x)$

GF Coupling on the Lattice

Discretization:

- Action
  - Tree-level Symanzik improved gauge action S_{LW}:  , 

    [M. Lüscher, P. Weisz, Commun.Math.Phys. 97 (1985)]
  - $O(a)$ improved Wilson fermions
- Flow equation
  - Wilson-flow: use Wilson's plaquette action S_W: 

$$a^2 \left[\partial_t V_\mu(t,x)\right] V_\mu^\dagger(t,x) = -g_0^2 \partial_{x,\mu} S_W[V]$$

  - Symanzik improved "Zeuthen-flow"

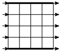
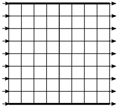
    [A. Ramos, S. Sint, Eur.Phys.J. C76 (2016)]

$$a^2 \left[\partial_t V_\mu(t,x)\right] V_\mu^\dagger(t,x) = -g_0^2 \left(1 + \frac{a^2}{12} \nabla_\mu^* \nabla_\mu\right) \partial_{x,\mu} S_{LW}[V]$$

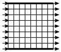
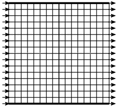
- Action density
  - Clover, 
  - Symanzik improved S_{LW}  , 

Computing Step Scaling Functions

Instead of $\beta(u)$, compute: $\sigma(u) = \bar{g}^2(\mu/2)|_{u=\bar{g}^2(\mu)}$

$m_0^{(1)}, g_0^{(1)}$:  $\overset{\text{same } a^{(1)}}{\leftrightarrow}$  $= \Sigma(u, \frac{a^{(1)}}{L})$

$\updownarrow$ same L, $\bar{g}^2(L)$

$m_0^{(2)}, g_0^{(2)}$:  $\overset{\text{same } a^{(2)}}{\leftrightarrow}$  $= \Sigma(u, \frac{a^{(2)}}{L})$

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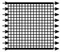
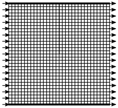
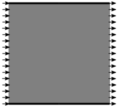
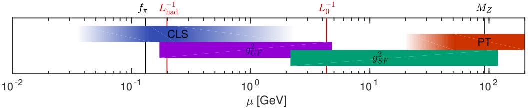
$m_0^{(3)}, g_0^{(3)}$:  $\overset{\text{same } a^{(3)}}{\leftrightarrow}$  $= \Sigma(u, \frac{a^{(3)}}{L})$

$\downarrow$cont. limit

$\bar{g}^2 = u, \ \overline{m} = 0$  $= \sigma(u)$

Overall Strategy



- Define two scales: $L_{\rm had}$ and L_0 implicitly via

$$\bar{g}^2_{\rm GF}(L_{\rm had}) \equiv 11.31\,, \qquad \bar{g}^2_{\rm SF}(L_0) \equiv 2.012$$

- Compute step scaling functions
  - GF coupling: $\sigma_{\rm GF}(u)$ in the range $u \in [\bar{g}^2_{\rm GF}(2L_0), \bar{g}^2_{\rm GF}(L_{\rm had})]$

    LW-action, SF boundaries, no background field
  - SF coupling: $\sigma_{\rm SF}(u)$ in the range $u \in [\bar{g}^2_{\rm SF}(L_{\rm PT}), \bar{g}^2_{\rm SF}(L_0)]$

    plaquette action, SF boundaries, background field: $\eta = \nu = 0$
- Nonperturbative scheme matching at L_0: compute $\bar{g}^2_{\rm GF}(2L_0)$
- Large volume simulations (CLS): obtain $1/\sqrt{t_0}$ in GeV

  LW-action, open boundaries in time
- Relate the scales $L_{\rm had}$ and $\sqrt{t_0}$

$$\Lambda^{(3)}_{\overline{\rm MS}} = \underbrace{\frac{f^{\rm PDG}_{\pi K}}{f_{\pi K}\sqrt{t_0}}}_{\text{scale setting}} \times \underbrace{\frac{\sqrt{t_0}}{L_{\rm had}}}_{\text{connection to CLS}} \times \underbrace{\frac{L_{\rm had}}{2L_0}}_{\text{GF running}} \times \underbrace{\frac{2L_0}{L_0}}_{\text{change of schemes}} \times \underbrace{\Lambda^{(3)}_{\overline{\rm MS}} L_0}_{\text{SF running}}$$

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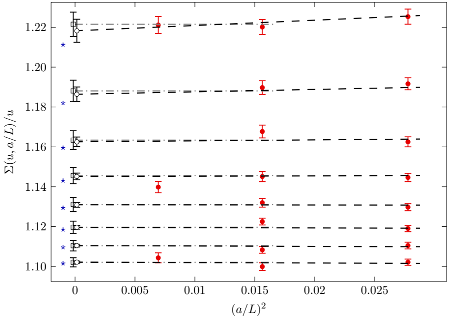
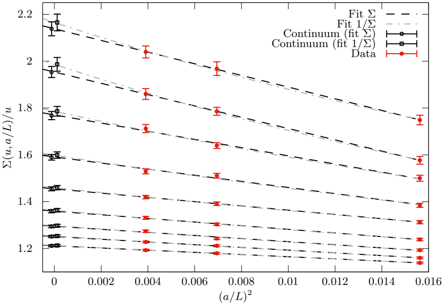
SF-coupling

[M. Dalla Brida, P. Fritzsch, T. K., A. Ramos, S. Sint, R. Sommer, PRL 117 (2016)]



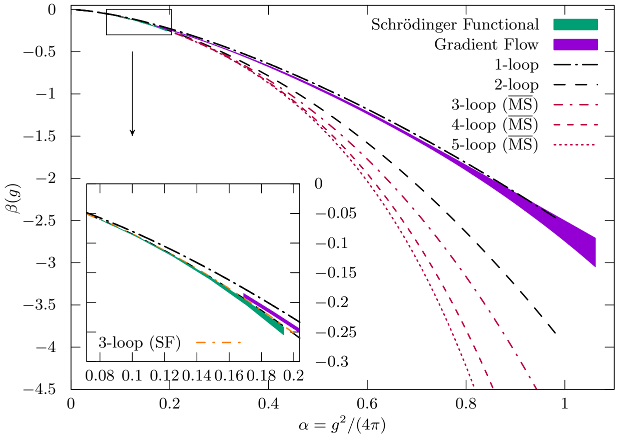
GF-coupling

[M. Dalla Brida, P. Fritzsch, T. K., A. Ramos, S. Sint, R. Sommer, PRD 95 (2017)]



Relation $\sigma \leftrightarrow \beta$:

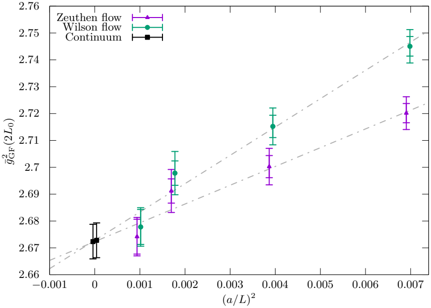
$$\ln(2) = -\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{dx}{\beta(x)}$$



$$\Lambda_{\overline{\mathrm{MS}}}^{(3)} = \underbrace{\frac{f_{\pi K}^{\mathrm{PDG}}}{f_{\pi K}\sqrt{t_0}}}_{\text{scale setting}} \times \underbrace{\frac{\sqrt{t_0}}{L_{\mathrm{had}}}}_{\text{connection to CLS}} \times \underbrace{\frac{L_{\mathrm{had}}}{2L_0}}_{\text{GF running}} \times \underbrace{\frac{2L_0}{L_0}}_{\text{change of schemes}} \times \underbrace{\Lambda_{\overline{\mathrm{MS}}}^{(3)} L_0}_{\text{SF running}}$$

[M. Dalla Brida, P. Fritzsch, T. K., A. Ramos, S. Sint, R. Sommer, PRD 95 (2017)]

- $\bar{g}^2_{\mathrm{SF}}(L_0) = 2.012$
- compute: $\bar{g}^2_{\mathrm{GF}}(2L_0)$
  - $\Phi(u, a/L) = \bar{g}^2_{\mathrm{GF}}(2L)\Big|_{\bar{g}^2_{\mathrm{SF}}(L)=u}$
  - $\phi(u) = \lim_{a\to 0} \Phi(u, a/L)$



Finite Volume Results

$$\Lambda_{\overline{\rm MS}}^{(3)} = \underbrace{\frac{f_{\pi K}^{\rm PDG}}{f_{\pi K}\sqrt{t_0}}}_{\text{scale setting}} \times \underbrace{\frac{\sqrt{t_0}}{L_{\rm had}}}_{\text{connection to CLS}} \times \underbrace{\frac{L_{\rm had}}{2L_0}}_{\text{GF running}} \times \underbrace{\frac{2L_0}{L_0}}_{\text{change of schemes}} \times \underbrace{\Lambda_{\overline{\rm MS}}^{(3)} L_0}_{\text{SF running}}$$

Final Results

$$\bar{g}^2_{\rm GF}(2L_0) \overset{\text{matching}}{=} 2.6723(64) \quad , \quad \frac{L_{\rm had}}{L_0} \overset{\text{GF running}}{=} 21.86(42)$$

$$L_0\Lambda_{\rm SF}^{(3)} \overset{\text{SF running}}{=} 0.0303(8) \quad \leftrightarrow \quad L_0\Lambda_{\overline{\rm MS}}^{(3)} = 0.0791(21)$$

together

$$L_{\rm had}\Lambda_{\overline{\rm MS}}^{(3)} = 1.729(57)$$

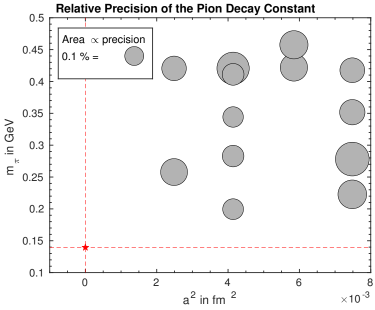
Scale Setting

$$\Lambda_{\overline{MS}}^{(3)} = \underbrace{\frac{f_{\pi K}^{\rm PDG}}{f_{\pi K}\sqrt{t_0}}}_{\text{scale setting}} \times \underbrace{\frac{\sqrt{t_0}}{L_{\rm had}}}_{\text{connection to CLS}} \times \underbrace{\frac{L_{\rm had}}{2L_0}}_{\text{GF running}} \times \underbrace{\frac{2L_0}{L_0}}_{\text{change of schemes}} \times \underbrace{\Lambda_{\overline{MS}}^{(3)} L_0}_{\text{SF running}}$$

We use CLS ensembles

[M.Bruno et al., JHEP 1502 (2015)]+newer runs

- LW action + clover fermions, open boundaries, $N_f = 2+1$, $Lm_\pi > 4$
- Chiral trajectories with bare masses $m_u + m_d + m_s =$ constant



Scale Setting

The experimental input is

- $m_\pi = 134.8(3)$ MeV, $m_K = 494.2(3)$ MeV

  [FLAG Working Group, Phys. J. C (2017)]
- $f_{\pi K} \equiv \frac{2}{3} f_K + \frac{1}{3} f_\pi = 147.6(5)$ MeV

  [Particle Data Group, Chin.Phys. C38 (2014)]

  has a weaker quark mass dependence than f_π or f_K
  (along our chiral trajectory)

  [W. Bietenholz et al., Phys.Lett. B690 (2010)]

We use t_0 as “intermediate scale” and compute

- $\phi_2 = 8 t_0 m_\pi^2$ $\sim \overline{m}_{u,d}$
- $\phi_4 = 8 t_0 \left(m_K^2 + \frac{m_\pi^2}{2} \right)$ $\sim \overline{m}_u + \overline{m}_d + \overline{m}_s$
- $\sqrt{t_0} f_{\pi K}$
- The derivatives of all these with respect to the bare masses
  $\Rightarrow$ allows for small shifts $\mathcal{O}(m') = \mathcal{O}(m) + (m' - m) \frac{d\mathcal{O}}{dm}$

Scale Setting Strategy

[M.Bruno, T.K, S.Schaefer Phys.Rev. D95 (2017)]

- Use mass-derivatives to shift chiral trajectory to $\phi_4 = \phi_4^{\text{phys}}$
- Compute ϕ_2 and $\sqrt{t_0} f_{\pi K}$ on all shifted ensembles
- Combined chiral/continuum extrapolation: $\sqrt{t_0} f_{\pi K} = f(\phi_2, a)$
- At the physical point: read off $\sqrt{t_0^{\text{phys}}} = f(\phi_2^{\text{phys}}, 0)/f_{\pi K}^{\text{PDG}}$

minor complication: ϕ_4^{phys} contains t_0^{phys} $\quad \rightarrow \quad$ iterate

1. Taylor around the flavor-symmetrical point ϕ_2^{sym}: linear term vanishes

   [W. Bietenholz et al., Phys.Lett. B690 (2010)]

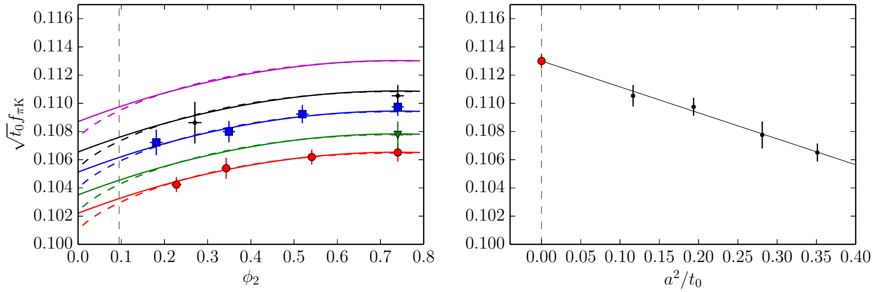
   Fit: $$f(\phi_2, a) = c_0 + c_1(\phi_2 - \phi_2^{\text{sym}})^2 + c_2 \frac{a^2}{t_0^{\text{sym}}}$$

2. NLO Chiral perturbation theory

   [O. Bär, M. Golterman, Phys.Rev. D89 (2014)], [J. Gasser, H. Leutwyler, Nucl.Phys. B250 (1985)]

   Fit: $$f(\phi_2, a) = (\sqrt{t_0} f_{\pi K})^{\text{sym}} \left[1 - \frac{7(L_\pi - L_\pi^{\text{sym}})}{6} - \frac{4(L_K - L_K^{\text{sym}})}{3} - \frac{L_\eta - L_\eta^{\text{sym}}}{2} \right] + c_4 \frac{a^2}{t_0^{\text{sym}}}$$

   logarithms: $$L_x = \frac{m_x^2}{(4\pi f)^2} \ln \left[\frac{m_x^2}{(4\pi f)^2} \right]$$



Scale t_0^*

$$\sqrt{8t_0^{\text{phys}}} = 0.415(4)(2)\ \text{fm}$$

Alternatively: $\sqrt{t_0} f_{\pi K} \to \sqrt{t_0^*} f_{\pi K}$ in the extrapolations, where t_0^* is defined at the (unphysical) mass-point $m_u = m_d = m_s$ and $\phi_4 = 1.1100$

Final Result

$$\sqrt{8t_0^*} = 0.413(5)(2)\ \text{fm}$$

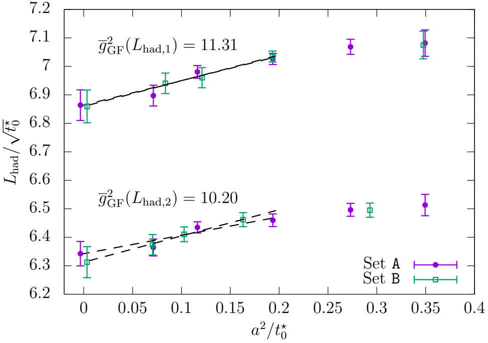
Moreover t_0^*/a^2 measured directly also on fifth lattice spacing $a \approx 0.039$ fm

Connecting SF to Large Volume

$$\Lambda_{\overline{\mathrm{MS}}}^{(3)} = \underbrace{\frac{f_{\pi K}^{\mathrm{PDG}}}{f_{\pi K}\sqrt{t_0^*}}}_{\text{scale setting}} \times \underbrace{\frac{\sqrt{t_0^*}}{L_{\mathrm{had}}}}_{\text{connection to CLS}} \times \underbrace{\frac{L_{\mathrm{had}}}{2L_0}}_{\text{GF running}} \times \underbrace{\frac{2L_0}{L_0}}_{\text{change of schemes}} \times \underbrace{\Lambda_{\overline{\mathrm{MS}}}^{(3)} L_0}_{\text{SF running}}$$

- From large volume simulations
  - t_0^* known in fm
  - t_0^*/a^2 known at $\beta \in \{3.4, 3.46, 3.55, 3.7, 3.85\}$ (massive theory)
  - Corresponds to $\beta \in \{3.3985, 3.4587, 3.549, 3.6992, 3.8494\}$ (massless)
- From gradient flow running
  - L_{had}/a for $\beta \in \{3.3998, 3.5498, 3.6867, 3.8, 3.9791\}$ (massless)
- Interpolate L_{had}/a to large-volume β's (or other way around)
- Continuum extrapolate: $\frac{L_{\mathrm{had}}/a}{\sqrt{t^*/a^2}}$

Connecting SF to Large Volume



Final Result

$$\frac{L_{\mathrm{had}}}{\sqrt{t_0^*}} = 6.825(47)$$

Perturbative Decoupling

Decoupling:

$$\bar{g}^{N_f}(\mu) = \bar{g}^{N_f+1}(\mu) \times \xi\left(g^{N_f}(\mu), \overline{m}_h/\mu\right) \quad + \quad O(\overline{m}_h^{-2})$$

Equivalently a relation $\Lambda^{(N_f)}/\Lambda^{(N_{f+1})}$

- $O(\overline{m}_h^{-2})$ are very small already for $\overline{m}_h = \overline{m}_c$

  [M. Bruno, J. Finkenrath, F. Knechtli, B. Leder, R. Sommer, Phys.Rev.Lett. 114 (2015)]

  [F. Knechtli, T.K., B. Leder, G. Moir, arXiv:1706.04982 (2017)]

  See also talk by F.Knechtli, Wed. 10:20, Seminarios 1+2
- ξ known in perturbation theory to 4 loops

  [K. Chetyrkin, J. Kühn, C. Sturm, Nucl. Phys. B744 (2006)]

  [Y. Schröder, M. Steinhauser, JHEP 01, 051 (2006)]
- Perturbation theory looks surprisingly well-behaved already at $\mu = \overline{m}_c$

n (loops)	$\alpha_{\overline{MS}}^{(N_f=5)}$	$\alpha_n - \alpha_{n-1}$
1	0.11699	
2	0.11827	0.00128
3	0.11846	0.00019
4	0.11852	0.00006

conservative error (within PT):
$\alpha_4 - \alpha_2 \approx 0.0003$

Everything Together

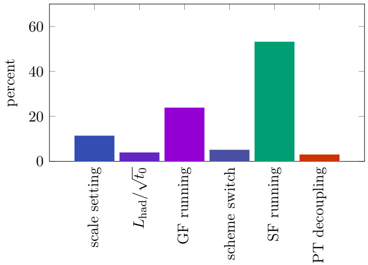
Final Result

$$\Lambda^{(3)}_{\overline{\text{MS}}} = 341(12)\ \text{MeV}$$

$$\Lambda^{(5)}_{\overline{\text{MS}}} = 215(10)(03)\ \text{MeV} \qquad \text{pert. decoupling}$$

$$\alpha_{\overline{\text{MS}}}(M_Z) = 0.1185(8)(3)$$

0.1174(16) PDG non-lattice



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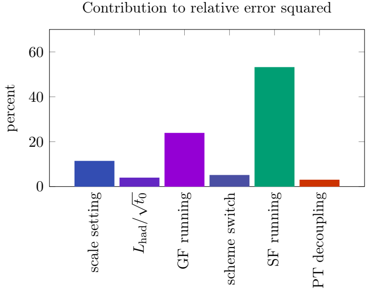

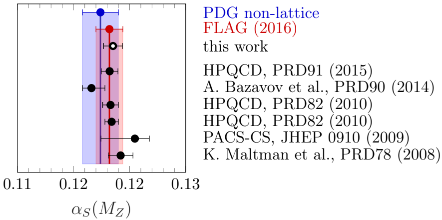

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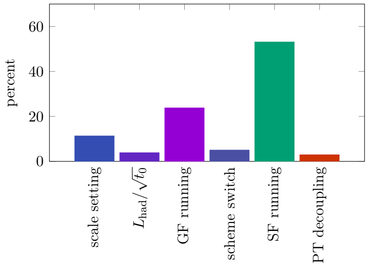
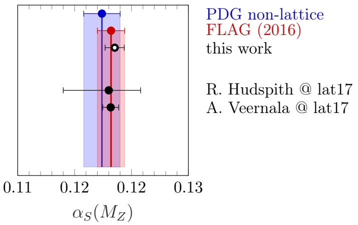
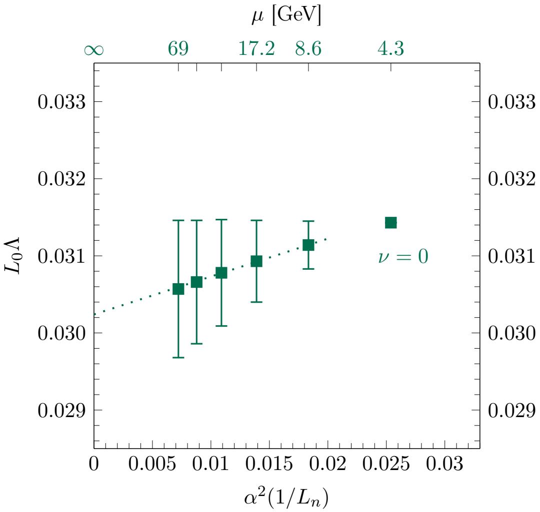
Contribution to relative error squared





Warning 1: Accuracy of Perturbation Theory

- $L_n = 2^{-n} L_0, \qquad \alpha = \bar{g}_\nu^2/(4\pi)$
- Use 3-loop PT at $\alpha(1/L_n) \Rightarrow$ Residual error $O(\alpha^2)$



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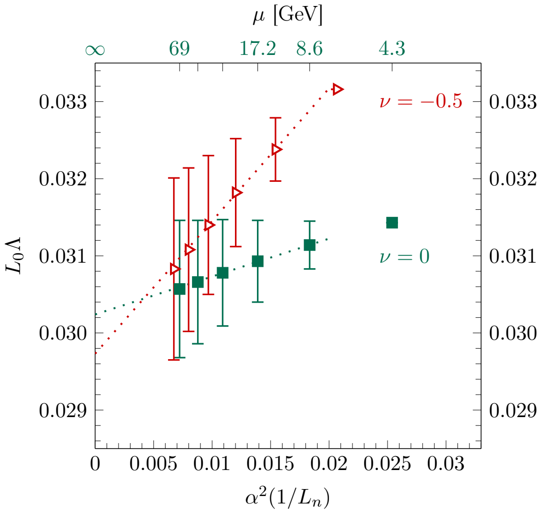

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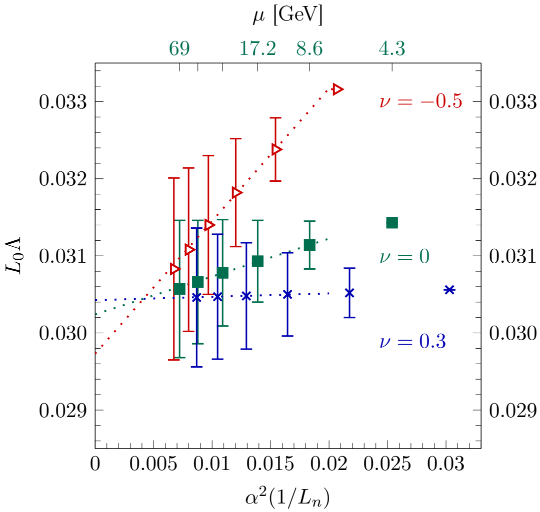

Warning 1: Accuracy of Perturbation Theory

- Similarly: α_{qq} = coupling from the static force
- at $\alpha_{\mathrm{qq}} < 0.22$: $\frac{\Lambda_{\overline{\mathrm{MS}}}^{\text{4-loop}} - \Lambda_{\overline{\mathrm{MS}}}}{\Lambda_{\overline{\mathrm{MS}}}} = 7.0(5)\alpha_{\mathrm{qq}}^3$

poster by N. Husung, P. Krah, M. Koren and R. Sommer (presenter) @lat17